



iTHEMS<sup>°</sup>

RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences

## HEP seminar

# Phenomenological Constraints on the Leptophilic Zee Model

Presented by: Le Duc-Truyen

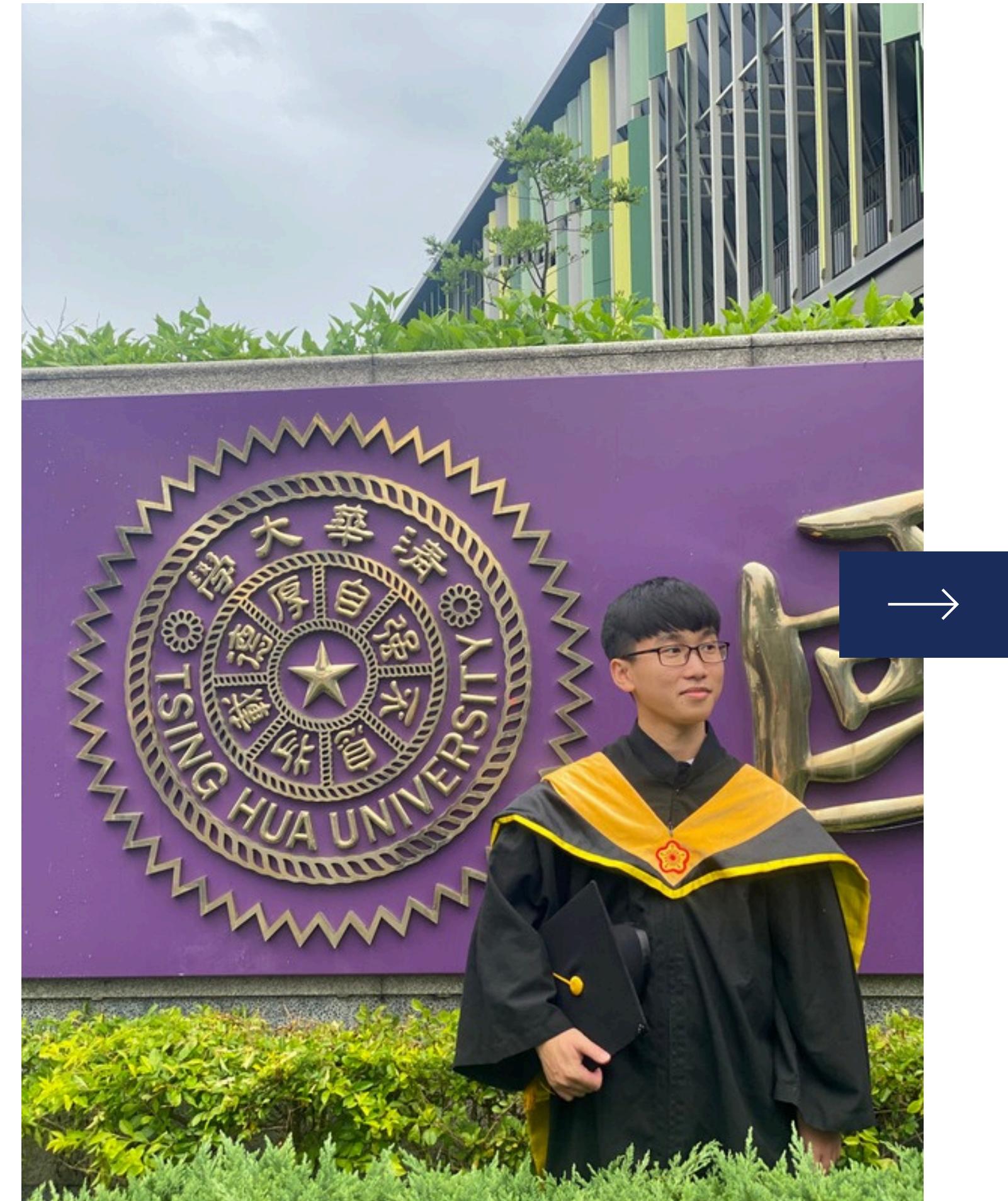
Collaboration: Prof. Chang We-Fu

# Hi! I'm Le Duc Truyen (黎德傳)

from National  
Tsing Hua  
University



“To move forward you need to leave something behind”  
Interstellar Quote inspired by Newton's third law





Where  
I am



# What I Do During My Free Time

Things that keep me occupied usually on weekends

## TRAVELING



## READING BOOK

Philosophy, History, Math,  
Mythology, Tourism, Cooking

## MUSICS

Listen and play  
musical instruments

## BOARD GAME

Play, analyse and create  
a new board game



## SPORT

Soccer, Baminton,  
Swimming, Ping pong, and  
any outdoor activities



## WACHING MOVIE

Film criticism

# What Topics Keep Me Interested

Research topics



## HIGH ENERGY PHYSIC

- Building Beyond Standard Model
- Dark Matter and Anomalies
- Computational application for High Energy Physics

## QUANTUM COMPUTATION AND QUANTUM INFORMATION

- Quantum algorithms and software, specifically applying for High Energy Physics
- Black Hole Quantum Information
- Quantum AI
- Quantum Optimization and Simulation

ALL THINGS EFT

GUT

SMEFT

SM

QED

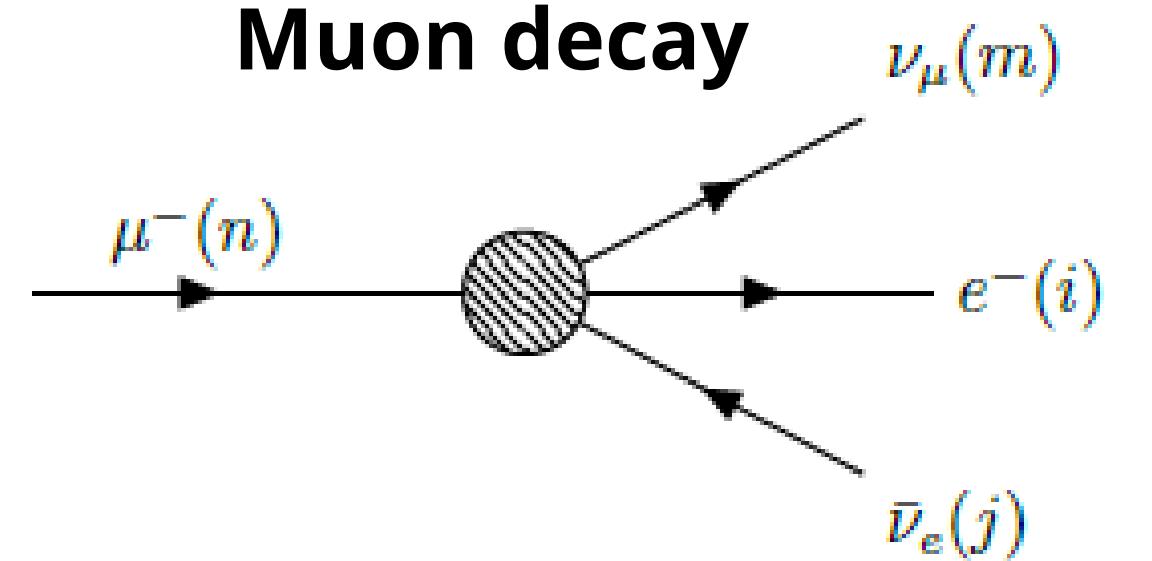
# ALL THINGS EFT

GUT

SMEFT

SM

Muon decay



$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F \bar{e}_L \gamma^\mu \nu_e \bar{\nu}_\mu \gamma_\mu \mu_L$$

QED

# ALL THINGS EFT

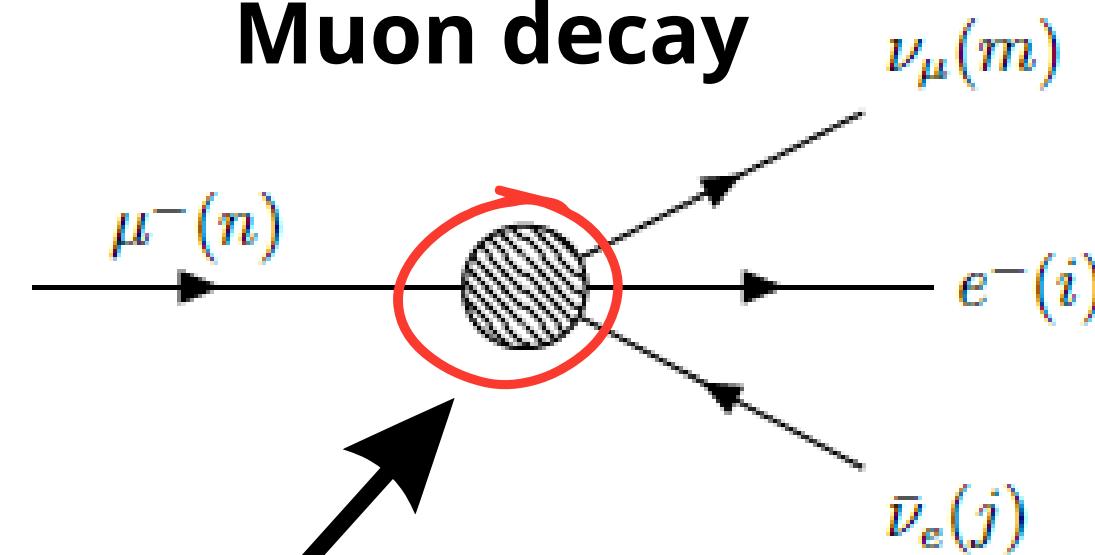
GUT

SMEFT

SM

QED

Muon decay



$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F \bar{e}_L \gamma^\mu \nu_e \bar{\nu}_\mu \gamma_\mu \mu_L$$

# ALL THINGS EFT

GUT

SMEFT

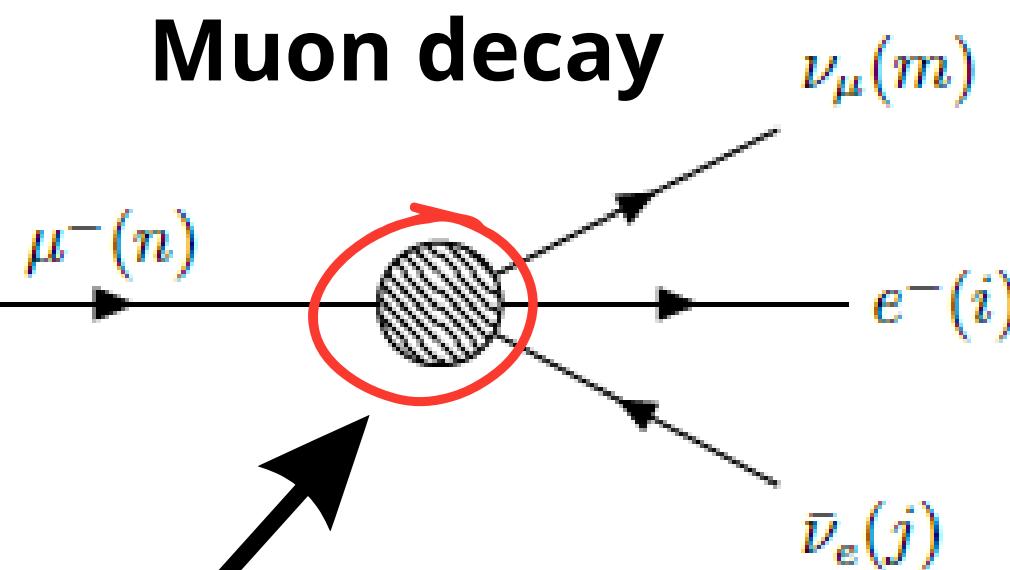
SM

LVF decays

Muon g-2

Other lepton decays

QED



$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F \bar{e}_L \gamma^\mu \nu_e \bar{\nu}_\mu \gamma_\mu \mu_L$$

# ALL THINGS EFT

GUT

SMEFT

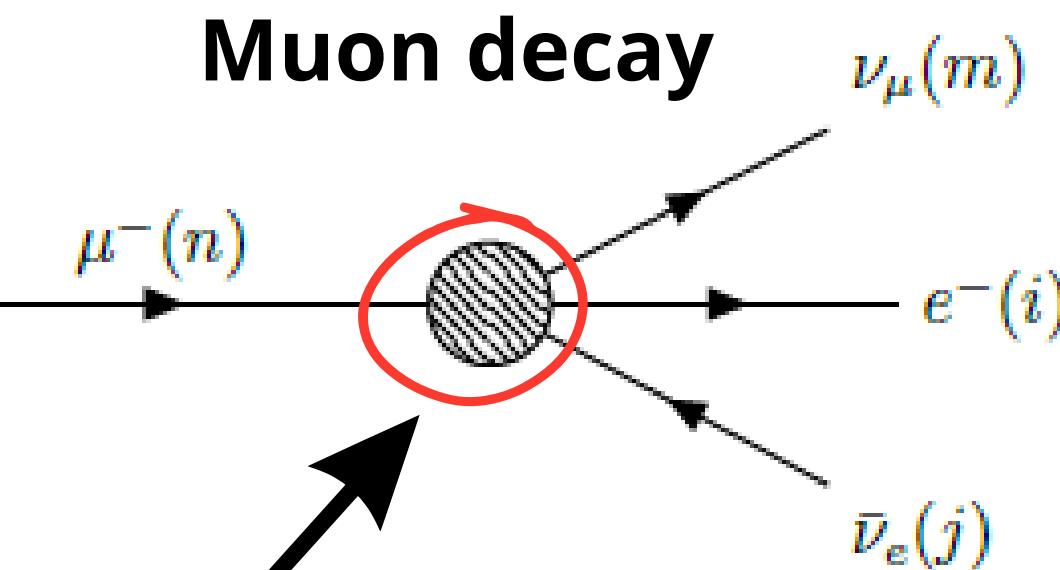
SM

LVF decays

Muon g-2

Other lepton decays

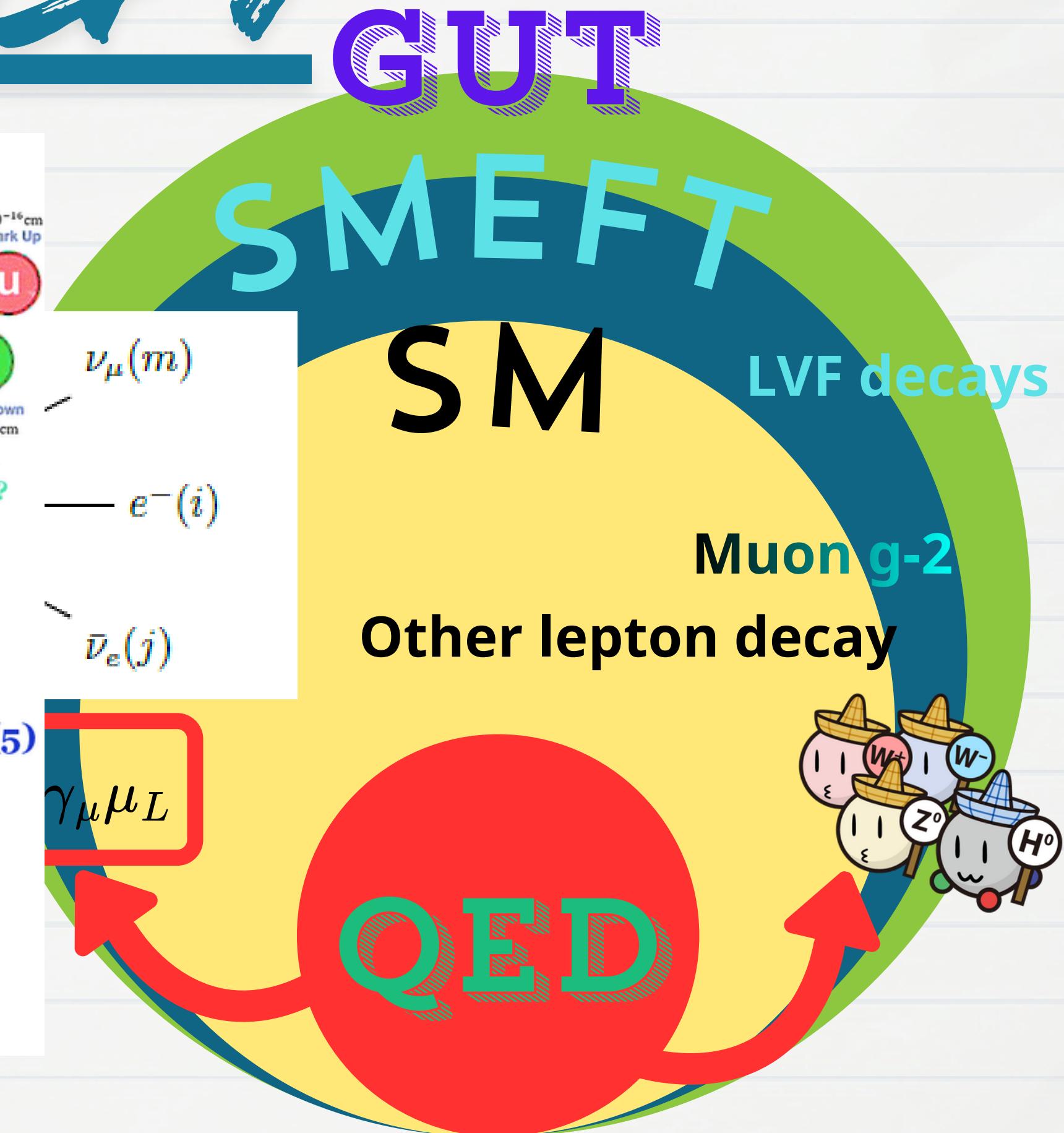
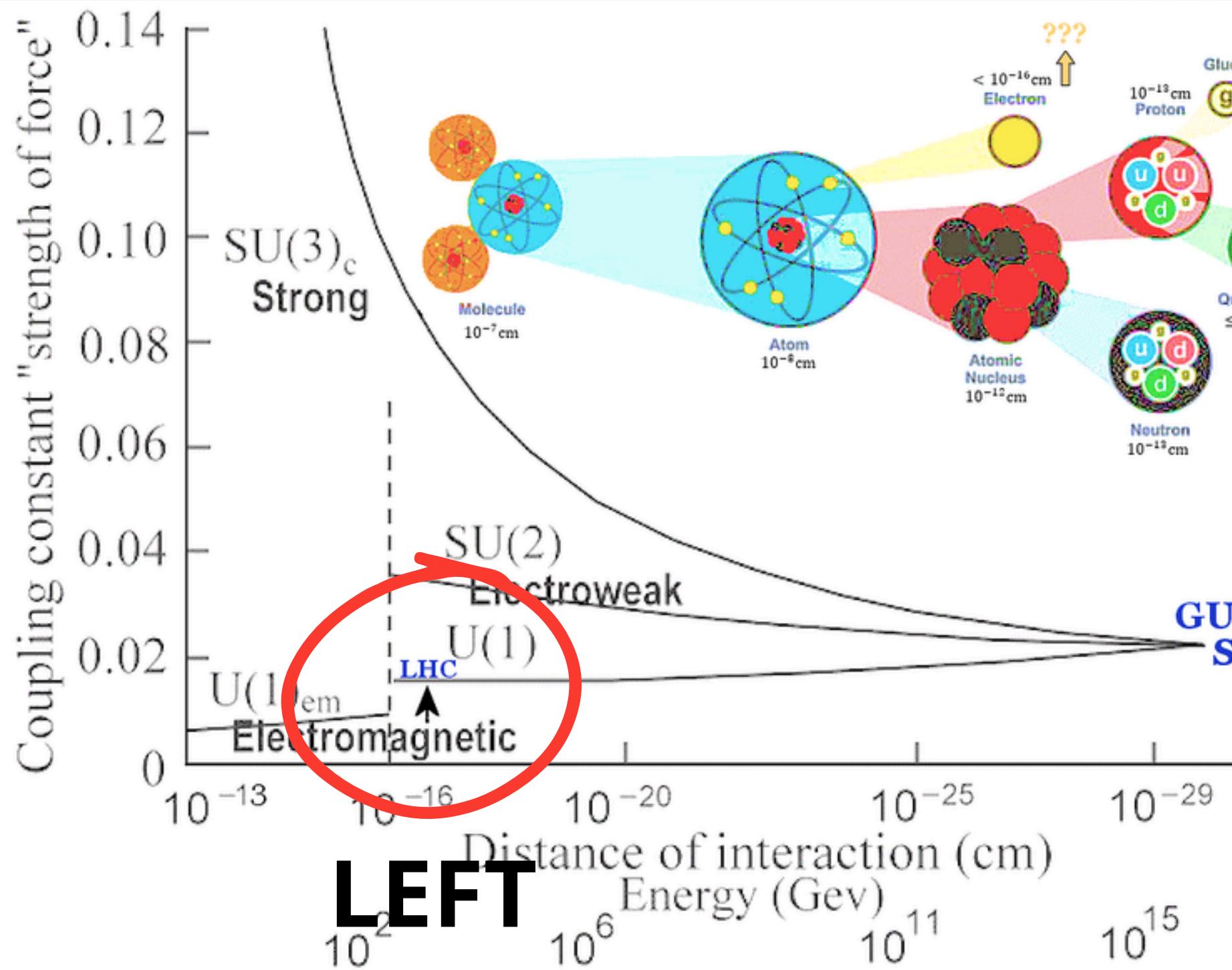
QED



$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F \bar{e}_L \gamma^\mu \nu_e \bar{\nu}_\mu \gamma_\mu \mu_L$$



# ALL THINGS EFT

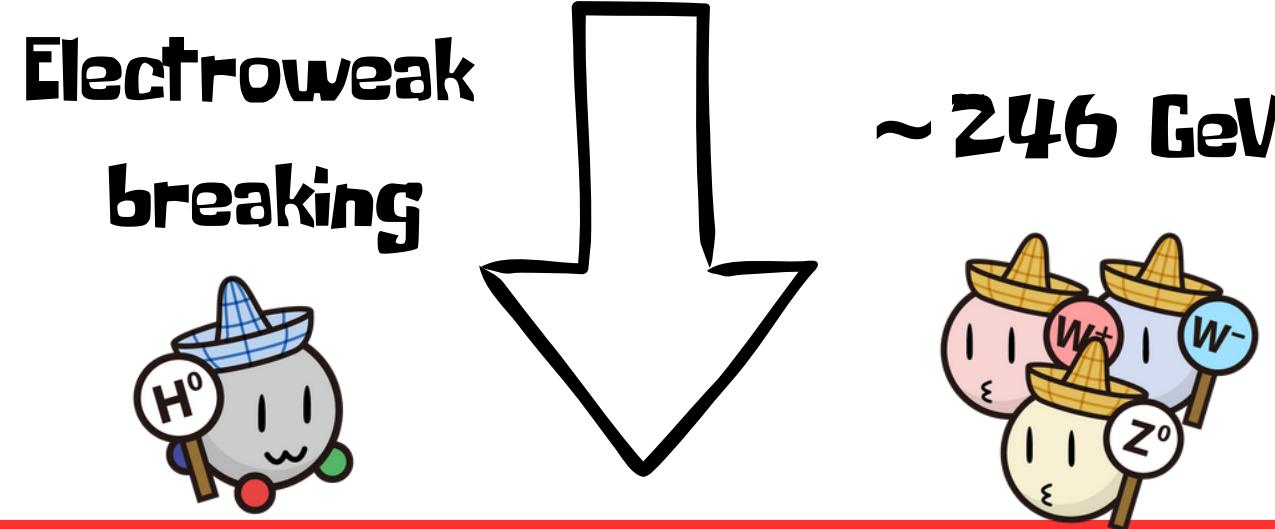


# STANDARD Model

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + |D_\mu \phi|^2 - V(\phi) + \bar{\psi}_L \hat{Y} \phi \psi_R + \text{h.c.}.$$

# STANDARD Model

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + |D_\mu \phi|^2 - V(\phi) + \bar{\psi}_L \hat{Y} \phi \psi_R + \text{h.c.}$$



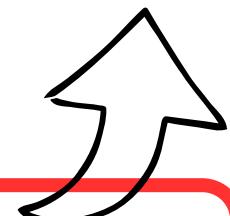
$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$

$\mathcal{L}_{\text{Fermi}}$

$$\mathcal{L}_{WW\gamma} \sim A_{\mu\nu} W^{+\mu} W^{-\nu} + \dots$$



$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{\nu}_l \gamma^\mu \ell^L W_\mu^+ + \text{h. c.}$$



# STANDARD Model

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + |D_\mu \phi|^2 - V(\phi) + \bar{\psi}_L \hat{Y} \phi \psi_R + \text{h.c.}$$

UV models

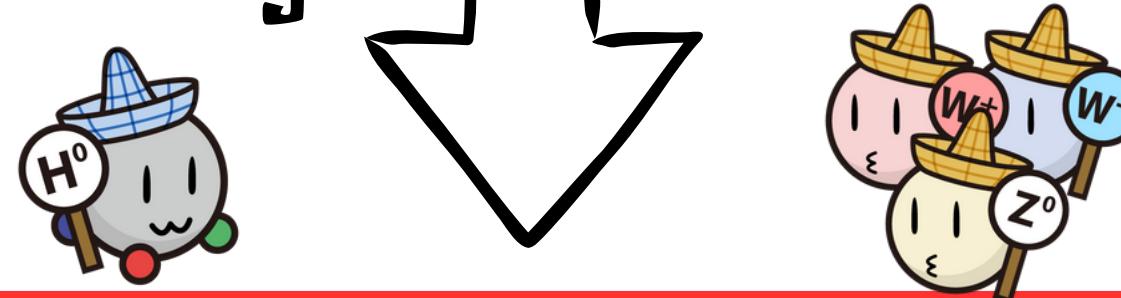
LVF decays

Muon g-2

Neutrino mass

Electroweak  
breaking

~ 246 GeV



$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$

$$\mathcal{L}_{WW\gamma} \sim A_{\mu\nu} W^{+\mu} W^{-\nu} + \dots$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{\nu}_l \gamma^\mu \ell^L W_\mu^+ + \text{h. c.}$$

$\mathcal{L}_{\text{Fermi}}$



# LOW-ENERGY EFT

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + |D_\mu \phi|^2 - V(\phi) + \bar{\psi}_L \hat{Y} \phi \psi_R + \text{h.c.}$$

