Nonlinear Optimization
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HW 2: Conjugate Gradient

volba počáteční aproximace $\boldsymbol{x}_0$

výpočet počátečního rezidua $\boldsymbol{r}_0 = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_0$

nastavení počátečního směrového vektoru $\boldsymbol{s}_0 = \boldsymbol{r}_0$

iterace $k = 0, 1, \ldots$

$$\alpha_k = \frac{\boldsymbol{r}_k^T \boldsymbol{r}_k}{\boldsymbol{s}_k^T \boldsymbol{A} \boldsymbol{s}_k}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \boldsymbol{s}_k$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \alpha_k \boldsymbol{A} \boldsymbol{s}_k$$

pokud $\|\boldsymbol{r}_{k+1}\| < \varepsilon$, konec iterace

$$\beta_k = \frac{\boldsymbol{r}_{k+1}^T \boldsymbol{r}_{k+1}}{\boldsymbol{r}_k^T \boldsymbol{r}_k}$$

$$\boldsymbol{s}_{k+1} = \boldsymbol{r}_{k+1} + \beta_k \boldsymbol{s}_k$$

Exercise:

Implement a Gradient Descent algorithm and plot the solution

Příklad. Soustava rovnic

$$\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

má řešení

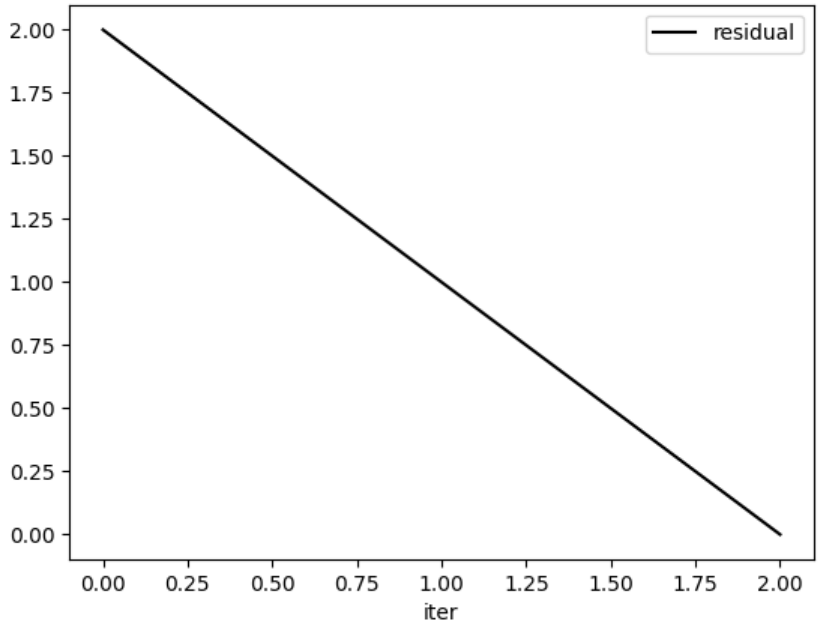
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

For following experiments, initial guess is (x, y) = (0, 0) the termination condition parameter epsilon is set to 0.001.

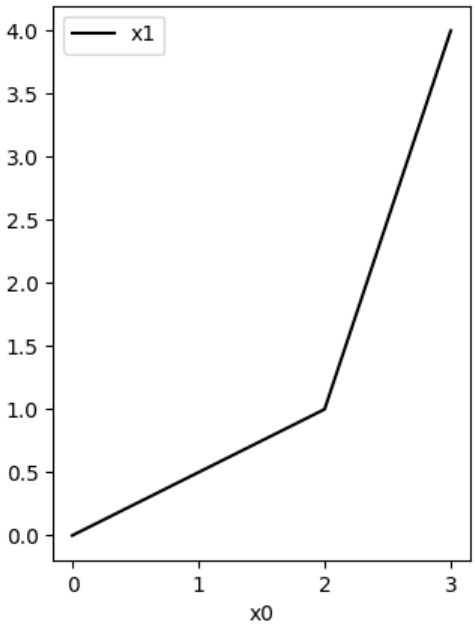
Solution:

To optimize the computation cost in each iteration we store the intermediate values of $\boldsymbol{r}_k^T \boldsymbol{r}_k$ and $\boldsymbol{r}_{k+1}^T \boldsymbol{r}_{k+1}$ since their values are used twice in the iteration.

Plot of the iteration step against residual norm reveals converging behavior in 2 iterations compared to 20 iterations of Gradient Descent. Gradient Descent is less effective because it does not use the step direction vector to estimate the best step to take and instead it takes small steps to arrive to the goal.



Plot of the actual solution converges to (x0, x1) = (3, 4)


x1
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
x0