Chapter 3: Dealing with Uncertainty

Outline

- Sources of uncertainty
- Bayesian updating
- Certainty theory
- Fuzzy set & fuzzy logic

Certainty Theory

- Bayesian updating → probability → statistical data → much data
- What is data are insufficiently much?

Certainty Theory

- Why statistics required?
  - Mathematic foundation
- Computer science is new → it can setup it own foundation occasionally
  - Less mathematical correctness but more practical usage

Certainty Value

- Given a hypothesis *H*
  - $C(H) = 1.0$ *H* is true
  - $C(H) = 0.0$ *H* is unknown
  - $C(H) = -1.0$ *H* is false

Certainty Value

- Given a hypothesis *H*
  - $C(H) = 1.0$ *H* is true — $P(H) = 1.0$
  - $C(H) = 0.0$ *H* is unknown — $P(H) = P(H)$ <what?>
  - $C(H) = -1.0$ *H* is false — $P(H) = 0.0$

Certainty Factor

IF evidence *E* THEN hypothesis *H* WITH certainty factor *CF*

Certainty Factor

IF evidence *E* THEN hypothesis *H* WITH certainty factor *CF*

E is not certain?

Certainty Factor

IF evidence *E* THEN hypothesis *H* WITH certainty factor *CF*

$CF' = CF \bullet C(E)$

Certainty Updating

Updating $C(H)$, given evidence E

$C(H) \leftarrow C(H|E)$

Certainty Updating

Updating *C*(*H*), given evidence *E*

C(*H*) ← *C*(*H*|*E*)

CF' =?

if *C*(*H*) ≥ 0 and *CF'* ≥ 0: C(*H*|*E*)=*C*(*H*)+[*CF'*•(1-*C*(*H*))]

if *C*(*H*) ≥ 0 and *CF'* ≥ 0: *C*(*H*|*E*)=*C*(*H*)+[*CF'*•(1+*C*(*H*))]

if *C*(*H*) ≥ and *CF'* ≥ 0:

$$C(H \mid E) = \frac{C(H) + CF'}{1 - \min(|C(H)|, |CF'|)}$$

Meanings behind

- $-1 \leq C(H|E) \leq 1$
- $C(H)$ or CF' is 1(-1), so is $C(H|E)$
- $C(H) = -CF' \rightarrow C(H|E) = 0$
- $C(H) = 0 \rightarrow C(H|E) = CF'$
- $C(E) = 1 \rightarrow CF' = CF$

Evidence Logical Combination

- Conjunction

  IF E_1 AND E_2 THEN H WITH CF

  $C(E_1 \text{ AND } E_2) = \min[C(E_1), C(E_2)]$

- Disjunction

  IF E_1 OR E_2 THEN H WITH CF

  $C(E_1 \text{ OR } E_2) = \max[C(E_1), C(E_2)]$

- Negation

  IF NOT E THEN H WITH CF

  $C(\sim E) = -C(E)$

Example

- Read for yourself

Possibility theory

- Probability: likelihood of the hypothesis
- Possibility: meaning of the hypothesis

Possibility theory

- Probability: likelihood of the hypothesis
- Possibility: meaning of the hypothesis
  - Fuzzy set

Possibility theory

- Probability: likelihood of the hypothesis
- Possibility: meaning of the hypothesis
  - Fuzzy set
  - Fuzzy logic

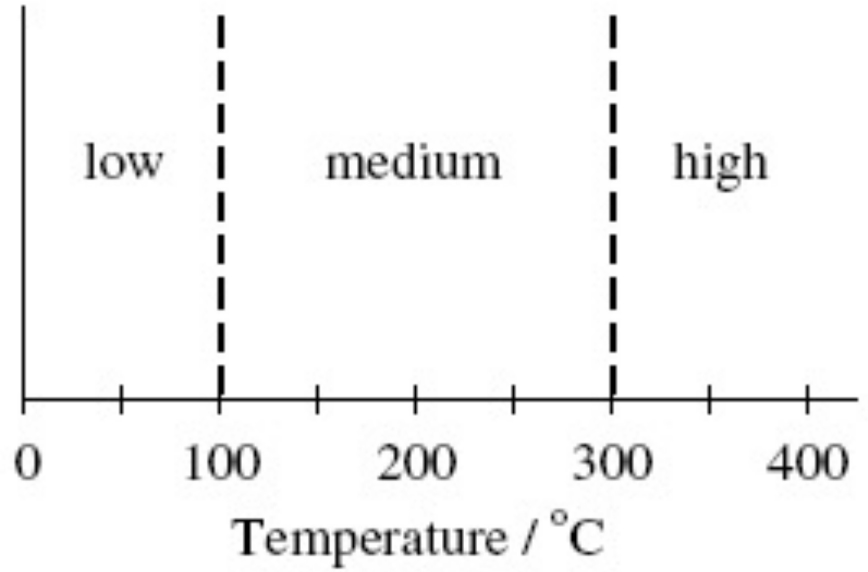
Crisp set and fuzzy set

- Vague in languages:
  - Water level is *low*
  - Temperature is *high*
- Crisp set:

  $\{Low\} \cap \{High\} = \emptyset$
- Fuzzy set:

  $\{Low\} \cap \{High\} \neq \emptyset$

Crisp set



Fuzzy set

- F is a fuzzy set, x is a value

$$x \in F\ ?$$

Fuzzy set

- F is a fuzzy set, x is a value

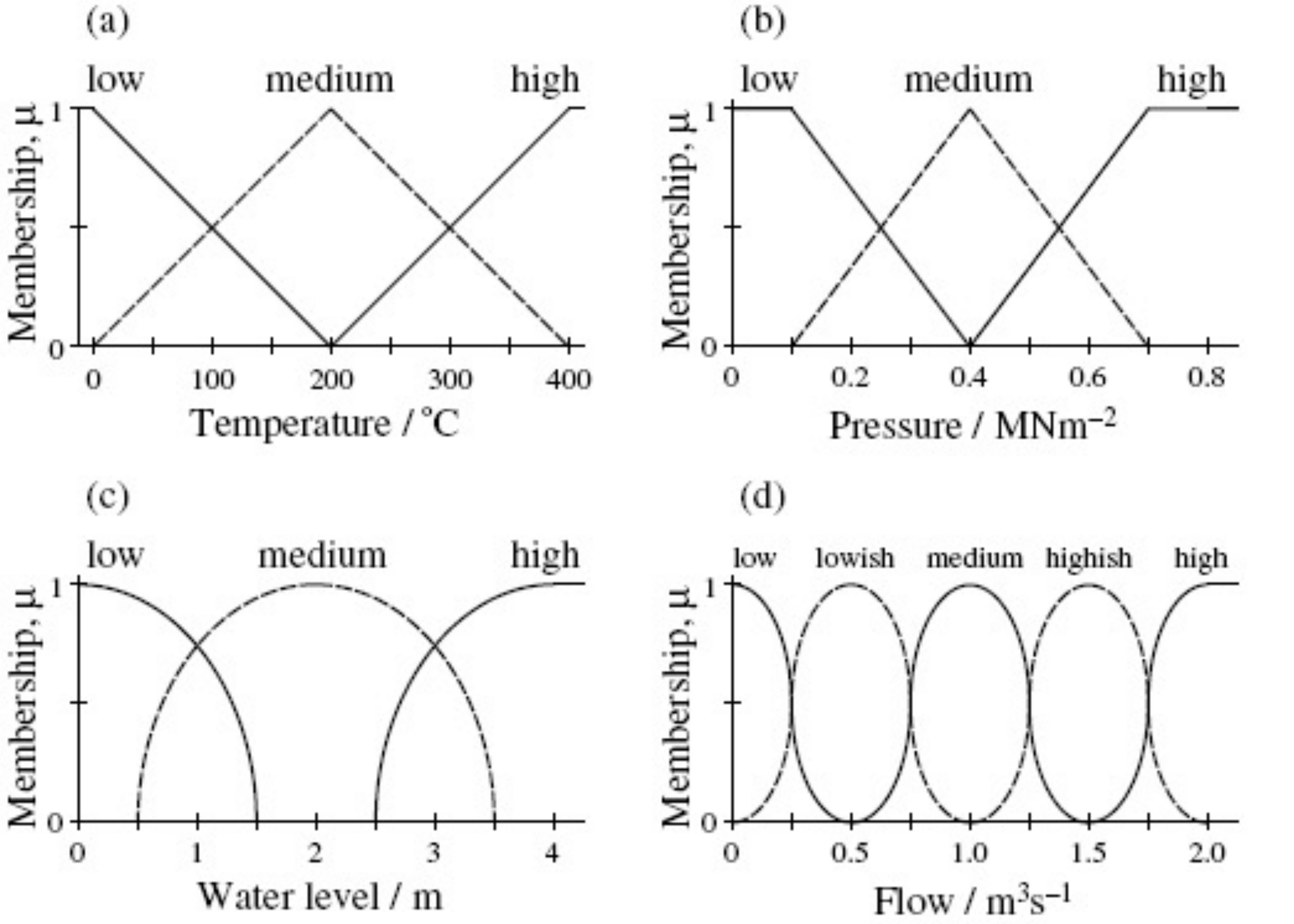
$$x \in F\,? \Leftrightarrow \mu_F(x)$$

Fuzzy set

- F is a fuzzy set, x is a value

$$x \in F\ ? \Leftrightarrow \mu_F(x)$$

  - $\mu_F(x)$: membership value of x in F
  - $0 \leq \mu_F(x) \leq 1$


(a)
low
medium
high
Membership, μ
1
0
0
100
200
300
400
Temperature / °C
(b)
low
medium
high
Membership, μ
1
0
0
0.2
0.4
0.6
0.8
Pressure / MNm−2
(c)
low
medium
high
Membership, μ
1
0
0
1
2
3
4
Water level / m
(d)
low
lowish
medium
highish
high
Membership, μ
1
0
0
0.5
1.0
1.5
2.0
Flow / m3s−1

Fuzzy rules

