TWO U(1)'S AND ϵ CHARGE SHIFTS

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If new particles are gauged by a new U(1) then their electromagnetic charges may be shifted by a calculable amount.

Suppose a theory has just one U(1) gauge factor and consider two fermions with charges in an integer ratio. This charge ratio will be a property of the effective theory at any scale, since a Ward identity ensures that renormalization of charge only arises from the wave function renormalization of the gauge field. But a theory which has two or more U(1) gauge factors can have nondiagonal wave function renormalization in the space of U(1) gauge fields. As we will see, this implies that charges which are integer multiples at one scale need not be integer multiples in the effective theory at another scale. Charges can be shifted by some amount ϵ.

We note that the physics which is responsible for ϵ charge shifts may be occurring at arbitrarily high energy scales, since we are discussing a property of the renormalizable part of the effective theory. But we will find that ϵ charge shifts with respect to a massless U(1) will persist down to arbitrarily low energies only if two U(1) gauge fields remain massless.

We consider two abelian gauge symmetries, $U_1(1)$ and $U_2(1)$. Fermions will carry subscripts 1, 2, 12, and 0 depending on whether they carry only charge 1, only charge 2, both charges or neither charge respectively. For example, if we want to associate the photon with $U_1(1)$ then all known fermions are either of type f_1 or f_0. But f_{12} fermions can contribute to the off-diagonal vacuum polarization diagram in fig. 1. We will illustrate how a nonvanishing contribution arises due to mass splittings among f_{12} fermions. The result is an effective interaction between an f_1 and an f_2. Intuitively, virtual f_{12} pairs around an f_1 fermion induce an effective "2" charge, and vice versa. In the end we will see that the

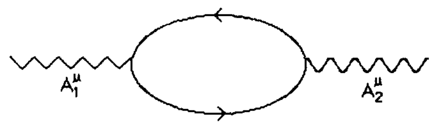

Fig. 1.

abelian fields can always be redefined such that ϵ charge shifts occur with respect to just one of the U(1)'s.

Consider a toy model with four fermions f_1, f_2, f_{12}, f'_{12} having charges $(e_1, 0)$, $(0, e_2)$, (e_1, e_2), $(e_1, -e_2)$ under a vector $U_1(1) \times U_2(1)$ gauge symmetry. Assume that $m_{12'} > m_{12} > m_1 \approx m_2$. From the first inequality and the charge assignments the diagram in fig. 1 with f_{12} and f'_{12} contributions is nonvanishing and finite. To study its effect we can define the theory to have conventional gauge field kinetic terms at some scale $\Lambda' > m_{12'}$. Then we can consider the effective theory at some scale Λ, $m_{1,2} < \Lambda < m_{12}$, in terms of the same fields. The gauge field kinetic terms take the form

$$-4L_{\text{kin}}(\Lambda) = \chi_1 (F_1^{\mu\nu})^2 + \chi_2 (F_2^{\mu\nu})^2 + 2\chi F_1^{\mu\nu} F_{2\mu\nu} . \quad (1)$$

For illustration we can arrange the couplings and mass scales so that the lowest order result in the couplings e_1 and e_2 approximates well the small deviations from $L_{\text{kin}}(\Lambda')$. $\chi_1 - 1$ and $\chi_2 - 1$ are the usual diagonal vacuum polarizations involving logs of ratios of Λ', Λ, $m_{12'}$, and m_{12}, while

$$\chi = (e_1 e_2 / 6\pi^2) \ln (m_{12'}/m_{12}) . \quad (2)$$

We can define new gauge fields $A_1'^\mu$ and $A_2'^\mu$ to regain diagonal kinetic terms with conventional normalization. The two sets of fields are related as follows where matrix notation is in the 1,2 space:

$$A'^{\mu} = D^{1/2} O^{\mathrm{T}} A^{\mu}, \quad A^{\mu} = O D^{-1/2} A'^{\mu}. \tag{3}$$

Here O and D are orthogonal and diagonal matrices respectively defined by

$$\begin{pmatrix} \chi_1 & \chi \\ \chi & \chi_2 \end{pmatrix} = O D O^{\mathrm{T}}. \tag{4}$$

After integrating out f_{12} and f'_{12} the effective theory at Λ has fermions f_1 and f_2 which couple to A_1^{μ} and A_2^{μ} respectively. But we see from the second equation in (3) that these two gauge fields are two nonorthogonal combinations of the primed fields. In other words, the gauge field radiated by f_1 has a component of the gauge field which is radiated by f_2, and vice versa. This gives the interaction between f_1 and f_2.

Note that any pair of orthonormal combinations of the A'^{μ} fields will have conventional kinetic terms. Let $A_1''^{\mu}$ and $A_2''^{\mu}$ be the orthonormal fields which respectively do and do not couple to f_1. Then $A_1''^{\mu} \propto A_1^{\mu}$ and

$$e_1 A_1^{\mu} = e_1 (O D^{-1/2} A'^{\mu})_1 = e_1 \chi_1^{-1/2} A_1''^{\mu} \equiv e_1'' A_1''^{\mu}. \tag{5}$$

The middle equality is good to lowest order in the small quantities $\chi_1 - 1$, $\chi_2 - 1$, and χ. Similarly

$$\begin{aligned} e_2 A_2^{\mu} &= e_2 (O D^{-1/2} A'^{\mu})_2 \\ &= e_2 \chi_2^{-1/2} (A_2''^{\mu} \cos\theta - A_1''^{\mu} \sin\theta) \\ &\equiv e_2'' (A_2''^{\mu} \cos\theta - A_1''^{\mu} \sin\theta), \end{aligned} \tag{6}$$

where $\sin\theta = \chi$ to lowest order. Thus $A_1''^{\mu}$ couples to f_2 and f_1 with couplings in the ratio of $\epsilon = -e_2''\chi/e_1''$. Of course we could have done things the other way around and identified the orthonormal fields which respectively did and did not couple to f_2. But the end result for interaction between f_1 and f_2 would be the same.

We have noted that the conventional relations $e_1'' = e_1 \chi_1^{-1/2}$ and $e_2'' = e_2 \chi_2^{-1/2}$ hold to lowest order. This implies that the two β-functions $\beta_{1,2}(E)$ are smooth functions of E as E is decreased and fig. 1 turns on. In a proper treatment the evolution of $e_1''(E)$ and $e_2''(E)$ would deviate from the case of no mixing as $\sin\theta(E)$ grew.

If there were two massless U(1) fields in the real world then we can always define the photon as we defined $A_1''^{\mu}$ in the above example. The orthonormal partner to the photon, the paraphoton, would by definition not couple to known fermions. But other fermions which did couple to the paraphoton end up either with a new induced coupling to the photon (like f_2) or having their original photon couplings altered (like f_{12}).

Is it possible that the paraphoton can gain a large mass and still leave the photon coupling to ϵ charges? The answer is no. The operator whose vacuum expectation value breaks the symmetry obviously has the charge of the broken U(1) and no charge under the unbroken U(1). Orthonormal gauge fields are then chosen with respectively do and do not couple to this charge. The unbroken U(1) is analogous to the $A_2''^{\mu}$ field which did not couple to the f_1 charge. Thus the unbroken U(1), as for $A_2''^{\mu}$, does not end up with ϵ charge shifts.

We now consider different possibilities for the generation of ϵ shifts of ordinary charge. Of course the proton to electron charge ratio remains unaffected since only fermions coupling to a paraphoton receive charge shifts.

(i) We have mentioned that the mixing between U(1)'s can occur at arbitrarily high energies. We may consider $U_Y(1)$ hypercharge becoming mixed with a second U(1). The fermions responsible for the mixing would have to couple to both U(1)'s and according to the above example they must be massive. Then if they are not to break the gauge symmetries their U(1) couplings must be vectorial. This implies exotic fermions which for example have a gauged $B - L$ but no τ_{R}^3. This situation would lead to hypercharge shifts for those fermions coupling to the second U(1). Then when $SU(2) \times U_Y(1)$ breaks to electromagnetism we could be left with electromagnetic charge shifts for low energy fermions. This is an example of how exotic physics at very high energies can affect the low energy world.

(ii) There is another way that U(1)'s can mix without involving massive fermions. Consider a gauge group $G_1 \times G_2 \times G_3$ where the G_i's are simple nonabelian groups. It is possible in this situation that a condensate which is nontrivial under all three G_i's can break the gauge group down to $H \times U_1(1) \times U_2(1)$ where H has no abelian factors. The U(1) fields can be written in terms of G_i fields, but which pair of orthonormal combinations of G_i fields one chooses is rather arbitrary. If the three G_i couplings are different then in general it is not possible to choose the U(1) fields such that they have orthogonal generators. That is, the diagram in fig. 1 has a nontrivial contribution from the remaining massless fermions of the theory. It is always possible to choose one of the U(1)'s, say $U_1(1)$, such that all $U_1(1)$ charges are integer multiples of the smallest $U_1(1)$ charge. For fermions which only carry $U_1(1)$ charge this will continue to hold. But fermions with $U_2(1)$ charge will now experience $U_1(1)$ charge shifts.

(iii) It is probably least exotic to consider new families of fermions with standard $SU(3) \times SU(2) \times U(1)$ quantum numbers and in addition a new vectorial paraphoton charge. A simple way to cancel anomalies is to have two such families with paraphoton charge +1 and −1 respectively. Mass splittings among the parafermions can yield a mixing between the photon and the paraphoton via fig. 1. Then parafermions experience ϵ charge shifts with respect to the standard photon. All parafermion masses are bounded from above by the weak scale. Paraneutrinos can in fact be quite light and have escaped detection since they only carry ϵ charge.

Paraneutrinos present in cosmic rays and interacting with the earth could conceivably produce detectable events in underground detectors. Paraneutrinos travel $1/\epsilon$ times as far as electrons or protons with the same energy in the galactic magnetic field before being significantly deflected. For a source at a distance approaching the gyroradius, the directions of incoming paraneutrinos would appear to be smeared out around the source direction. If this effect was confirmed in the Cygnus X-3 data [1] then we would require $\epsilon \approx 10^{-7} E$ [TeV] where E is the upper range of paraneutrino energies. (The paraneutrino would also have to have stronger than weak interactions with ordinary matter to have been detected.)

The paraphoton does not have renormalizable couplings to ordinary matter, but nonrenormalizable couplings are expected to be induced through loops. Such couplings must be considered for the possible detection of solar and cosmic paraphotons [2].

Finally, there is the possibility that parafermions also carry technicolor. Then the paraphoton becomes the techniphoton and as such it has been considered previously. The techniphoton symmetry and the family structure it entails has been connected with the suppression of flavor changing neutral currents [3]. The techniphoton symmetry is also of interest for a viable heavy neutrino, both in providing a suitable decay mode and in connection with a suitable neutrino mass matrix [4].

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