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MATHEMATICAL MODELING (CO2011)

Assignment (Semester: 233, Duration: 04 weeks)

“Cutting stock problem”

(Version 0.1, *in Preparation*)

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Abstract. The Cut Stock Problem (CSP) is a classic optimization problem in the field of Operations Research and Industrial Engineering. It involves determining the most efficient way to cut a set of smaller lengths (orders) from a given larger length (stock), such that waste is minimized. This problem arises in various industries such as paper, steel, and textile, where materials are produced in standard lengths and need to be cut into smaller sizes to meet customer demand. The complexity of the problem increases with the number of different order sizes and the requirement to fulfill all demands exactly. Various solution approaches have been proposed for the CSP, including dynamic programming, column generation, and heuristic methods.

Team Assignment Goals: Learn concepts, ideas and methods to

a) Literature Review: Conduct a comprehensive literature review on the Cut Stock Problem. This should include understanding the problem's history, its significance in various industries, and the different mathematical models used to represent it.

b) Algorithm Analysis: Analyze different algorithms and heuristics that have been proposed to solve the Cut Stock Problem. Understand their strengths, weaknesses, and the conditions under which they perform best.

c) Case Studies: Identify and study real-world cases where the Cut Stock Problem is relevant. Understand how these problems were addressed and what solutions were implemented.

d) Tool Development: Develop a tool or software that can solve the Cut Stock Problem. This could involve implementing one or more of the algorithms studied in goal 2.

e) Performance Evaluation: Evaluate the performance of the developed tool on both synthetic and real-world data. Compare its performance with existing tools.

f) Future Work: Identify gaps in the current state of research and propose future work. This could involve developing new algorithms, improving existing ones, or exploring new application areas for the Cut Stock Problem.

1 Introduction to Cutting-stock problem

In operations research, the cutting-stock problem is the problem of cutting standard-sized pieces of stock material, such as paper rolls or sheet metal, into pieces of specified sizes while minimizing material wasted. It is an optimization problem in mathematics that arises from applications in industry. Regarding computational complexity, the problem is an NP-hard problem reducible to the knapsack problem. The problem can be formulated as an integer linear programming problem.

Firstly, the simplification relationships should follow a straight line. Hence, the term linear. Secondly, all the values need to be subject to constraints, which may be in numerical or in terms or properties. And lastly, the solution has to optimize (i.e., maximize or minimize, depending on the problem) the quantity of a given variable.

Cutting-stock problems can be classified in several ways. One way is the dimensionality of the cutting: the above example illustrates a one-dimensional (1D) problem; other industrial applications of 1D occur when cutting pipes, cables, and steel bars. Two-dimensional (2D) problems are encountered in furniture, clothing and glass production. When either the master item or the required parts are irregular-shaped (a situation often encountered in the leather, textile, and metals industries), this is referred to as the nesting problem.

Not many three-dimensional (3D) applications involving cutting are known; however, the closely related 3D packing problem has many industrial applications, such as packing objects into shipping containers (see, e.g. containerization: the related sphere packing problem has been studied since the 17th century (Kepler conjecture)).

2 One-dimensional cutting-stock problem

The one-dimensional cutting-stock problem is a classic problem in operations research and industrial engineering. It involves determining the most efficient way to cut a set of smaller lengths (orders) from a given larger length (stock) such that waste is minimized. This problem arises in various industries such as paper, steel, and textile, where materials are produced in standard lengths and need to be cut into smaller sizes to meet customer demand. The complexity of the problem increases with the number of different order sizes and the requirement to fulfil all demands exactly.

♦ **EXAMPLE 1.** A company produces metal rods that are 100 units long. They receive orders for rods of lengths 45, 36, 30, and 15 units. The respective demands for these lengths are 7, 5, 8, and 9.

Goal: The goal is to determine how to cut the 100-unit rods to fulfil these orders while minimizing waste.

One possible solution:

- Cut 7 rods into lengths of 45 and 55 units. Use the 55-unit pieces to fulfill the demand for 30-unit and 25-unit rods.
- Cut 5 rods into lengths of 36 and 64 units. Use the 64-unit pieces to fulfill the demand for 30-unit and 34-unit rods.
- Cut 8 rods into lengths of 30 and 70 units. Use the 70-unit pieces to fulfill the demand for 15-unit rods.
- Cut 9 rods into lengths of 15 and 85 units. Use the 85-unit pieces to fulfill the remaining demand for 15-unit rods.