- IF temperature high THEN pressure high
- IF temperature medium THEN pressure medium
- IF temperature low THEN pressure low

$t = 200^{o}C$?

$t = 300^{o}C$?

$t = 400^{o}C$?

Condition Combination

- IF temperature high AND water NOT low THEN pressure high

- IF temperature high THEN pressure high
- IF water high THEN pressure high

- IF temperature high OR water high THEN pressure high

Fuzzy Logic

- $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$
- $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$
- $\mu_{\sim A}(x) = 1 - \mu_A(x)$

Defuzification

- t = 350°C

  $\mu_{HT}(x) = 0.75$ $\mu_{MT}(x) = 0.25$

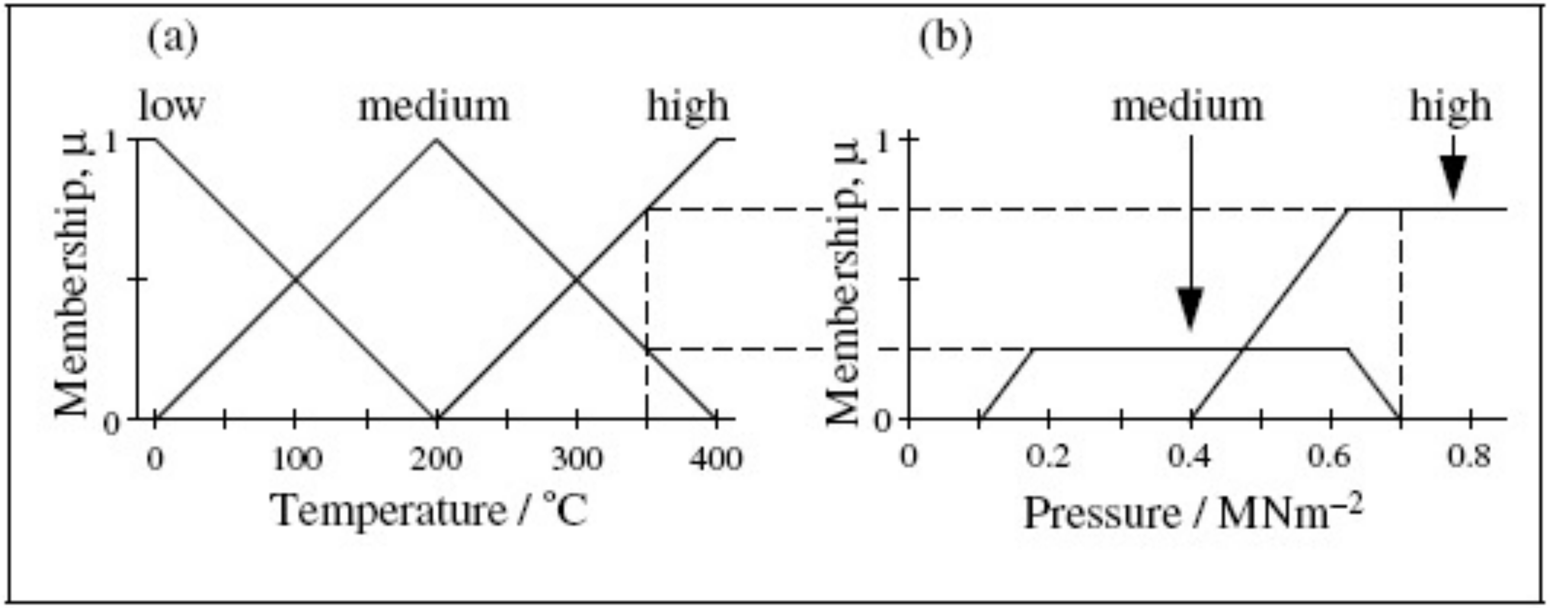
- IF temperature high THEN pressure high
- IF temperature medium THEN pressure medium

- Pressure in terms of MNn^2???

