Nonlinear Optimization
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HW 1: Gradient Descent

Algoritmus metody největšího spádu

volba počáteční aproximace $\boldsymbol{x}_0$

výpočet počátečního rezidua $\boldsymbol{r}_0 = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_0$

iterace $k = 0, 1, \ldots$

$$\alpha_k = \frac{\boldsymbol{r}_k^T \boldsymbol{r}_k}{\boldsymbol{r}_k^T \boldsymbol{A} \boldsymbol{r}_k}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \boldsymbol{r}_k$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{r}_k - \alpha_k \boldsymbol{A} \boldsymbol{r}_k$$

pokud $\|\boldsymbol{r}_{k+1}\| > \varepsilon$, další krok, jinak konec

Exercise:

Implement a Gradient Descent algorithm and get the solution

Příklad. Soustava rovnic

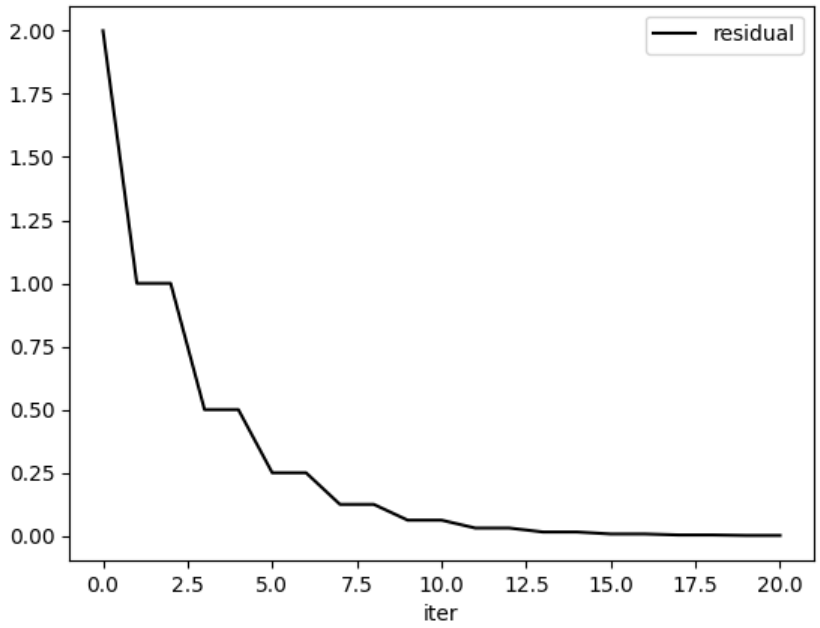
$$\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

má řešení

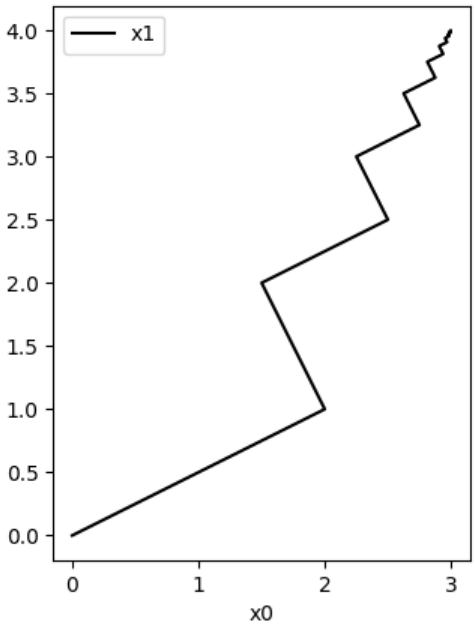
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

For following experiments, initial guess is (x, y) = (0, 0) the termination condition parameter epsilon is set to 0.001.

Plot of the iteration step against residual norm reveals converging behavior in 20 iterations.



Plot of the actual solution converges to (x0, x1) = (3, 4)


x1
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
x0