EXACT ELECTRON-MUON ELASTIC SCATTERING IN QED AT NEXT-TO-LEADING ORDER: SOFT-PHOTON CONTRIBUTION

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ABSTRACT

Recently, the new experiment MUonE has been proposed to measure precisely the running of the fine-structure constant at space-like momenta via the electron-muon elastic scattering. The motivation is to resolve the current 3-sigma anomaly on the muon (g-2) measurement. For the MUonE measurement, we have to calculate the cross section of electron-muon elastic scattering very precisely, at the level of 10 ppm relative accuracy. This requires the electron and muon masses have to be kept and next-to-next-to-leading order QED corrections must be included. Already at next-to-leading order, the precise calculation keeping the electron mass is numerically challenging due to different scales involved. Here we report on our attempt to calculate the cross section of electron-muon elastic scattering in QED at next-to-leading order in the soft-photon approximation keeping the electron mass. We will present some numerical results for a MUonE experiment setup.

MUONE EXPERIMENT

The MUonE has been proposed [1] to measure precisely the running of the fine-structure constant at space-like momenta via the electron-muon elastic scattering. In this experiment, planned to be at CERN, a high energy muon beam ($E_\mu^{beam} = 150 GeV$ in the Lab frame) scatters on atomic electrons of low-Z target. For this experiment we have to calculate the cross section of electron-muon elastic scattering very precisely, at the level of 10 ppm relative accuracy [2]. At this level of precision, the electron mass has to be kept.
From the theoretical side, the full next-to-leading order (NLO) QED corrections have been calculated in [3] and attempts to calculate the next-to-next-to-leading order (NNLO) keeping the electron mass have been reported in [4], [5]. A recent theoretical review can be found in [2].

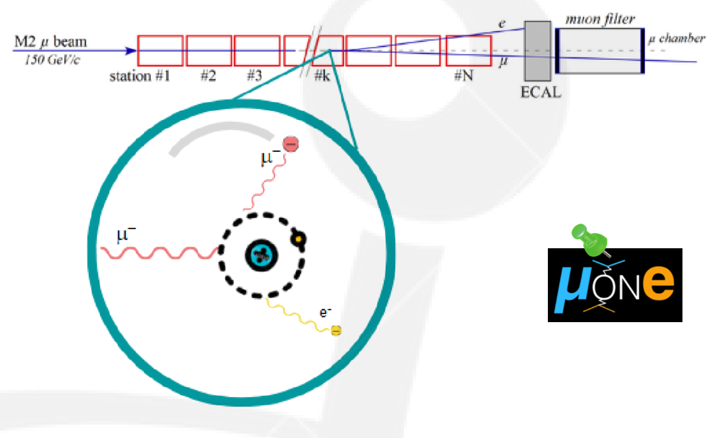

Figure 1: The MUonE experiment.

LEADING ORDER

For the $e^-\mu^- \to e^-\mu^-$ scattering, we obtain the following differential cross section at leading order (LO):

$$\frac{d\sigma}{d\cos\theta} = 4\pi\frac{\alpha^2}{t^2 s}\left[\frac{s^2}{4} + \frac{u^2}{4} + (m_\mu^2 + m_e^2)t - \frac{(m_\mu^2+m_e^2)^2}{2}\right], \quad (1)$$

$$\begin{cases} t = -2|\vec{p}|^2(1-\cos\theta), \\ u = 2(m_\mu^2+m_e^2) - s - 2|\vec{p}|^2(1-\cos\theta), \\ |\vec{p}| = -\frac{1}{2}\left[m_e^2 + m_\mu^2 - \frac{1}{2}\left(\frac{(m_e-m_\mu)^2}{s} + s\right)\right]. \end{cases} \quad (2)$$

The energy of the incoming muon, we require $E_\mu = 150$ GeV corresponding to Center of Mass Frame (CMF) colliding energy $\sqrt{s} = \sqrt{m_e^2 + E_\mu^2 + 2m_e E_\mu} = 0.405541158$ GeV.

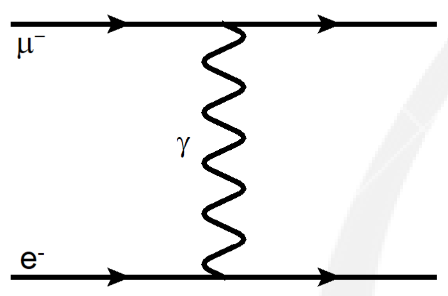

Figure 2: Electron muon elastic scattering diagram

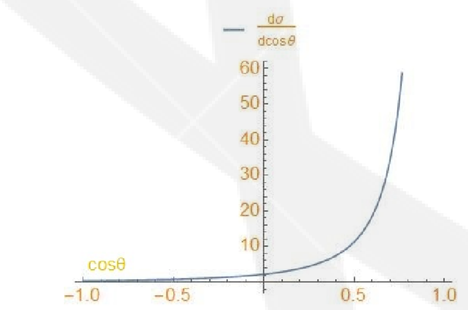

Figure 3: Differential cross section distribution in CMF

$\mu^-e^- \to \mu^-e^-$		
$\cos\theta$	$d\sigma/d\cos\theta$ $[\mu b]$	
	$m_e \neq 0$	$m_e = 0$
-0.9	0.44659304	0.44659234
-0.5	0.8012993	0.8012969
0	2.2195627	2.2195538
0.5	11.336969	11.336919
0.9	346.85751	346.85598

Table 1: Differential cross section at different values of $\cos\theta$

In Lab frame (where the inital electron is at rest), we request : $\theta_e, \theta_\mu < 100$ mrad and $E_e > 0.2$ GeV(corresponding to $\cos\theta \lesssim 0.997$ in CMF) for the final-state particles [3].

$\mu^-e^- \to \mu^-e^-$		
Cross section	Analytical result using Mathematica	Monte-Carlo simulation [3]
$\sigma_{LO}^{QED}(m_e \neq 0)$	1265.0603541	1265.060312(7)
$\sigma_{LO}^{QED}(m_e = 0)$	1264.9381128	(-)

Table 2: Leading order cross section for the MUonE experiment.

REFERENCES

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NEXT-TO-LEADING ORDER

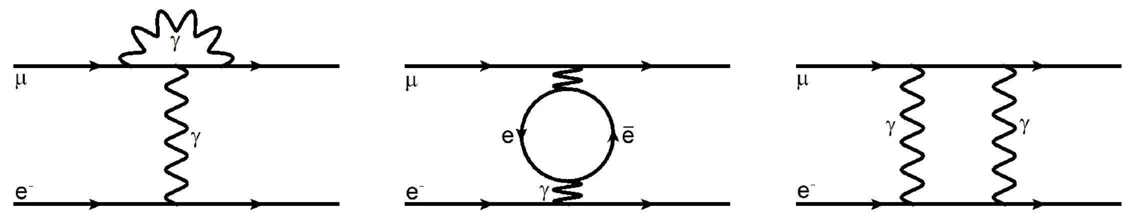
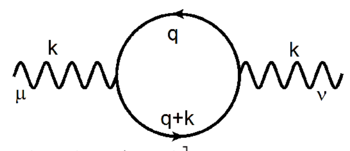
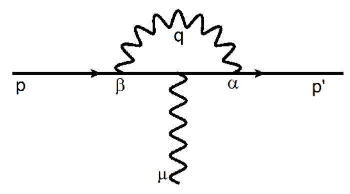


Figure 4: One-loop amputated Feynman diagrams

To increase our precision, we have to include higher order corrections. At the NLO level, we have to solve the problems of UV and IR divergences.



$$\Sigma_{\mu\nu}^{AA}(k) = -ie^2\int_{-\infty}^{+\infty}\frac{d^4q}{(2\pi)^4}\frac{Tr\left[\gamma_\mu(\not{q}+m)\gamma_\nu(\not{q}+\not{k}+m)\right]}{(q^2-m^2)[(q+k)^2-m^2]} \to \infty \text{ when } q^\mu \to \infty \text{ (UV-divergence).}$$



$$\Lambda_\mu(p,p') = -ie^2\int_{-\infty}^{+\infty}\frac{d^4q}{(2\pi)^4}\frac{\gamma_\alpha(\not{q}+\not{p}'+m)\gamma_\mu(\not{q}+\not{p}+m)\gamma^\alpha}{q^2[(q+p)^2-m^2][(q+p')^2-m^2]} \to \infty \text{ when } q^\mu \to 0 \text{ (IR-divergence).}$$

Ultraviolet Divergence

Loop momentum reaches to infinity

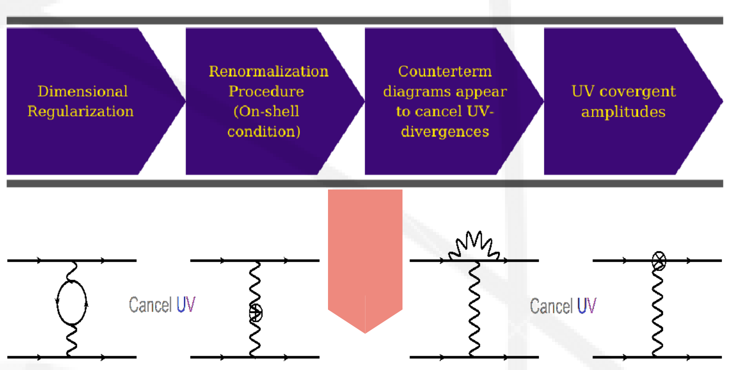


Figure 5: UV-divergence cancellation of the one-loop diagrams

Infrared Divergence

Loop momentum reaches to zero

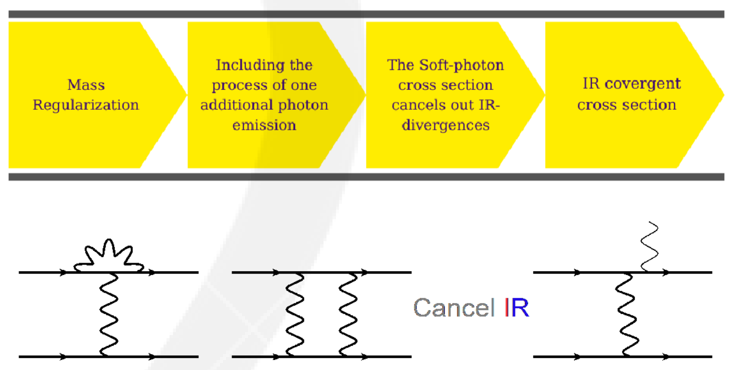


Figure 6: IR-divergence cancellation of the one-loop diagrams

$$d\sigma_{NLO} = d\sigma_{LO} + d\sigma_{Virt} + d\sigma_{real}(\alpha^3), \quad (3)$$
$$d\sigma_{real}(\alpha^3) = d\sigma_{soft}(\alpha^3, \triangle E) + d\sigma_{hard}(\alpha^3, \triangle E). \quad (4)$$

where the soft-photon region is defined by $E_\gamma \leq \triangle E$ with $\triangle E$ being a cutoff parameter. The value of $\triangle E$ must be very small compared to the colliding energy. Below, we will choose $\triangle E = 10^{-3}\sqrt{s}/2$ or $\triangle E = 10^{-4}\sqrt{s}/2$ for numerical results. Details of the calculation be found in [6] (without the $d\sigma_{hard}$ term).

In the first step, we have successfully calculated the NLO cross section with soft-photon corrections, namely :

$$d\sigma_{NLO}^{soft} = d\sigma_{LO} + d\sigma_{Virt} + d\sigma_{soft}. \quad (5)$$

The hard-photon correction has not been calculated and is left for future work. Our analytical result for Eq. (5) is too long to be presented here, it can be found in [6] instead. Here we would like to present some numerical results and a cross check with the program Formcalc-6.2 [7] (which uses the QED model file from the program FeynArt-3.4 [8]).

$e^-\mu^- \to e^-\mu^-$			
$\cos\theta$	$(d\sigma/d\cos\theta)_{NLO}^{soft}$ $[\mu b]$		
	FORM ($\triangle E = 10^{-4}\sqrt{s}/2$)	FORM ($\triangle E = 10^{-3}\sqrt{s}/2$)	FormCalc ($\triangle E = 10^{-3}\sqrt{s}/2$)
-0.9	0.289960	0.335339	0.335339
-0.5	0.467283	0.561601	0.561601
0	1.251400	1.523037	1.523037
0.5	6.56325	7.90200	7.90200
0.9	224.142	258.710	258.710

Table 3: NLO differential cross section in $\cos\theta$ (in the CMF) for the MUonE experiment

The one-loop quantum correction is defined as :

$$\delta = \frac{d\sigma_{NLO} - d\sigma_{LO}}{d\sigma_{LO}}.$$

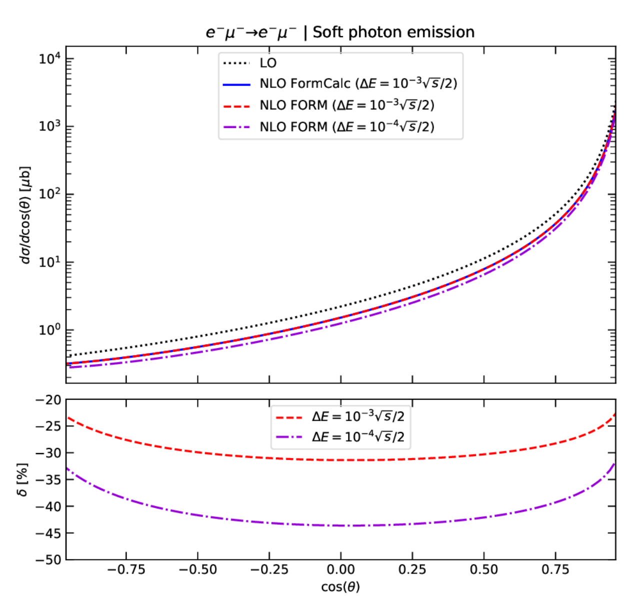


Figure 7: LO and NLO differential cross sections in $\cos\theta$ for the MUonE experiment

CONCLUSION AND OUTLOOK

We have successfully calculated the NLO QED corrections with soft-photon emission to the eletron-muon elastic scattering. We have checked that the result is free of UV and IR divergences. Full analytical results have been obtained and cross checked numerically with the program FormCalc.
The next step is to include the missing hard photon corrections. After that, we will include the electroweak corrections in the Standard Model.