The Mooring Dynamics of Floating Wind Turbines

MEKSP100

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Introduction

The problem at hand is to model the motion of a moored floating spar turbine through steady state force considerations, with tower specifications provided by the project supervisor Tor Anders NYGAARD. Of the major obstacles with floating offshore wind turbines, the cost of materials is the greatest. We should therefore explore alternatives to the steel chains being used today, perhaps the two most relevant being nylon and polyester. These materials do of course have very nonlinear unsteady behavior in strain, but modeling this is beyond the scope of the present work—we will strictly be interested in the tower dynamics, not that of the ropes.

The methodology follows from that outlined in LEE *et al.*,[1] where, among other criteria, the necessity of period of above 40 s is outlined. Although no derivations are included in the cited paper, the expressions for the spring stiffnesses derived below do concur with those found there.

Mooring systems

We wish to model a floating spar wind turbine which is moored by three taut mooring lines equiangularly secured to the seabed of depth h. Imagining the turbine interacting with the wind whilst neglecting the mooring lines, the net force acting in the direction coinciding with one of the mooring lines, as is shown in figure 1, the two other mooring lines will resist this motion. We assume a very simple system, where we imagine the mooring lines being made of a fiber that can be assumed to deform elastically like a spring, with a spring constant

$$k = \frac{\mathrm{E}A}{L}\cos\alpha,$$

where α is the angle between the mooring line and plane water surface.

[1] [2] LEE *et al.* (2024)

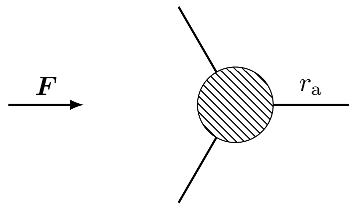

Figure 1: Top-down view of wind mooring system.

The net force acting in the direction of that one mooring line will induce a reactionary in the two others, inversely proportional to the cosine of the half angle between them. Since the three mooring lines are distributed evenly, this half angle must be $\pi/3$, whose cosine is $1/2$. The inverse proportionality is due to the trigonometry of the problem—the mooring line is the hypotenuse of a right triangle in which the net force is the adjacent side length. Thus the effective spring constant is twice that of the value above, considering the horizontal contribution only. The resulting resistance from the mooring line will also have to account for the decline at which it works in the transversal direction, yielding another cosine term. Finally, the spring constant for the system in the horizontal direction is given by

$$k_{\mathrm{H}} = \frac{2\mathrm{E}A}{L}\cos^2\alpha, \qquad \alpha = \arcsin\left(h/L\right).$$

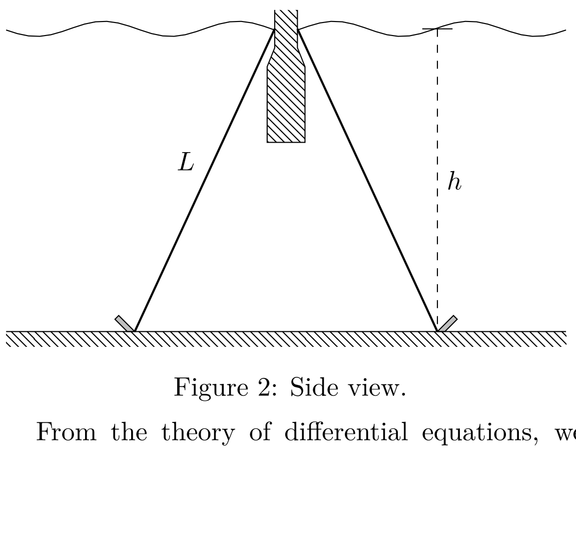

Figure 2: Side view.

From the theory of differential equations, we know the period of such an oscillation will be

$$T = 2\pi\sqrt{\frac{m + m_{11}}{k_{\mathrm{H}}}}, \qquad m_{11} = m,$$

where m is the mass of the tower, and m_{11} is its added mass. The spar we use here floats in the water, meaning the displaced volume of water must weigh the same as the tower, whence the added mass.

We now imagine some wave field producing heave motions in the tower. From the previous assignment we saw that the restoring forces for heave result in a coefficient $\varrho g S$, where S is the water line area. The resulting spring coefficient will then be the sum of this restoring force coefficient and three times the sine of the mooring line spring coefficient,

$$k_{\mathrm{V}} = \frac{3\mathrm{E}A}{L}\cos\alpha\sin\alpha + \varrho g S.$$

The factor three here comes from the three mooring lines being spaced equiangularly.

For the determination of the vertical added mass we may assume the tower to be a circular disk whose normal vector coincides with that of the heave. We may do this as the added mass is not concerned with the surfaces orthogonal to its mode, meaning the cylindrical shell will not contribute to the added mass. This is a simplification, as we suspect the conal shape of the spar may contribute to the added mass. Either way, the period will be given in a similar manner as in surge, with the added mass of such a disk, halved, as we only consider half the disk,[1]

$$T = 2\pi\sqrt{\frac{m + m_{22}}{k_{\mathrm{V}}}}, \qquad m_{22} = \frac{2\pi {r_2}^3}{3},$$

where r_2 is the bottom radius of the spar.

Implementation and Results

The specifications for the wind turbine are set to be $r_1 = 5\,\mathrm{m}$, $r_2 = 9\,\mathrm{m}$, and $m = 2.3846 \times 10^7\,\mathrm{kg}$. The axial stiffness is given by $\mathrm{E}A = 6 \times 10^8\,\mathrm{kg\,s^{-2}}$, and the depth of the ocean is assumed to be $h = 320\,\mathrm{m}$. Based on these initial parameters, we want to find an optimal anchor radius r_{a} such that the periods in heave and surge are less than $20\,\mathrm{s}$ and $40\,\mathrm{s}$, respectively. To do this, we implement the stiffnesses and periods as methods in the class `Moor`, and perform a search through different r_{a}. As is apparent, we have not chosen an explicit length of the morring lines L, since it may be calculated $L = \sqrt{{r_{\mathrm{a}}}^2 + h^2}$, and as such we may search for the anchor radius explicitly.

[1] [1] Kennard, p.393

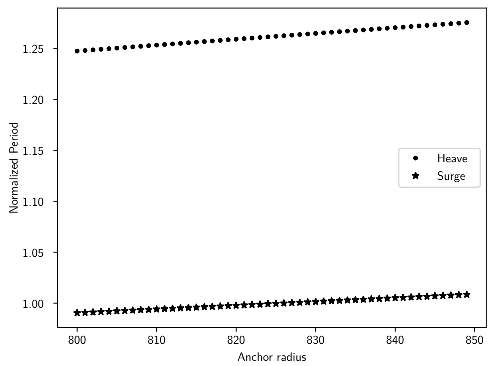

Figure 3: Result from searching in the interval $r_{\mathrm{a}} \in [800, 850]$.

When plotting the results, we have normalized the period with respect to the target periods, meaning the search should aim for a normalized period of unity. We see above in figure 3 that we surpass the $40\,\mathrm{s}$ period in surge at about $r_{\mathrm{a}} = 830\,\mathrm{m}$.

References

[1] Kennard, Earle H. *Irrotational Flow of Frictionless Fluids, Mostly of Invariable Density.* U.S. Government Printing Office, 1967.

[2] Lee, Chern Fong, Fjermedal, Sindre, and Ong, Muk Chen. “Design and Analysis of Taut Mooring Systems for a Combined Floating Offshore Wind and Wave Energy System at Intermediate Water Depth”. In: *Ocean Engineering* 312 (2024).