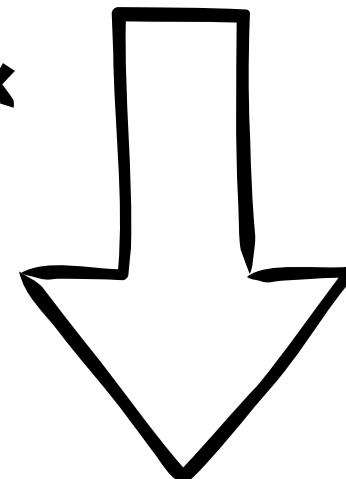
UV models

LVF decays

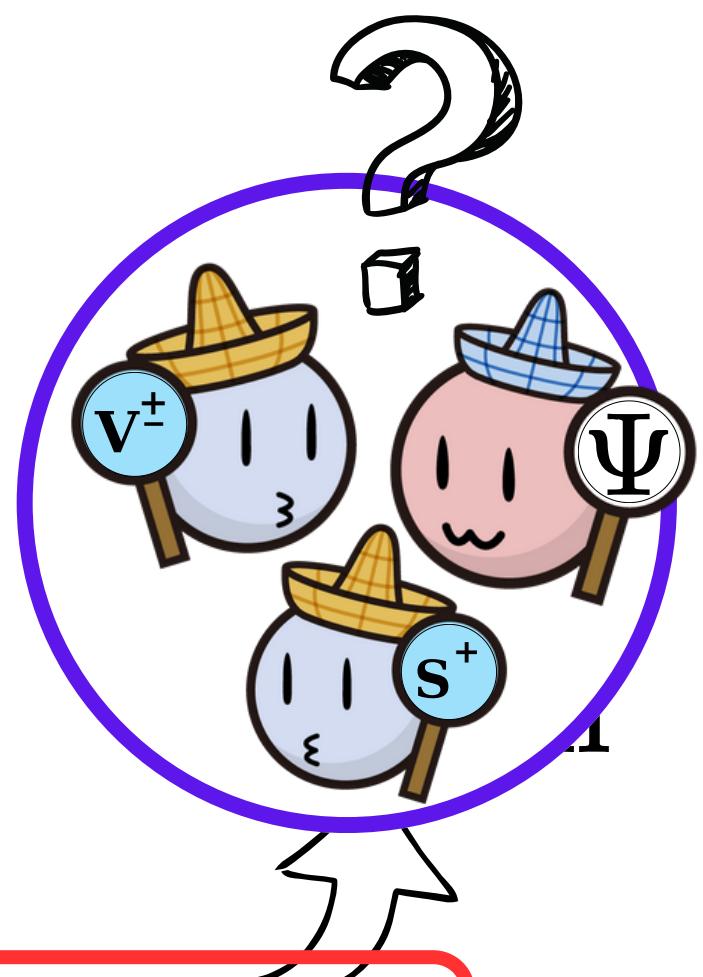
Muon g-2

Neutrino mass

Electroweak  
breaking



~ 246 GeV



$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$

$$\mathcal{L}_{WW\gamma} \sim A_{\mu\nu} W^{+\mu} W^{-\nu} + \dots$$



$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{\nu}_l \gamma^\mu \ell^L W_\mu^+ + \text{h. c.}$$

# LOW-ENERGY EFT

$$\mathcal{L}_S = a_\phi |D_\mu \phi|^2 - b_\phi m_\phi^2 |\phi|^2 \quad \mathcal{L}_{F-S} = \bar{\psi}_i \left( Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R} \right) \psi_j \phi + \text{h.c.}$$



$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$



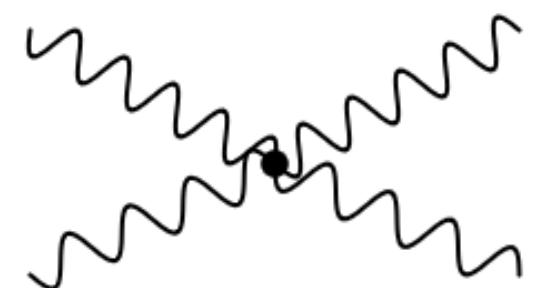
# LOW-ENERGY EFT

$$\mathcal{L}_S = a_\phi |D_\mu \phi|^2 - b_\phi m_\phi^2 |\phi|^2 \quad \mathcal{L}_{F-S} = \bar{\psi}_i \left( Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R} \right) \psi_j \phi + \text{h.c.}$$

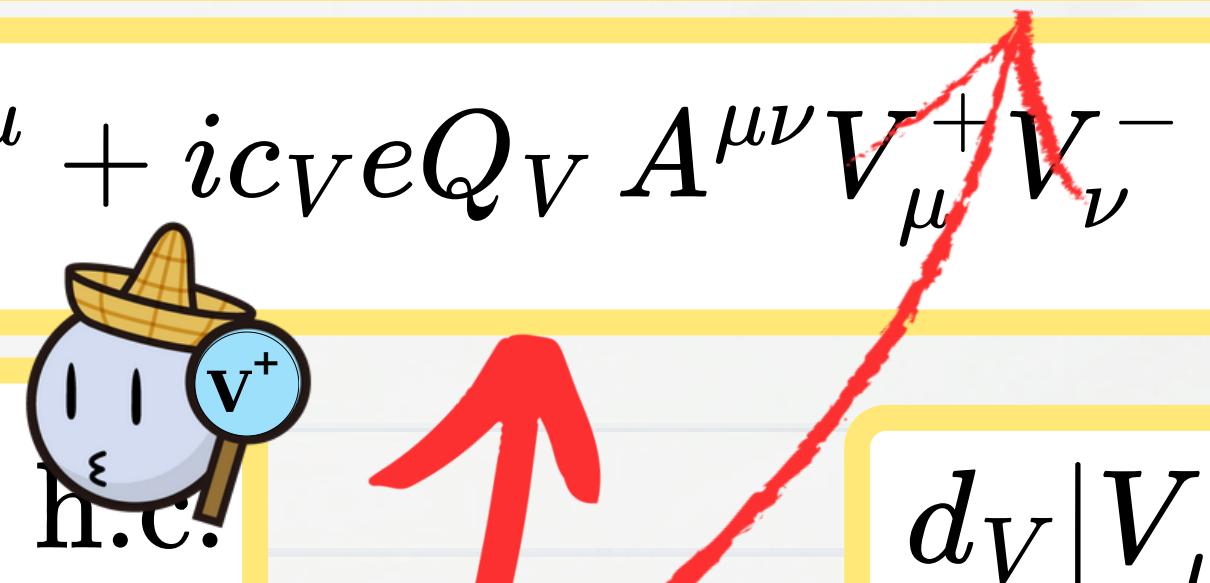
$$\mathcal{L}_{V_\pm} = -\frac{a_V}{2} \tilde{V}_{\mu\nu}^+ \tilde{V}^{-\mu\nu} + b_V m_V^2 V_\mu^+ V^{-\mu} + i c_V e Q_V A^{\mu\nu} V_\mu^+ V_\nu^-$$

$$\mathcal{L}_{F-V^\pm} = \bar{\psi}_i \left( V_{ij}^L \hat{L} + V_{ij}^R \hat{R} \right) \gamma^\mu \psi_j V_\mu^\pm + \text{h.c.}$$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$



$$d_V |V_\mu^\pm|^4$$



# LOW-ENERGY EFT

$$\mathcal{L}_S = a_\phi |D_\mu \phi|^2 - b_\phi m_\phi^2 |\phi|^2 \quad \mathcal{L}_{F-S} = \bar{\psi}_i \left( Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R} \right) \psi_j \phi + \text{h.c.}$$

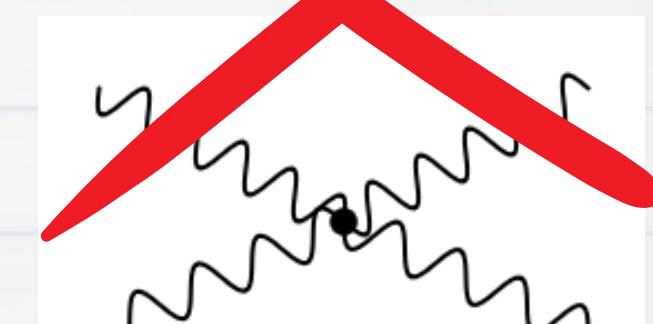
$$\mathcal{L}_{V_\pm} = -\frac{a_V}{2} \tilde{V}_{\mu\nu}^+ \tilde{V}^{-\mu\nu} + b_V m_V^2 V_\mu^+ V^{-\mu} + i c_V e Q_V A^{\mu\nu} V_\mu^+ V_\nu^-$$

$$\mathcal{L}_{F-V^\pm} = \bar{\psi}_i \left( V_{ij}^L \hat{L} + V_{ij}^R \hat{R} \right) \gamma^\mu \psi_j V_\mu^\pm + \text{h.c.}$$

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$



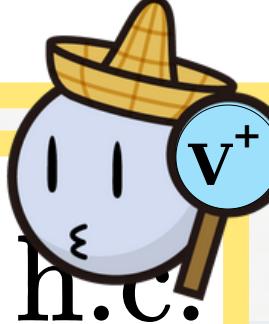
~~$d_V |V_\mu^\pm|^4$~~

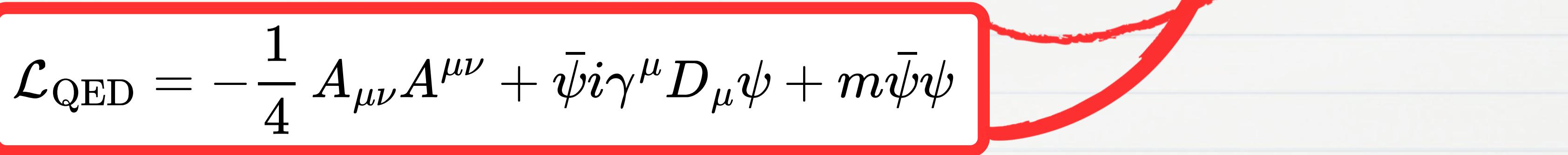


# LOW-ENERGY EFT

$$\mathcal{L}_S = \cancel{a_\phi} |D_\mu \phi|^2 - \cancel{b_\phi} m_\phi^2 |\phi|^2 \quad \mathcal{L}_{F-S} = \bar{\psi}_i \left( Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R} \right) \psi_j \phi + \text{h.c.}$$


$$\mathcal{L}_{V_\pm} = -\frac{\cancel{a_V}}{2} \tilde{V}_{\mu\nu}^+ \tilde{V}^{-\mu\nu} + \cancel{b_V} m_V^2 V_\mu^+ V^{-\mu} + i c_V e Q_V A^{\mu\nu} V_\mu^+ V_\nu^-$$

$$\mathcal{L}_{F-V^\pm} = \bar{\psi}_i \left( V_{ij}^L \hat{L} + V_{ij}^R \hat{R} \right) \gamma^\mu \psi_j V_\mu^\pm + \text{h.c.}$$


$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$


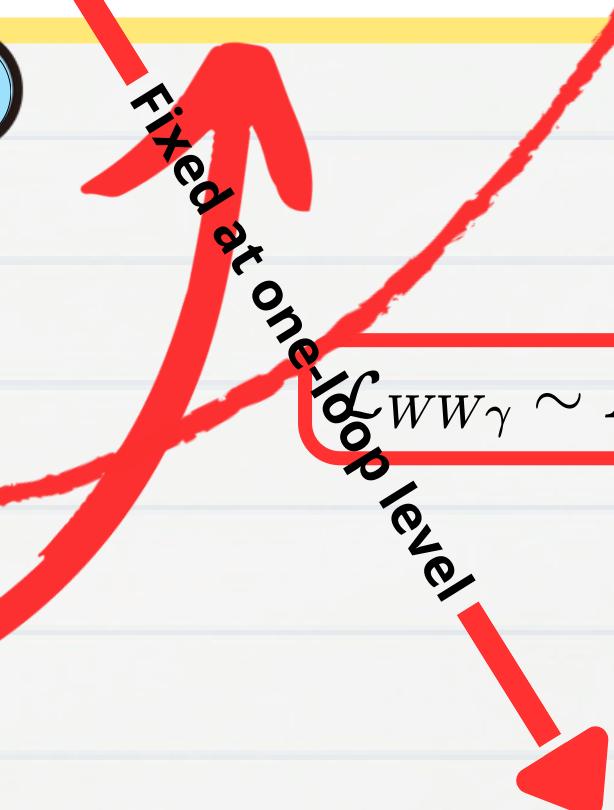
Klein-Gordon Eq. —→  $a_\phi = b_\phi = a_V = b_V = 1$

# LOW-ENERGY EFT

$$\mathcal{L}_S = \cancel{a_\phi} |D_\mu \phi|^2 - \cancel{b_\phi} m_\phi^2 |\phi|^2 \quad \mathcal{L}_{F-S} = \bar{\psi}_i \left( Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R} \right) \psi_j \phi + \text{h.c.}$$


$$\mathcal{L}_{V^\pm} = -\frac{\cancel{a_V}}{2} \tilde{V}_{\mu\nu}^+ \tilde{V}^{-\mu\nu} + \cancel{b_V} m_V^2 V_\mu^+ V^{-\mu} + \cancel{i c_V e Q_V} A^{\mu\nu} V_\mu^+ V_\nu^-$$

$$\mathcal{L}_{F-V^\pm} = \bar{\psi}_i \left( V_{ij}^L \hat{L} + V_{ij}^R \hat{R} \right) \gamma^\mu \psi_j V_\mu^\pm + \text{h.c.}$$



$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi + m \bar{\psi} \psi$$

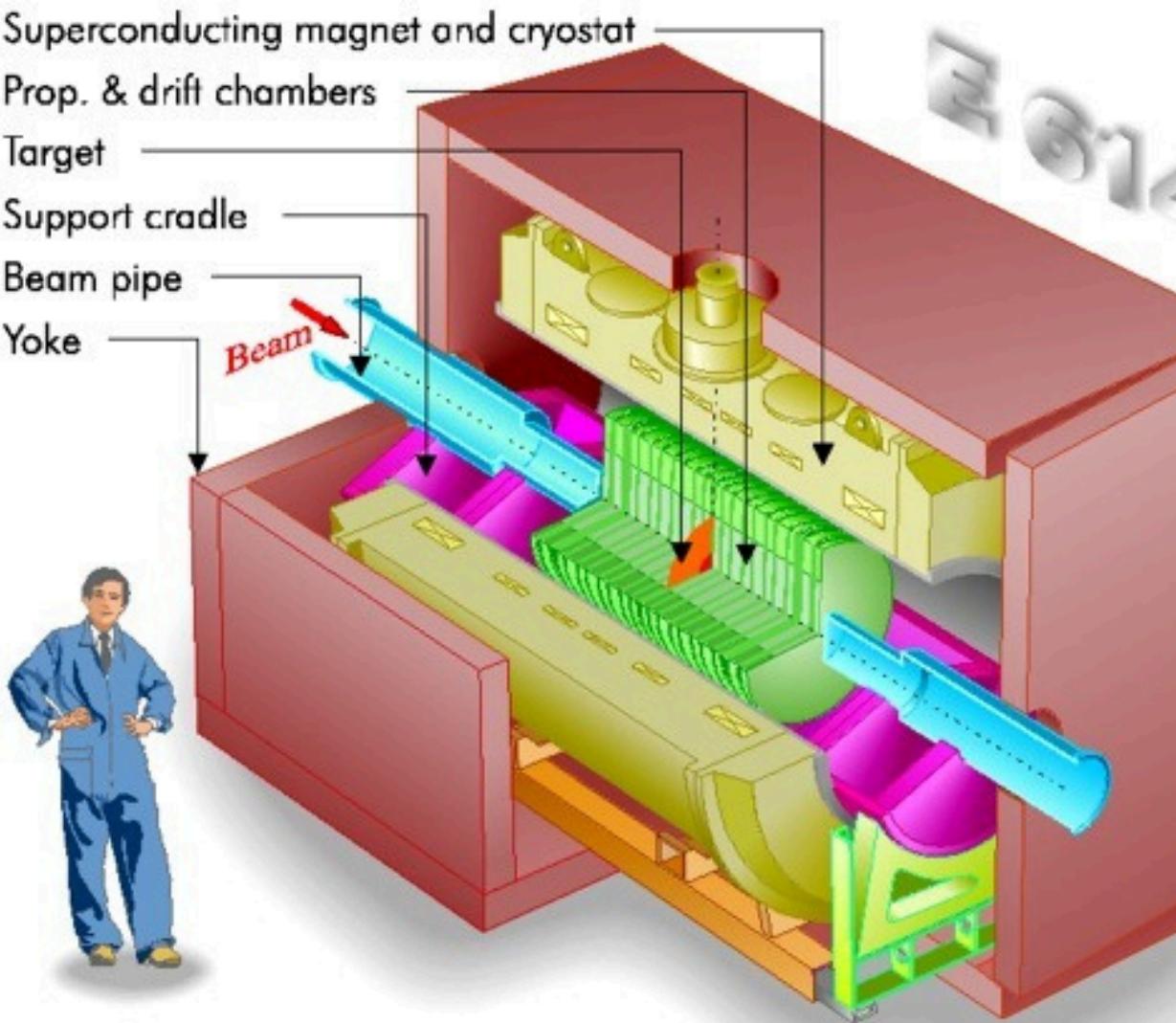
Klein-Gordon Eq. —→  $a_\phi = b_\phi = a_V = b_V = 1$

$$c_{V^\pm} = 1$$

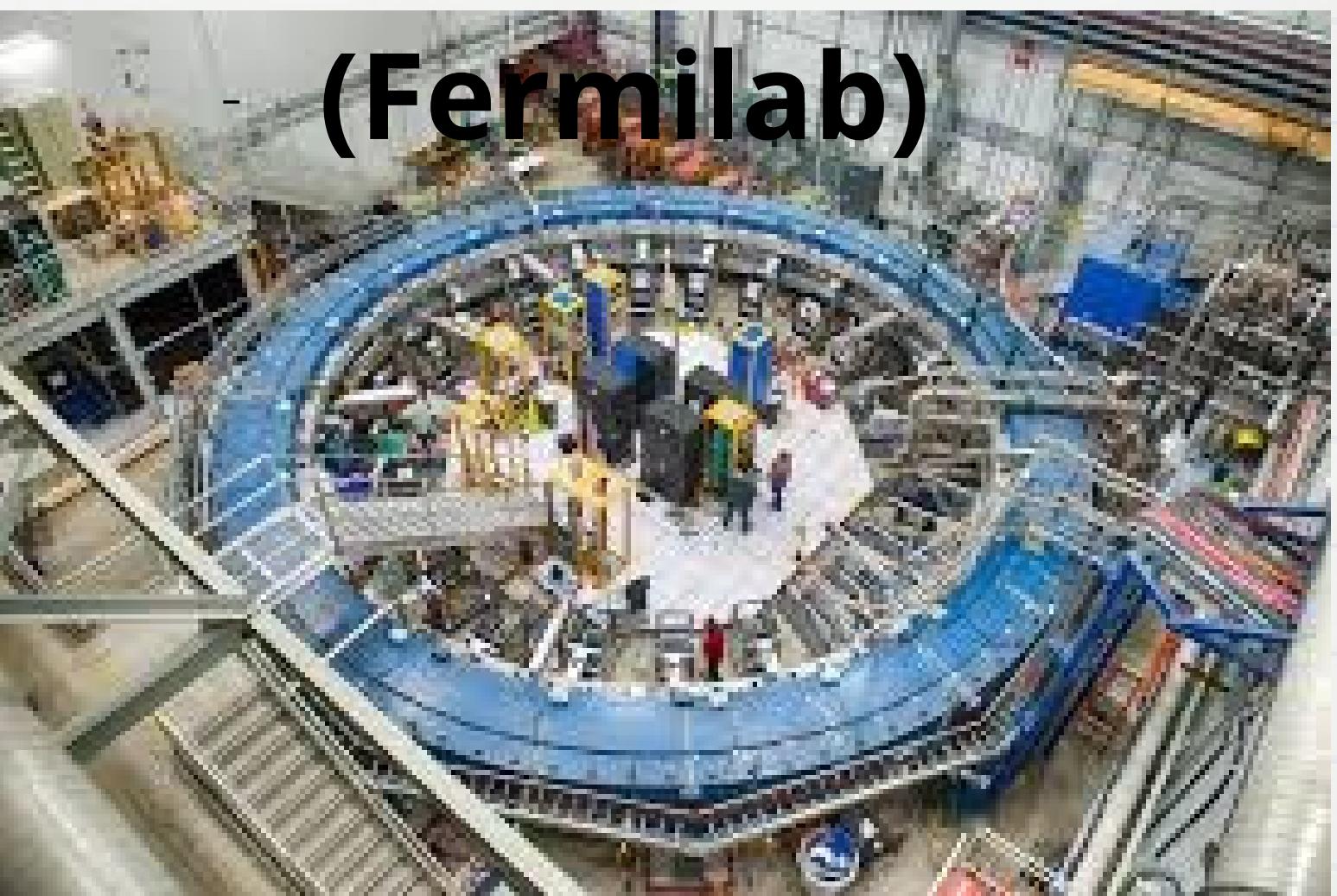
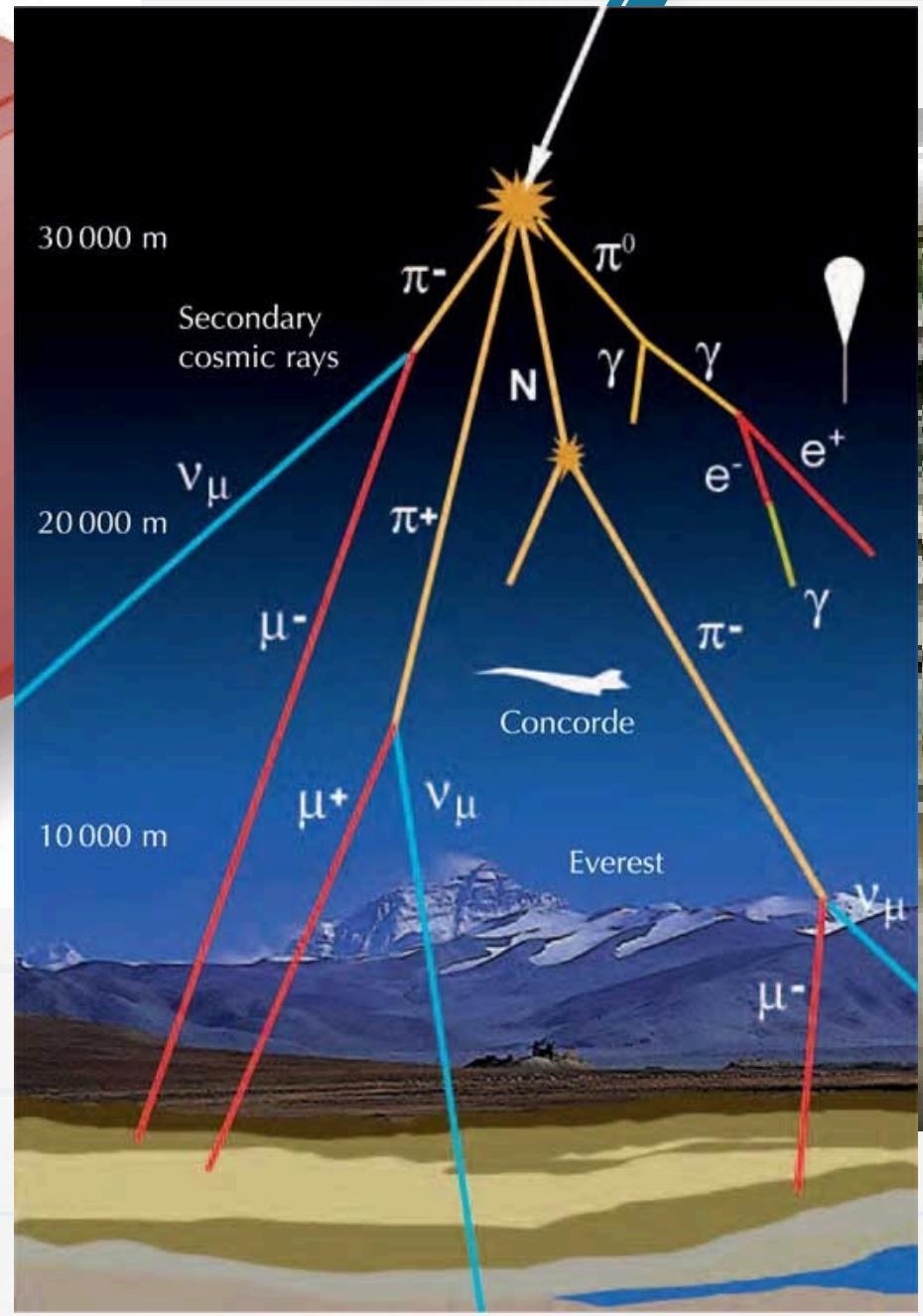
$$\mathcal{L}_{WW\gamma} \sim A_{\mu\nu} W^{+\mu} W^{-\nu} + \dots$$