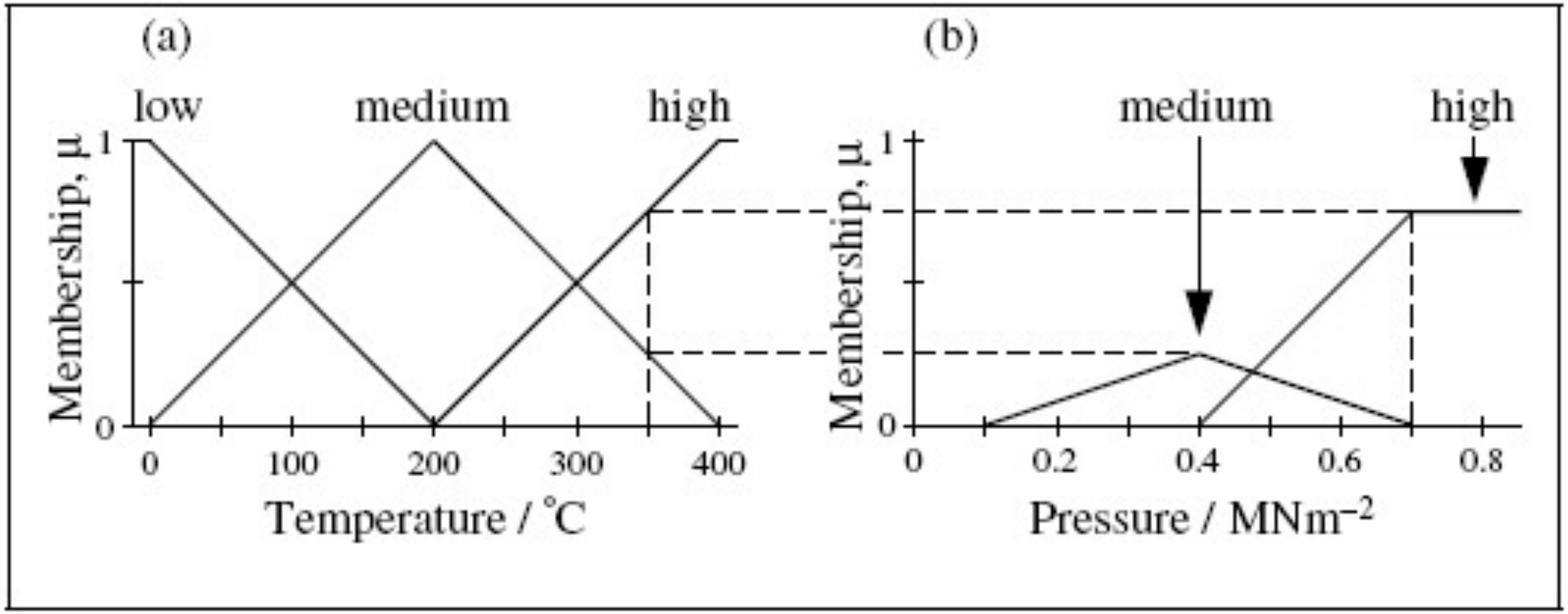
Defuzification

- Scaling membership function
- Finding combined centroid

Scaling membership function – Mamdami's method



Scaling membership function – Larson's method



Finding combined centroid

- Formula << textbook
- What is the formula?

Worked Example

- IF x is A_1 AND y is B_1 THEN z is C_1
- IF x is A_2 AND y is B_2 THEN z is C_2

$\mu_{A1}(x) = (x\text{-}2)/3$ $(2 \le x \le 5)$ $(8\text{-}x)/3$ $(2 < x \le 5)$

$\mu_{A2}(x) = (x\text{-}3)/3$ $(3 \le x \le 6)$ $(9\text{-}x)/3$ $(6 < x \le 9)$

