The Standard Model

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Plan

- A mini-review of quantum field theory (QFT)
- Constructing the Standard Model (SM), one generation
- Why three generations? CKM matrix
- Feynman rules
- Precision tests of the SM
- Perturbative QCD, Higgs production at the LHC
- Helicity amplitude method

The little particles

Fermions: spin 1/2			
Leptons	ν_e e	ν_μ μ	ν_τ τ
Quarks	u d	c s	t b

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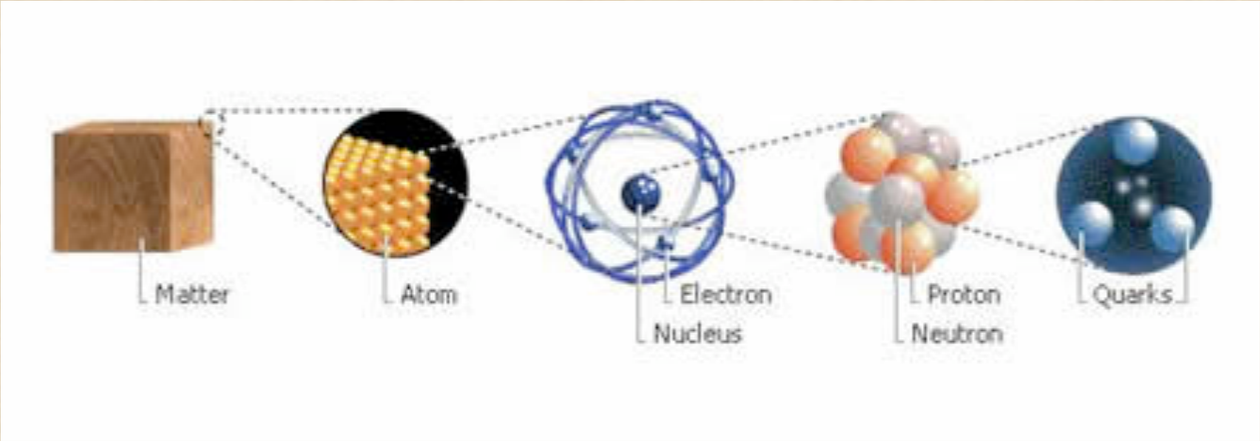
What are these particles?

Why are they needed?

How do they interact?

The structure of matter



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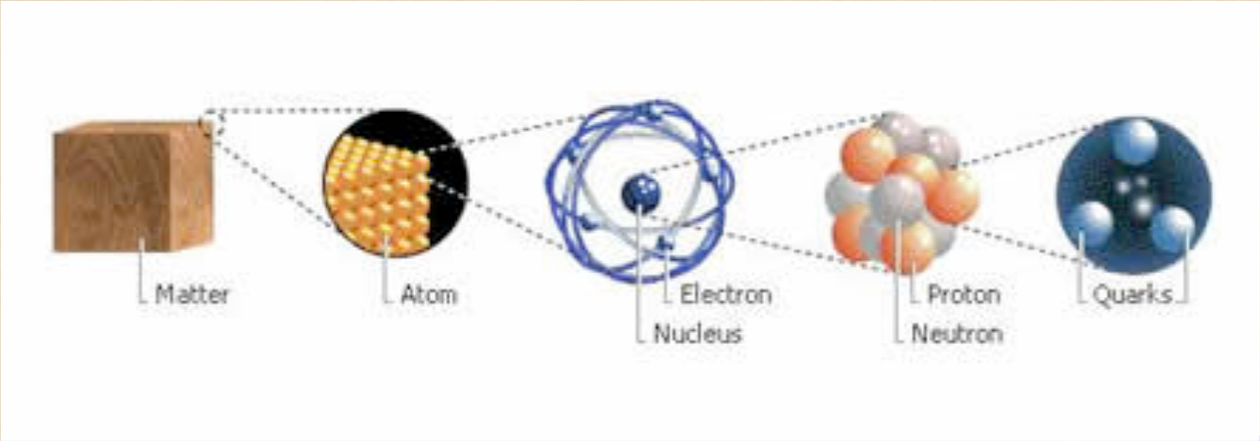

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, …

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

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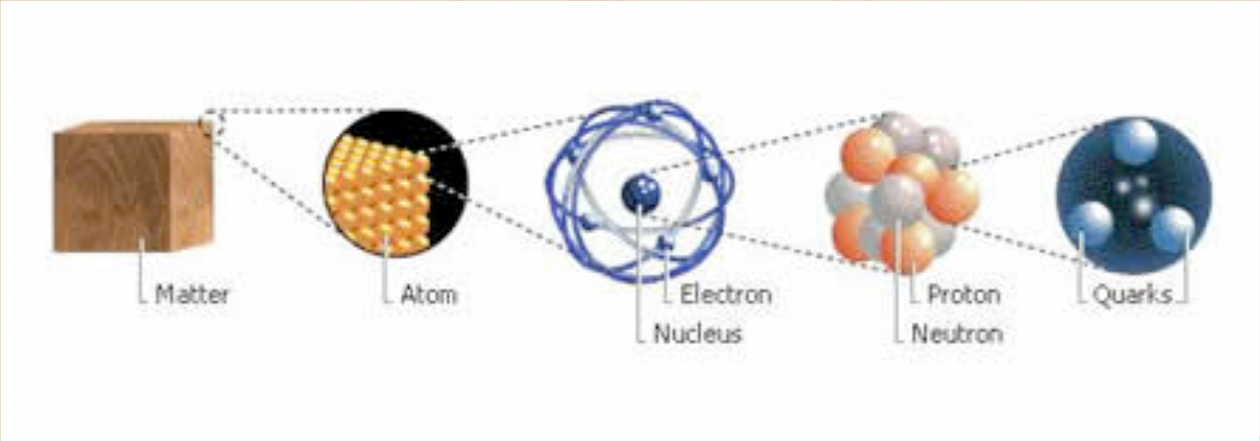

where are νs, μ, τ, t?

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- Positron: 1932, Carl D. Anderson found the anti-electron predicted by Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0$$

- Muon: 1936, Carl D. Anderson and S. Neddermeyer.Who ordered that?
- Neutrinos:ν_e (1956), ν_μ (1940), ν_τ (2000). Postulated in 1930 by W. Pauli to preserve the p-E conservation in beta decay: $n \to p + e + \bar{\nu}_e$.
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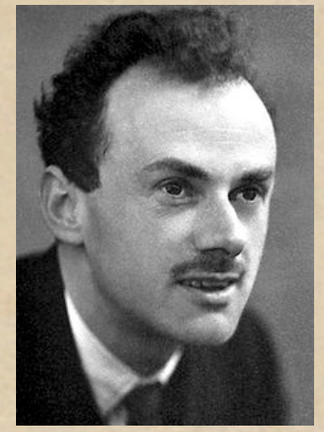

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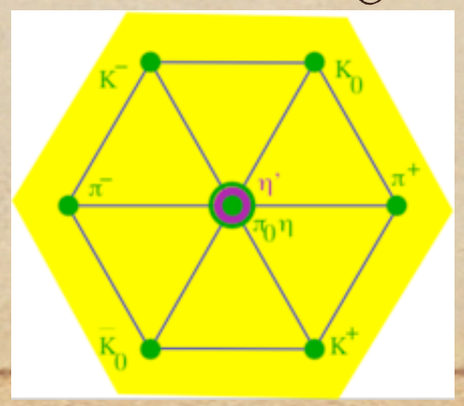
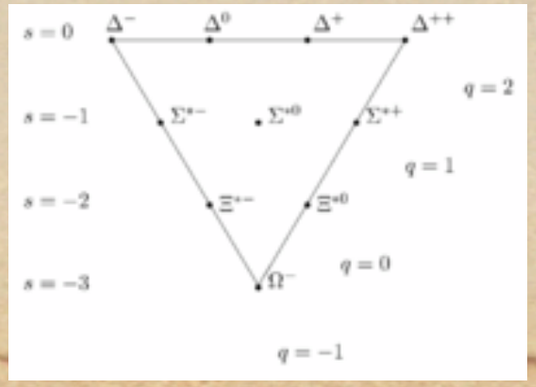
- Neutron: 1932, J. Chadwick. Neutral penetrating rays. How do neutrons interact with e and p ?
- 1940-1960: many mesons and new baryons were discovered.





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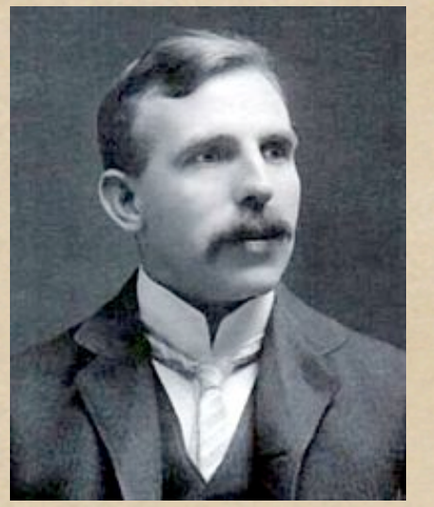
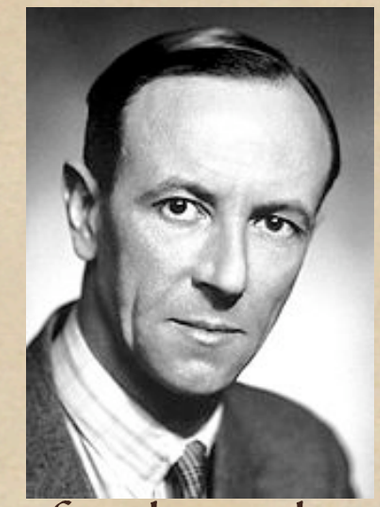

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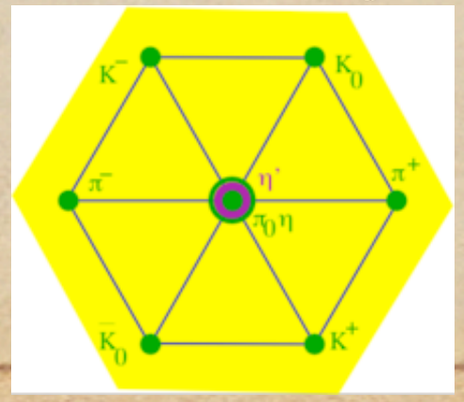

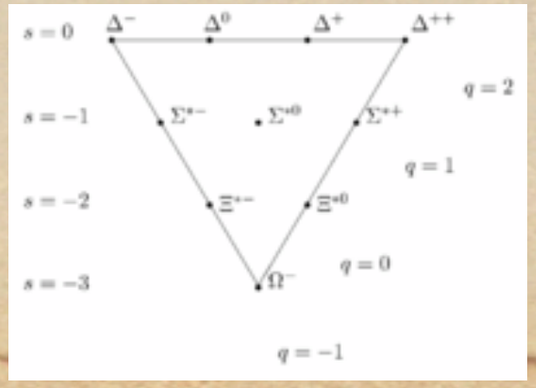

Quarks

Remarks: mesons and baryons can be classified as representations of a SU(3) group. The fundamental representation was missing.

Gell-Mann and Zweig: $\{u, d, s\}$ as the fundamental representation of the flavor $SU(3)_F$ group. $u\bar{d}(\pi^+)$, $u\bar{s}(K^+)$, $uud(p)$, $udd(n)$, ... Quarks: spin 1/2, $Q_u = 2/3e$, $Q_d = Q_s = -1/3e$.

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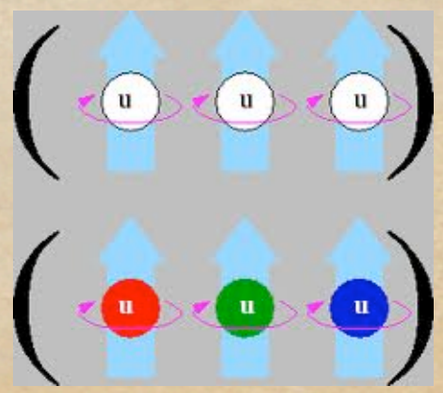
$\Delta^{++}(s_z = 1/2) = (u^\uparrow u^\uparrow u^\uparrow)$ contradicts Fermi-Dirac statistics. Color charge:

Y. Nambu $\quad \Delta^{++}(s_z = 1/2) = \frac{1}{\sqrt{6}} \epsilon_{ijk} u_i^\uparrow u_j^\uparrow u_k^\uparrow; \quad i, j, k = 1, 2, 3.$



Each quarks have 3 colors.

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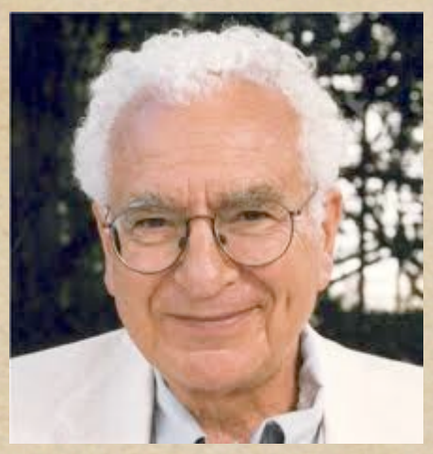

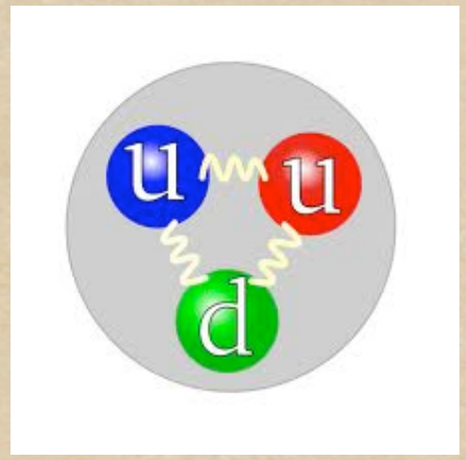

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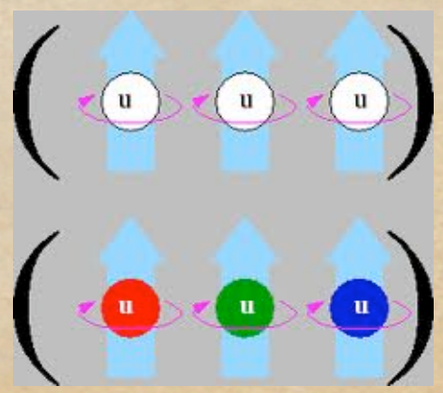

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Quantum field theory

A mini review

Classical fields

- scalar fields $\phi(x)$: spin-0 particles
- vector fields $A_\mu(x)$: spin-1 particles
- spinor fields $\psi(x)$: spin-1/2 particles

Lagrangian formalism

$$\mathcal{L} = \mathcal{L}(x, \dot{x}) \rightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x))$$

$$\begin{aligned} S[\phi] &= \int d^4x \mathcal{L} \\ \delta S &= S[\phi + \delta\phi] - S[\phi] = 0 : \quad \text{Hamilton's principle} \end{aligned}$$

Euler-Lagrange equation:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

Natural units: $\hbar = c = 1$
Action is dimensionless: $[\mathcal{L}] = 4$ $([E] = 1, [x] = -1)$

Free quantum fields: scalar

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \, \phi^\dagger \phi$$

Klein–Gordon equation $(\Box + m^2)\, \phi = 0\,.$

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 k}{2k^0} \left[a(k)\, e^{-ikx} + b^\dagger(k)\, e^{ikx} \right]$$ → operator

- a, $a^\dagger$: annihilation and creation operators for particle states.
- b, $b^\dagger$: annihilation and creation operators for anti-particle states.