# MUON Q<sub>EDM</sub> Experiments

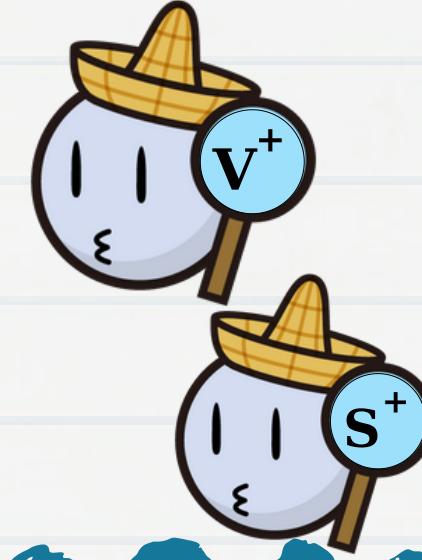
G-2



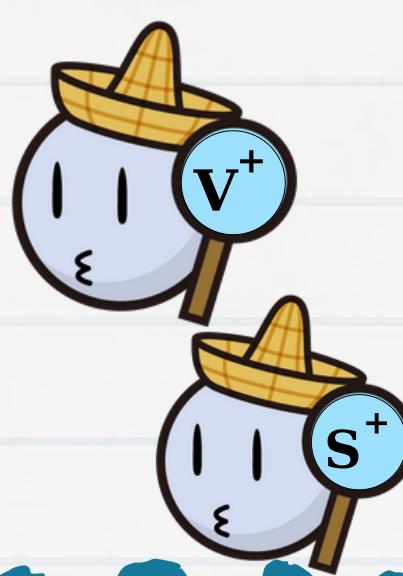
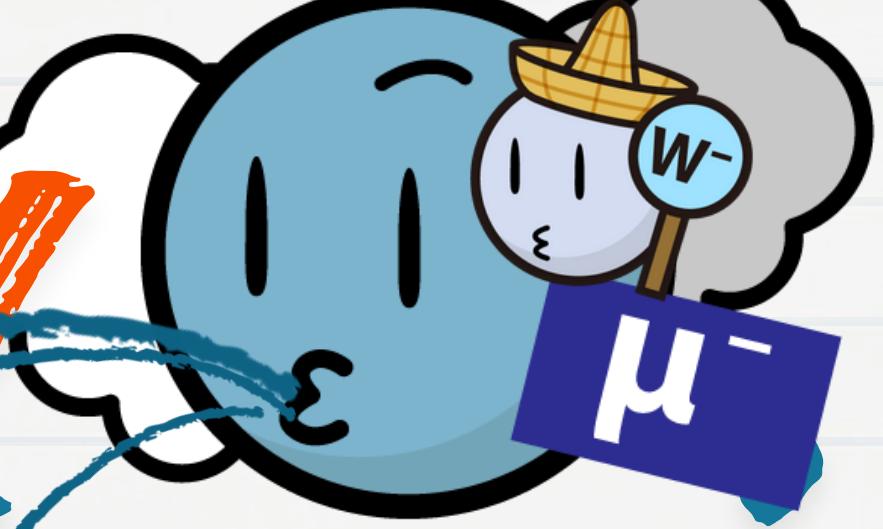
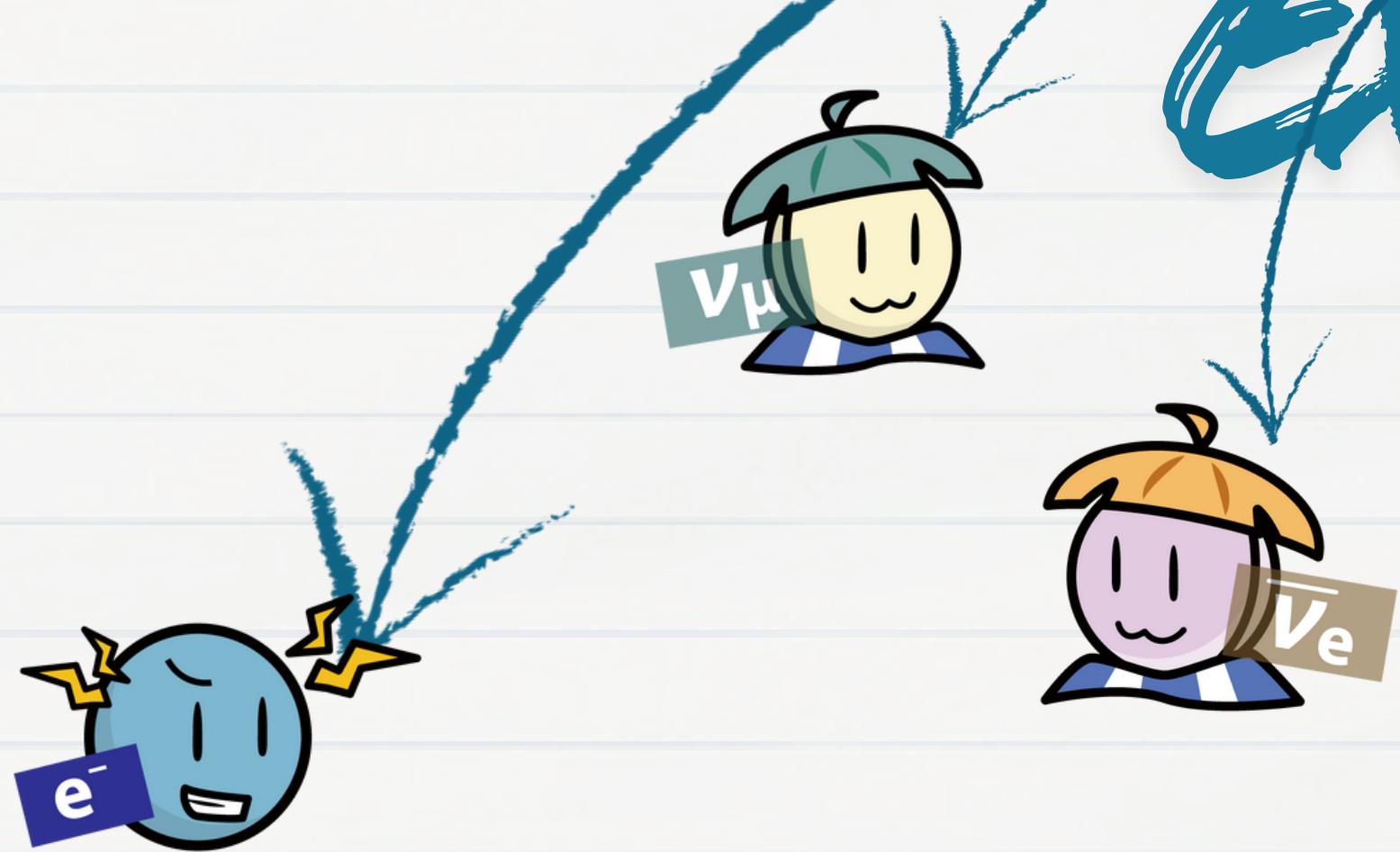
**TWIST**  
**(TRIUMF)**



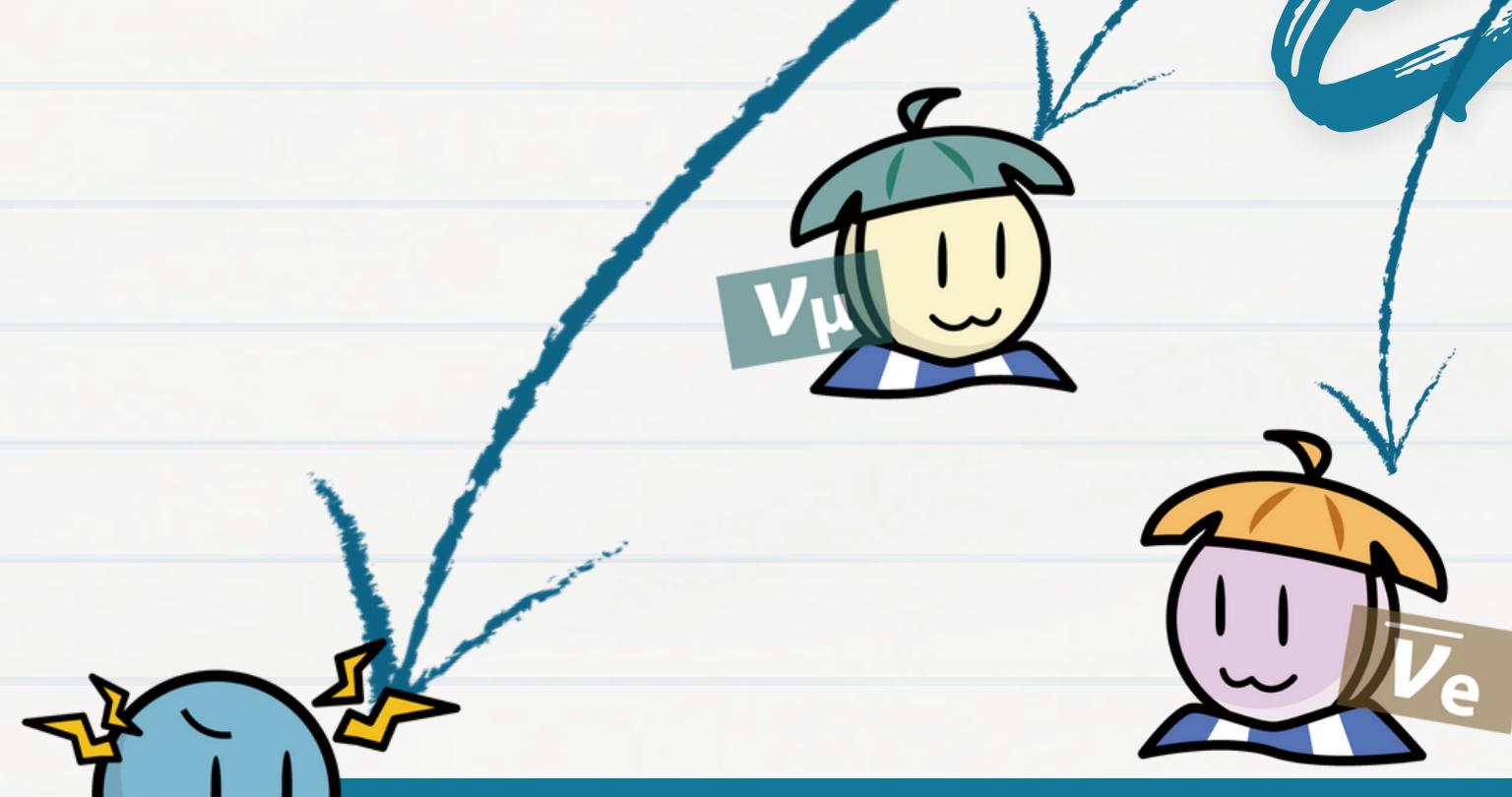
**(Fermilab)**



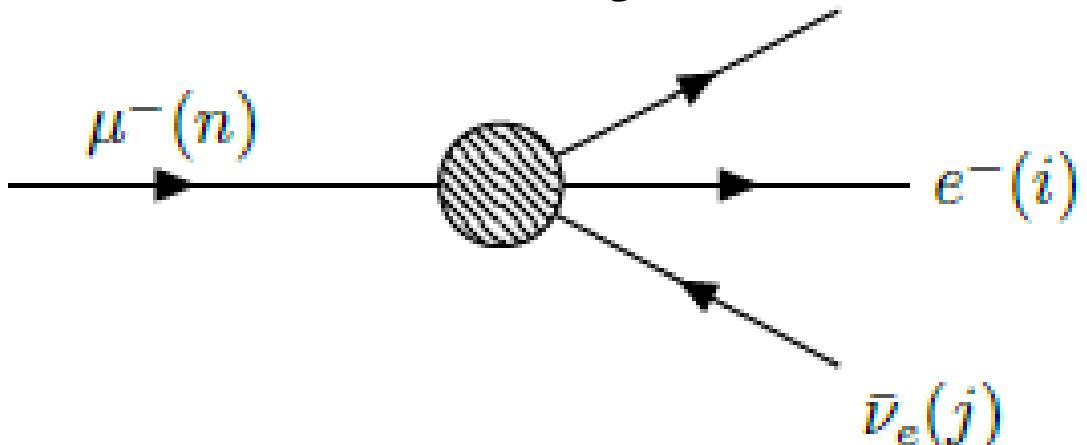
# Muon Experiments



# Muon Experiments

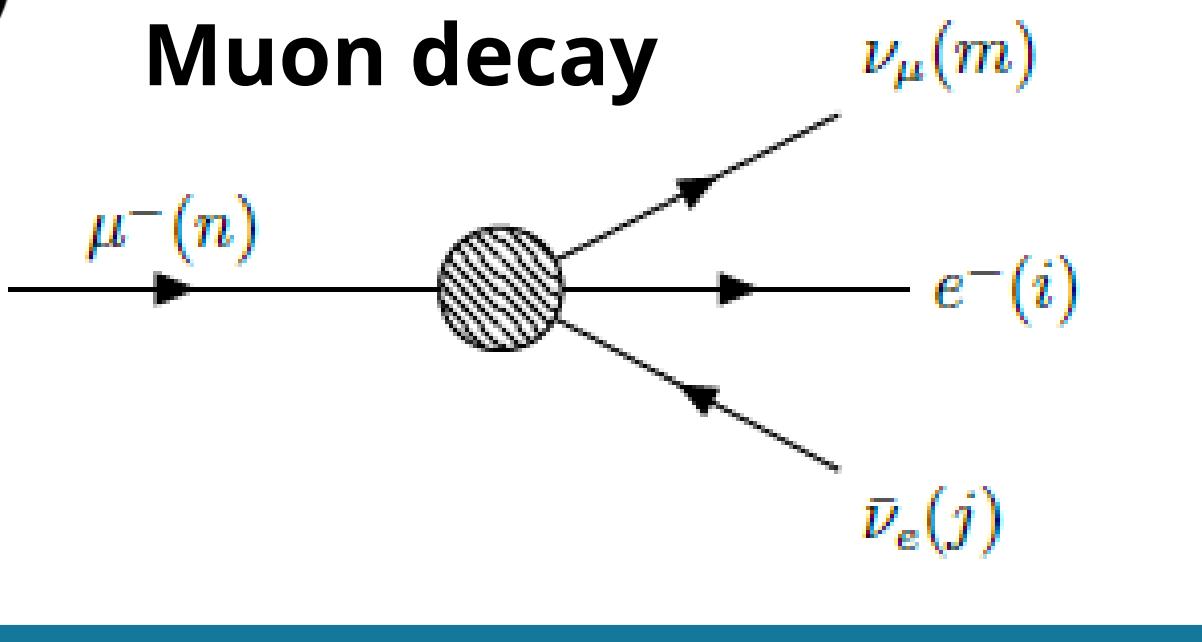
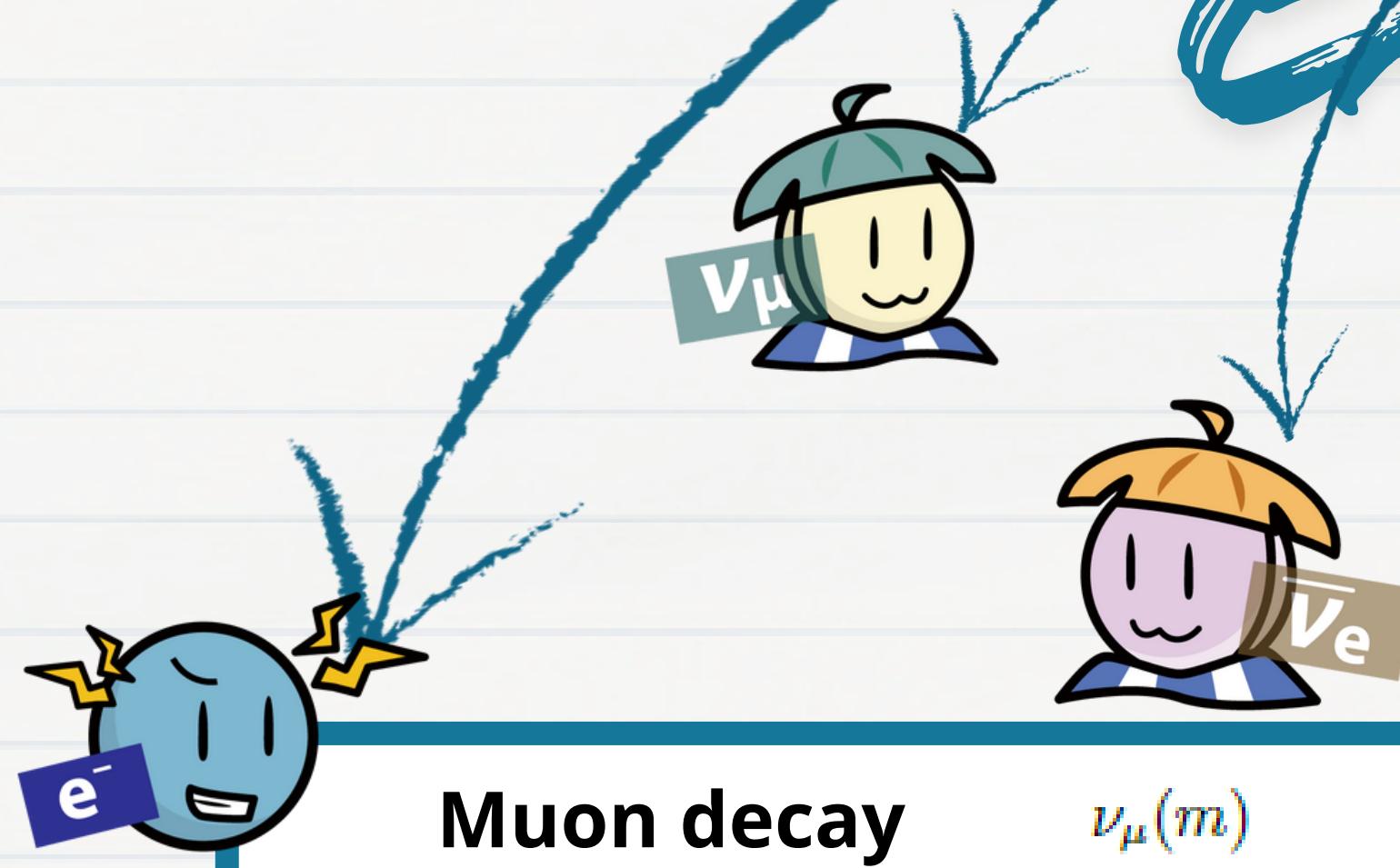


**Muon decay**

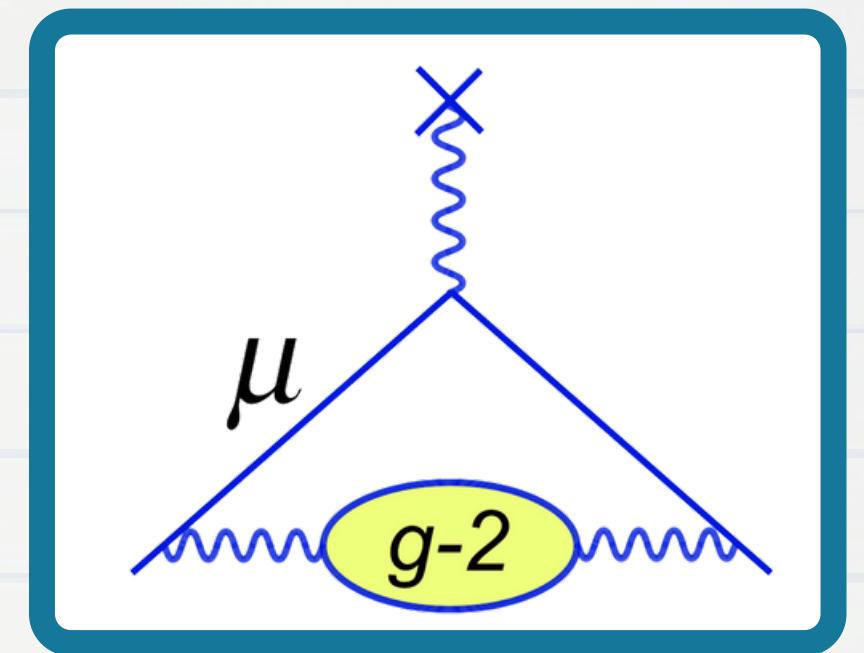


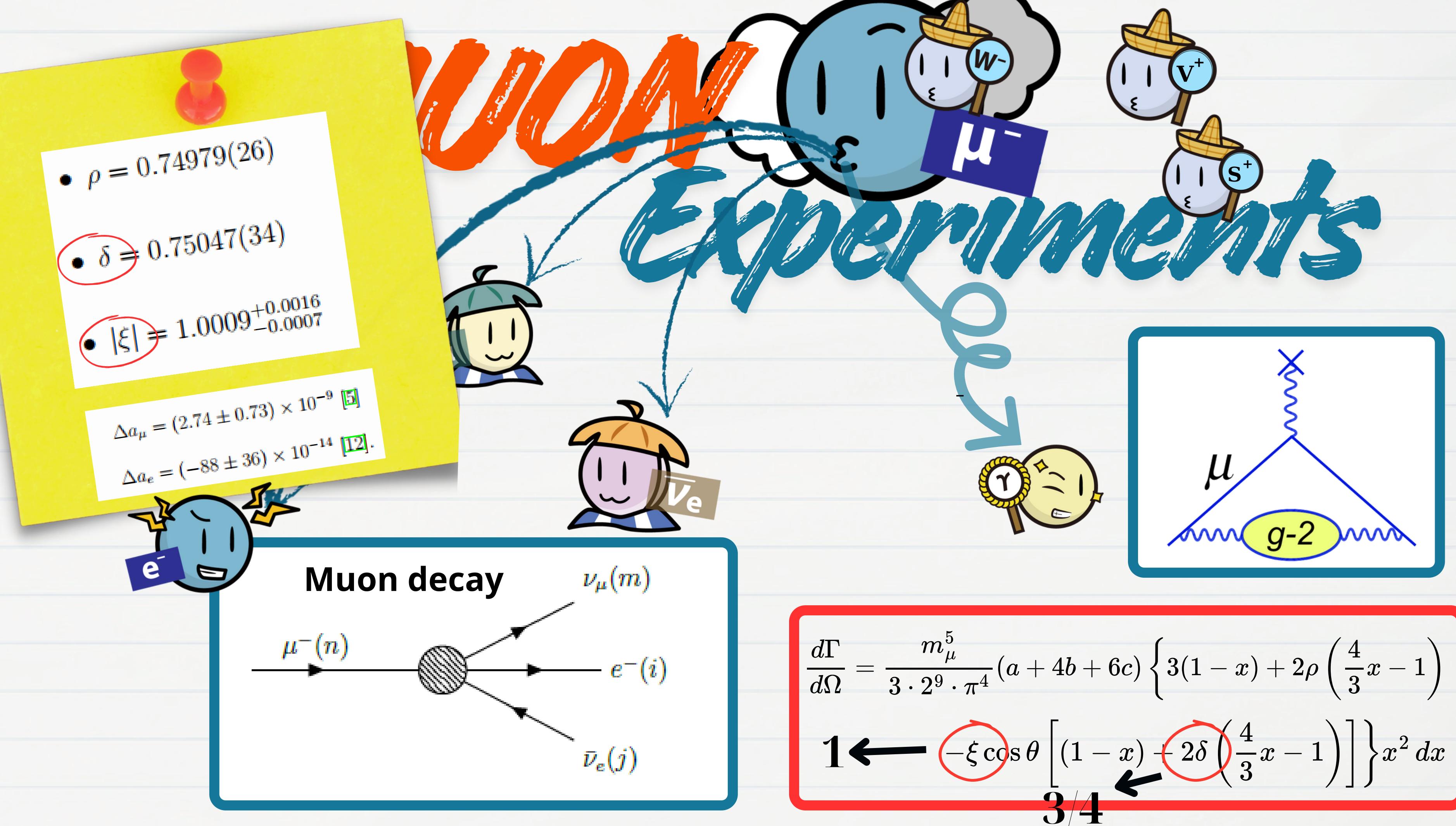
$$\frac{d\Gamma}{d\Omega} = \frac{m_\mu^5}{3 \cdot 2^9 \cdot \pi^4} (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

# Muon Experiments



$$\frac{d\Gamma}{d\Omega} = \frac{m_\mu^5}{3 \cdot 2^9 \cdot \pi^4} (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$





# EFT framework

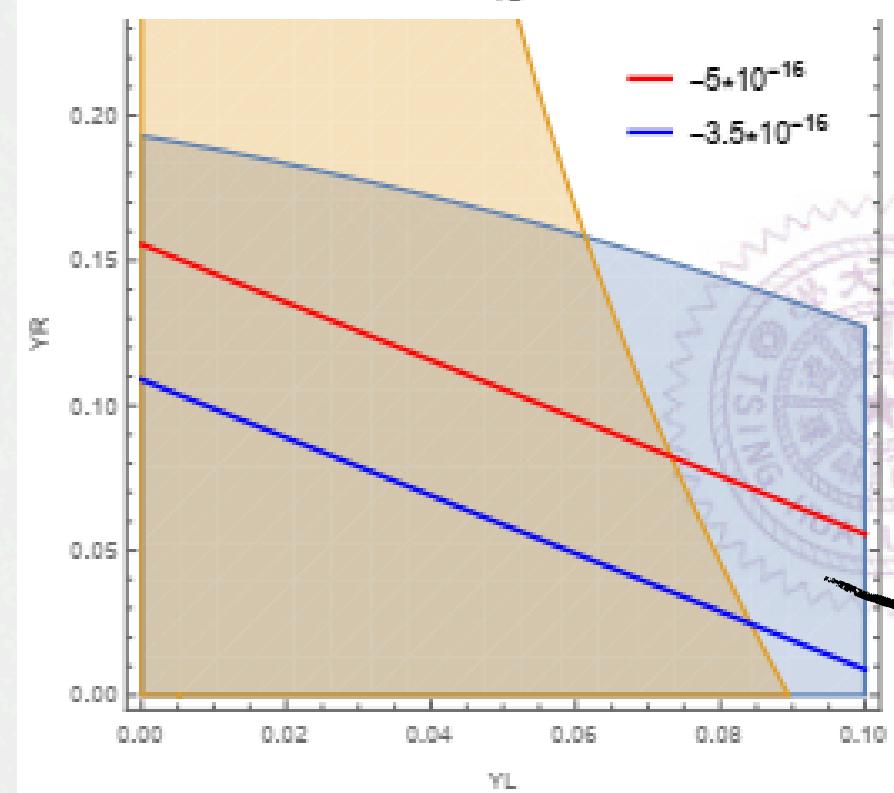
$$\begin{aligned}\mathcal{L}_S &= |D_\mu \phi|^2 - m_\phi^2 |\phi|^2, \\ \mathcal{L}_F &= i\bar{\psi} D\psi - m_f^2 \bar{\psi}\psi, \\ \mathcal{L}_{V^\pm} &= -\frac{1}{2} \tilde{V}_{\mu\nu}^\pm \tilde{V}^{-\mu\nu} + m_V^2 V_\mu^+ V_\nu^- + ie Q_V a_F \mu^\mu V_\mu^+ V_\nu^-, \\ \mathcal{L}_{F,S} &= \bar{\psi}_i (Y_{ij}^L \hat{L} + Y_{ij}^R \hat{R}) \psi_j \phi + h.c., \\ \mathcal{L}_{F,V^\pm} &= \bar{\psi}_i (V_{ij}^L \hat{L} + V_{ij}^R \hat{R}) \gamma^\mu \psi_j V_\mu^- + h.c.\end{aligned}$$

Negative contribution

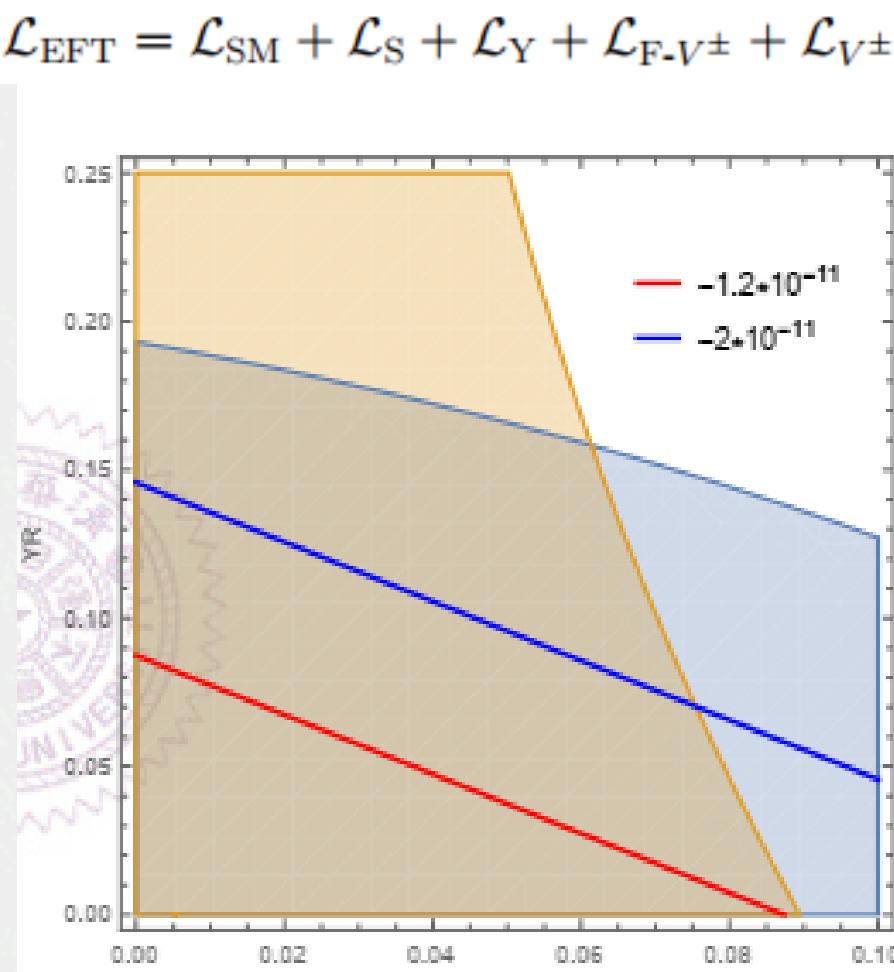
!! Possible solution comes from neutral boson with chiral enhancement

