Name:________________________ **Exam 2 (11/2/18)** **Tufts ID:**__________________

50 minutes. No notes, books or devices allowed. Please SHOW YOUR WORK and/or explain answers for full credit!

1. (10 pts) Let $L = \{a, b, c, d, r\}$.

   - Decide whether or not each of the following relations on L is an equivalence relation. Prove your answer.
   - If it is an equivalence relation, describe and/or draw the partition.

   (a) $R = \{(x, y) : x$ and y appear the same number of times in the word "abracadabra."$\}$

   (b) $R = \{(x, y) : x$ and y appear next to each other in the word "abracadabra."$\}$

2. SHORT ANSWER (5 pts each, 30 pts total)

   (a) Compute $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \ldots + \binom{10}{10}$.
   Hint: There are fast and slow ways to do this.

   (b) Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$. For each description below, choose a final ordered pair to obtain a relation from A to B which satisfies the description.

   i. $f = \{(1, 5), (2, 5), (3, 6), (\_\_\_, \_\_\_)\}$ where the relation f is not a function.

   ii. $f = \{(1, 5), (2, 5), (3, 6), (\_\_\_, \_\_\_)\}$ where the relation f is a function from A to B but is not onto B.

   iii. $f = \{(1, 5), (2, 5), (3, 6), (\_\_\_, \_\_\_)\}$ where the relation f is a function from A to B and is onto B.

(c) You are visiting an ice cream store which has 7 different flavors. How many ways can you choose a quadruple cone with 4 different flavors (order matters)?

Simplify your answer.

(d) You are visiting an ice cream store which has 7 different flavors. How many ways can you choose a quadruple cone with 4 scoops (order matters and repetition is allowed)?

You do not need to simplify your answer.

(e) You are visiting an ice cream store which has 7 different flavors. How many ways can you choose a sampler bowl with 4 different flavors (order **doesn't** matter)?

Simplify your answer.

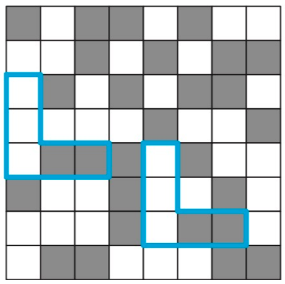
(f) Why does $\binom{10}{4}$ tell you the coefficient of x^4y^6 in $(x+y)^{10}$? (Don't just cite a theorem; explain in a sentence or two the justification behind this theorem.)

PROOFS: You must choose THREE of the following FOUR proofs. If you attempt all four, the highest three scores will count.

3. (20 pts) Prove using contradiction that the sum of a rational number and an irrational number is irrational. *Recall: A number is rational if it can be written as* $\frac{a}{b}$*, where* $a, b \in \mathbb{Z}$ *and* $b \neq 0$*. If this is not possible, the number is called irrational.*

4. (20 pts) Prove using some form of induction that for all $n \in \mathbb{N}$, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

5. (20 pts) Suppose the squares of an 8×8 board are colored black or white at random. For this problem, an L-region is a collection of 5 squares in the shape of a capital L (namely, a corner square with two squares above and two squares to the right; see two examples in the picture below). Use the Pigeonhole Principle to prove that no matter how we color the board, there must be two L-regions that are colored identically (as illustrated by the two L-regions in the picture.)



6. (20 pts) Use a combinatorial proof to prove that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$