Vietnam National University - Ho Chi Minh City, University of Science, Faculty of Mathematics and Computer Science

FVM: Practical Assignment 2

Le Dinh Tan - MSSV: 1411263

April 8, 2017

[1]tanld996@gmail.com

Problem

We discretize the Laplace equation with Robin boundary condition by finite volume method:

$$\begin{cases} -u_{xx}(x) = f(x) \quad in \;\; \Omega \\ u'(0) - \lambda_0 u(0) = u'(1) + \lambda_1 u(1) = 0 \end{cases} \tag{1}$$

1. Solve equation (1) with regular grid and the control point be midpoint of control volume $\left(1/2x_{i-1/2} + 1/2x_{i+1/2}\right)$.
2. Solve equation (1) with regular grid and the control point be 1/3 from the left of each control volume $\left(x_1 = 2/3x_{i-1/2} + 1/3x_{i+1/2}\right)$.
3. Solve equation (1) with singular grid (not uniform grid).
4. Find the somw approximations to error between the exact and discrete solutions in discrete H_0^1 norm be 2 order for the equations such that Laplace equation with Dirichlet, Neumann, Robin boundary conditions.

Solution

Discrete similar problem for the case of homogeneous dirichlet boundary, we have:

$$\begin{cases} \alpha_i u_{i-2} + \beta_i u_i + \gamma_i u_{i+1} = f_i \quad \forall i \in \overline{1, N} \\ u'(0) - \lambda_0 u(0) = u'(1) + \lambda_1 u(1) = 0 \end{cases}$$

where,

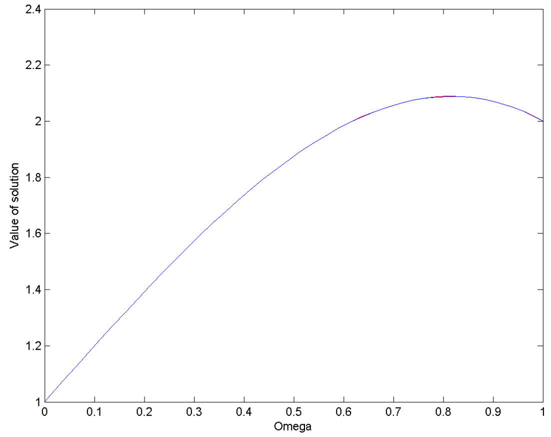
$$\alpha_i = \frac{-1}{|D_{i-1/2}||T_i|}$$
$$\beta_i = \frac{1}{|D_{i+1/2}||T_i|} + \frac{1}{|D_{i-1/2}||T_i|}$$
$$\gamma_i = \frac{-1}{|D_{i+1/2}||T_i|}$$

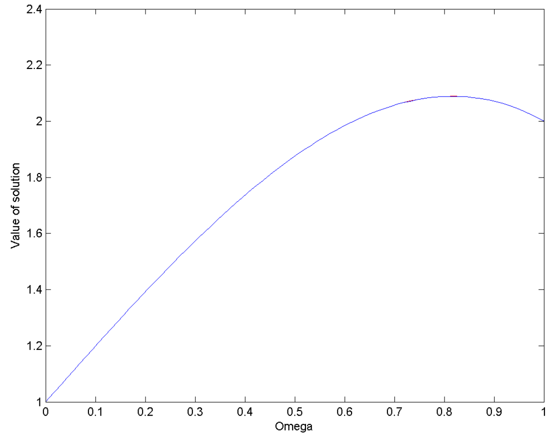
We have linear system for the scheme

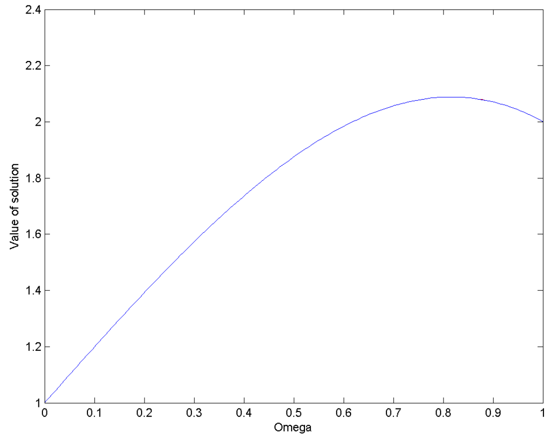
$$\begin{cases} i = 1, \left(\frac{\alpha_1}{1+\lambda_0|D_{1/2}|} + \beta_1\right) u_1 + \gamma_1 u_2 & = f_1 \\ i = 2, \quad \alpha_2 u_1 + \beta_2 u_2 + \gamma_2 u_3 & = f_2 \\ i = 3, \quad \alpha_3 u_2 + \beta_3 u_3 + \gamma_3 u_4 & = f_4 \\ \quad \ldots & \\ i = N-1, \quad \alpha_{N-1} u_{N-2} + \beta_{N-1} u_{N-1} + \gamma_{N-1} u_N & = f_{N-1} \\ i = N, \quad \alpha_N u_{N-1} + \left(\frac{\gamma_N}{1+\lambda_1|D_{N+1/2}|} + \beta_N\right) u_N & = f_N \end{cases}$$