Charged Scalar

(a) Electron MDM

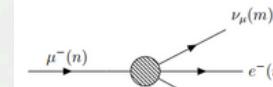


(b) Muon MDM



# MUON DECAY

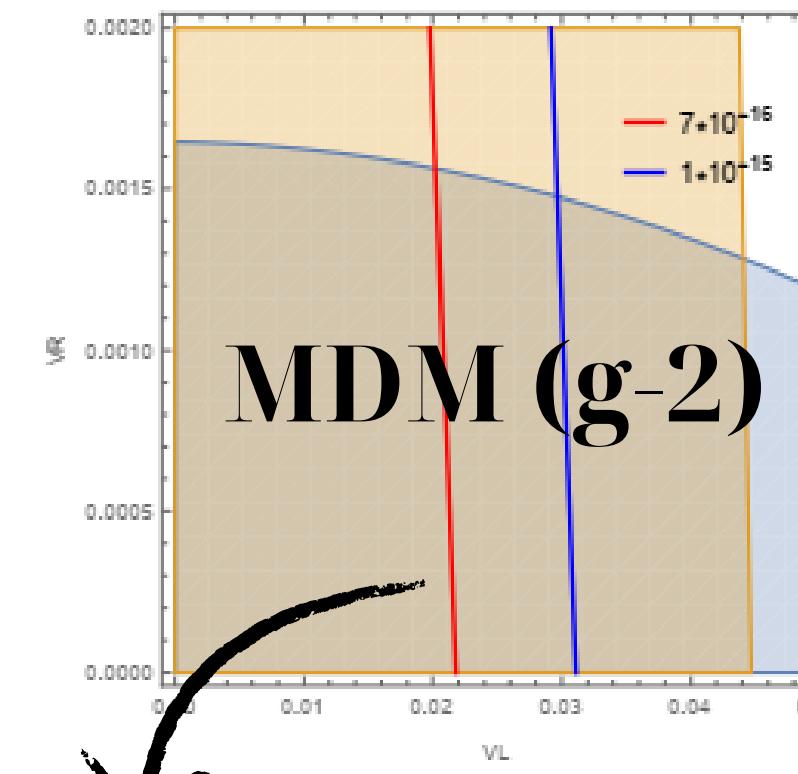
$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) \right.$$



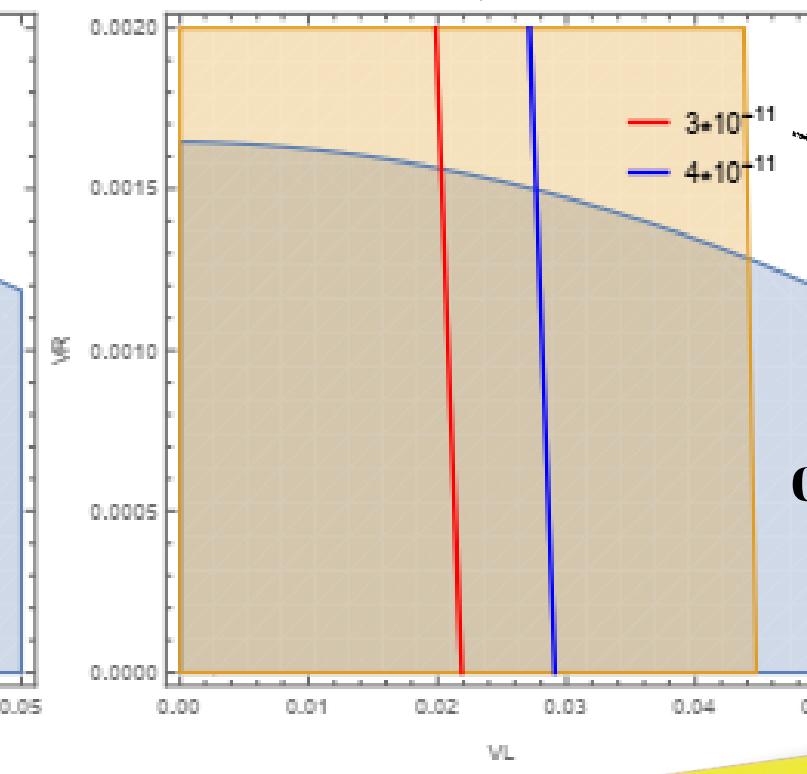
$$\left. -\xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx,$$

1

3/4



(a) Electron MDM



(b) Muon MDM

Charged Vector

The mass scale for the SM-like  $W'$  with only left-handed and only right-handed are 3.3 TeV and 637 GeV, respectively. These limits are compared to the 2016 PDG values of 3.7 TeV for left-handed and 715 GeV for right-handed

- $\rho = 0.74979(26)$

- $\delta = 0.75047(34)$

- $|\xi| = 1.0009^{+0.0016}_{-0.0007}$

$\Delta a_\mu = (2.74 \pm 0.73) \times 10^{-9}$  [5]

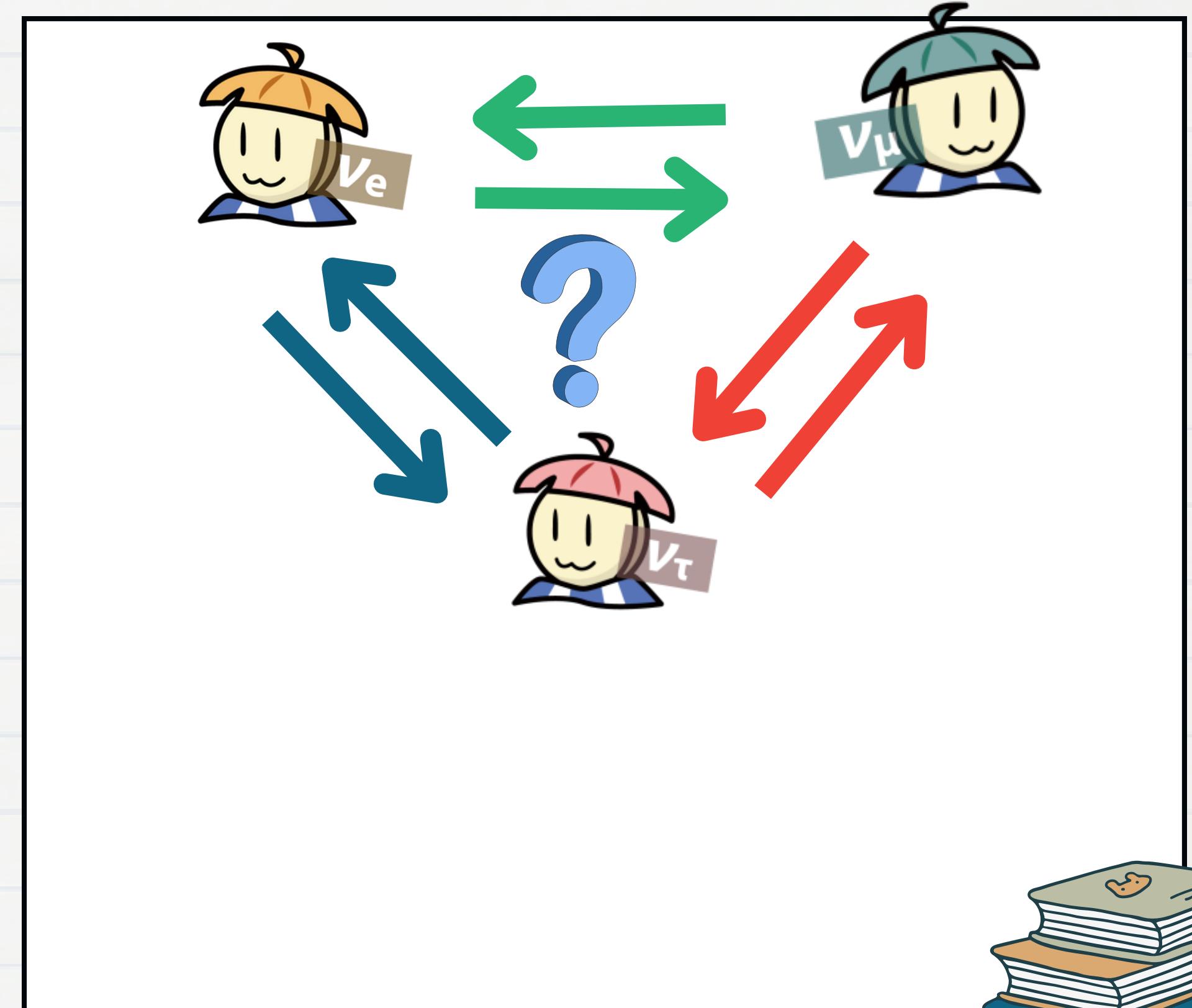
$\Delta a_e = (-88 \pm 36) \times 10^{-14}$  [12]

# Overview

- Neutrino Oscillation
- Zee model
- LFV decays
- Muon decays
- Conclusions



## Phenomenological Constraints on the Leptophilic Zee Model

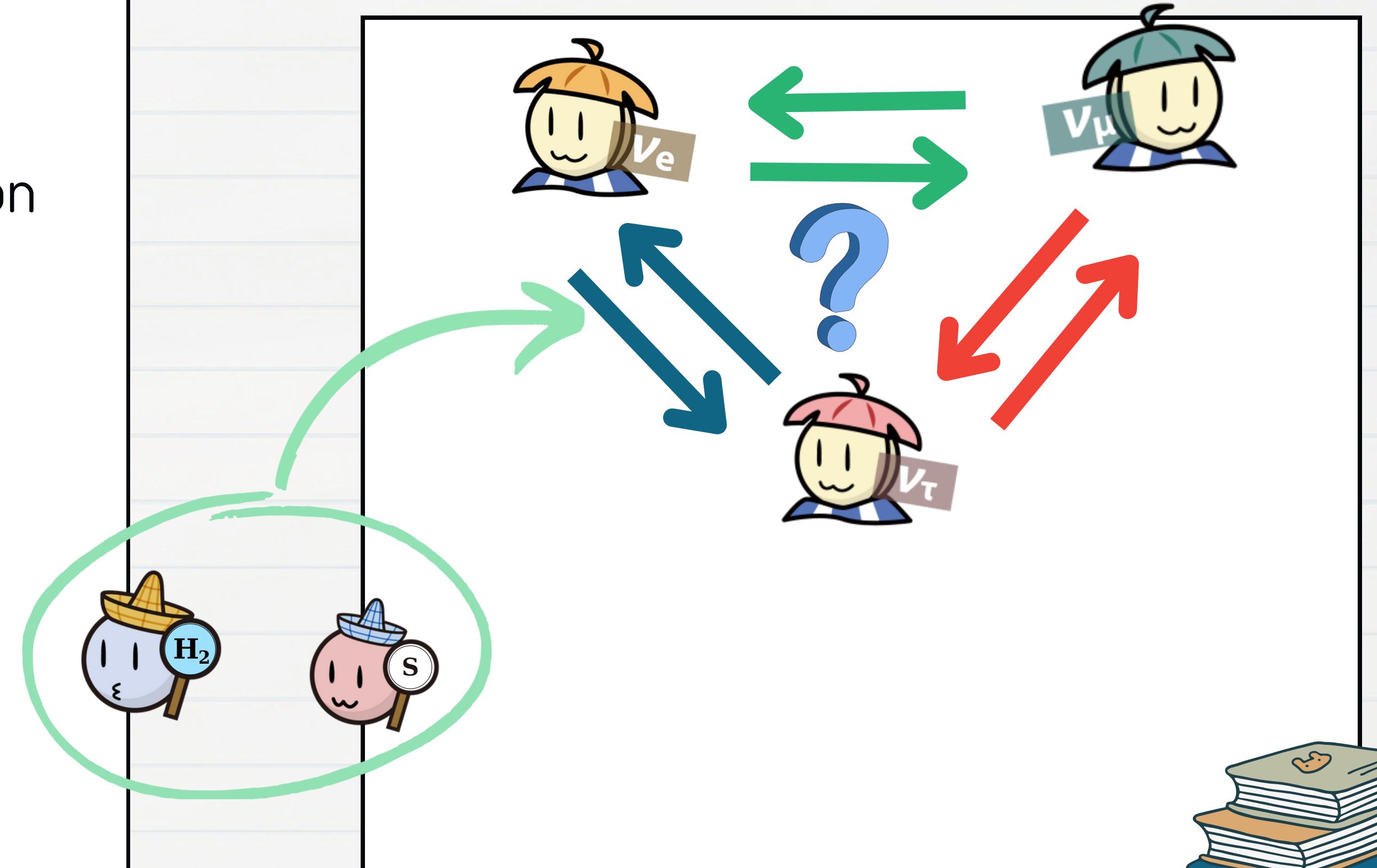


# Overview

- Neutrino Oscillation
- Zee model
- LFV decays
- Muon decays
- Conclusions



## Phenomenological Constraints on the Leptophilic Zee Model

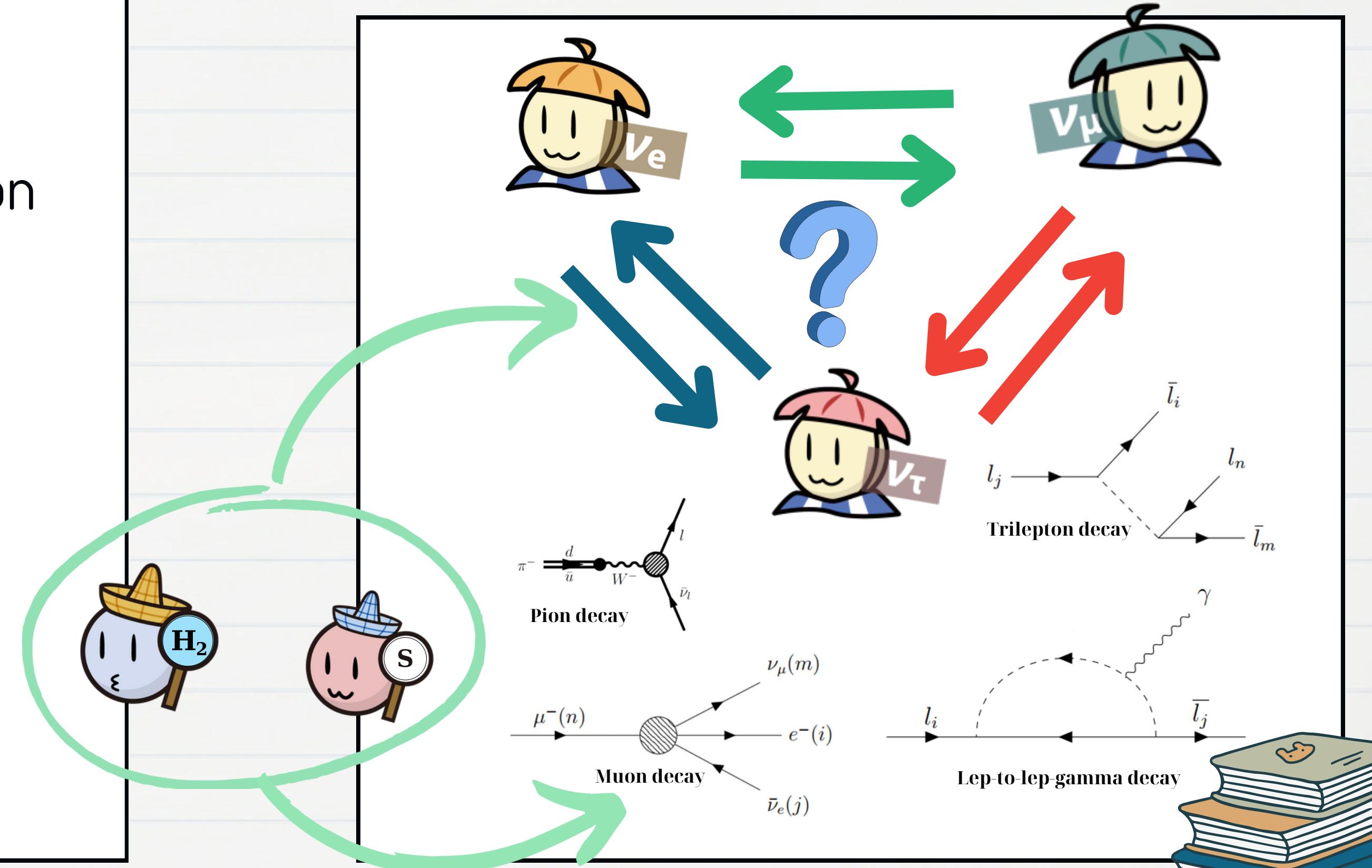


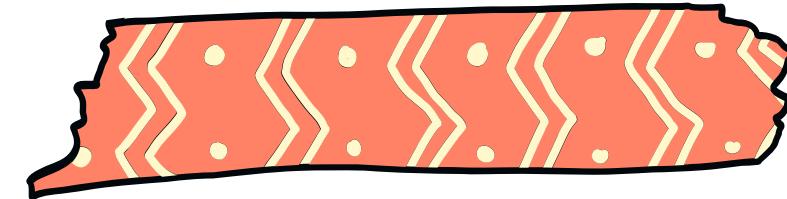
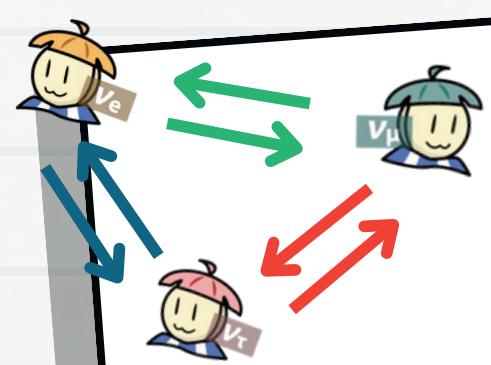
# Overview

- Neutrino Oscillation
- Zee model
- LFV decays
- Muon decays
- Conclusions



## Phenomenological Constraints on the Leptophilic Zee Model





# Neutrino Oscillation

**“Neutral” and “ino” → “neutrino”!**

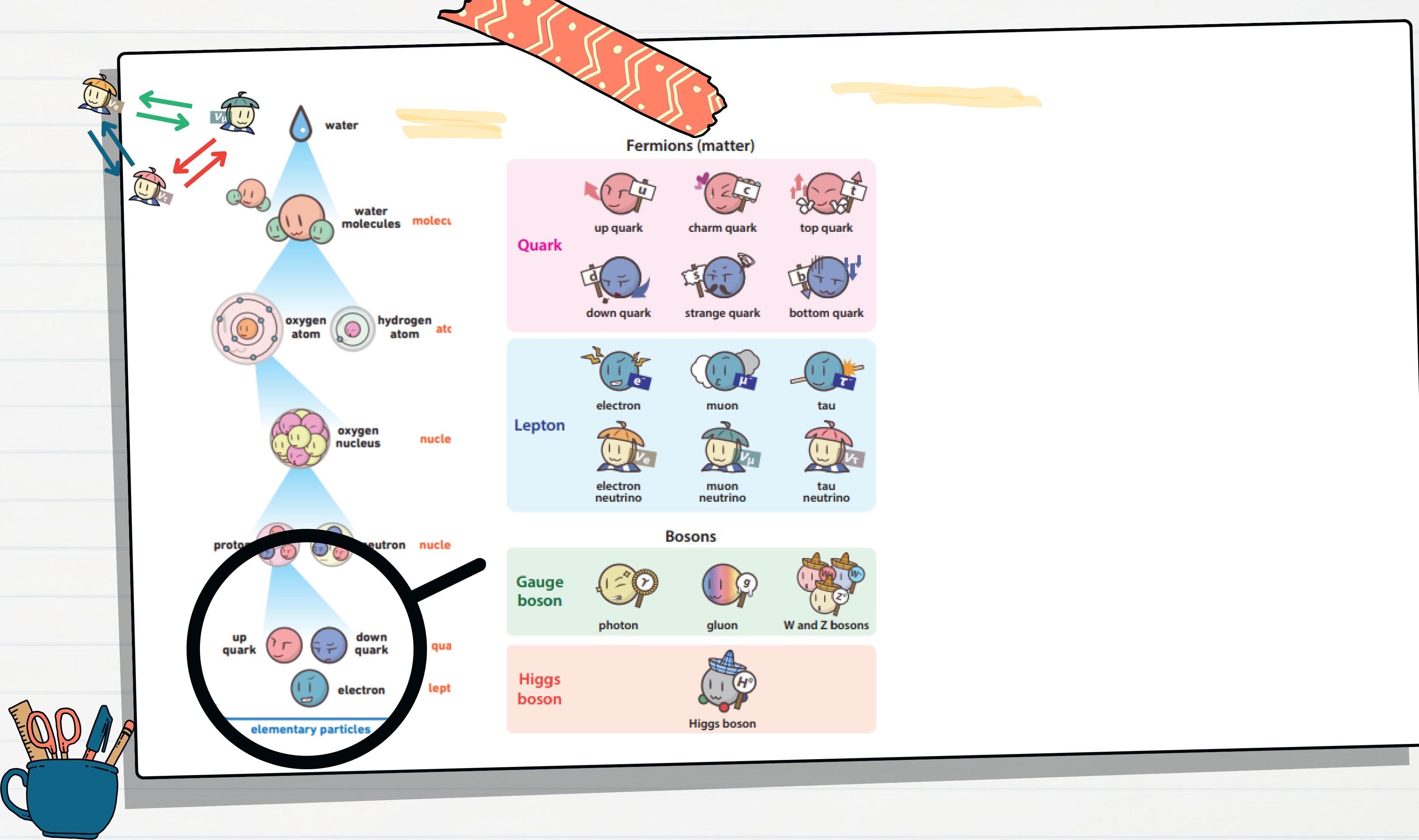
Neutrino means “Neutral = no electric charge” and “-ino = small (in Italian)”

Oscillation: Neutrinos change flavor state as they travel

- Discovered in 1998 by the SuperKamiokande group through their analysis of atmospheric neutrinos
- Challenging our traditional understanding embedded within the Standard Model framework
- Sources of neutrinos: Solar, Atmospheric, Reactor, Accelerator

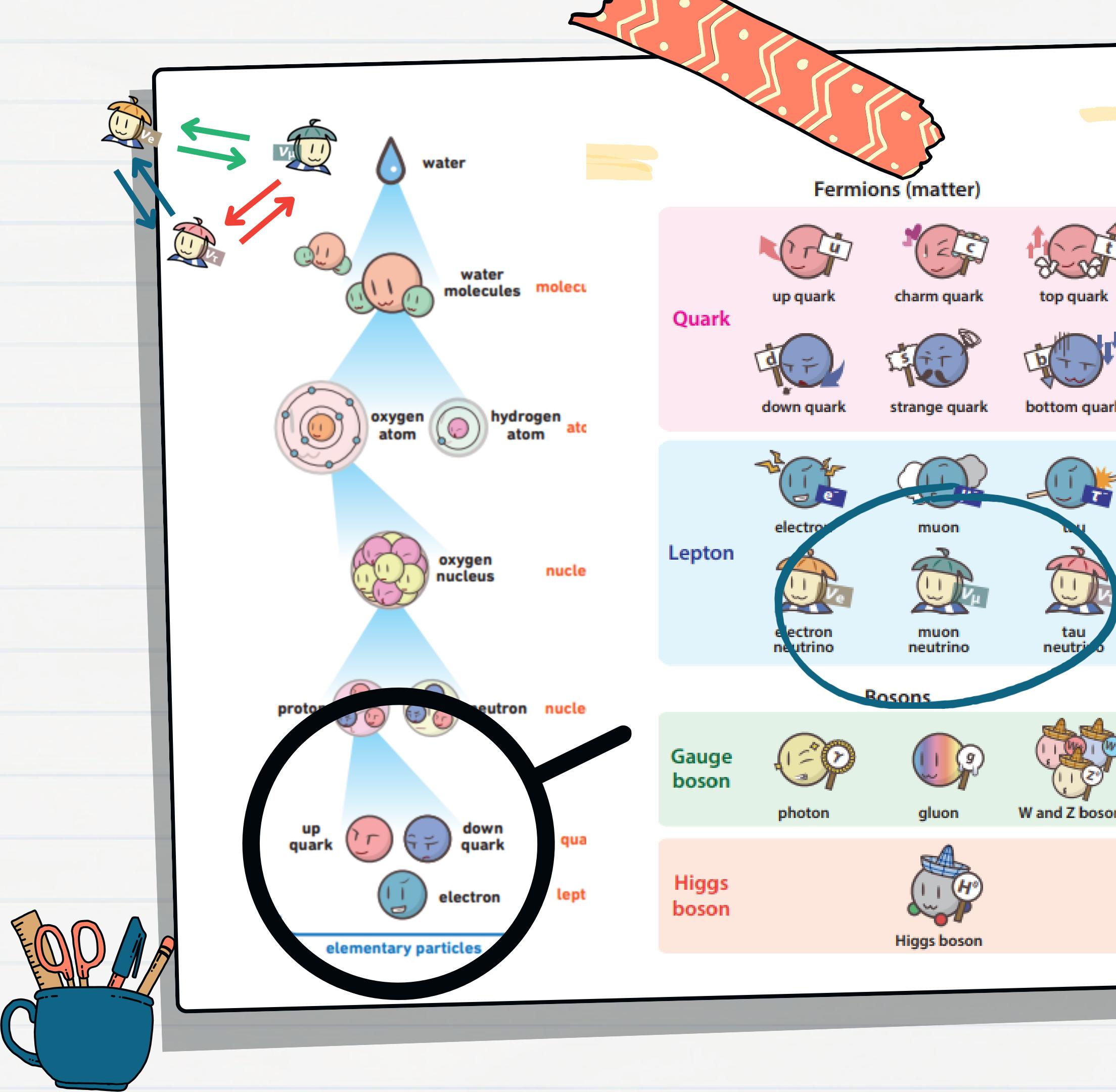
Why is it a fundamental puzzle in our current understanding?





# Neutrinos in Standard Model

- A fundamental neutral particle interact weakly with anything (through weak and gravity force)
- “Catch it” indirectly via Cherenkov effects when neutrinos interact with the substance
- Beta decay ( first evidence for the existence of neutrinos)  
⇒ Only left-handed neutrinos involved
- Therefore **no mass term** generated through Higgs mechanism  
⇒ No neutrino oscillation



# Neutrinos in Standard Model

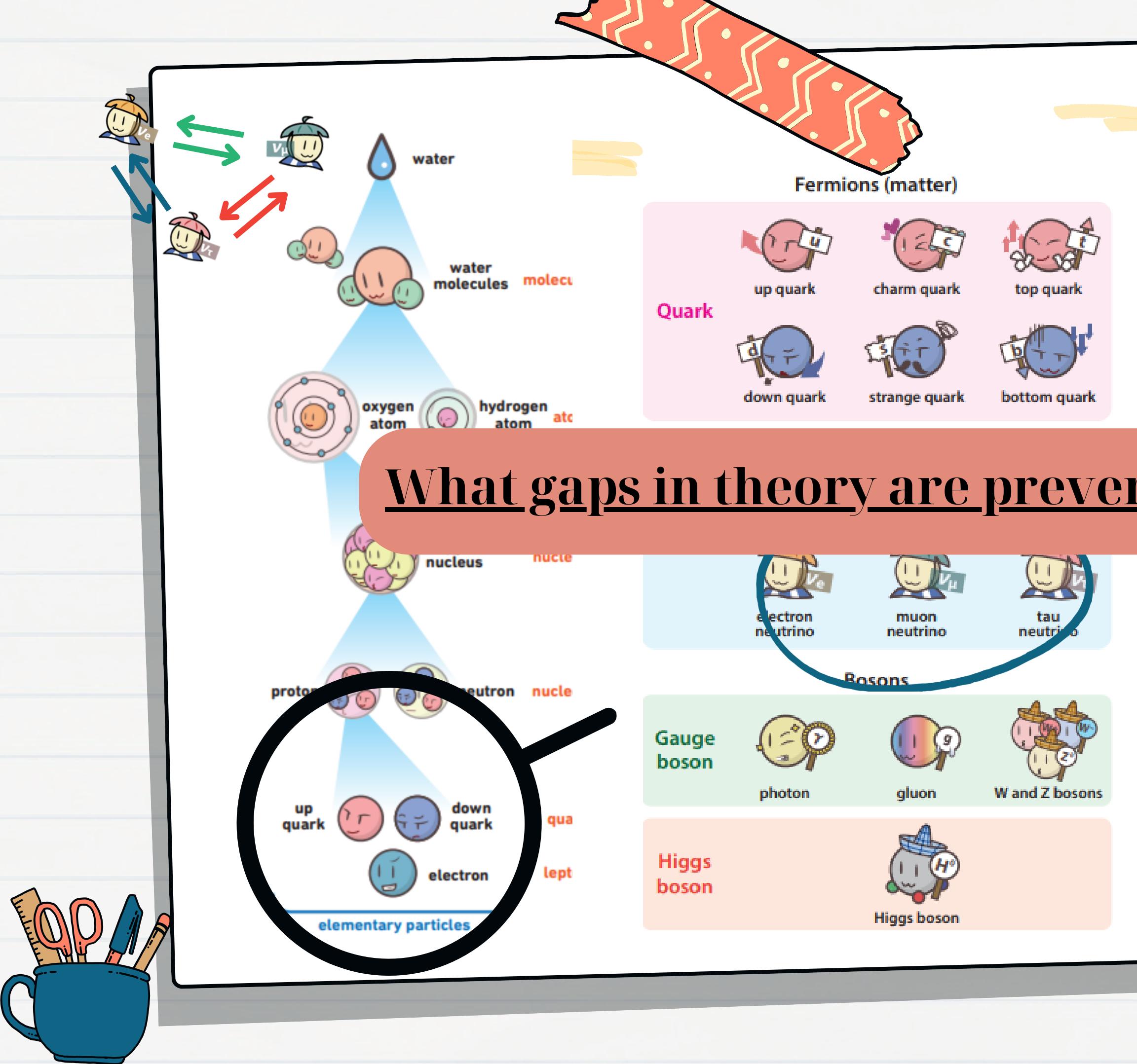
- A fundamental neutral particle interact weakly with anything (through weak and gravity force)
- “Catch it” indirectly via Cherenkov effects when neutrinos interact with the substance

What gaps in theory are preventing a resolution?

⇒ Only left-handed neutrinos involved

- Therefore **no mass term** generated through Higgs mechanism

⇒ No neutrino oscillation



# Oscillation



$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin((X_{ij})^2)$$



Difference between interacting state and mass state

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}^D} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}^D} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \text{Diag}\left(1, e^{i\frac{\mu_1}{2}}, e^{i\frac{\mu_2}{2}}\right)$$

**PMNS matrix**



# Oscillation



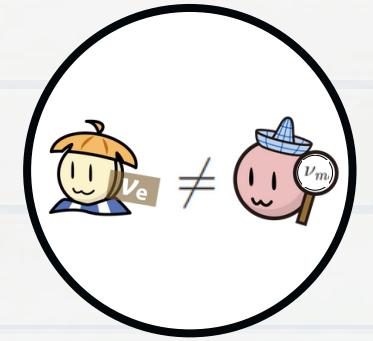
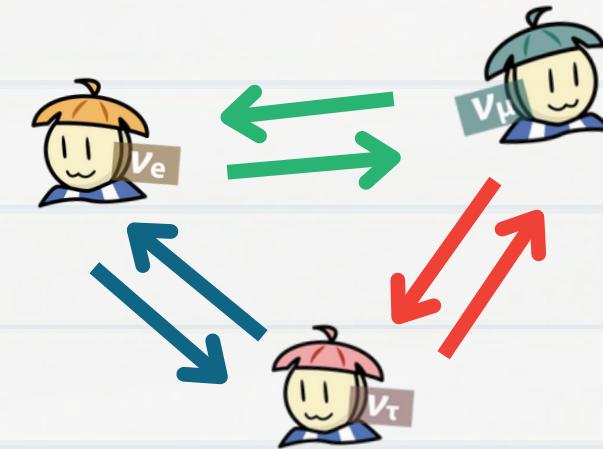
$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin(X_{ij})^2$$



Mass



Mass difference



Difference between interacting state and mass state

$$X_{ij} \equiv \frac{m_i^2 - m_j^2}{4E} L$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}^D} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}^D} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \text{Diag}(1, e^{i\frac{\mu_1}{2}}, e^{i\frac{\mu_2}{2}})$$

Atmosphere

Reactor

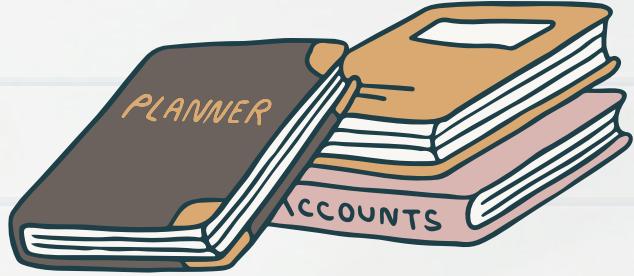
Solar

Neutrinoless Double beta decay

**PMNS matrix**



# Oscillation



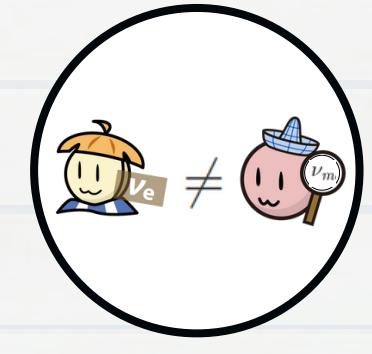
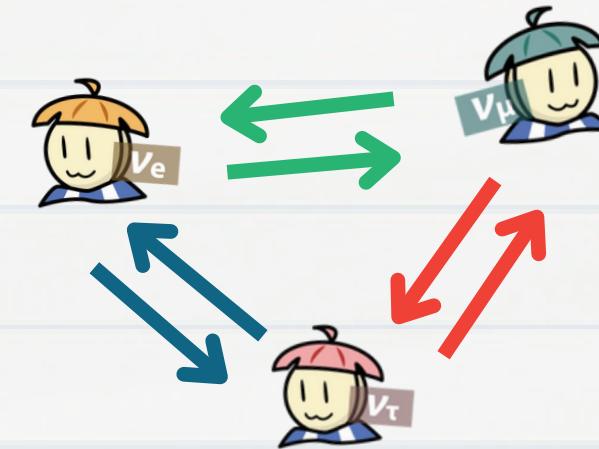
$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4 \sum_{i>j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin(X_{ij})^2$$



Mass



Mass difference



Difference between interacting state and mass state

$$X_{ij} \equiv \frac{m_i^2 - m_j^2}{4E} L$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}^D} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}^D} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \text{Diag}(1, e^{i\frac{\mu_1}{2}}, e^{i\frac{\mu_2}{2}})$$

Atmosphere

Reactor

Solar

Neutrinoless Double beta decay

**PMNS matrix**

**Mass term for neutrino in new model !!**



# Neutrino mass generation

$$\mathcal{L}_{M_\nu} = -M_D^{ij} \overline{\nu_i^R} \nu_j^L - \frac{1}{2} M_M^{ij} \overline{(\nu_i^L)^c} \nu_j^L + h.c.$$



$m_\nu \sim 1\text{eV}$

# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

Naturalness problem!!



$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L - \frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Dirac mass term      Majorana mass term



$$m_\nu \sim 1\text{eV}$$

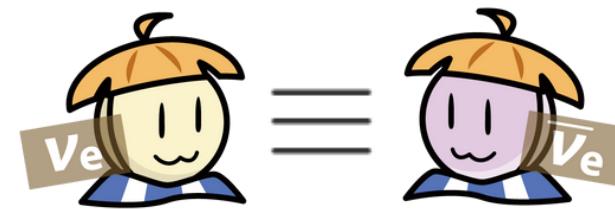
# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

Naturalness problem!!



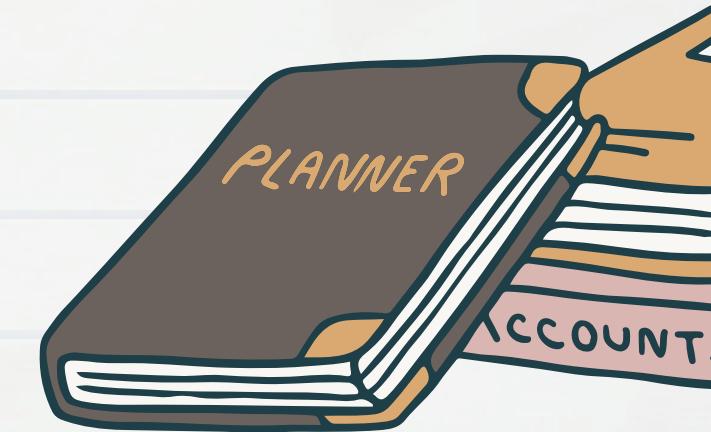
Majorana particle



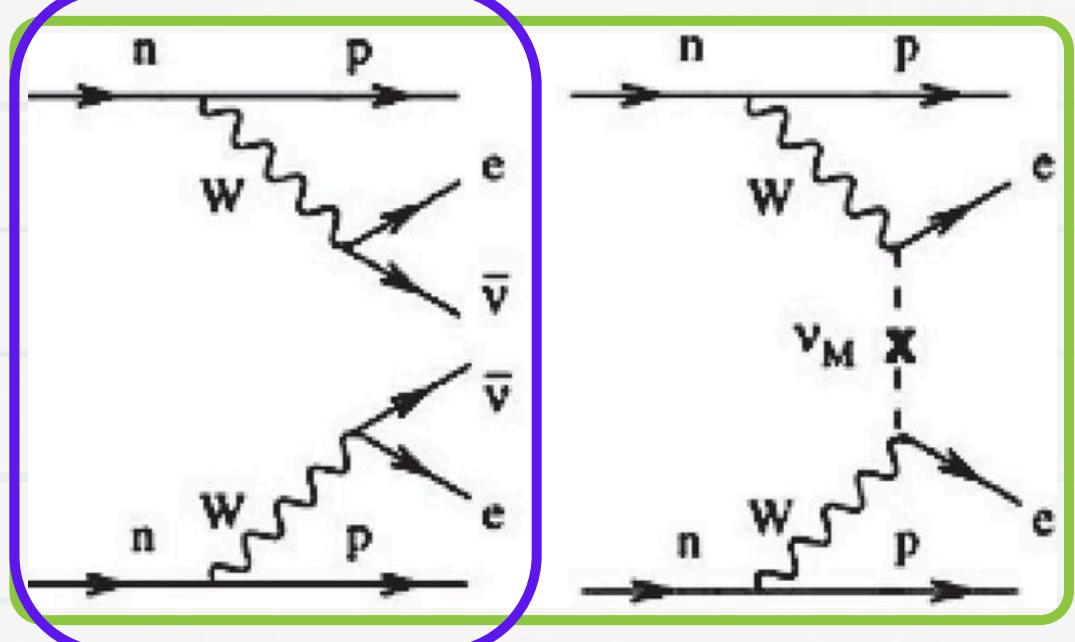
$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L - \frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Dirac mass term

Majorana mass term



Dirac particle



Neutrinoless double beta decay.

$m_\nu \sim 1\text{eV}$

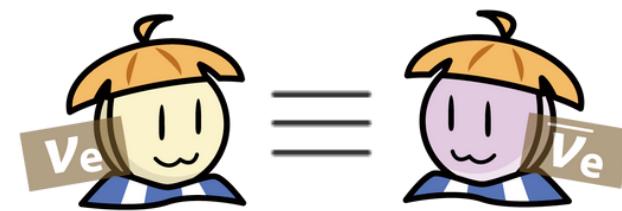
# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

Naturalness problem!!

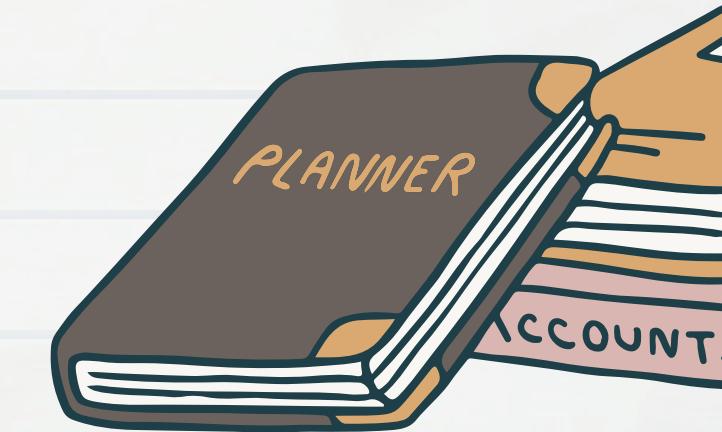


Majorana particle

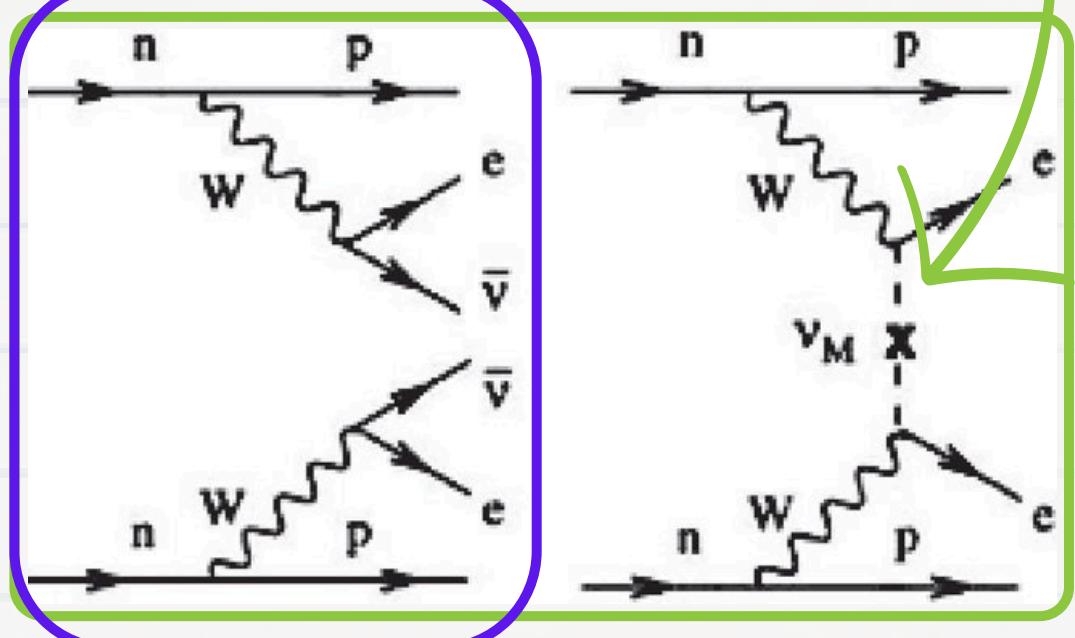


$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L - \frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Dirac mass term      Majorana mass term



Dirac particle



Neutrinoless double beta decay.

$m_\nu \sim 1\text{eV}$

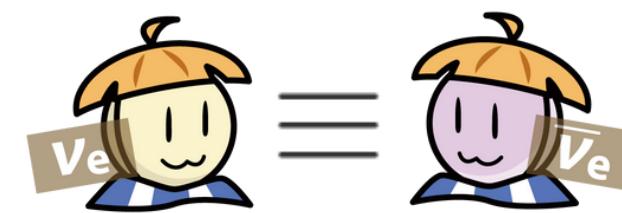
# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

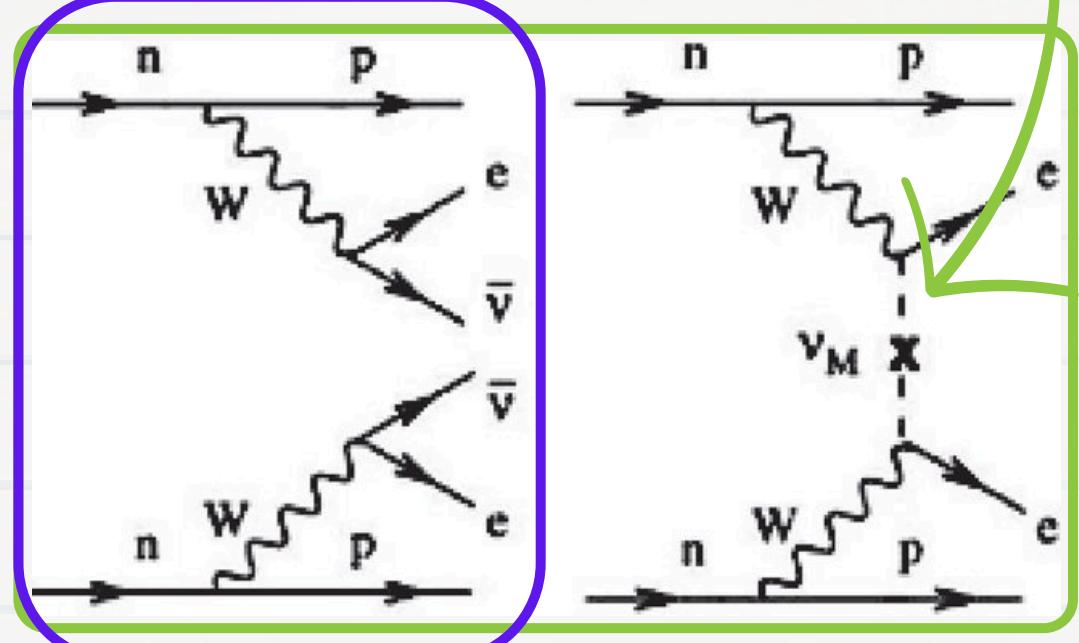
Naturalness problem!!



Majorana particle



Dirac particle



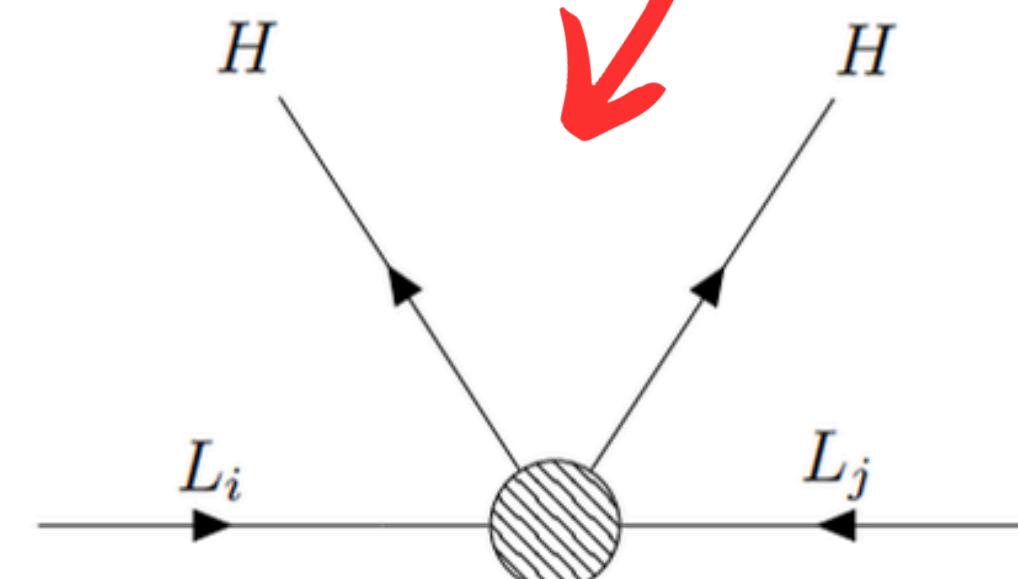
Neutrinoless double beta decay

$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L$$

Dirac mass term

$$\frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Majorana mass term



$$\mathcal{L}_5 = \frac{c_{ij}}{\Lambda} (\tilde{L}_i H) (\tilde{H}^\dagger L_j) \xrightarrow[\langle H \rangle = \frac{v}{\sqrt{2}}]{\text{After SSB}} \mathcal{L}_5 \supset \frac{c_{ij}}{\Lambda} \frac{v^2}{2} (\bar{\nu}_i^L)^c \nu_j^L,$$

Effective dim-5 Weinberg operator

$m_\nu \sim 1\text{eV}$

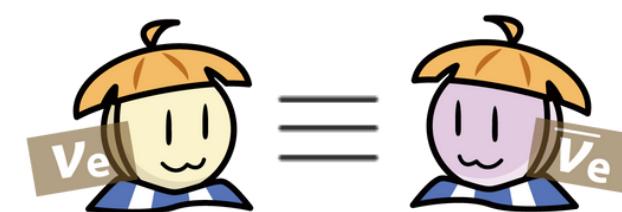
# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

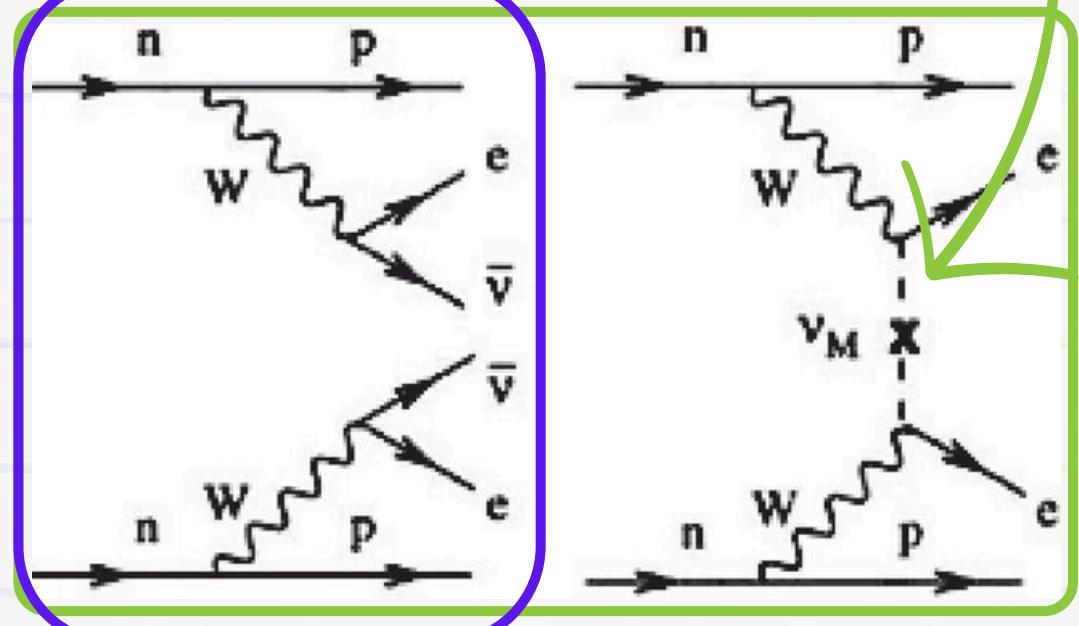
Naturalness problem!!



Majorana particle



Dirac particle



Neutrinoless double beta decay

$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L$$

Dirac mass term

$$\frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Majorana mass term



NP scale  $\Lambda \simeq \frac{\langle H \rangle^2}{m_\nu} \simeq 10^{13} \text{ GeV}$

$$\mathcal{L}_5 = \frac{c_{ij}}{\Lambda} (\bar{L}_i H) (\tilde{H}^\dagger L_j) \xrightarrow{\langle H \rangle = \frac{v}{\sqrt{2}}} \mathcal{L}_5 \supset \frac{c_{ij}}{\Lambda} \frac{v^2}{2} (\bar{\nu}_i^L)^c \nu_j^L,$$

Effective dim-5 Weinberg operator

$m_\nu \sim 1\text{eV}$

# Neutrino mass generation

$$Y^\nu \sim 10^{-11} \ll Y^{l,u,d}$$

Naturalness problem!!

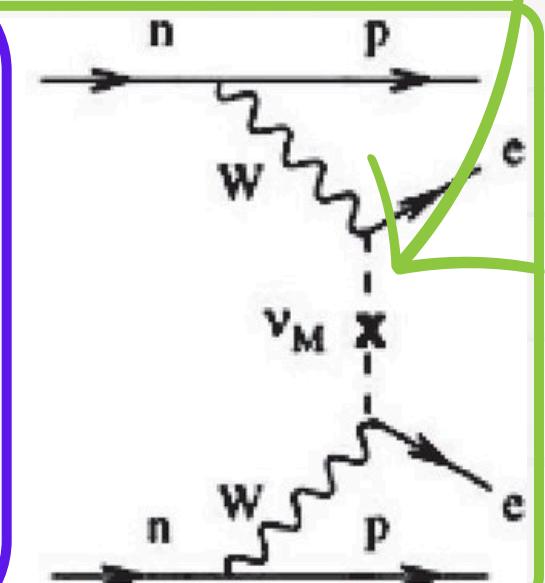
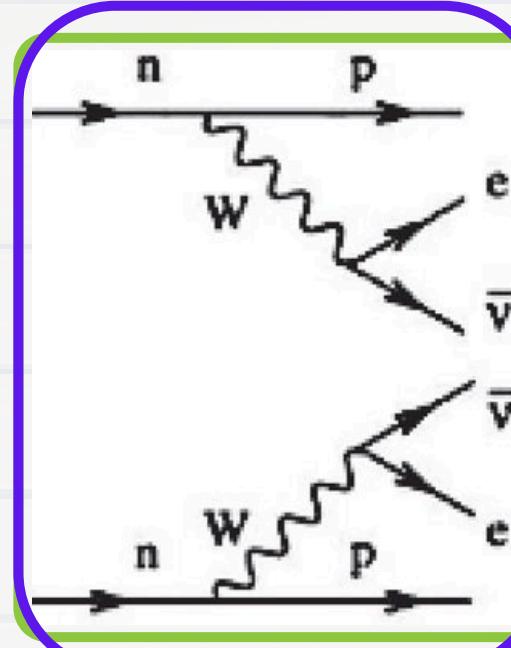


Majorana particle



New fields shoul be showed up

Dirac particle



Neutrinoless double beta decay

$$\mathcal{L}_{M_\nu} = -M_D^{ij} \bar{\nu}_i^R \nu_j^L$$

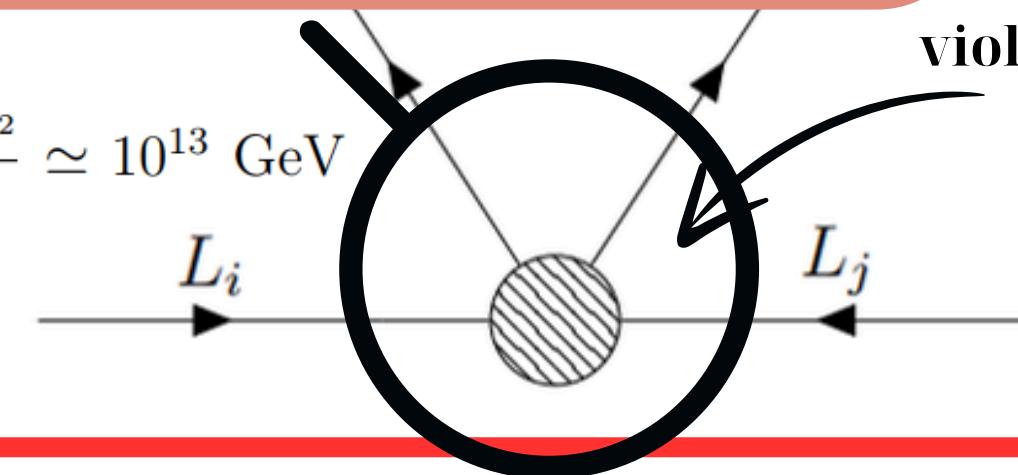
Dirac mass term

$$\frac{1}{2} M_M^{ij} (\bar{\nu}_i^L)^c \nu_j^L + h.c.$$

Majorana mass term



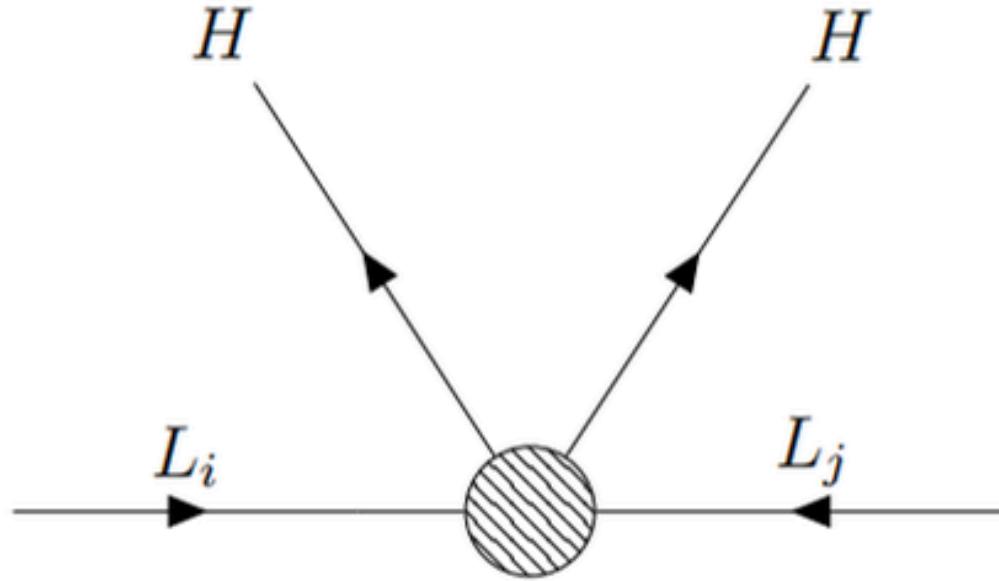
NP scale  $\Lambda \simeq \frac{\langle H \rangle^2}{m_\nu} \simeq 10^{13} \text{ GeV}$



$$\mathcal{L}_5 = \frac{c_{ij}}{\Lambda} (\bar{L}_i H) (\tilde{H}^\dagger L_j) \xrightarrow{\langle H \rangle = \frac{v}{\sqrt{2}}} \text{After SSB} \rightarrow \mathcal{L}_5 \supset \frac{c_{ij}}{\Lambda} \frac{v^2}{2} (\bar{\nu}_i^L)^c \nu_j^L,$$

Effective dim-5 Weinberg operator

# Effective dim-5 Weinberg operator



$$\mathcal{L}_5 = \frac{c_{ij}}{\Lambda} (\tilde{L}_i H) (\tilde{H}^\dagger L_j) \xrightarrow[\langle H \rangle = \frac{v}{\sqrt{2}}]{\text{After SSB}} \mathcal{L}_5 \supset \frac{c_{ij}}{\Lambda} \frac{v^2}{2} (\bar{\nu}_i^L)^c \nu_j^L,$$

Effective dim-5 Weinberg operator

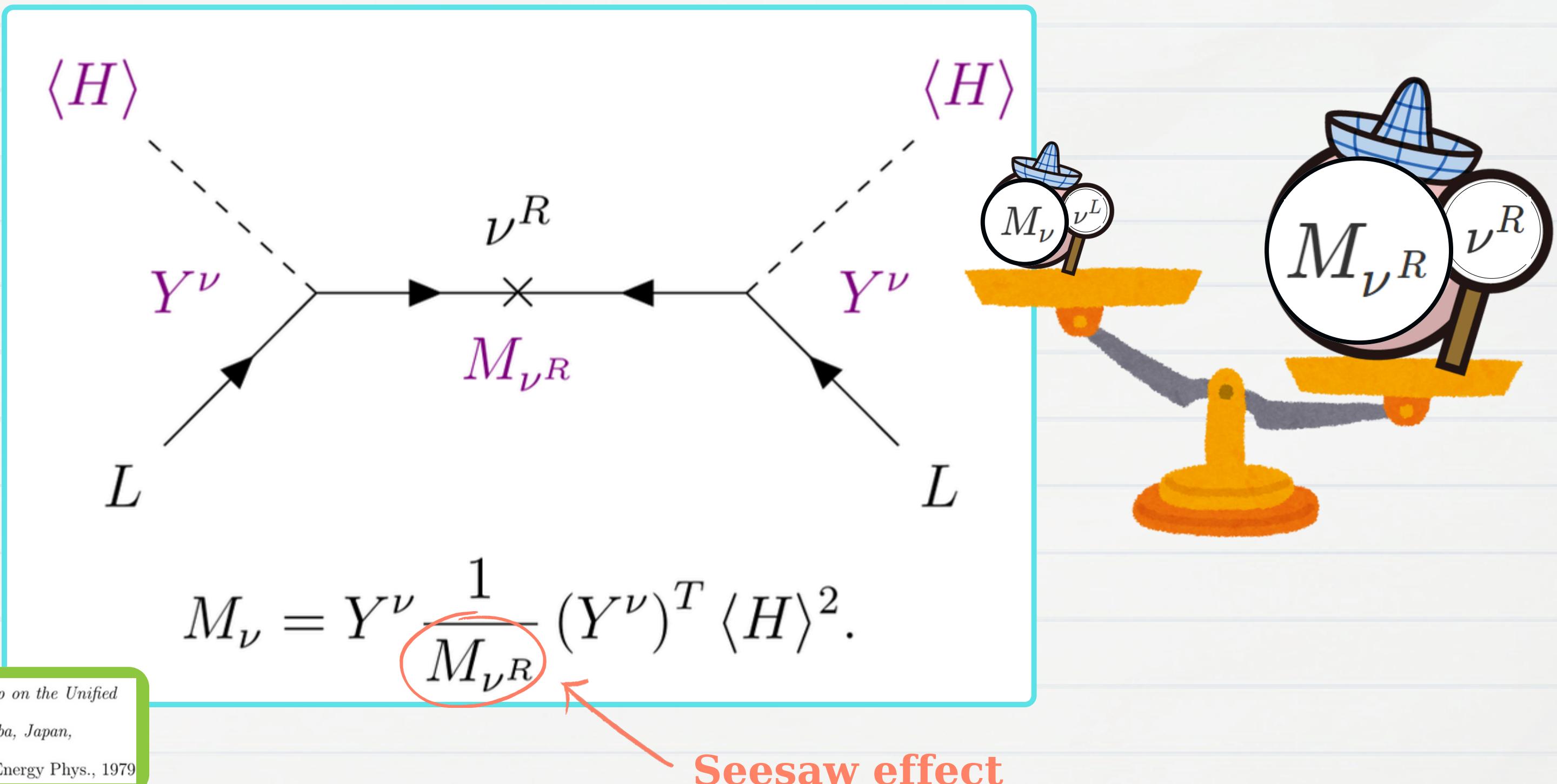
**SM gauge invariance  $SU(2) \times U(1)$**

At minimal extension, to form a singlet  $SU(2)$  creating the neutrino's mass from SM materials, e.g. doublet lepton  $L$  and scalar  $H$ , we get

$$2 \otimes 2 = 1 \oplus 3$$

# New fermion field with Hypercharge $Y = 0$

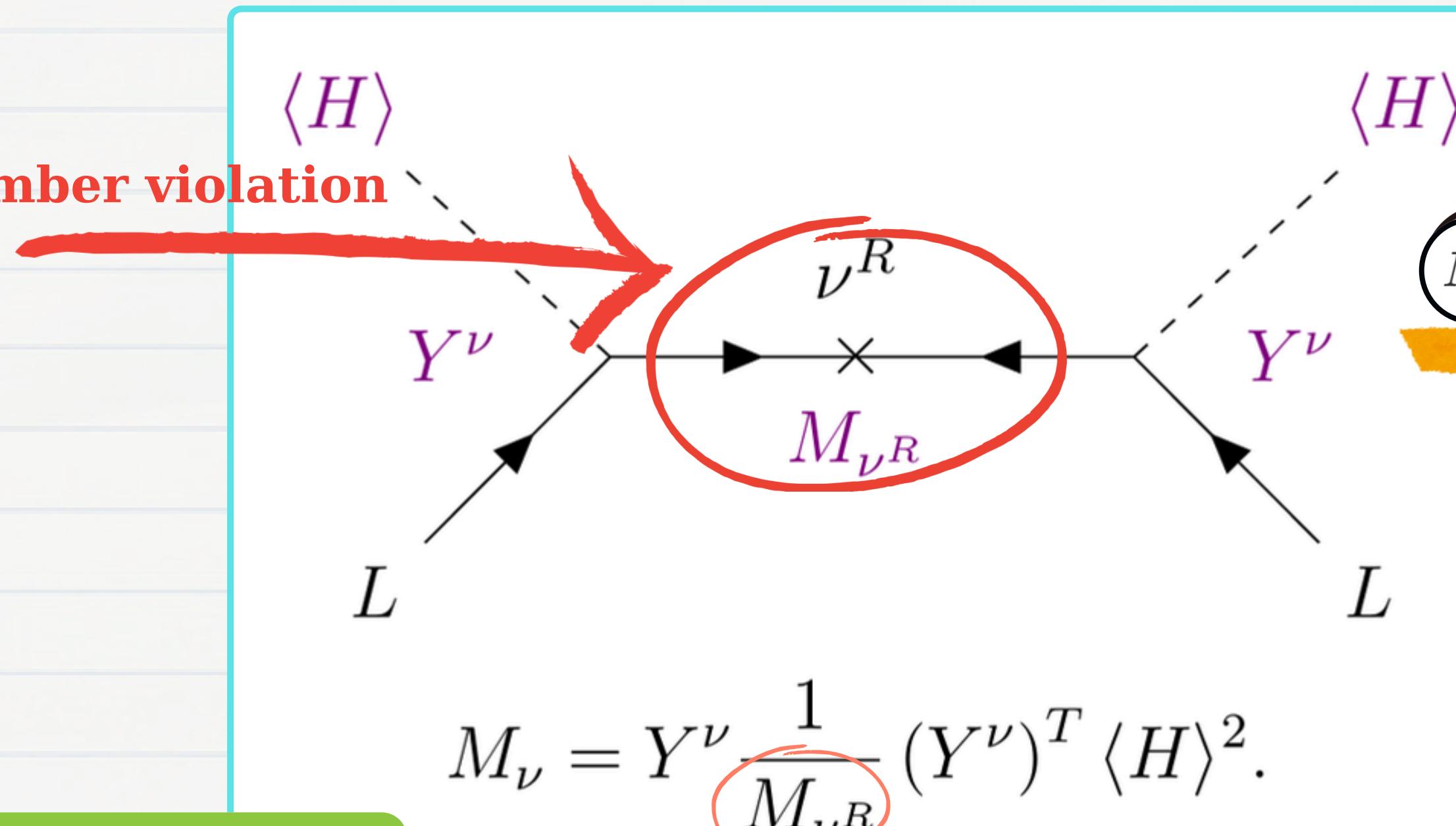
Singlet fermion  $\nu^R$ :  $Y_{ij}^\nu \bar{\nu}_i^R \tilde{H}^\dagger L_j$  (e.g. Type-I Seesaw sterile neutrino)



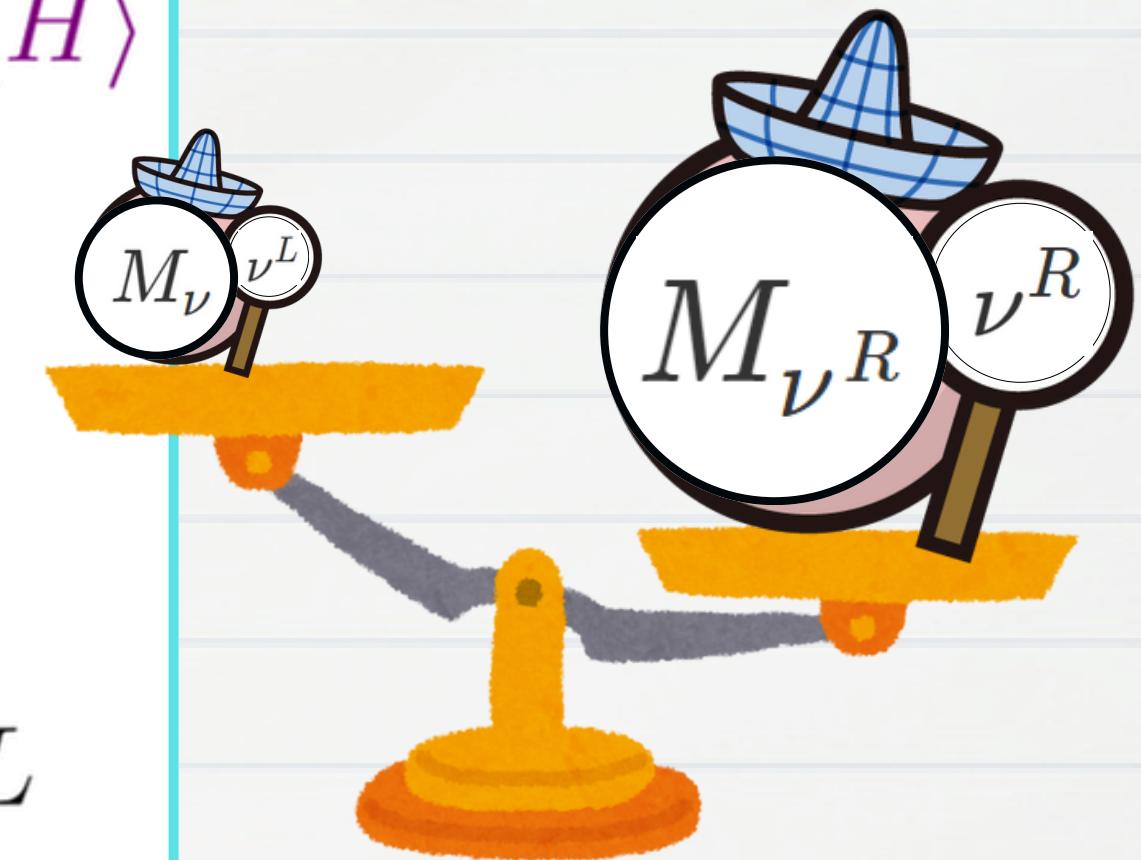
# New fermion field with Hypercharge $Y = 0$

Singlet fermion  $\nu^R$ :  $Y_{ij}^\nu \bar{\nu}_i^R \tilde{H}^\dagger L_j$  (e.g. Type-I Seesaw sterile neutrino)

**Lepton number violation**

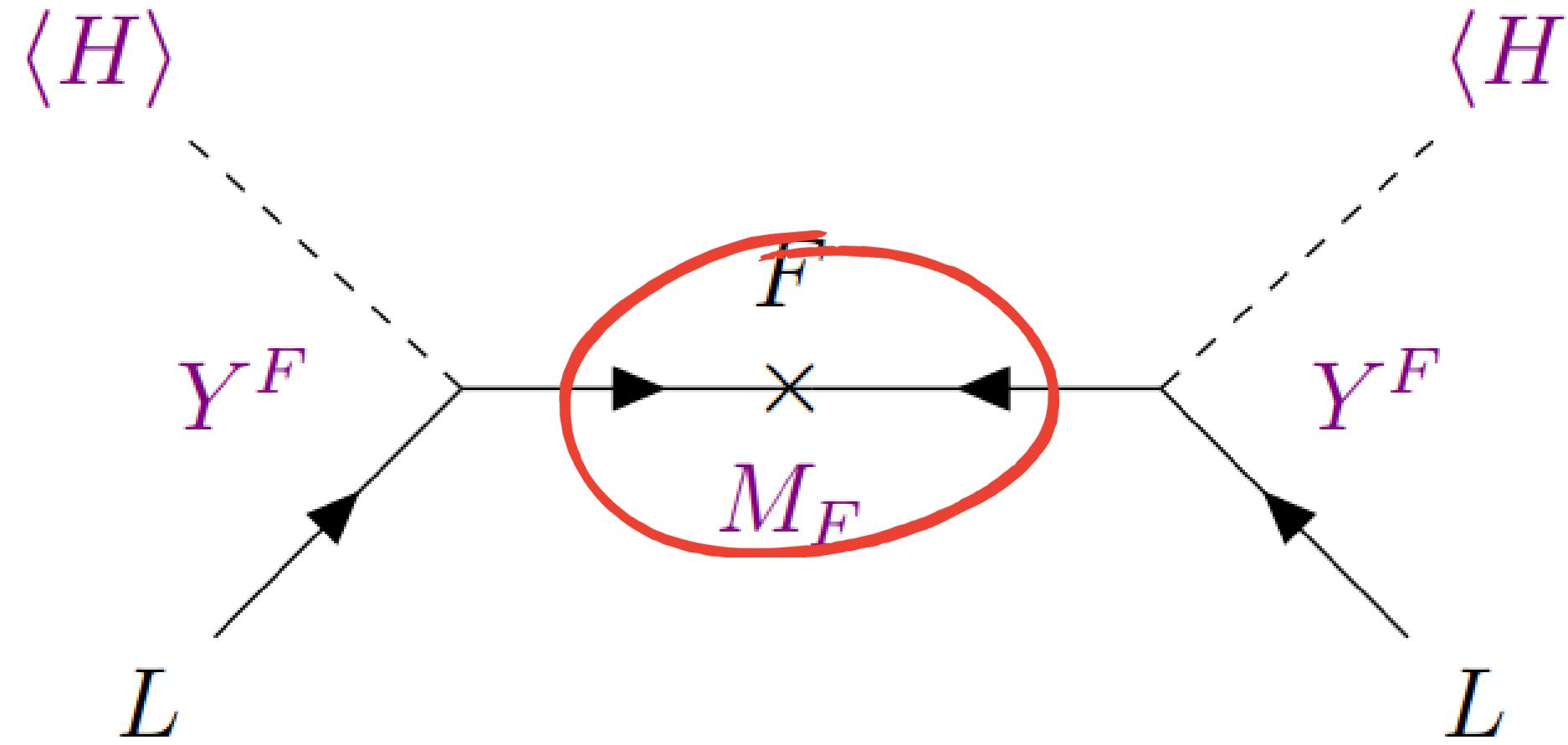


**Seesaw effect**

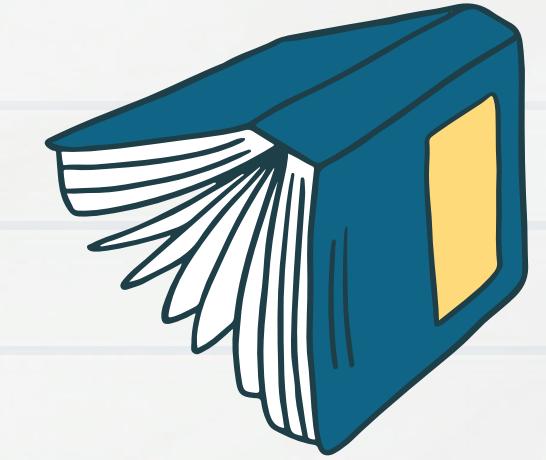


# New fermion field with Hypercharge $Y = 0$

Triplet fermion  $F$ :  $Y_{ij}^F \tilde{H}^\dagger \overline{F}_i L_j$  (e.g. Type-III Seesaw)



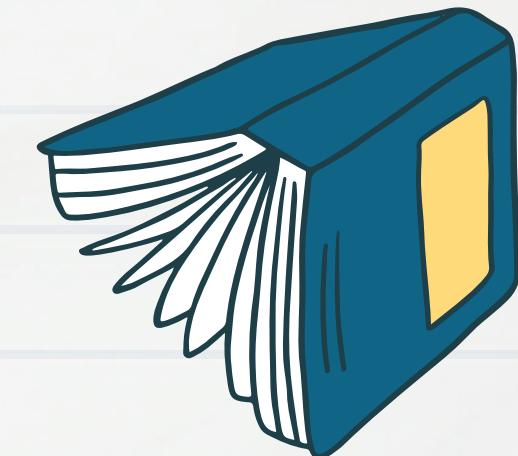
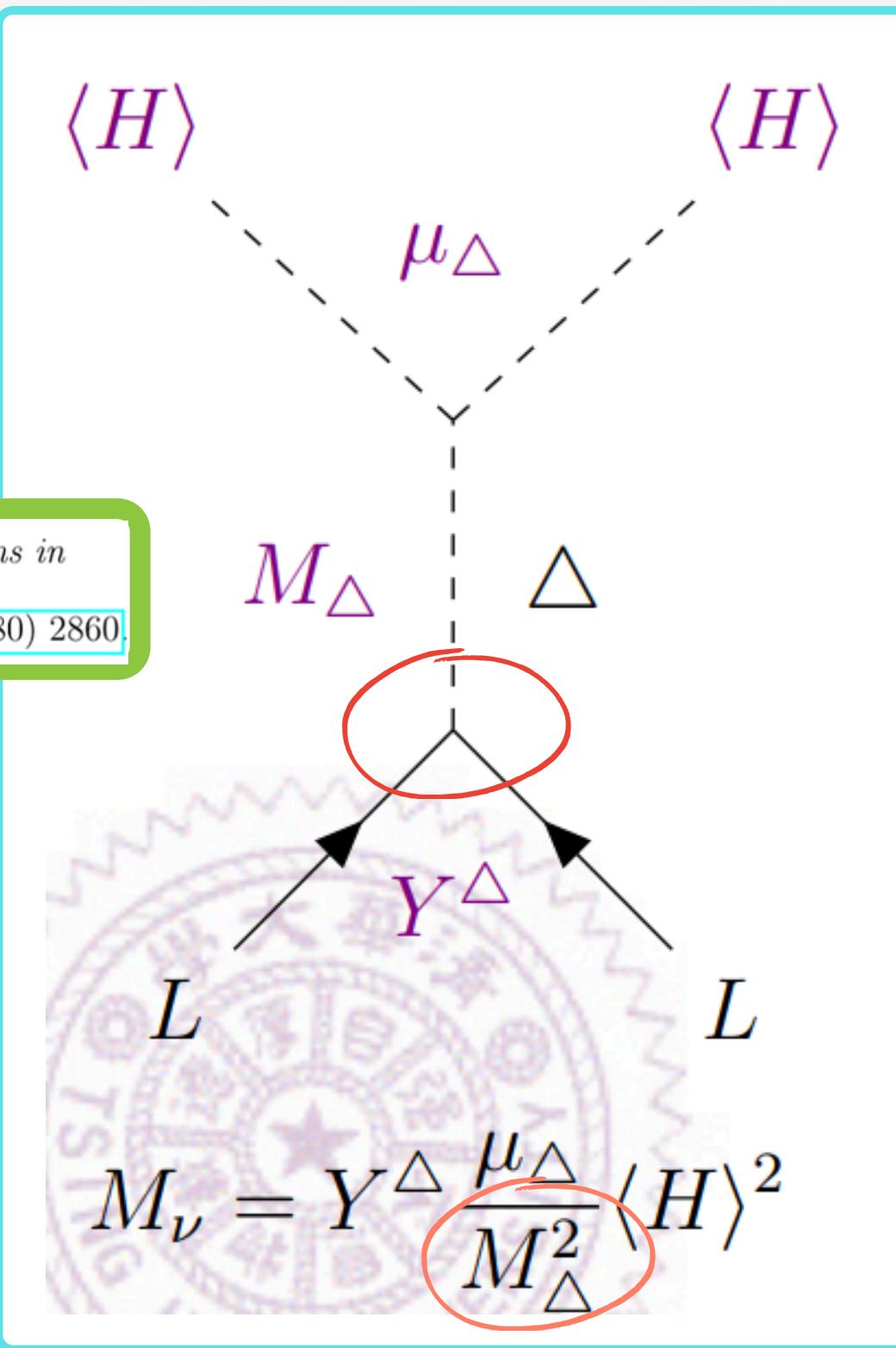
$$M_\nu = Y^F \frac{1}{M_F} (Y^F)^T \langle H \rangle^2$$



# New scalar field with Hypercharge $Y = 2$

Scalar triplet  $\Delta$ :  $Y_{ij}^{\Delta} \overline{L}_i^c i\sigma_2 \Delta L_j$  (e.g. Type-II Seesaw)

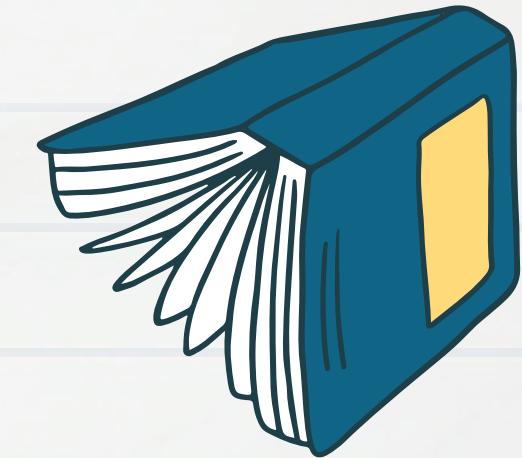
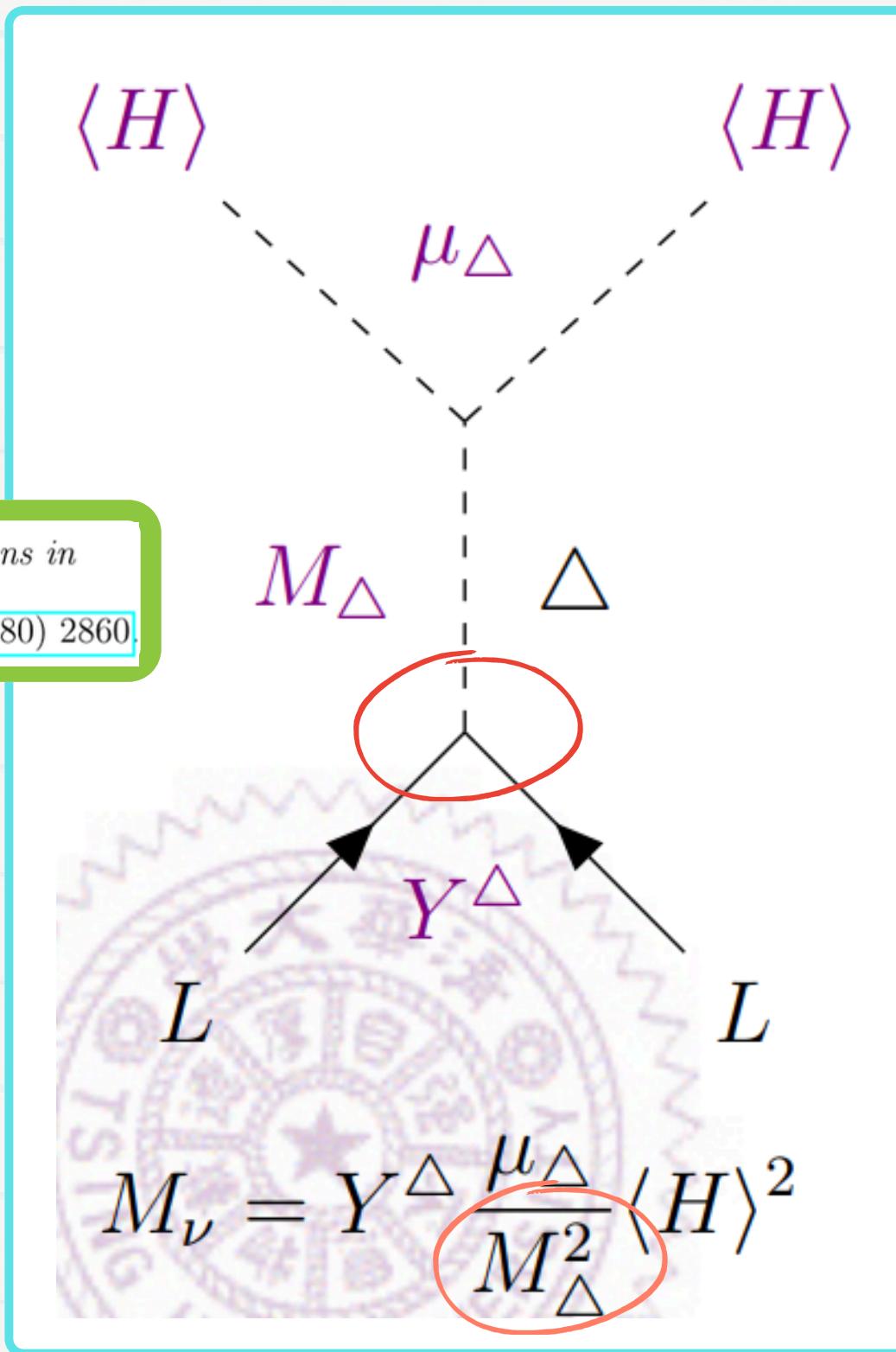
T. P. Cheng and L.-F. Li, *Neutrino masses, mixings, and oscillations in  $su(2)xu(1)$  models of electroweak interactions*, *Phys. Rev. D* **22** (1980) 2860.



# New scalar field with Hypercharge $Y = 2$

Scalar triplet  $\Delta$ :  $Y_{ij}^{\Delta} \overline{L}_i^c i\sigma_2 \Delta L_j$  (e.g. Type-II Seesaw)

T. P. Cheng and L.-F. Li, *Neutrino masses, mixings, and oscillations in  $su(2)xu(1)$  models of electroweak interactions*, *Phys. Rev. D* **22** (1980) 2860.



**Custodial symmetry**

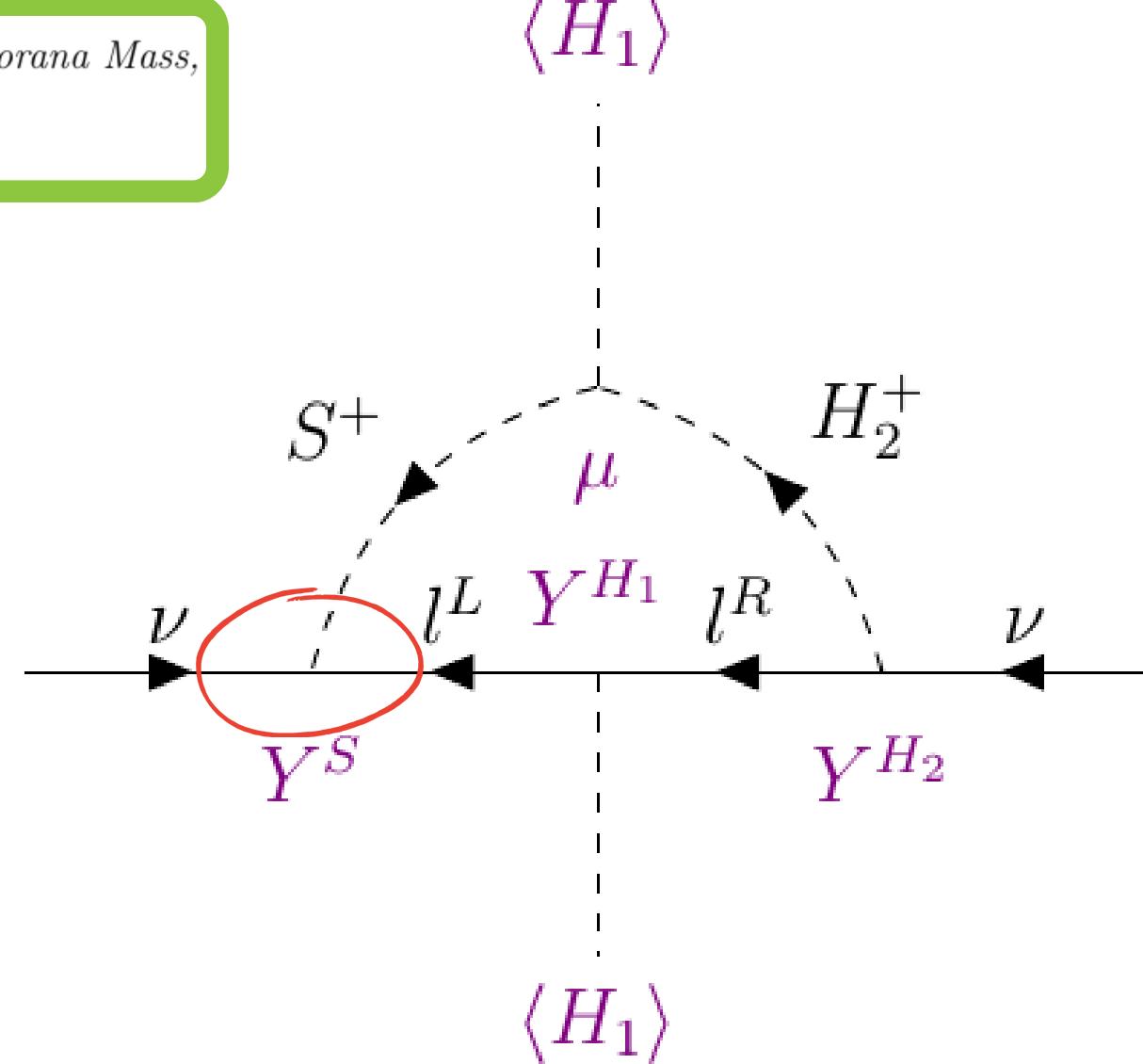
$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} \approx 1$$

$$4I_3(I_3 + 1) = 3Y^2$$

# New scalar field with Hypercharge $Y = 2$

Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

A. Zee, *A Theory of Lepton Number Violation, Neutrino Majorana Mass, and Oscillation*, *Phys. Lett. B* **93** (1980) 389.

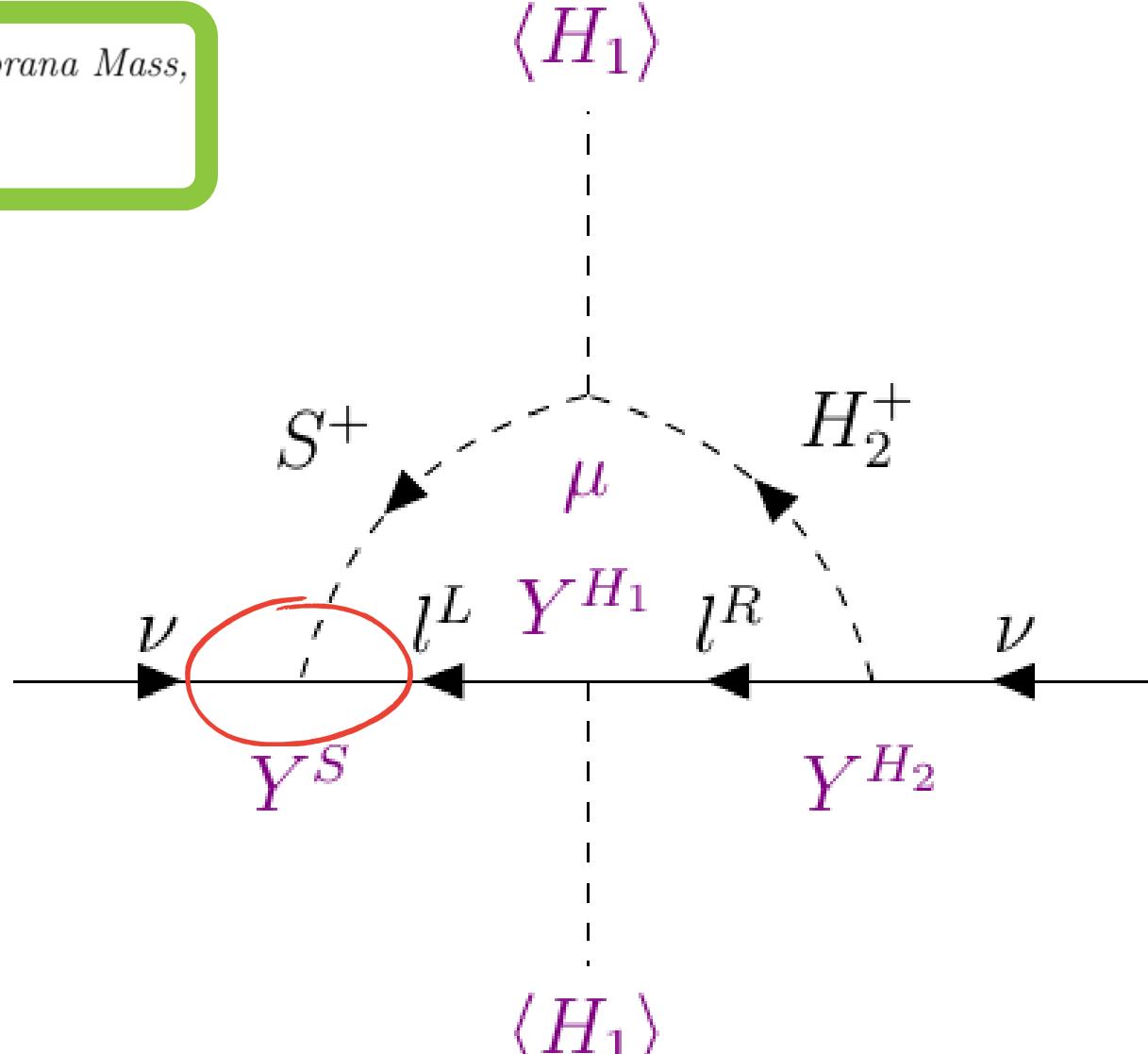


$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S (\langle H_1 \rangle Y^{H_1}) (Y^{H_2})^T + Y^{H_2} (\langle H_1 \rangle Y^{H_1}) (Y^S)^T \right]$$

# New scalar field with Hypercharge $Y = 2$

Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

A. Zee, *A Theory of Lepton Number Violation, Neutrino Majorana Mass, and Oscillation*, [Phys. Lett. B 93 \(1980\) 389](#).



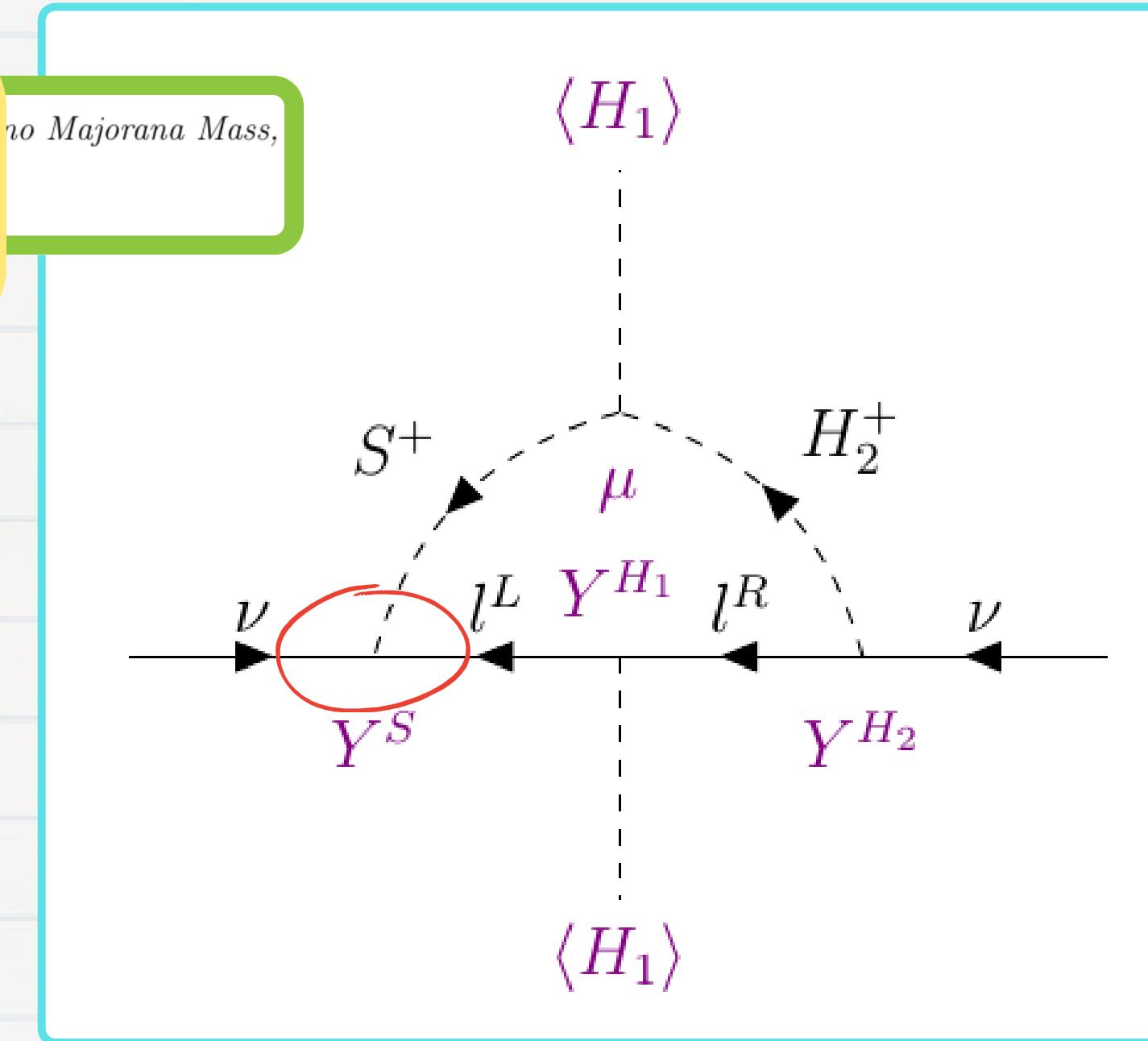
No need new high massive particles

$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S (\langle H_1 \rangle Y^{H_1}) (Y^{H_2})^T + Y^{H_2} (\langle H_1 \rangle Y^{H_1}) (Y^S)^T \right]$$

# New scalar field with Hypercharge $Y = 2$

Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

An acceptable solution  
of the neutrino mass  
hierarchy



No need new high massive particles

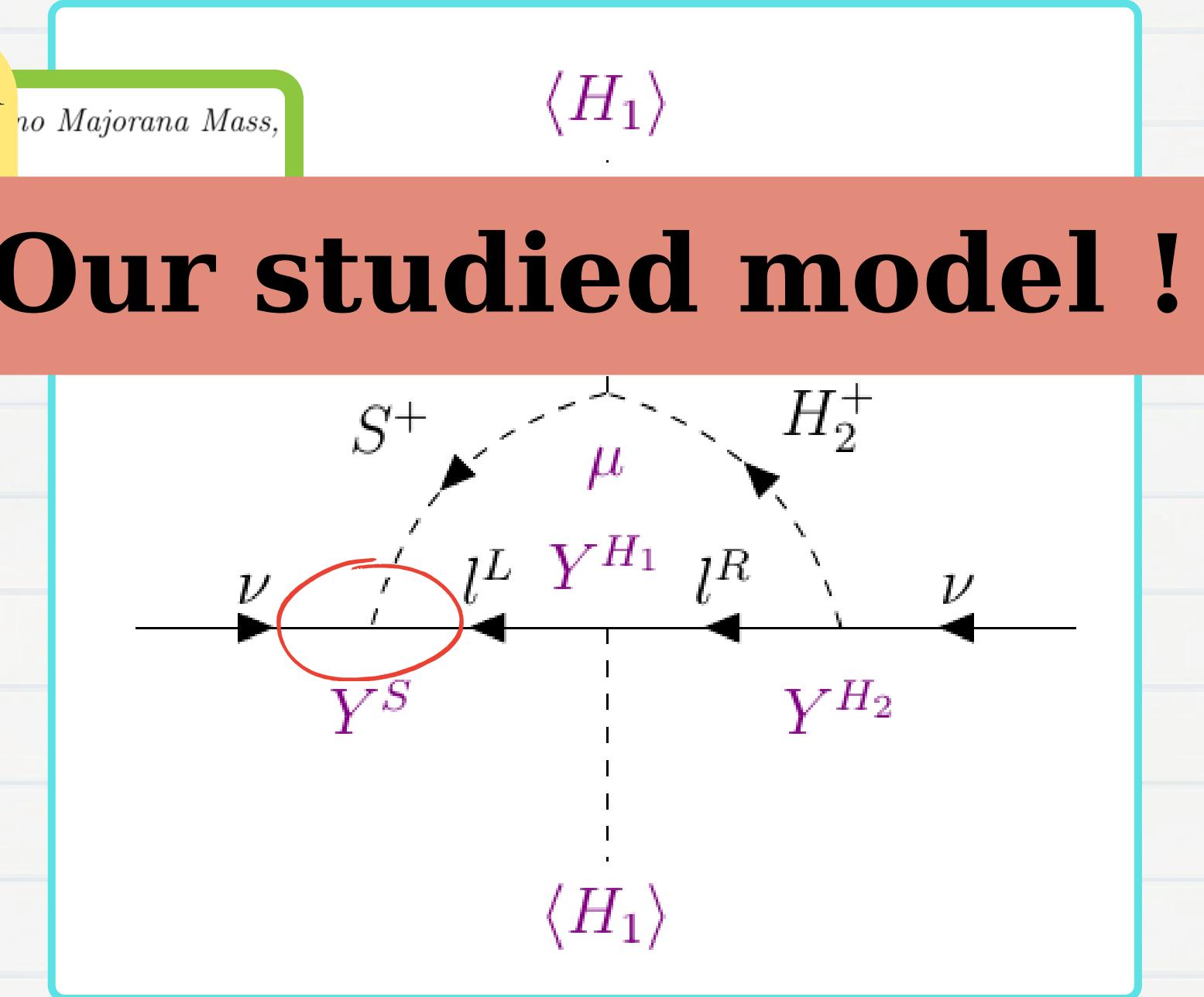
$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S \left( \langle H_1 \rangle Y^{H_1} \right) (Y^{H_2})^T + Y^{H_2} \left( \langle H_1 \rangle Y^{H_1} \right) (Y^S)^T \right]$$

# New scalar field with Hypercharge $Y = 2$

Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

An acceptable solution  
of the neutrino mass  
hierarchy

Our studied model !!



No need new high massive  
particles



$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S \left( \langle H_1 \rangle Y^{H_1} \right) (Y^{H_2})^T + Y^{H_2} \left( \langle H_1 \rangle Y^{H_1} \right) (Y^S)^T \right]$$

# New scalar field with Hypercharge $Y = 2$

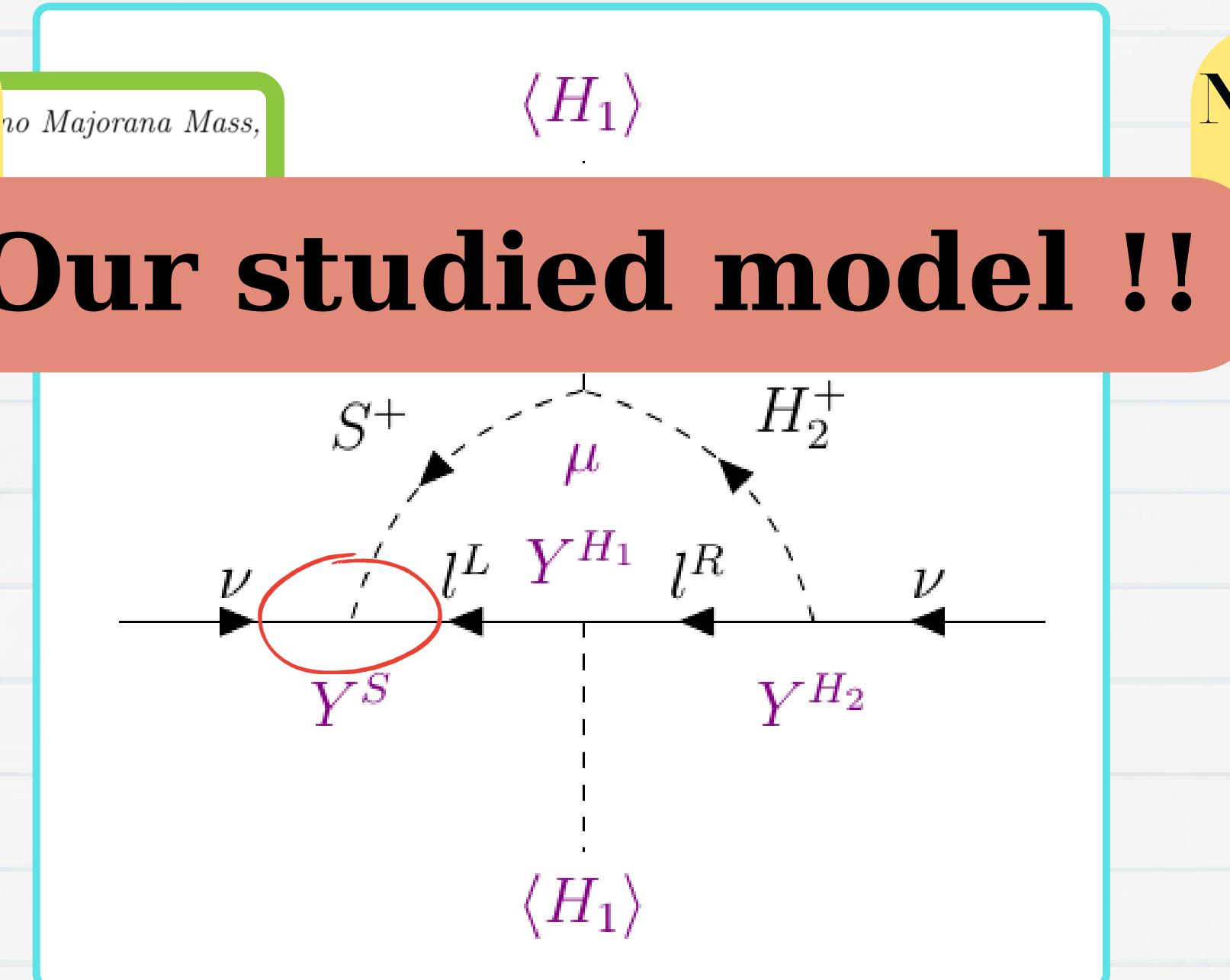
Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

An acceptable solution  
of the neutrino mass  
hierarchy

Our studied model !!