Dimension: $[\phi] = 1$

Free quantum fields: spinor

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \qquad \overline{\psi} = \psi^\dagger\,\gamma^0$$

Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0\,.$$

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \sum_\sigma \int \frac{\mathrm{d}^3 k}{2k^0} \left[c_\sigma(k)\, u_\sigma(k)\, e^{-ikx} \,+\, d^\dagger_\sigma(k)\, v_\sigma(k)\, e^{ikx} \right]$$

$$(\not{p} - m)u(p) = 0\,, \qquad (\not{p} + m)v(p) = 0$$

Dimension: $[\psi] = 3/2$

Vector fields

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu \quad \text{with} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Proca equation: $\left[(\Box + m^2)\, g^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\nu = 0$

Photon: $m = 0$, impose $\partial^\mu A_\mu = 0 \rightsquigarrow \Box A_\mu = 0$ Maxwell equation

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \sum_\lambda \int \frac{\mathrm{d}^3 k}{2k^0} \left[a_\lambda(k)\, \epsilon_\mu^{(\lambda)}(k)\, e^{-ikx} + a_\lambda^\dagger(k)\, \epsilon_\mu^{(\lambda)}(k)^*\, e^{ikx}\right]$$

$$\epsilon^{(\lambda)} \cdot k = 0, \quad \epsilon^{(\lambda)*} \cdot \epsilon^{(\lambda')} = -\delta_{\lambda\lambda'}$$

$m \neq 0 \quad : \quad \sum_{\lambda=1}^{3} \epsilon_\mu^{(\lambda)*} \epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$: 3 polarization states

$m = 0 \quad : \quad \sum_{\lambda=1}^{2} \epsilon_\mu^{(\lambda)*} \epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k \cdot n}$ (gauge $n^\mu \epsilon_\mu = 0$) : 2 polarization states

$n \quad : \quad$ arbitrary reference vector with $n^2 = 0, k \cdot n \neq 0$

Dimension: $[A^\mu] = 1$

Interactions

Electromagnetic Interaction

e, μ and qs have electric charge: they interact with photons (QED)

$$\mathcal{L}_{QED} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{free photon field}} + \underbrace{\bar{\psi}_f(i\partial\!\!\!/ - m_f)\psi_f}_{\text{free fermion field}} - \underbrace{eQ_f\bar{\psi}_f\gamma_\mu\psi_f A^\mu}_{\text{interaction}}.$$

- m_f: fermion mass, $p\!\!\!/ = p^\nu\gamma_\nu$
- e: coupling constant, $Q_e = -1$, $Q_u = 2/3$, $Q_d = -1/3$

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How to find this?

Symmetry

$$\mathcal{L}_{kin} = i\bar{\psi}_f \partial\!\!\!/ \psi_f$$

Global symmetry: $\psi_f(x) \to e^{i\Lambda}\psi_f(x), \quad \bar{\psi}_f(x) \to e^{-i\Lambda}\bar{\psi}_f(x)$

$$\partial\!\!\!/ \psi_f \to e^{i\Lambda}\partial\!\!\!/ \psi_f: \text{ invariant.}$$

Make it local: $\Lambda = \Lambda(x)$

$$\partial\!\!\!/ \psi_f \to e^{i\Lambda(x)}\partial\!\!\!/ \psi_f + e^{i\Lambda(x)}[\partial\!\!\!/ \Lambda(x)]\psi_f: \text{ not invariant.}$$

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How to make $\mathcal{L}_{kin}$ invariant under the local symmetry?

$$\partial\!\!\!/ \to D\!\!\!/, \quad D\!\!\!/ \psi_f \to e^{i\Lambda(x)} D\!\!\!/ \psi_f$$

$$D\!\!\!/ = \partial\!\!\!/ + X\!\!\!/, \quad X_\mu \to X'_\mu$$

Interaction and Symmetry

$$D_\mu = \partial_\mu + ieQ_f A_\mu, \quad A_\mu \to A_\mu - \frac{1}{eQ_f}\partial_\mu \Lambda(x)$$

$$\begin{aligned}\mathcal{L}_{inv} &= i\bar{\psi}_f \not{D}\psi_f \\ &= i\bar{\psi}_f \not{\partial}\psi_f - eQ_f \underbrace{\bar{\psi}_f \gamma^\mu \psi_f}_{J_V^\mu} A_\mu\end{aligned}$$

- A_μ: a massless vector field (gauge field).

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global symmetries -> local symmetries -> interactions

Axial gauge boson?

- Indeed, $\mathcal{L}_{kin}$ has another global symmetry $U(1)_A$:
  $F \to e^{i\gamma_5\lambda(x)}$, $f_R \to e^{i\gamma_5\lambda(x)}$.
  If this axial symmetry is made local, there would be a neutral gauge boson associated with it. This gauge boson has not been observed.

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- In gauge theories, the conservation of the axial vector current $J_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ is incompatible with gauge invariance [Peskin and Schroeder book].

$$\partial_\mu J_A^\mu = -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}.$$

Adler-Bell-Jackiw anomaly

The axial current is only conserved at tree level and is violated at loop level in gauge theories.

Covariant derivative

$$D^\mu = \partial^\mu + ieQA^\mu$$

$$\begin{aligned}
\psi(x) &\rightarrow e^{i\Lambda(x)}\psi(x) \\
D^\mu\psi(x) &\rightarrow e^{i\Lambda(x)}D^\mu\psi(x) \\
D^{\mu_1}\ldots D^{\mu_N}\psi(x) &\rightarrow e^{i\Lambda(x)}D^{\mu_1}\ldots D^{\mu_N}\psi(x)
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$$[D^\mu, D^\nu]\psi(x) \rightarrow e^{i\Lambda(x)}[D^\mu, D^\nu]\psi(x)$$

$$[D^\mu, D^\nu]\psi(x) = (D^\mu D^\nu - D^\nu D^\mu)\psi(x) = ieQ\underbrace{(\partial^\mu A^\nu - \partial^\nu A^\mu)}_{F^{\mu\nu}}\psi(x)$$

$$\Longrightarrow F^{\mu\nu}\text{: invariant!}$$

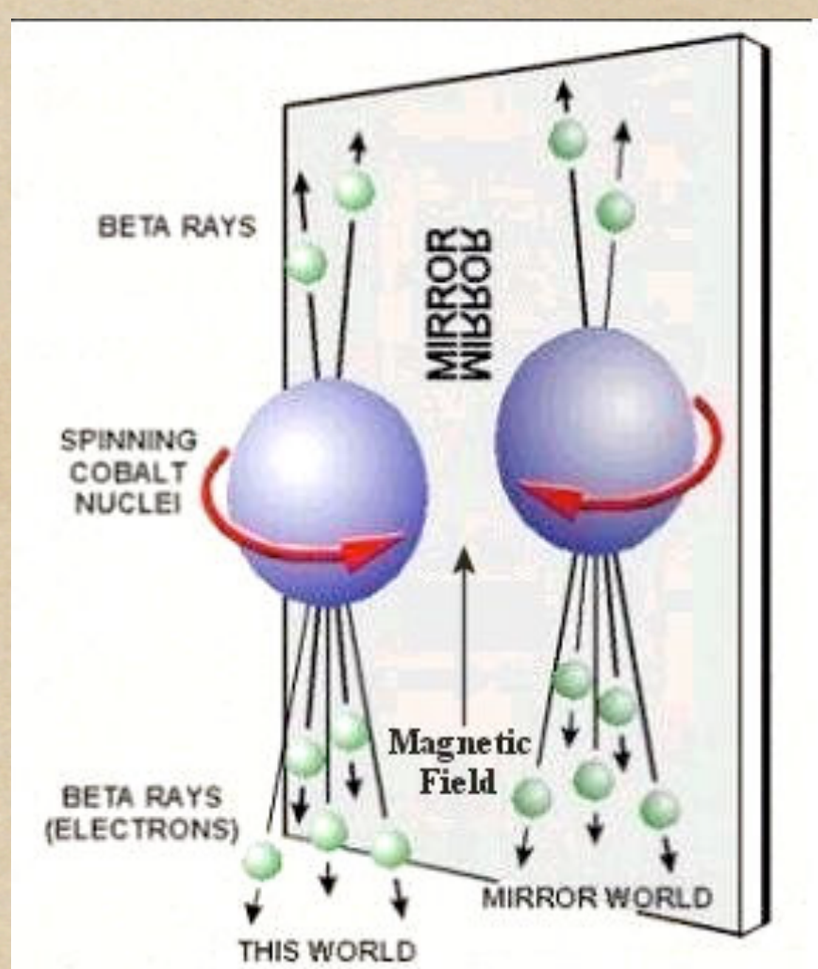
$$\mathcal{L}_{gauge} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Weak interaction