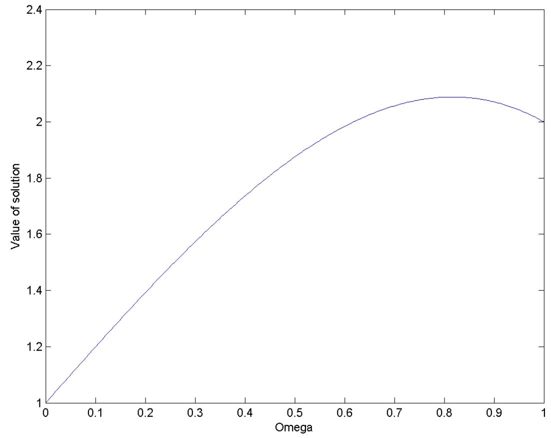
We set up with the following exact solution u and function f

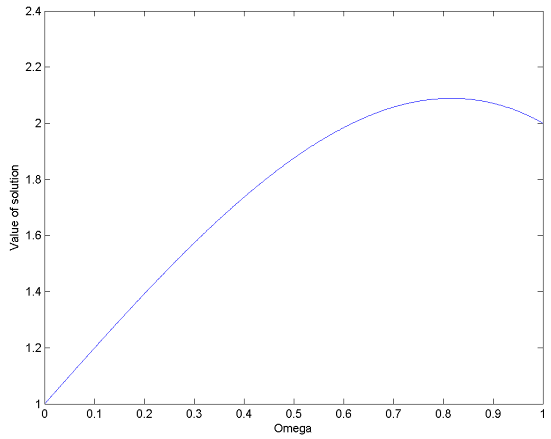
$$\begin{cases} u(x) = -x^3 + 2x + 1 \\ f(x) = 6x \\ u'(0) - 2u(0) = u'(1) + \frac{1}{2}u(1) = 0 \end{cases}$$

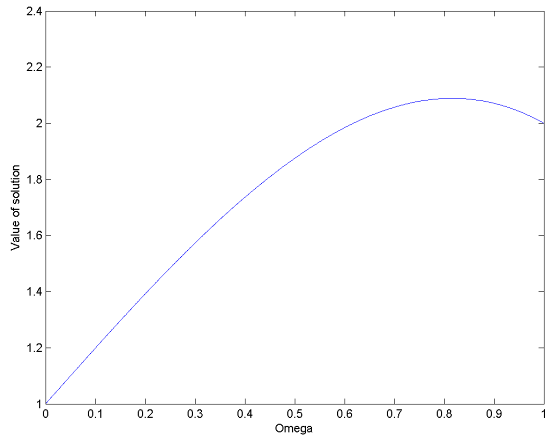

Value of solution
Omega

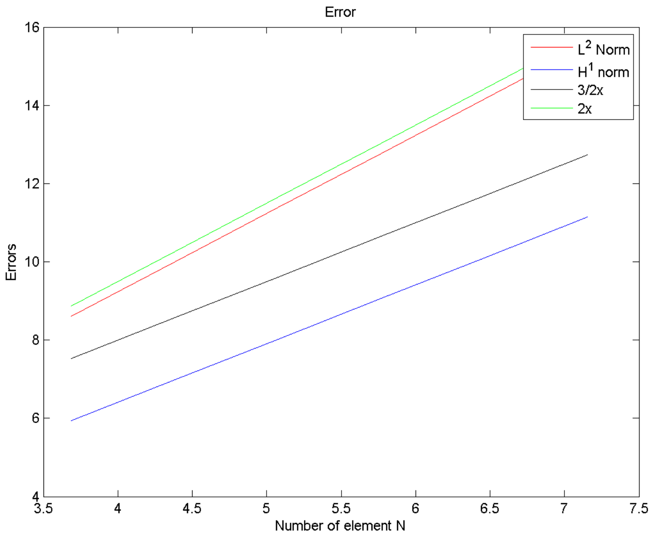

Value of solution
Omega


Value of solution
Omega


Value of solution
Omega

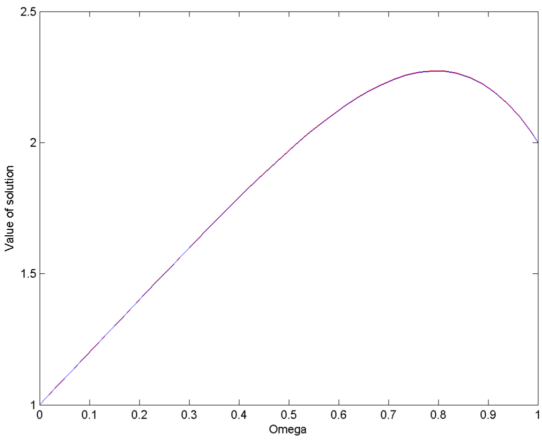

Value of solution
Omega

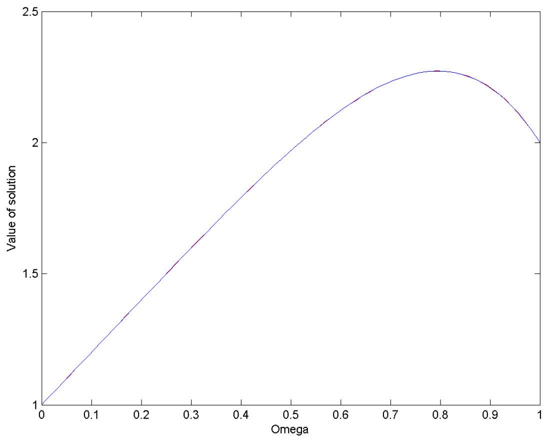

Value of solution
Omega

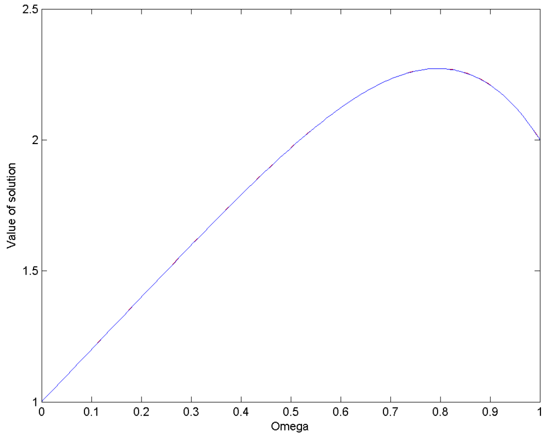


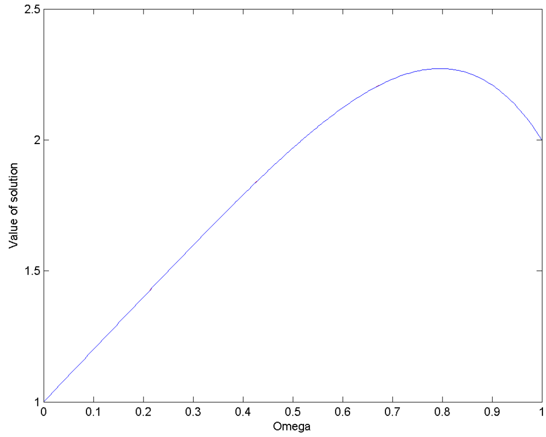
With the following exact solution u and function f

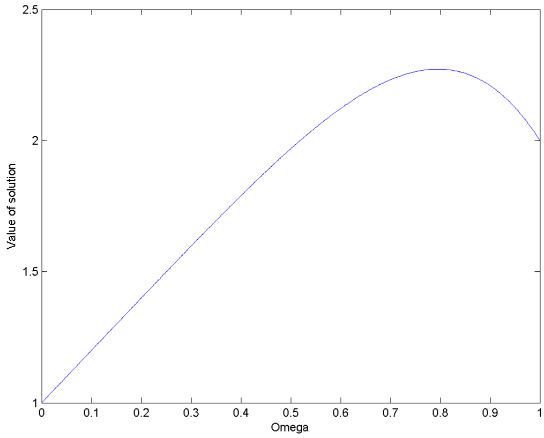
$$\begin{cases} u\left(x\right) = -x^5 + 2x + 1 \\ f\left(x\right) = 20x^3 \\ u'\left(0\right) - 2u\left(0\right) = u'\left(1\right) + \frac{3}{2}u(1) = 0 \end{cases}$$

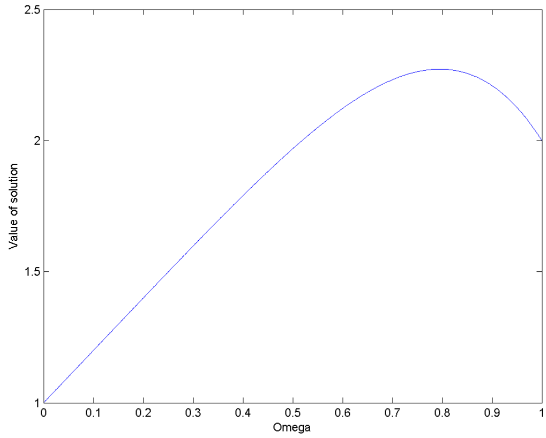


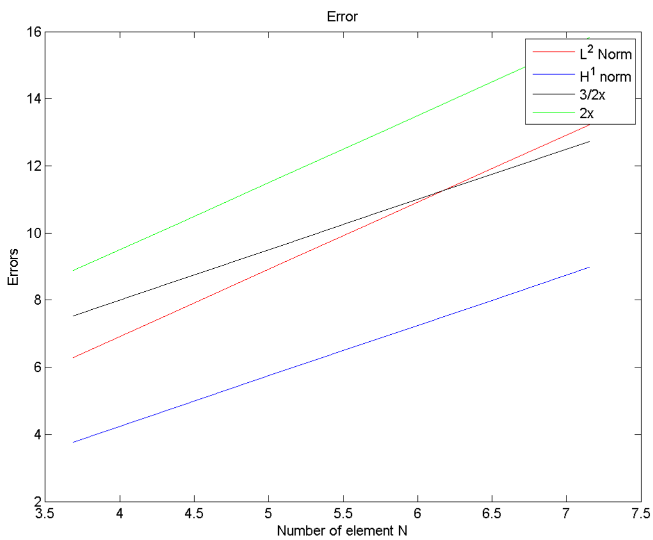



Value of solution
Omega


Value of solution
Omega

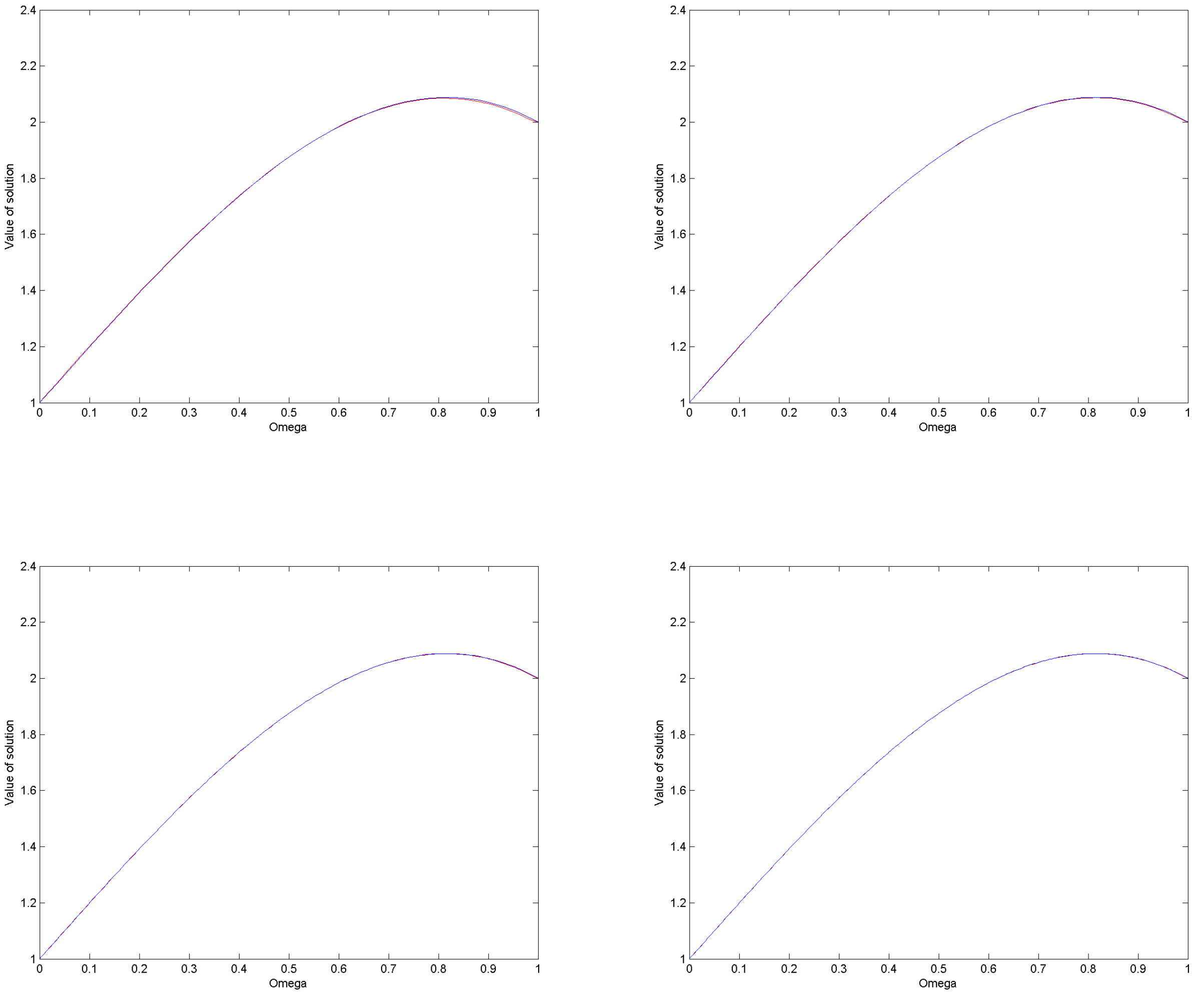

Value of solution
Omega

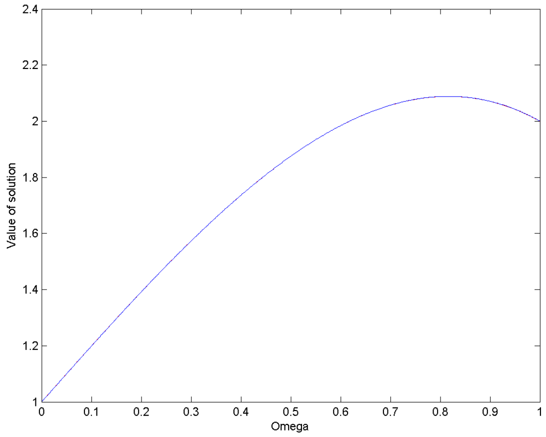

Value of solution
Omega


Error
L^2 Norm
H^1 norm
3/2x
2x
Errors
Number of element N

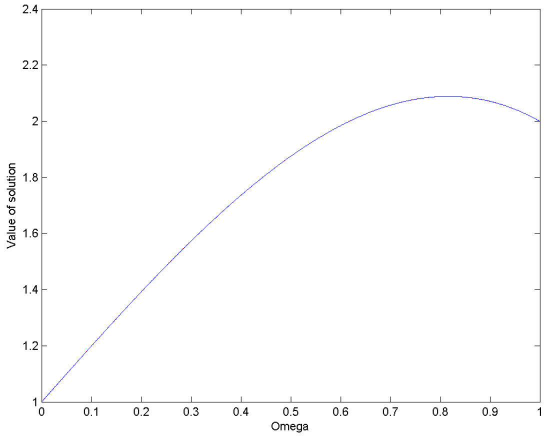
2. We set up with the following exact solution u and function f

$$\begin{cases} u\left(x\right) = -x^3 + 2x + 1 \\ f\left(x\right) = 6x \\ u'\left(0\right) - 2u\left(0\right) = u'\left(1\right) + \frac{1}{2}u(1) = 0 \end{cases}$$

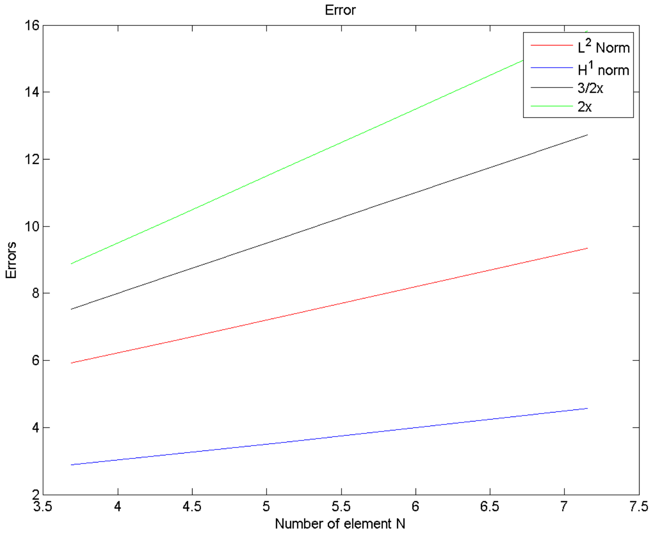




Value of solution
Omega



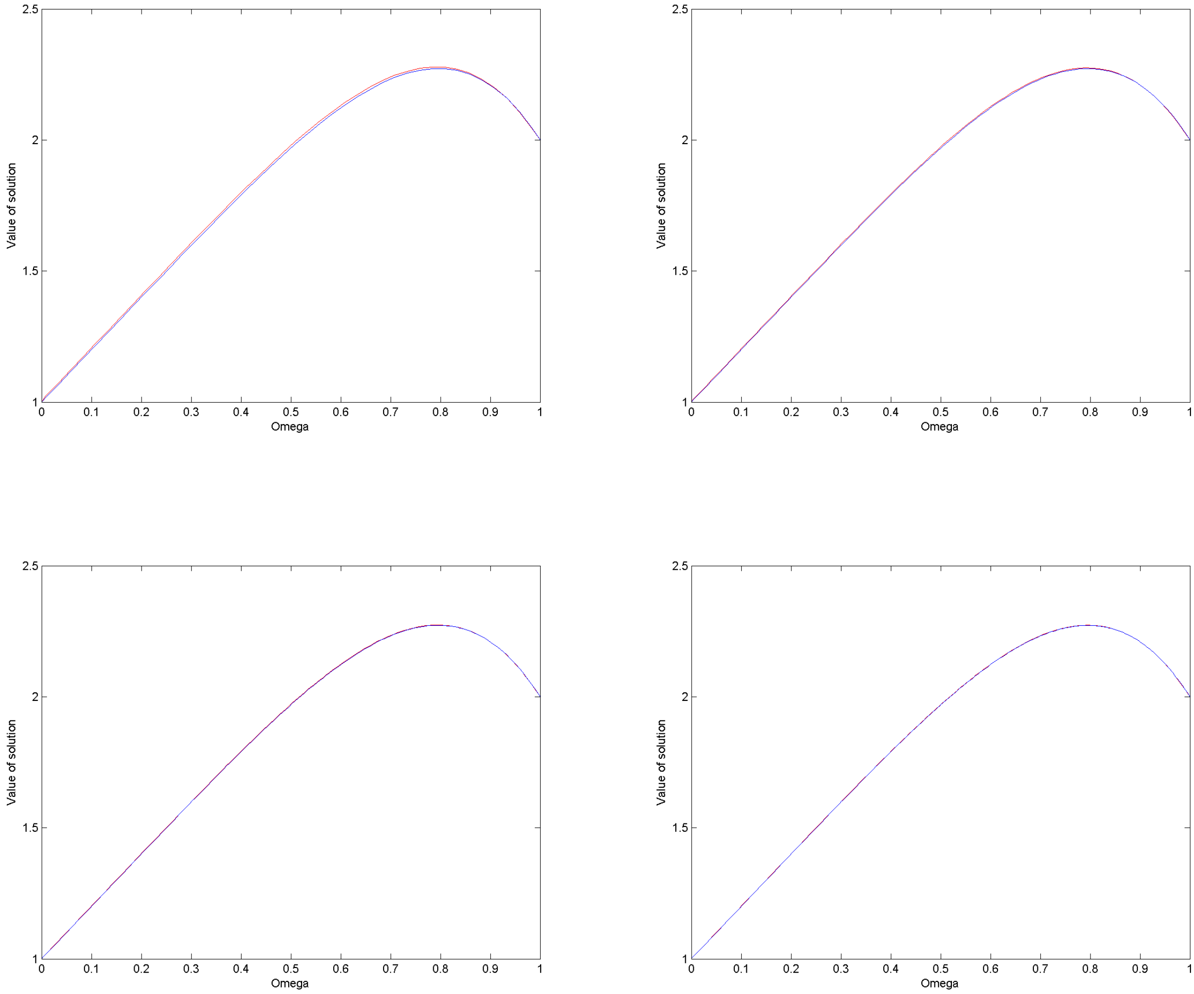
Value of solution
Omega

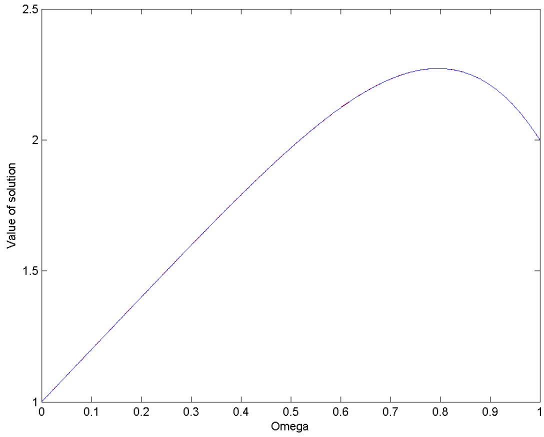


Error
L^2 Norm
H^1 norm
3/2x
2x
Errors
Number of element N

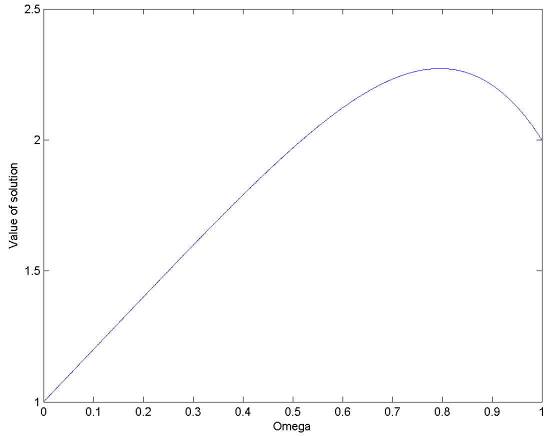
With the following exact solution u and function f

$$\begin{cases} u\left(x\right) = -x^5 + 2x + 1 \\ f\left(x\right) = 20x^3 \\ u'\left(0\right) - 2u\left(0\right) = u'\left(1\right) + \frac{3}{2}u(1) = 0 \end{cases}$$

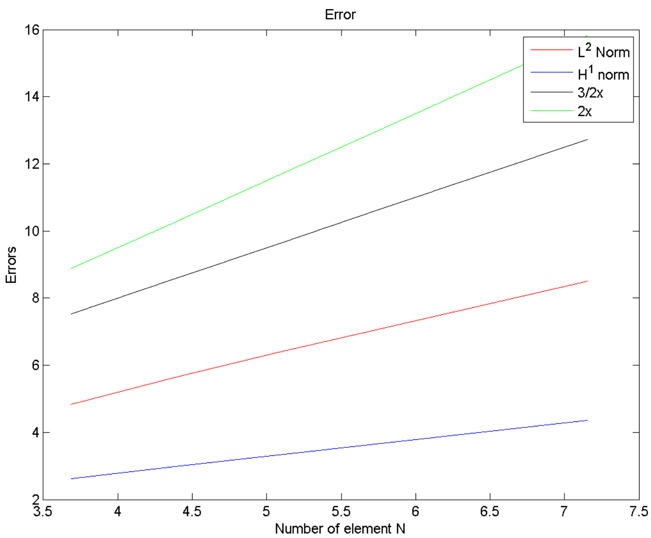




Value of solution
Omega



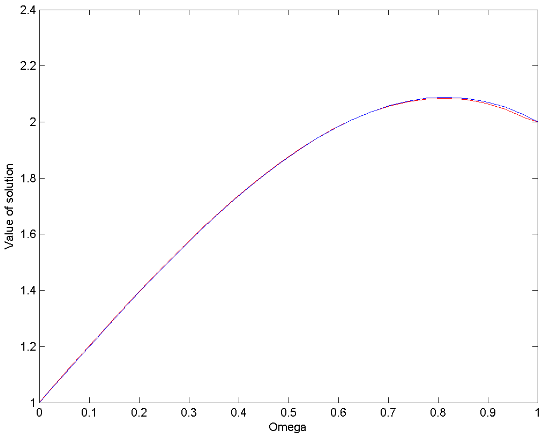
Value of solution
Omega

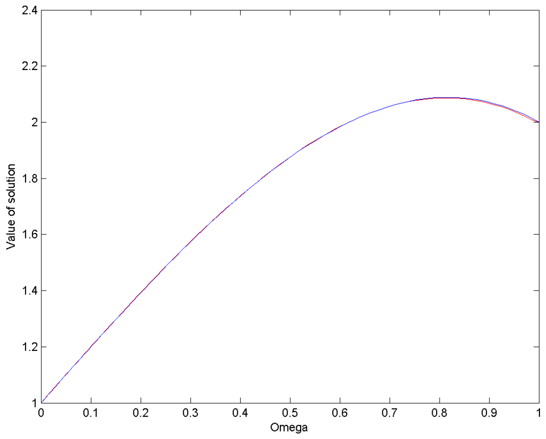


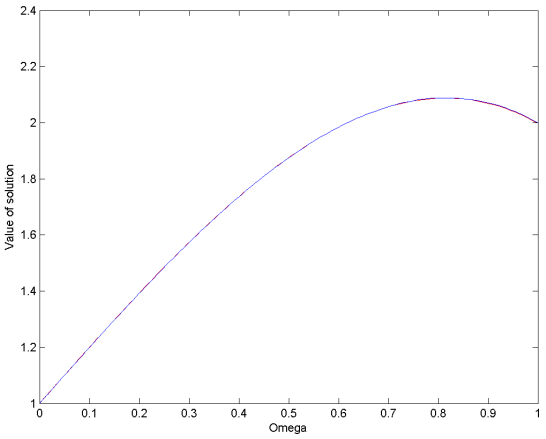
Error
L^2 Norm
H^1 norm
3/2x
2x
Errors
Number of element N

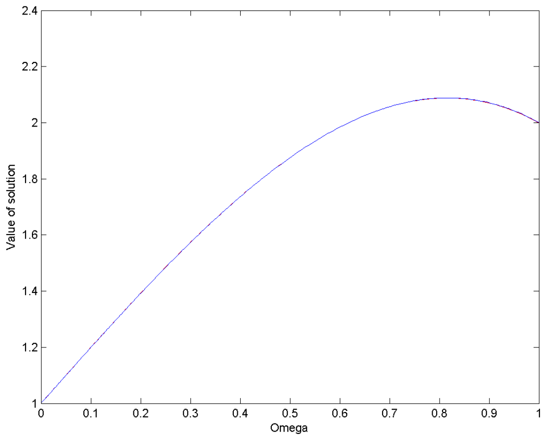
3. We set up with the following exact solution u and function f

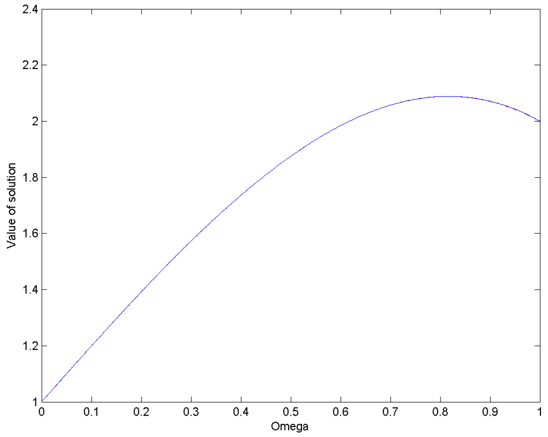
$$\begin{cases} u\left(x\right) = -x^3 + 2x + 1 \\ f\left(x\right) = 6x \\ u'\left(0\right) - 2u\left(0\right) = u'\left(1\right) + \frac{1}{2}u(1) = 0 \end{cases}$$