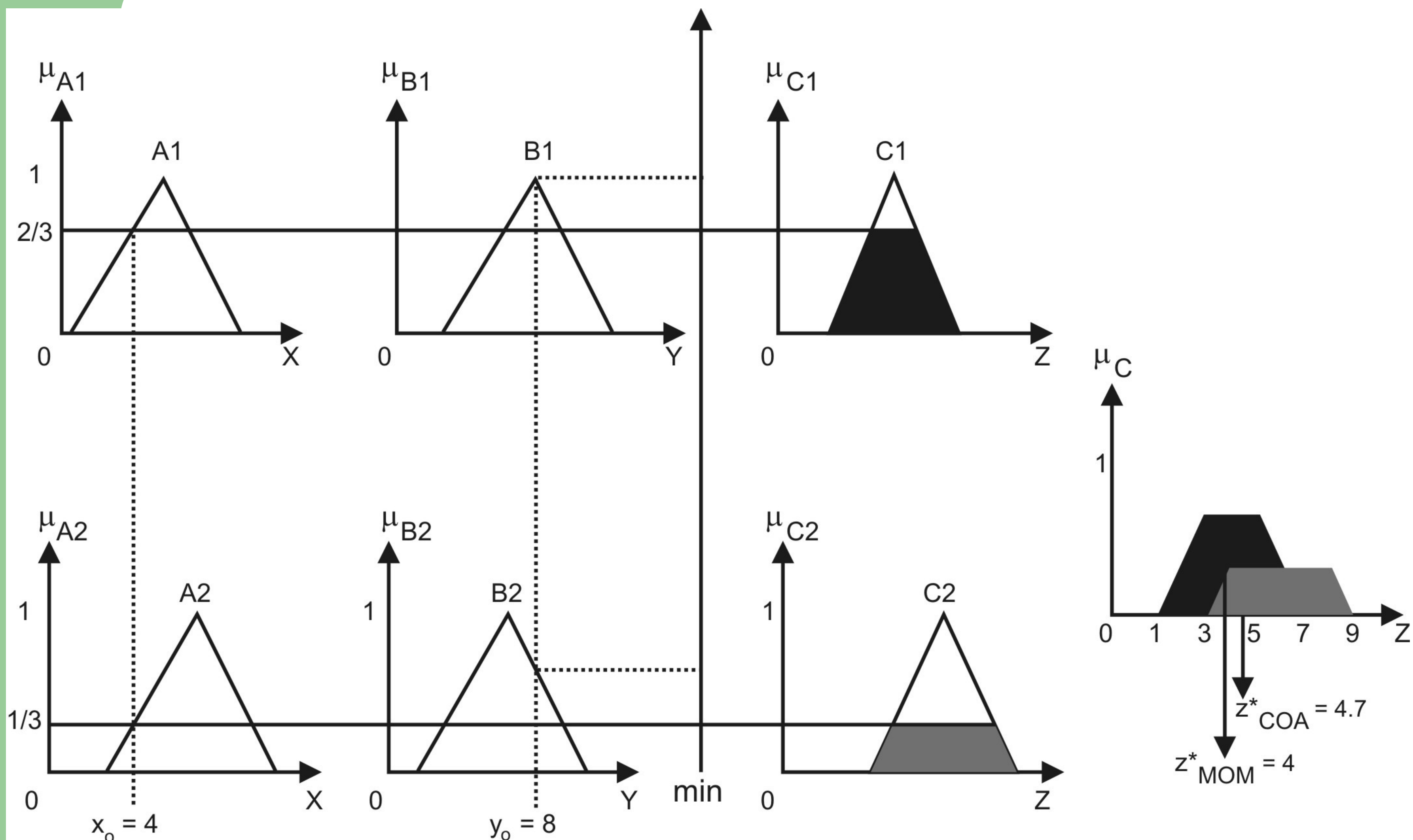
$\mu_{B1}(y) = (y\text{-}5)/3$ $(5 \le y \le 8)$ $(11\text{-}y)/3$ $(8 < y \le 11)$

$\mu_{B2}(y) = (y\text{-}4)/3$ $(4 \le y \le 7)$ $(10\text{-}y)/3$ $(7 < y \le 10)$

$\mu_{C1}(z) = (z\text{-}1)/3$ $(1 \le z \le 4)$ $(7\text{-}z)/3$ $(4 < z \le 7)$

$\mu_{C2}(z) = (z\text{-}3)/3$ $(3 \le z \le 6)$ $(9\text{-}z)/3$ $(6 < z \le 9)$

$x = 4$, $y = 8$, $z = ?$



μ_{A1}
A1
1
2/3
0
X
μ_{B1}
B1
0
Y
μ_{C1}
C1
0
Z
μ_C
1
0 1 3 5 7 9 Z
$z^*_{COA} = 4.7$
$z^*_{MOM} = 4$
μ_{A2}
A2
1
1/3
0
X
$x_o = 4$
μ_{B2}
B2
1
0
Y
$y_o = 8$
min
μ_{C2}
C2
1
0
Z