Assignment 1: Population Growth Models

In this exercise, you will model exponential and logistic population growth.

Exponential population growth (8 points):

To model exponential population growth, we will use the following (discrete-time) equation:

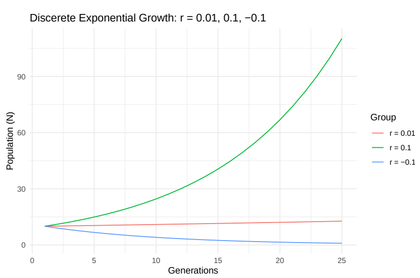
$$N_{t+1} = N_t e^r$$

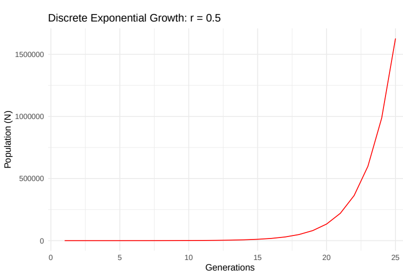
Where N_t and N_{t+1} are the population sizes at times t and $t+1$ and r is the intrinsic rate of increase. In R, the equation looks like `p[i] <- Nt * exp(r)`.

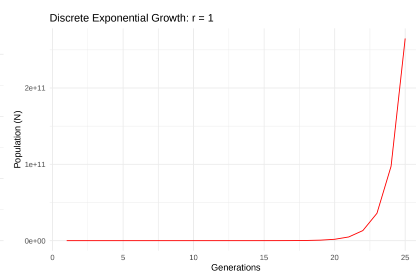
Start each model at $N_0 = 10$. Run the model for 25 time steps. Complete separate runs for each of the following r values:

1. $r = 0.01$
2. $r = 0.1$
3. $r = -0.1$
4. $r = 0.5$
5. $r = 1$

This tab should contain three graphs (N_t versus t): the first three populations (1-3) can be put in one graph, and the last two (4 and 5) on separate graphs. Your graphs should look like the following:







Logistic population growth (8 points):

For logistic population growth, we will be using the discrete form of the logistic equation:

$$N_{t+1} = N_t e^{r\left(\frac{K-N_t}{K}\right)}$$

Where N_t and N_{t+1} are the population sizes at times t and $t+1$, r is the intrinsic rate of increase, and K is the carrying capacity. In R, the equation looks like `p[i] <- Nt * exp(r * ((K-Nt)/K))`.

For the following exercises your carrying capacity will be [choose any number between 50 and 500]:

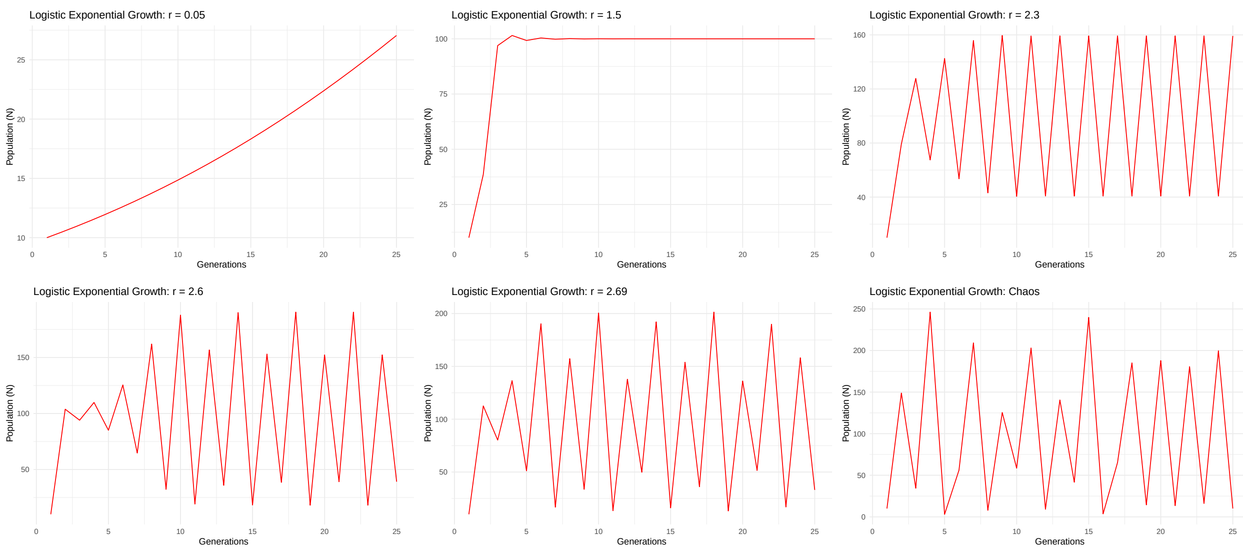
$$K =$$

Set N_0 to 20% of your K. For example, if your K is 340, your N_0 would be 68. For each simulation, calculate N_t over 100 timesteps, and graph N_t versus t. In the legend of the graph, include the value of r that you used. If the graph is a limit cycle, include the number of points in the title, and the N values for each of the "boundaries" of the cycle in the figure.

This tab should contain 6 graphs for each of the following r values:

1. $r = 0.5$
2. $r = 1.5$
3. $r = 2.3$
4. $r = 2.6$
5. $r = 2.69$
6. Chaos ($r =$? Can you find out this value by yourself?)

Your graphs should look like the following:



Modifications to the logistic population growth model (8 points):

The discrete form of the logisitic growth equation that we have used so far, $N_{t+1} = N_t e^{r(\frac{K-N_t}{K})}$, may not reflect the real population growth observed in nature. Ecologists these days use more refined versions by modifying the degree of density dependence, like this one:

$$N_{t+1} = N_t e^{r(1-[\frac{N_t}{K}]^\alpha)}$$

Different sets of α and r values will dampen or amplify population cycles. In R, the equation looks like `p[i] <- Nt * exp(r * (1 - (Nt/K)^\alpha))`. For each simulation, calculate N_t over 100 timesteps, and graph N_t versus t. Use the K value from the previous section and set N_0 to 20% of your K.

This tab should contain 6 graphs, one for each of the following values:

1. $r = 0.5, \alpha = 0.5$
2. $r = 1.5, \alpha = 0.5$
3. $r = 2.3, \alpha = 0.5$
4. $r = 0.5, \alpha = 1.5$
5. $r = 1.5, \alpha = 1.5$
6. $r = 2.3, \alpha = 1.5$

Your graphs should look like the following: