There are a number of texts on discrete-time signals and systems. We mention as examples the books by McGillem and Cooper (1984), Oppenheim and Willsky (1983), and Siebert (1986). Linear constant-coefficient difference equations are treated in depth in the books by Hildebrand (1952) and Levy and Lessman (1961).

The last topic in this chapter, on correlation of discrete-time signals, plays an important role in digital signal processing, especially in applications dealing with digital communications, radar detection and estimation, sonar, and geophysics. In our treatment of correlation sequences, we avoided the use of statistical concepts. Correlation is simply defined as a mathematical operation between two sequences, which produces another sequence, called either the *crosscorrelation sequence* when the two sequences are different, or the *autocorrelation sequence* when the two sequences are identical.

In practical applications in which correlation is used, one (or both) of the sequences is (are) contaminated by noise and, perhaps, by other forms of interference. In such a case, the noisy sequence is called a *random sequence* and is characterized in statistical terms. The corresponding correlation sequence becomes a function of the statistical characteristics of the noise and any other interference.

The statistical characterization of sequences and their correlation is treated in Chapter 12. Supplementary reading on probabilistic and statistical concepts dealing with correlation can be found in the books by Davenport (1970), Helstrom (1990), Peebles (1987), and Stark and Woods (1994).

Problems

2.1 A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

(b) Sketch the signals that result if we:

1. First fold $x(n)$ and then delay the resulting signal by four samples.
2. First delay $x(n)$ by four samples and then fold the resulting signal.

(c) Sketch the signal $x(-n+4)$.

(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

2.2 A discrete-time signal $x(n)$ is shown in Fig. P2.2. Sketch and label carefully each of the following signals.

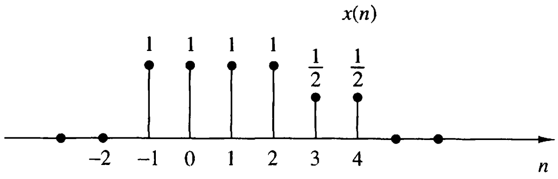

Figure P2.2

(a) $x(n-2)$ **(b)** $x(4-n)$ **(c)** $x(n+2)$ **(d)** $x(n)u(2-n)$ **(e)** $x(n-1)\delta(n-3)$
(f) $x(n^2)$ **(g)** even part of $x(n)$ **(h)** odd part of $x(n)$

2.3 Show that

(a) $\delta(n) = u(n) - u(n-1)$

(b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

2.4 Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

2.5 Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

2.6 Consider the system

$$y(n) = \mathcal{T}[x(n)] = x(n^2)$$

(a) Determine if the system is time invariant.

(b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied into the system.

(1) Sketch the signal $x(n)$.

(2) Determine and sketch the signal $y(n) = \mathcal{T}[x(n)]$.

(3) Sketch the signal $y_2'(n) = y(n-2)$.

(4) Determine and sketch the signal $x_2(n) = x(n-2)$.

(5) Determine and sketch the signal $y_2(n) = \mathcal{T}[x_2(n)]$.

(6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

(c) Repeat part (b) for the system

$$y(n) = x(n) - x(n-1)$$

Can you use this result to make any statement about the time invariance of this system? Why?

(d) Repeat parts (b) and (c) for the system

$$y(n) = \mathcal{T}[x(n)] = nx(n)$$

2.7 A discrete-time system can be