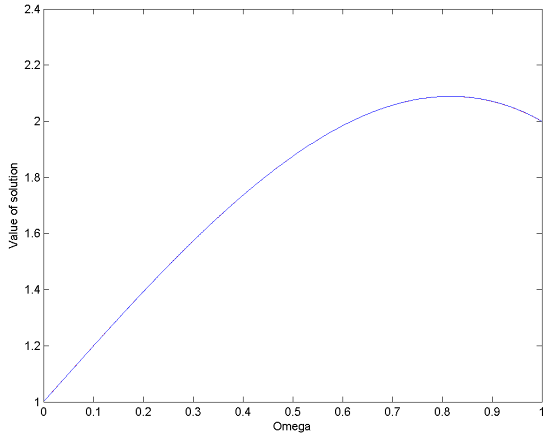




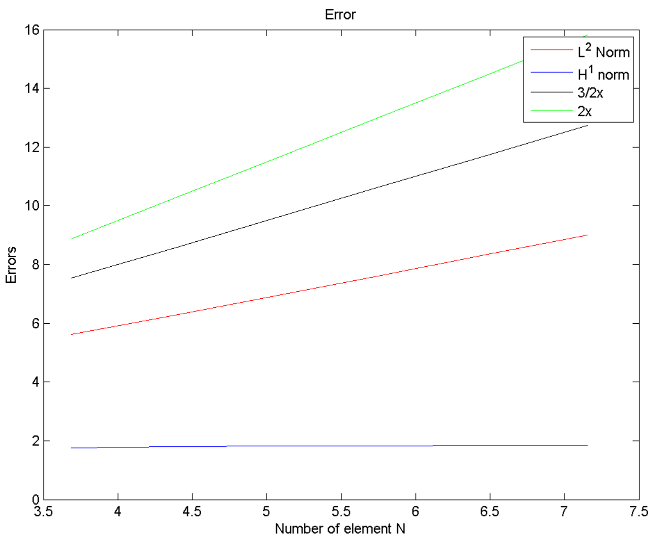




Value of solution
Omega



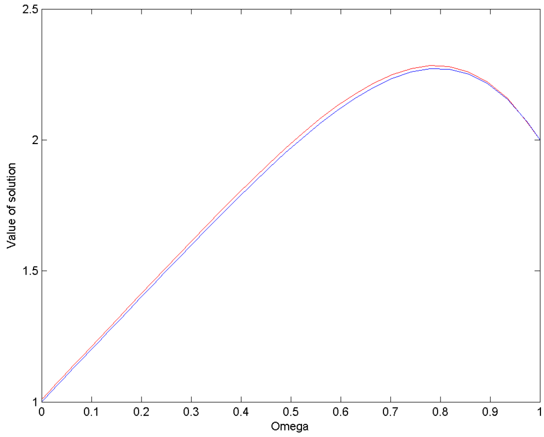
Value of solution
Omega

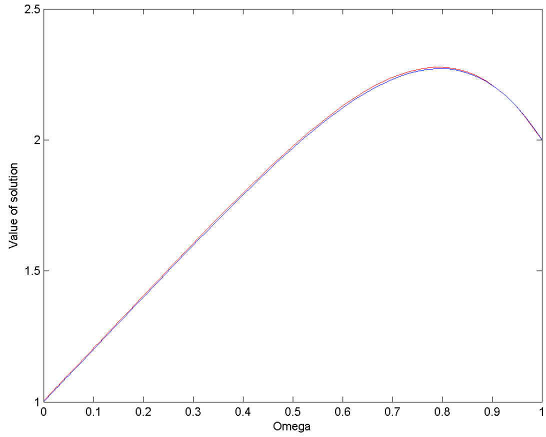


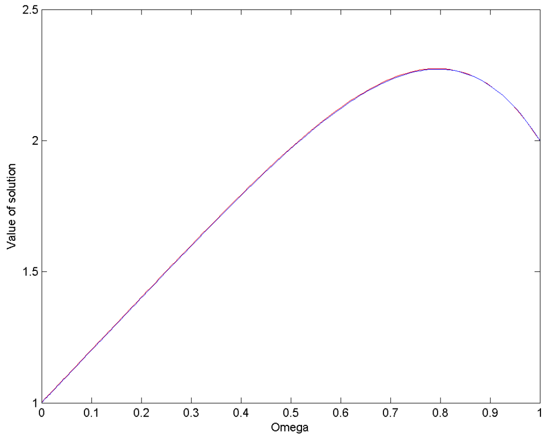
Error
L^2 Norm
H^1 norm
3/2x
2x
Errors
Number of element N

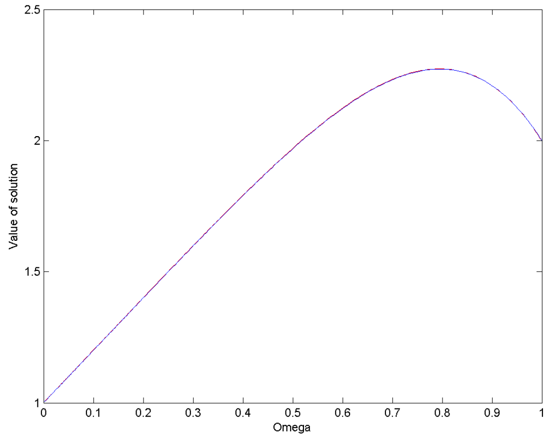
With the following exact solution u and function f

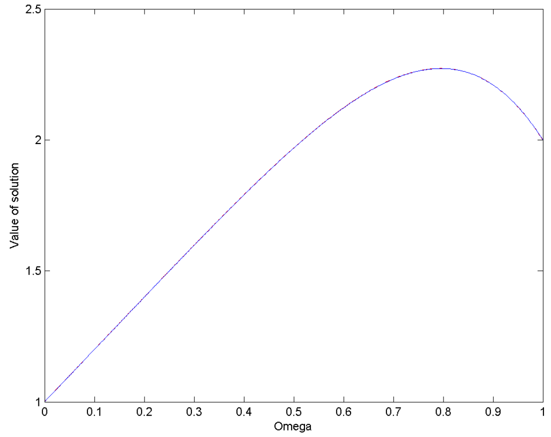
$$\begin{cases} u\left(x\right) = -x^5 + 2x + 1 \\ f\left(x\right) = 20x^3 \\ u'\left(0\right) - 2u\left(0\right) = u'\left(1\right) + \frac{3}{2}u(1) = 0 \end{cases}$$



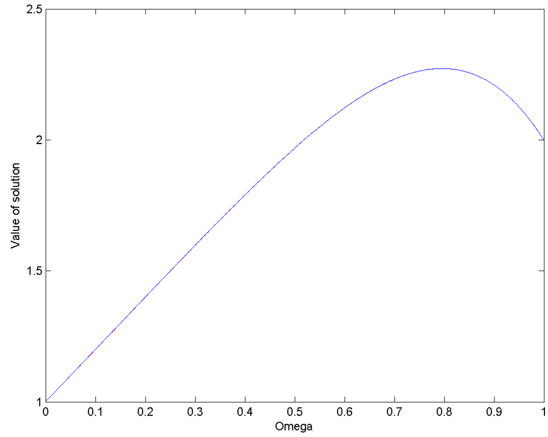




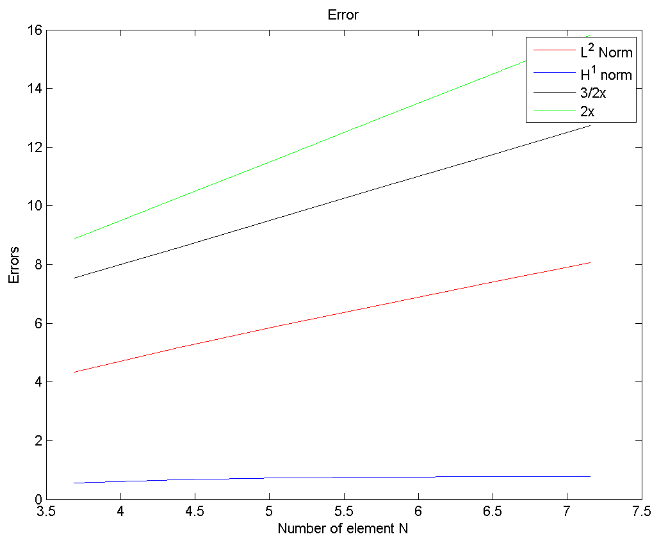




Value of solution
Omega



Value of solution
Omega



Error
L^2 Norm
H^1 norm
3/2x
2x
Errors
Number of element N