This solution minimizes the waste while fulfilling all the orders. However, it's important to note that this is just one possible solution. The optimal solution would depend on the specific demands and the cutting costs.

3 QUESTIONS for ASSIGNMENT 233

Students prepare a report consisting of the theoretic answer and computational solution following requests.

1. **Problem Formulation:** Formulate the one-dimensional cutting-stock problem as an integer linear programming problem. Define the decision variables, objective function, and constraints.
2. **Algorithm Analysis:** Research and write a brief summary of at least two algorithms or heuristics used to solve the one-dimensional cutting-stock problem. Discuss their advantages and disadvantages.
3. **Case Study:** Identify a real-world scenario where the one-dimensional cutting-stock problem is relevant. Describe the scenario and discuss how the problem could be solved in this context.
4. **Performance Evaluation:** Discuss how you would evaluate the performance of the algorithm used in Exercise 3. What metrics would you use? How would you gather the necessary data?

Hint: Students can refer to the paper [1].

4 Instructions and requirements

Students have to follow the instructions and comply with the requirements below. *Lecturers do not solve the cases arising because students do not follow the instructions or do not comply with the requirements.*

4.1 Instructions

Students must work closely with the other members of their team of 3-5 members.

All aspects of this assignment will be quizzed (about 4 - 5 of 25 multiple-choice questions) in the subject's final exam. Therefore, team members must work together so that all of you understand all aspects of the assignment. The team leader should organize the work team to meet this requirement.

If you have any questions about the assignment during work, **please post that question on the class forum on LMS.**

Regarding the background knowledge related to the topic, students should refer to all the references. However, it would be best to put all of them in the reference section of your report.

4.2 Requirements

- Deadline for submission: **August 10, 2024**. Students have to answer each question clearly and coherently.
- Write a report by using LaTeX in accordance with **the layout as in the template file** (you can find it on <https://www.overleaf.com/read/shszcwtrfhk#6e8f6f>).
- Each team when submitting its report **need to submit also a log file (diary)** in which clearly state: **weekly work progress for all 04 week(s)**, tasks, content of opinions exchanged of the members, ...
- Programming languages: C++/Python/Java/...(at your team preference)
- **Reporting requirements:** Your report should include the following sections:
 - **Introduction:** Introduce the problem and goals of the major exercise.
 - **Acknowledge**
 - **Algorithm**
 - **Modeling:** Describe how you modelled the problem as a linear programming problem. This should include defining variables, objective functions, and constraints.
 - **Analyze results:** Analyze and interpret model results, including explaining the optimal solution and its consequences for the company's operations.
 - **Conclusion:** Provide final comments and extensibility of the model and algorithms.
 - **References**

4.3 Submission

- Students must submit their team report via the BK-eLearning system; compress all necessary files (.pdf file, .tex file, coding file, ...) into a single file named "*Assignment-CO2011-CSE233-Team_name.zip*" and submit it to the assignment submission site.

- All of the meeting minutes of your group must be combined into one .txt file and consist of the % of each member's effort on the whole (total effort is 100%) on the first line.
- Noting that for each team, **only the leader will submit the report of the team.**

5 Evaluation and cheating treatment

5.1 Evaluation

Each assignment will be evaluated as in Table 1.

Table 1: Evaluation

Criteria	Score (%)
- Analyze, answer coherently, systematically, focus on the goals of the questions and requests	30%
- The programs are neatly written and executable	30%
- Correct, clear, and intuitive graphs & diagrams	20%
- Background section is well written, correct, and appropriate	15%
- Well written report and correct	5%

5.2 Cheating treatment

The assignment has been done by each group separately. Students in a group will be considered as cheating if:

- There is an unusual similarity among the reports (especially in the background section). In this case, ALL similar submissions are considered cheating. Therefore, the students of a group must protect their group's work.
- They do not understand the works written by themselves. You can consult from any source, but make sure that you know the meaning of everything you have written.

Students will be judged according to the university's regulations if the article is cheating.

6 COMPLEMENTS:

Few well-known 1D Cut Stock Problem and analysis are

1. **Extra Material: Cutting Stock**,
get from link <https://shorturl.at/0YrXd>.

References

- [1] G. R. Cerqueira, S. S. Aguiar, and M. Marques, "Modified greedy heuristic for the one-dimensional cutting stock problem," *Journal of Combinatorial Optimization*, vol. 42, no. 3, pp. 657–674, 2021.