- If we interpret A_μ as the photon then eQ_f must be the electric charge of the fermion, electromagnetic interaction.
- How do the neutrinos interact?
  (What are the local symmetries of the neutrinos?)
- First hint: beta decay $n \to p + e + \bar{\nu}_e$ indicates that ν_e interacts with e via weak interaction.
- Local (gauge) symmetry: there must be a new gauge boson associated with the weak interaction

$$\mathcal{L}_{weak} = \bar{\psi}_e \Gamma^\mu \psi_{\nu_e} B_\mu, \quad \Gamma^\mu = a\gamma^\mu + b\gamma^\mu\gamma^5.$$

Parity violation

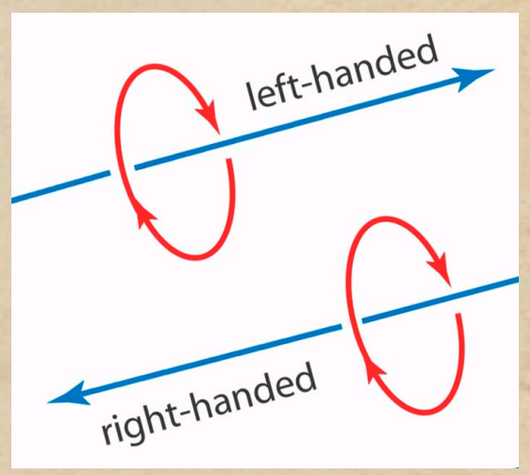


The ellipsoid on the left represents a large number of cobalt-60 nuclei, all with their spins in the same direction, and all emitting beta rays. (In reality, any one cobalt nucleus emits only one beta ray - transforming itself thereby into a nickel nucleus). On the right this process is seen in a mirror. The direction of spin is reversed, while the direction in which most beta rays are emitted remains unchanged. The mirror world is thus distinguishable from the real world.

The parity transformation of (x,y,z) to (-x,-y,-z) is completed by turning the mirror image upside down. The spins of the cobalt nuclei are thus returned to their original direction, but most beta rays are now emitted upward - contrary to experimental fact. The parity-transformed world is not identical with the real world; parity is not conserved.

Note that the asymmetry would be perfect with the beta rays emitting in one direction only (and none in the other direction), if all the cobalt nuclei are aligned perfectly with the magnetic field. In the actual experiment, the requirements of high vacuum and low temperature (~ 1/100°K) is very difficult to achieve.

Parity Violation in Weak Interaction



$$\psi_\nu = \psi_{\nu L} \equiv P_L \psi_\nu$$
$$P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

Yang and Lee: neutrino is purely left-handed.

Weak interaction

$$\psi = \psi_L + \psi_R, \quad \psi_{L,R} = P_{L,R}\psi$$

$$\mathcal{L}_{weak} = \bar{\psi}_e(a\gamma^\mu + b\gamma^\mu\gamma^5)\psi_{\nu_e L}B_\mu$$

$$= \bar{\psi}_{eL}(a\gamma^\mu + b\gamma^\mu\gamma^5)\psi_{\nu_e L}B_\mu$$

Weak interaction

$$\psi = \psi_L + \psi_R, \quad \psi_{L,R} = P_{L,R}\psi$$

$$\mathcal{L}_{weak} = \bar{\psi}_e(a\gamma^\mu + b\gamma^\mu\gamma^5)\psi_{\nu_e L}B_\mu$$

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Only e_L can interact with ν_{eL}.

This suggests that they belong to the same "family",
like the p and n are in a doublet of the isospin group $SU(2)$

Local symmetry for the weak interaction: $SU(2)_L$ with e_L
and ν_{eL} are in the same doublet.

$$L = \binom{\nu_{eL}}{e_L},\ e_R,\ Q = \binom{u_L}{d_L},\ u_R,\ d_R.$$

Weak interaction

For one generation:

$$\mathcal{L}_{kin} = \sum_{F=L,Q} i\bar{F}\partial\!\!\!/ F + \sum_{f=e,u,d} i\bar{f}_R \partial\!\!\!/ f_R.$$

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Local symmetries:

- $SU(2)_L$: Fs are doublets, f_Rs are singlets,
  $F \to e^{i\tau_a \alpha_a(x)} F$, $f_R \to f_R$, $\tau_a = \sigma_a/2$, $a = 1, 2, 3$.
- $U(1)_Y$: $F \to e^{iY\beta(x)} F$, $f_R \to e^{iY\beta(x)} f_R$, Y hypercharge operator.
- $SU(2)_L \otimes U(1)_Y$: $F \to e^{i\tau_a \alpha_a(x) + iY\beta(x)} F$,
  $f_R \to e^{iY\beta(x)} f_R$. $\quad Y \equiv 2Q + b\tau_3 \rightsquigarrow Y = 2(Q - \tau_3)$

Remarks

- Like QED, we have to introduce the covariant derivatives including gauge bosons to cancel the terms $\partial\alpha_a(x)$ $(a = 1, 2, 3)$ and $\partial\beta(x)$. Four gauge bosons (massless vector fields) are needed.

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\begin{aligned}
D^\mu f_R &= (\partial^\mu + i g_1 \frac{Y}{2} C^\mu) f_R \\
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c_W &= \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, s_W = \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}
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- Two difficulties: 4 massless vector bosons were not observed and the fermions are massive.

How to generate fermion masses?

$$m_f \bar{f} f = m_f (\bar{f}_R f_L + \bar{f}_L f_R).$$

These terms break $SU(2)_L \otimes U(1)_Y$ symmetry. How to couple f_R to f_L? Radiative corrections are impossible because the gauge interactions conserve chirality. We need something new here:

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Lorentz invariant

Gauge invariant

Renormalizable ($[g] \geq 0 \rightarrow [X] \leq 1$)

Scalar, fermion ($\bar{f}_L \gamma^\mu f_R = 0$)

SU(2) doublet

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Of course, we can introduce many (N) Higgs doublets.

N=1: SM, N=2: two-Higgs-doublet models.

The Higgs mechanism

An Abelian example, complex scalar fields

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\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi) \\
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V(\phi) &= -\mu^2\phi^*\phi + \frac{\lambda}{4}(\phi^*\phi)^2, \quad \mu^2 > 0, \lambda > 0 \\
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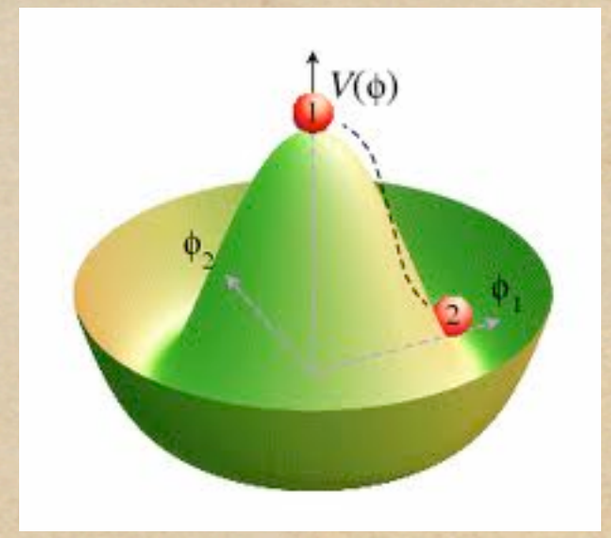
The vacuum (lowest energy state) is degenerate.



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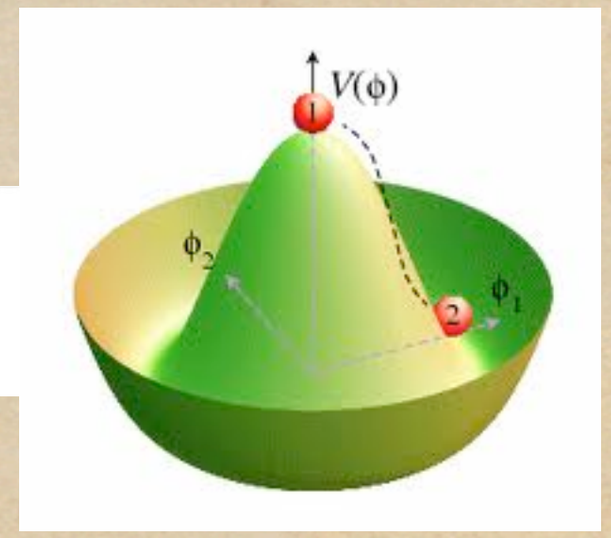

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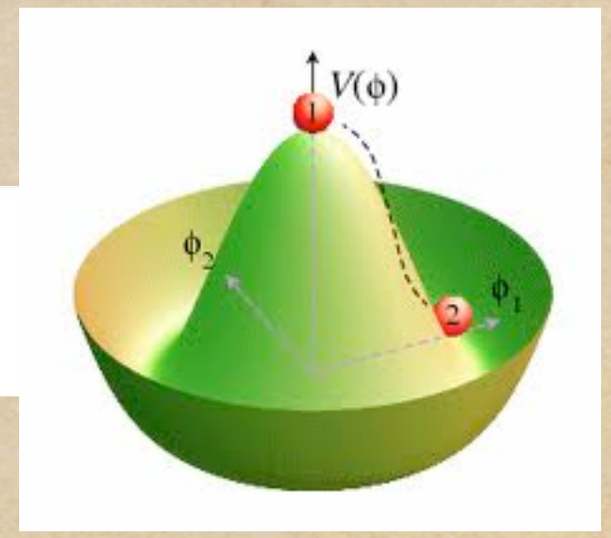

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Around this vacuum: $\phi(x) = \frac{1}{\sqrt{2}}[v + H(x) + i\phi_2(x)]$

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Physics is independent of this (hidden symmetry).

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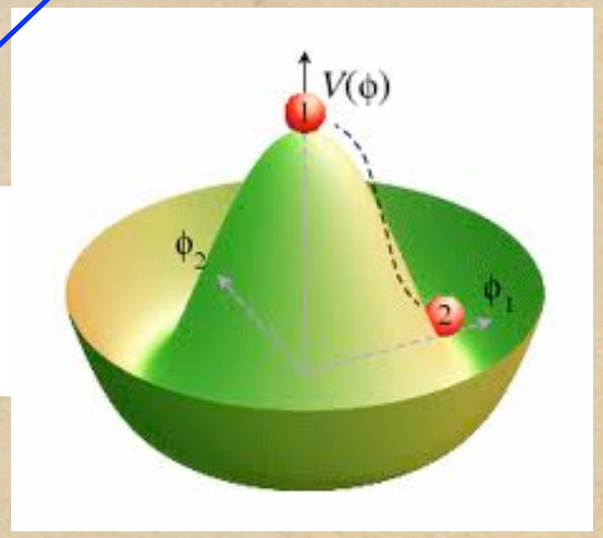

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After SSB: global

Global: $\Lambda(x) = \Lambda$, $A_\mu = 0$, $D_\mu = \partial_\mu$.

$$\mathcal{L} = \frac{1}{2}(\partial^\mu H)^2 + \frac{1}{2}(\partial^\mu \phi_2)^2 - \frac{1}{2}\underbrace{2\mu^2}_{M_H^2} H^2 + \ldots$$

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- A massive scalar appears, Higgs particle, $M_H = \sqrt{2}\mu$.
- A massless scalar appears, Goldstone boson.
- Both are physical states.

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- Counting physical states: before SSB 4 (massless A_μ, $\phi_{1,2}$), after SSB 4 (massive $\tilde{A}_\mu$, H).

SSB: SU(2)xU(1)

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SSB: $\phi^- = \phi^+ = 0$, $\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{2\mu^2}{\lambda}}$

The vacuum:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad e^{ieQ\Lambda(x)}\langle\Phi\rangle = \langle\Phi\rangle, \quad (Q\langle\Phi\rangle = 0)$$

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$$D^\mu\Phi = (\partial^\mu - ig_2\tau_a W_a^\mu + ig_1\frac{Y}{2}C^\mu)\Phi$$

$$V(\Phi) = -\mu^2\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2, \quad \mu^2 > 0, \lambda > 0$$

The vacuum state is degenerate: $\Phi^+\Phi = \phi^-\phi^+ + \phi^{0*}\phi^0 = \frac{2\mu^2}{\lambda} > 0$

SSB: $\phi^- = \phi^+ = 0$, $\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{2\mu^2}{\lambda}}$

The vacuum:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad e^{ieQ\Lambda(x)}\langle\Phi\rangle = \langle\Phi\rangle, \quad (Q\langle\Phi\rangle = 0)$$

The group $U(1)_Q$ is not broken: the photon remains massless.

$$SU(2)_L \otimes U(1)_Y \to U(1)_Q$$

(3+1) - 1 gauge bosons get a mass.

After SSB

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{2}[v + H(x) + i\chi(x)] \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{scalar} &= (D^\mu \Phi)^+ D_\mu \Phi - V(\Phi) \\ &= \frac{1}{2}\frac{g_2^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{1}{2}\frac{(g_1^2 + g_2^2)v^2}{4} Z_\mu Z^\mu - \frac{1}{2} 2\mu^2 H^2 + \ldots \end{aligned}$$

$$\boxed{M_W = \frac{1}{2} g_2 v, \quad M_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\, v, \quad M_H = \sqrt{2}\mu}$$

$$c_W = \frac{M_W}{M_Z}$$

The unphysical Goldstone bosons $\phi^\pm$, χ are massless.

Fermion masses

$$\mathcal{L}_{Yukawa} = -G_e \bar{L}\Phi e_R - G_d \bar{Q}\Phi d_R - G_u \bar{Q}\hat{\Phi} u_R + h.c.$$

$$\hat{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

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Show that this Lagrangian is invariant under $SU(2)_L \otimes U(1)_Y$ transformations.

Hint: using $e^{-i\theta\sigma\cdot n/2} = \cos(\theta/2) - i(n\cdot\sigma)\sin(\theta/2)$ where $n\cdot n = 1$.

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Remarks: The Yukawa couplings G_f can be complex. However, the phases can be removed by rephasing the fields as $f_R \to e^{i\alpha_f} f_R$.

Recap

- Gauge interactions: global $\rightarrow$ local symmetry, covariant derivative. Massless gauge bosons appear.
- SSB: to give mass to the gauge bosons and fermions. Massive Higgs boson appears.
- Parity violation: neutrinos are only left-handed.

Summary: 1 generation

Matter fields: $L = \binom{\nu_{eL}}{e_L}$, e_R, $Q = \binom{u_L}{d_L}$, u_R, d_R.

$$\mathcal{L}^1 = \mathcal{L}_{fermion} + \mathcal{L}_{self-gauge} + \mathcal{L}_{scalar} + \mathcal{L}^1_{Yukawa}$$

Gauge bosons: A^μ, $W^\pm_\mu$, Z^μ

Scalar bosons: H, $\phi^\pm$, χ → unphysical states

Parameters: e, M_W, M_Z, m_f, M_H

e → Electroweak unification

Remarks: $\mathcal{L}^1$ breaks parity by construction. However, CP is conserved.

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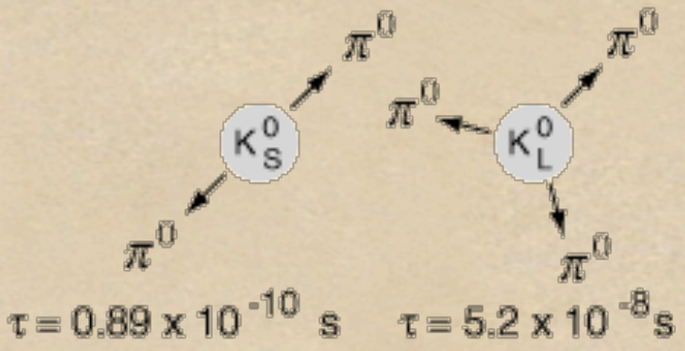
Ex:

- Calculate the $A\bar{f}f$, $Z\bar{f}f$, Wud, VVH, ... vertices.
- How do the gauge fields transform under $SU(2)_L \otimes U(1)_Y$?
- Calculate $\mathcal{L}_{self-gauge}$.

$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$



$$K^0_S = \frac{K^0 - \bar{K}^0}{\sqrt{2}}$$

$$K^0_L = \frac{K^0 + \bar{K}^0}{\sqrt{2}}$$

$$K^0 = d\bar{s},\ \bar{K}^0 = s\bar{d}$$

K^0_L also decays weakly into $\pi^0\pi^0$. Since pion parity is -1 this means parity is violated. For "neutral" particles it also means CP violation.

Cronin and Fitch 1964, Nobel Prize 1980

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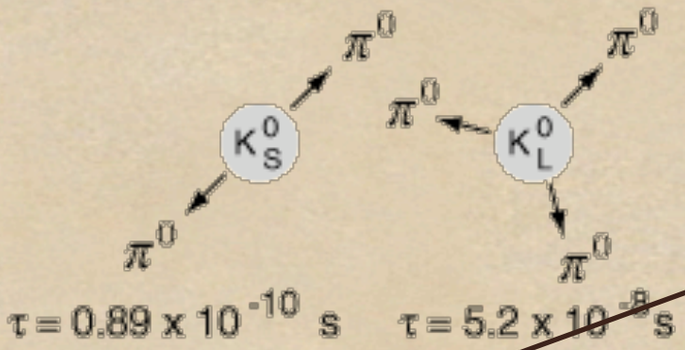

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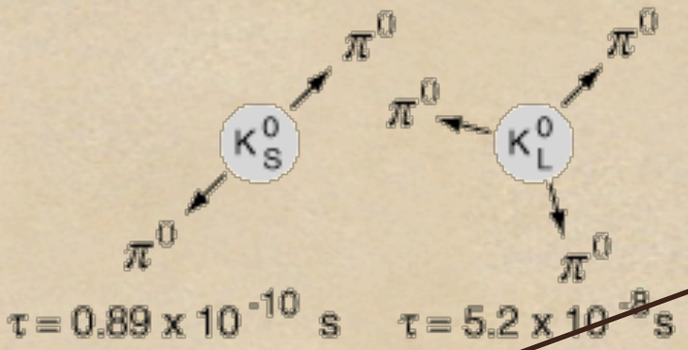

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Cronin and Fitch 1964, Nobel Prize 1980

CP violation can be explained if there are at least 3 generations of quarks.

Kobayashi and Maskawa 1973, Nobel Prize 2008

3 generations

$$\begin{aligned}\mathcal{L}^3_{Yukawa} &= -\bar{L}G_l l_R\Phi - \bar{Q}G_d d_R\Phi - \bar{Q}G_u u_R\hat{\Phi} + h.c.\\ f &= (f_1, f_2, f_3)^T, \quad F = (F_1, F_2, F_3)^T, \quad G_f = G_{fij},\ i,j = 1,2,3\end{aligned}$$

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After SSB, $\Phi = \frac{1}{\sqrt{2}}\binom{0}{v} + \ldots$

$$\begin{aligned}\mathcal{L}^{3,quark}_{Yukawa} &= -\bar{d}_L\frac{v}{\sqrt{2}}G_d d_R - \bar{u}_L\frac{v}{\sqrt{2}}G_u u_R + h.c.\\ &= -\bar{d}_L V_{dL}^\dagger \underbrace{V_{dL}\frac{v}{\sqrt{2}}G_d V_{dR}^\dagger}_{M_d^{diag}} V_{dR}d_R - \bar{u}_L V_{uL}^\dagger \underbrace{V_{uL}\frac{v}{\sqrt{2}}G_u V_{uR}^\dagger}_{M_u^{diag}} V_{uR}u_R + h.c.\\ &= -\bar{\tilde{d}}_L M_d^{diag}\tilde{d}_R - \bar{\tilde{u}}_L M_u^{diag}\tilde{u}_R + h.c.\end{aligned}$$

3 generations

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|mass> = V * |weak>

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$$= -\bar{\tilde{d}}_L M_d^{diag} \tilde{d}_R - \bar{\tilde{u}}_L M_u^{diag} \tilde{u}_R + h.c.$$

Rewrite the whole Lagrangian in terms of the mass eigenstates

$$\tilde{f}_L = V_{fL} f_L, \quad \tilde{f}_R = V_{fR} f_R$$

CKM matrix

Neutral currents:

$$\bar{f}_L \gamma^\mu f_L Z_\mu = \bar{f}_L V_{fL}^\dagger V_{fL} \gamma^\mu f_L Z_\mu = \bar{\tilde{f}}_L \gamma^\mu \tilde{f}_L Z_\mu$$

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Charged currents:

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$$V_{CKM} = V_{uL} V_{dL}^\dagger, \quad V_{CKM} V_{CKM}^\dagger = 1: \text{ unitary}$$

CKM: counting parameters

$N \times N$ unitary matrix has $2N^2 - N^2 = N^2$ real parameters.

$N = 2$: 1 rotation angle and 3 phases.
$N = 3$: 3 rotation angles and 6 phases.

There are $2N$ fields $\tilde{q}_{iL}$ which can be rephased as $\hat{q}_{iL} = e^{i\alpha_i}\tilde{q}_{iL}$ (same for $\tilde{q}_{iR}$). These $2N - 1$ relative phases can be chosen to cancel the phases in V_{CKM} (rewriting $\mathcal{L}$ in terms of $\hat{q}_{iL}$ and $\hat{q}_{iR}$).

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$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\theta_{12} = 13.04 \pm 0.05°$, $\theta_{13} = 0.201 \pm 0.011°$,
$\theta_{23} = 2.38 \pm 0.06°$, and $\delta_{13} = 1.20 \pm 0.08$.

$V_{CKM} \approx 1$

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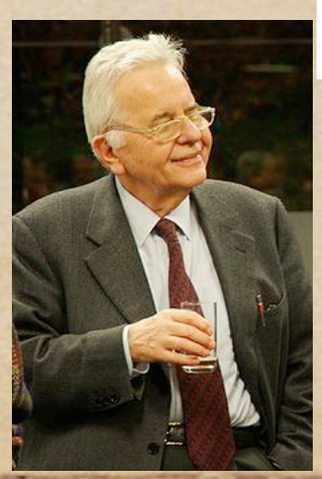

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CKM for leptons?

Asumming ν_{lR} exist. Like the quark sector, we have

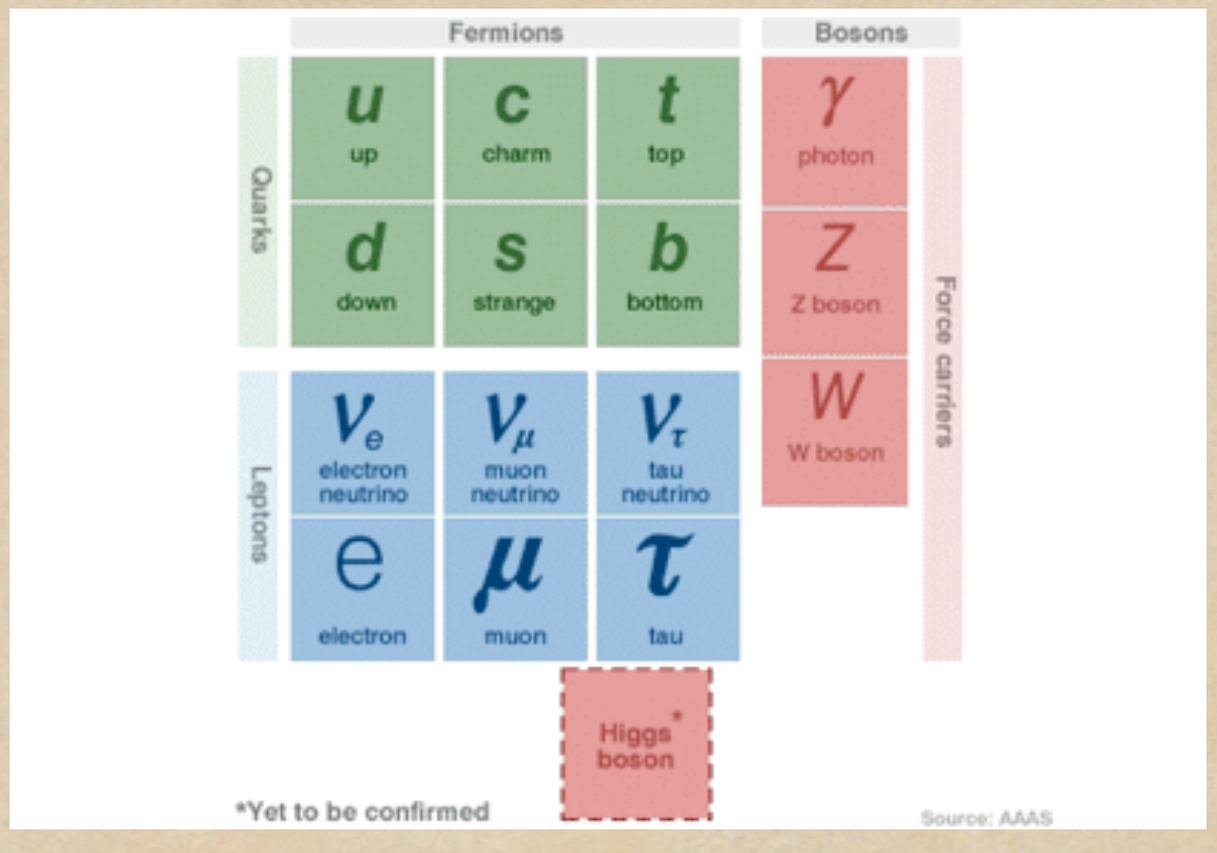
$$V_{CKM}^{lepton} = V_{\nu_l L} V_{lL}^{\dagger}, \quad G_{\nu_l}^{diag} = V_{\nu_l L} G_{\nu_l} V_{\nu_l R}^{\dagger}$$

$G_{\nu_l} = 0$ (since $\nu_R = 0$): we choose $V_{\nu_l L} = V_{lL}$

$$V_{CKM}^{lepton} = 1$$

If $\nu_R \neq 0$ (massive neutrinos) then there will be a nontrivial CKM matrix for the leptons.

Summary: 3 generations



Parameters: e, M_W, M_Z, m_f, M_H, CKM (3 angles, 1 phase).

$\nu_{lR} = 0$, Parity violation, CP violation (CKM is complex).

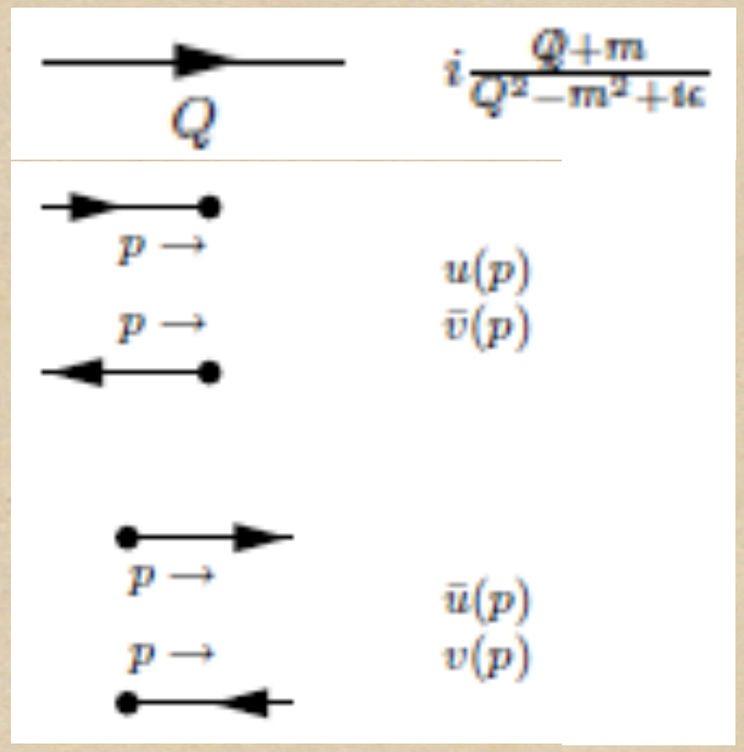
$$SSB : SU(2)_L \otimes U(1)_Y \to U(1)_Q$$

$$m_f = M_{W,Z} = 0 \qquad\qquad m_t \neq m_b \propto v,\ M_{W,Z} \propto v$$

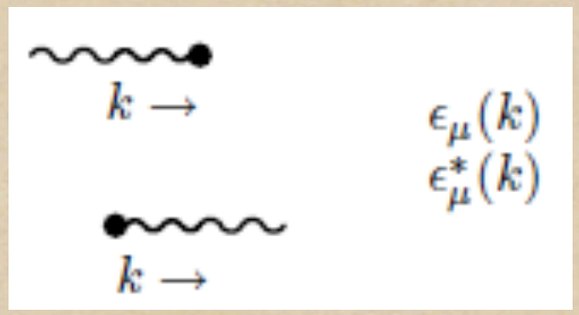
Feynman rules

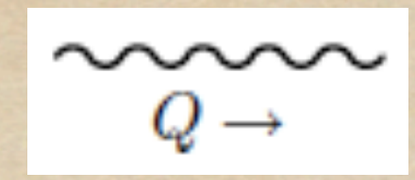
$$\mathcal{L}_{classical} = \mathcal{L}_{fermion} + \mathcal{L}_{self-gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}$$

Fermion



Vector





A_μ: $\frac{i}{Q^2+i\epsilon}[-g_{\mu\nu} + (1-\xi_A)\frac{Q_\mu Q_\nu}{Q^2}]$

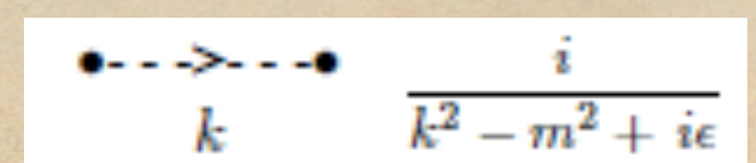
Z_μ: $\frac{i}{Q^2-M_Z^2+i\epsilon}[-g_{\mu\nu} + (1-\xi_Z)\frac{Q_\mu Q_\nu}{Q^2-\xi_Z M_Z^2}]$

$W_\mu^\pm$: $\frac{i}{Q^2-M_W^2+i\epsilon}[-g_{\mu\nu} + (1-\xi_W)\frac{Q_\mu Q_\nu}{Q^2-\xi_W M_W^2}]$

$\xi_V = \infty$: unitary gauge, no Goldstone (later)

$\xi_V = 1$: 't Hooft-Feynman gauge

Scalar



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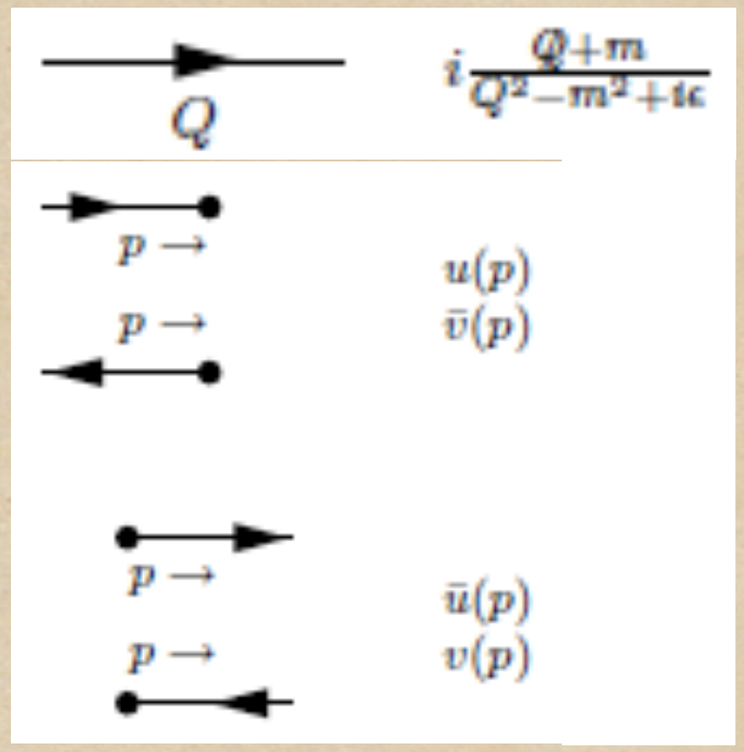

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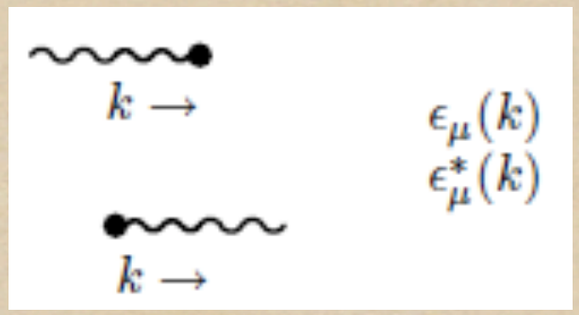

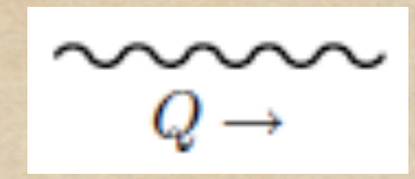

?

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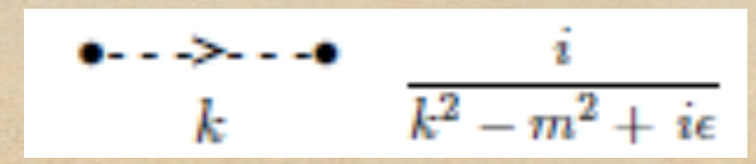

Gauge fixing: No SSB

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Vertex function: $\Gamma^{\mu\nu} = -ik^2(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})$
Propagator: $D_{\mu\nu}\Gamma^{\nu\rho} = -\delta^\rho_\mu$
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Gauge fixing: No SSB

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This problem is due to gauge invariance. To overcome this, we add to $\mathcal{L}$ a gauge-fixing regulator to break the gauge symmetry. Gauge invariance means that physical results are independent of this regulator.

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$$D_{\mu\nu} = \frac{i}{k^2}[-g_{\mu\nu} + (1-\xi_A)\frac{k_\mu k_\nu}{k^2}]$$ Ex: Prove this.

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Choose $\alpha(x) = \xi_A M_A \phi_2(x)$ to cancel the mixing term.

$$D_{\mu\nu} = \frac{i}{k^2 - M_A^2}[-g_{\mu\nu} + (1-\xi_A)\frac{k_\mu k_\nu}{k^2 - \xi_A M_A^2}]$$

Ex: Prove this.

$$D_{\phi_2} = \frac{i}{p^2 - \xi_A M_A^2}: \quad \text{Goldstone boson } (\xi_A \to \infty?)$$

Gauge fixing and Faddeev-Popov ghosts

$$
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\mathcal{L}_{GF} &= -\frac{1}{2}\left[(F^A)^2 + (F^Z)^2 + 2F^+F^-\right] \\
F^A &= \frac{1}{\sqrt{\xi_A}}\partial^\mu A_\mu \\
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F^\pm &= \frac{1}{\sqrt{\xi_W}}(\partial^\mu W_\mu^\pm \mp i\xi_W M_W \phi^\pm)
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With this gauge-fixing regulator, the mixings $Z^\mu \partial_\mu \chi$, $W_\mu^\pm \partial^\mu \phi^\mp$ are removed.

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We have to prove that physical results are independent of this $\mathcal{L}_{GF}$ regulator. This is correct at tree level.
It is however not true at one-loop level. The reason is that $\mathcal{L}_{GF}$ can introduce unphysical degrees of freedom to the virtual gauge boson propagators which can give unphysical contributions to the one-loop results. To cancel these contributions we introduce Faddeev-Popov ghosts $u^a(x)$ $(a = A, Z, \pm)$, which are fermionic scalar fields.

$$\mathcal{L}_{FP} = \bar{u}^a \frac{\delta F^a}{\delta\theta^b(x)} u^b(x)$$

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gauge transformation parameters

Unitarity and Gauge invariance

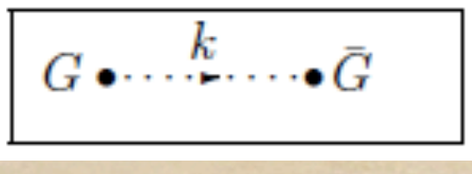
Unitarity: $\Gamma_Z = \frac{1}{M_Z} Im[\Sigma_{ZZ}(M_Z^2)]$

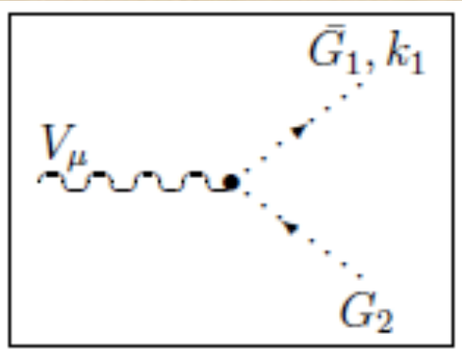
Ex: prove that $\mathcal{M}(e^+e^- \to \mu^+\mu^-)$ is independent of $\xi_{A,Z,W}$ at tree level, keeping $m_{e,\mu}$

Ghost Feynman rules

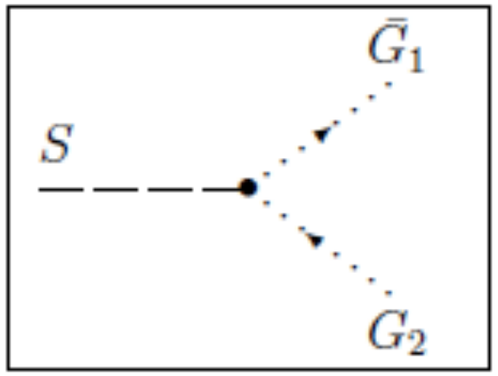
Faddeev-Popov ghosts $G = u^\gamma,\ u^Z,\ u^W$

$$\frac{i}{k^2 - \xi_G M_G^2} =$$





$= iek_{1,\mu}C$



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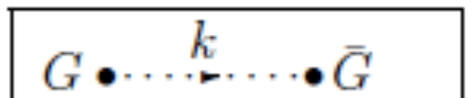

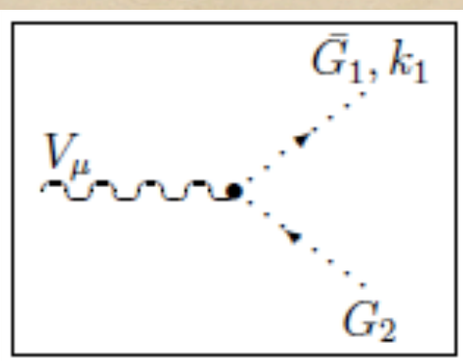

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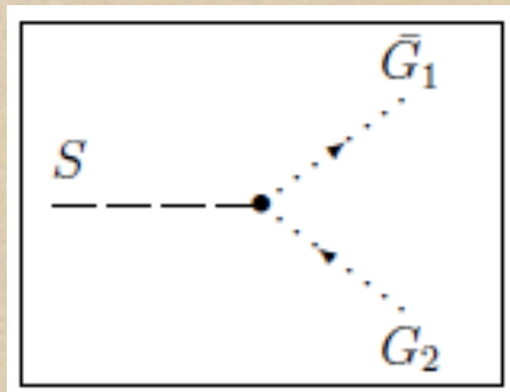

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For complete Feynman rules:
GRACE, hep-ph/0308080,
Denner, arXiv:0709.1075,
Boehm, Denner, Joos:
Gauge Theories of the strong
and Electroweak Interaction

SM Lagrangian

$$\mathcal{L}_{classical} = \mathcal{L}_{fermion} + \mathcal{L}_{self-gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{Yukawa}$$

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This Lagrangian is renormalizable and can be used to do precision calculations (quantum corrections) for physical processes.

=> precision tests of the SM