Based on  
alignment/decoupling  
limit

$H_1^0 \times H_2^0$



$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S \left( \langle H_1 \rangle Y^{H_1} \right) (Y^{H_2})^T + Y^{H_2} \left( \langle H_1 \rangle Y^{H_1} \right) (Y^S)^T \right]$$

No need new high massive  
particles

# New scalar field with Hypercharge $Y = 2$

Scalar singlet  $S^+$ :  $Y_{ij}^S \overline{L}_i^c i\sigma_2 L_j S^+$  (e.g. Zee-singlet scalar)

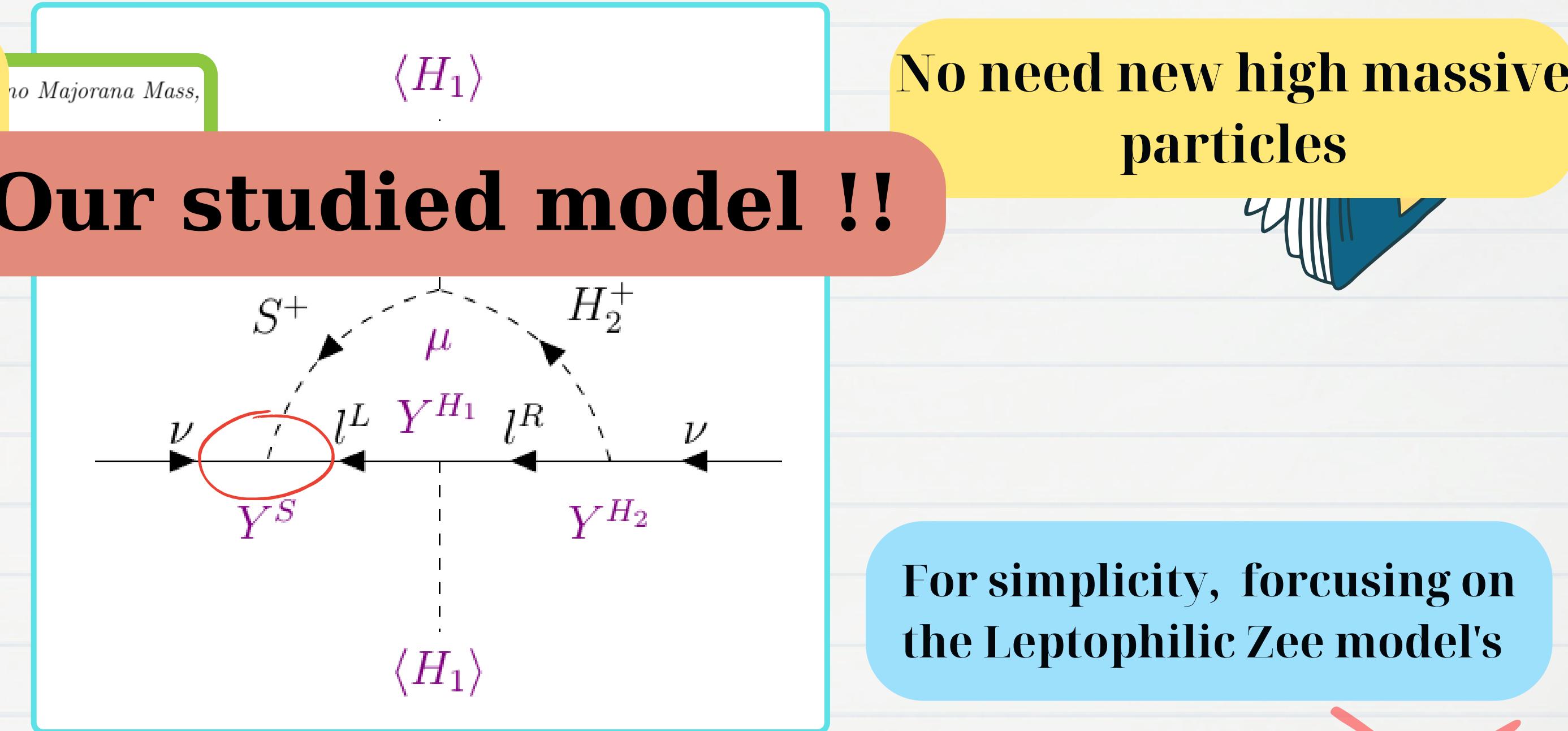
An acceptable solution  
of the neutrino mass  
hierarchy

Our studied model !!

Based on  
alignment/decoupling  
limit

$H_1^0 \times H_2^0$

$$M_\nu = \frac{1}{8\pi^2} \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S \langle \langle H_1 \rangle Y^{H_1} \rangle (Y^{H_2})^T + Y^{H_2} \langle \langle H_1 \rangle Y^{H_1} \rangle (Y^S)^T \right]$$



No need new high massive  
particles



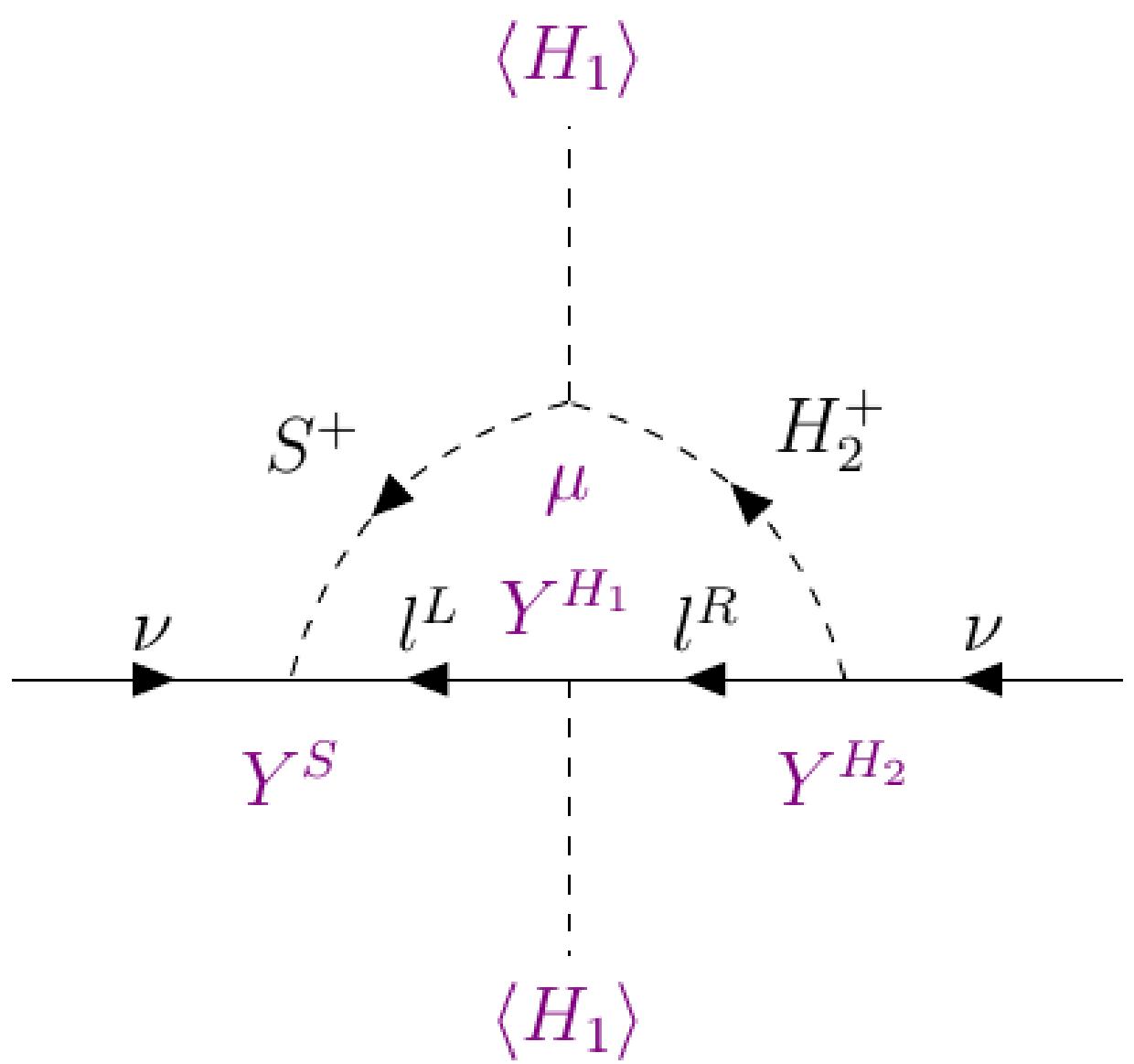
For simplicity, focusing on  
the Leptophilic Zee model's

$H_2 l \bar{l}$   $H_2 q \bar{q}$

# Leptophobic Zee model



# Zee model

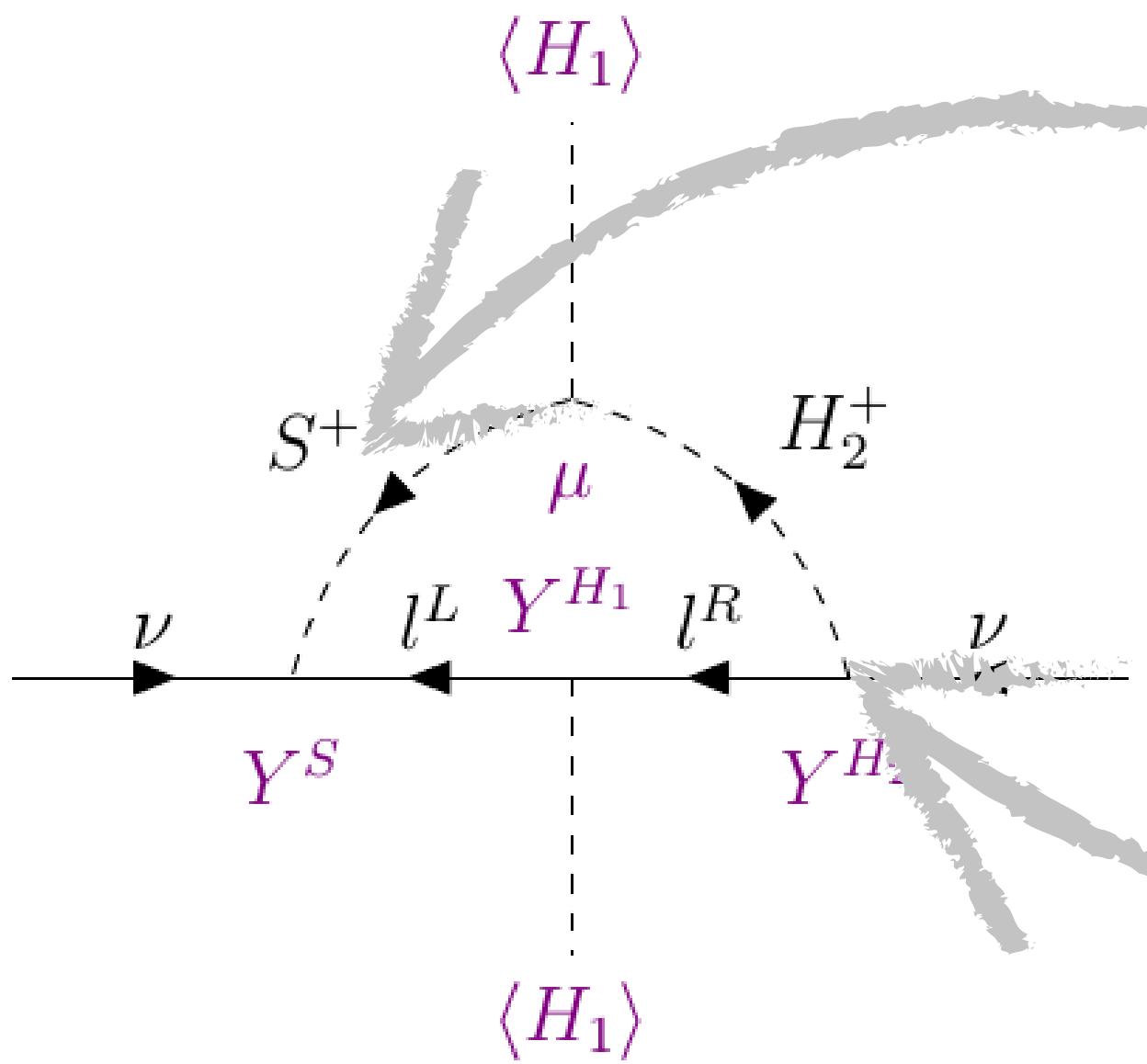


$$\mathcal{L}_{Yuk} \supset -Y_{\alpha\beta}^S \overline{L}^c_\alpha i\sigma_2 L_\beta S^+ - Y_{\alpha\beta}^{H_2} \overline{L}_\alpha H_2 l_\beta^R - Y_{\alpha\beta}^{H_1} \overline{L}_\alpha H_1 l_\beta^R + \text{h.c.}$$



$$M_\nu = \frac{s_\beta c_\beta}{8\pi^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S M_l (Y^{H_2})^\text{T} + Y^{H_2} M_l (Y^S)^\text{T} \right]$$

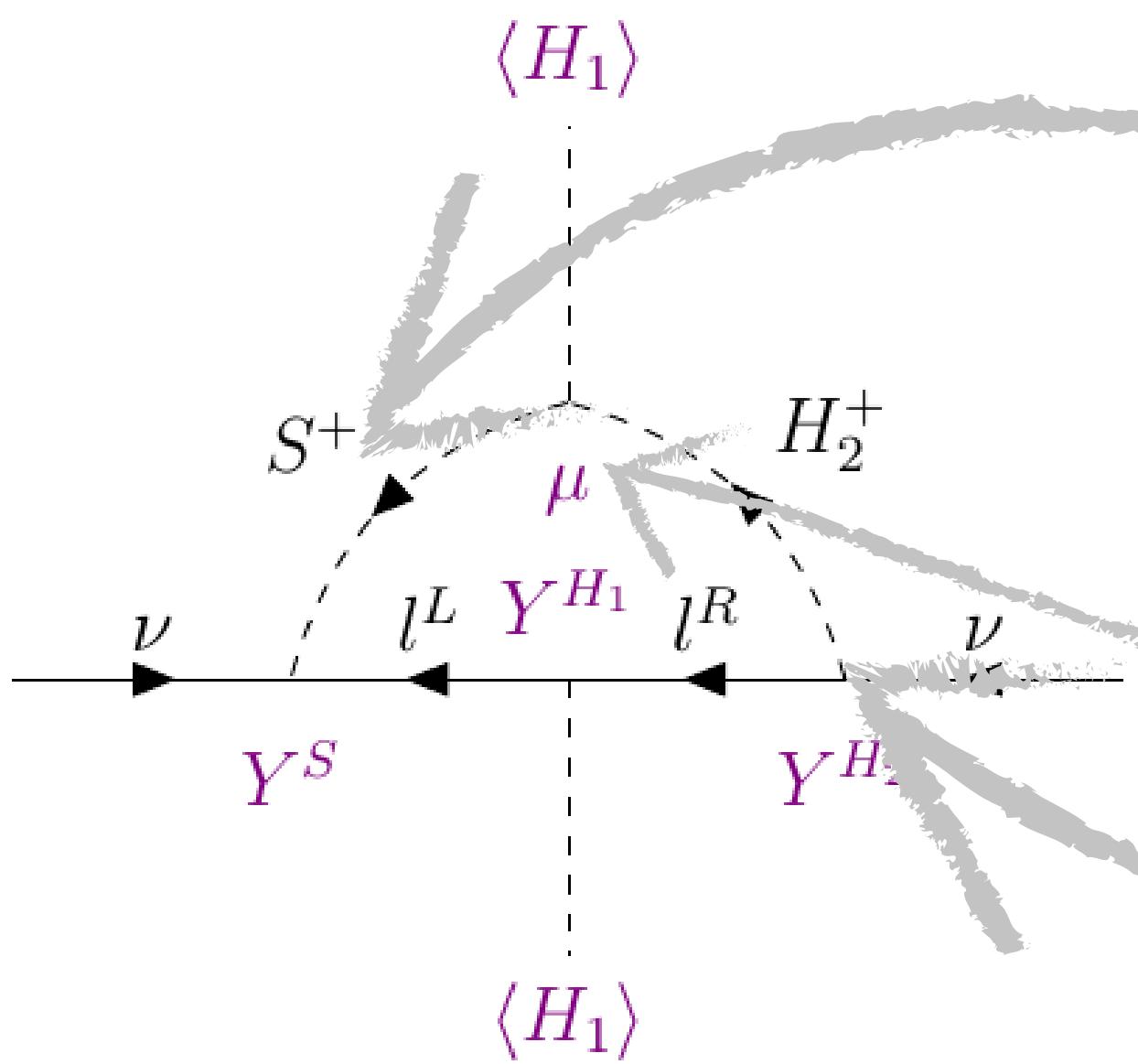
# Zee model



$$\mathcal{L}_{Yuk} \supset -Y_{\alpha\beta}^S \overline{L}{}^\alpha_\alpha i\sigma_2 L_\beta S^+ - Y_{\alpha\beta}^{H_2} \overline{L}{}^\alpha_\alpha H_2 l_\beta^R - Y_{\alpha\beta}^{H_1} \overline{L}{}^\alpha_\alpha H_1 l_\beta^R + \text{h.c.}$$

$$M_\nu = \frac{s_\beta c_\beta}{8\pi^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S M_l (Y^{H_2})^T + Y^{H_2} M_l (Y^S)^T \right]$$

# Zee model



$$M_\nu = \frac{s_\beta c_\beta}{8\pi^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) [Y^S M_l (Y^{H_2})^T - Y^{H_2} M_l (Y^S)^T]$$

$$\mathcal{L}_{Yuk} \supset -Y_{\alpha\beta}^S \overline{L^c}_\alpha i\sigma_2 L_\beta S^+ - Y_{\alpha\beta}^{H_2} \overline{L}_\alpha H_2 l_\beta^R - Y_{\alpha\beta}^{H_1} \overline{L}_\alpha H_1 l_\beta^R + \text{h.c.}$$

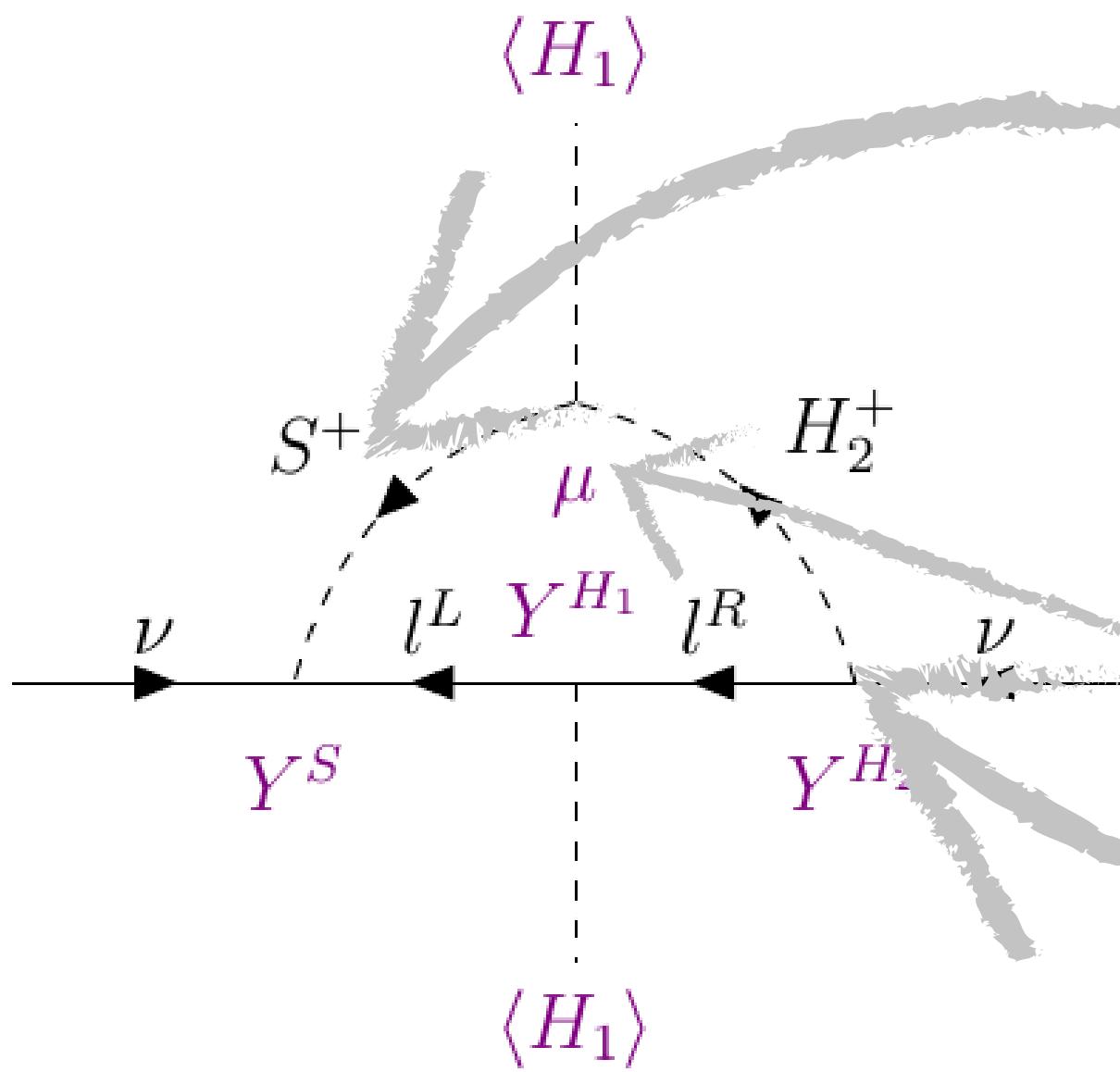
$$V(H_1, H_2, S^+) \supset \mu H_1^\text{T} i\sigma_2 H_2 S^-$$

The mixing angle  $c_\beta s_\beta = \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2}$

$$\begin{pmatrix} \eta^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} S^+ \\ H_2^+ \end{pmatrix}$$



# Zee model



$$\mathcal{L}_{Yuk} \supset -Y_{\alpha\beta}^S \overline{L^c}_\alpha i\sigma_2 L_\beta S^+ - Y_{\alpha\beta}^{H_2} \overline{L}_\alpha H_2 l_\beta^R - Y_{\alpha\beta}^{H_1} \overline{L}_\alpha H_1 l_\beta^R + \text{h.c.}$$

$$V(H_1, H_2, S^+) \supset \mu H_1^\text{T} i\sigma_2 H_2 S^-$$

$$\text{The mixing angle } c_\beta s_\beta = \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2}$$

$$\begin{pmatrix} \eta^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} S^+ \\ H_2^+ \end{pmatrix}$$

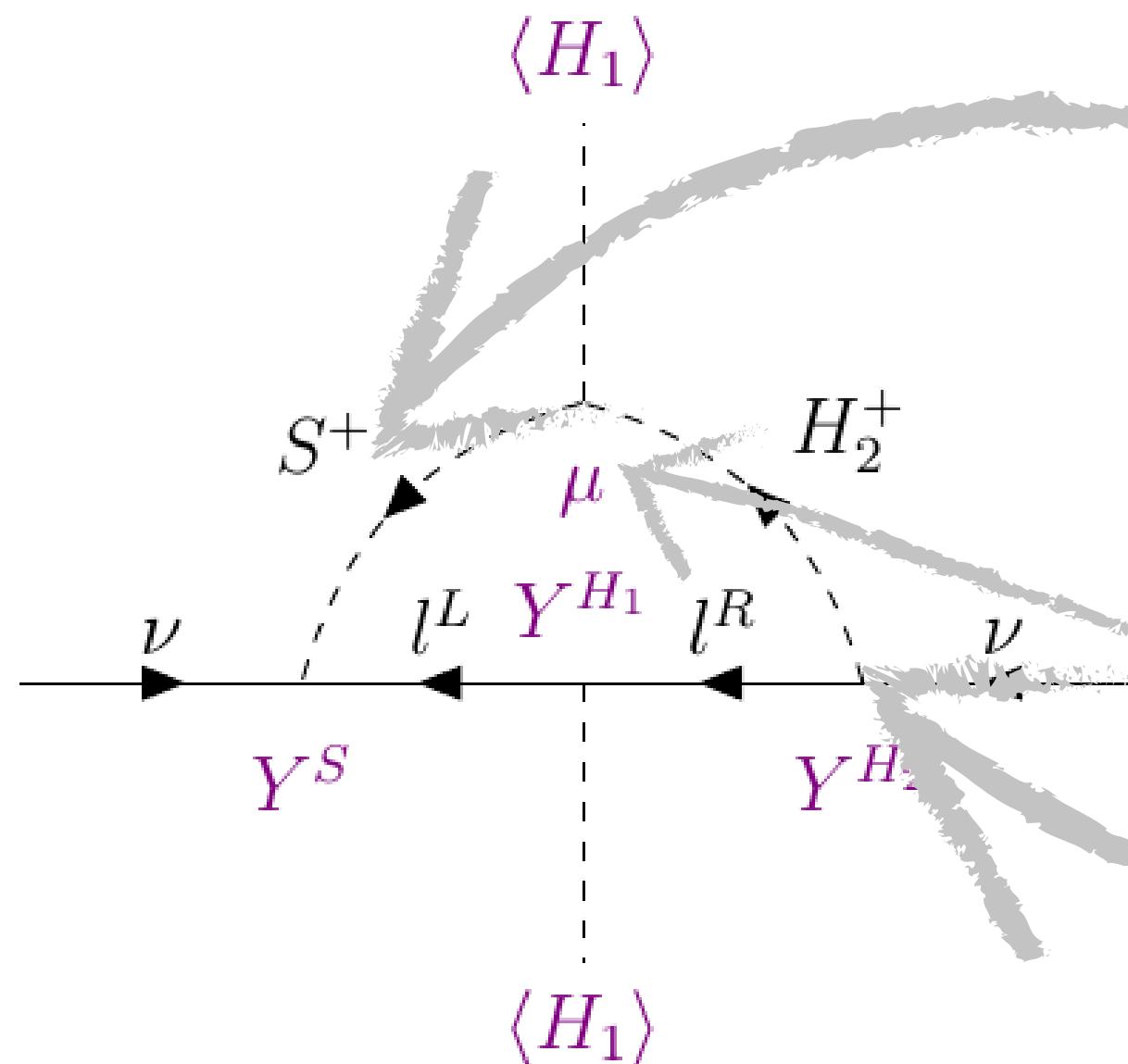
$$M_\nu = \frac{s_\beta c_\beta}{8\pi^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) \left[ Y^S M_l (Y^{H_2})^\text{T} + Y^{H_2} M_l (Y^S)^\text{T} \right]$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_0 \end{pmatrix}, \quad H_1 \begin{pmatrix} 0 \\ \frac{H_1^0 (\equiv H_{SM})}{\sqrt{2}} \end{pmatrix}$$

Higgs basis

Alignment/decoupling limit

# Zee model



$$\mathcal{L}_{Yuk} \supset -Y_{\alpha\beta}^S \bar{L}_\alpha^c i\sigma_2 L_\beta S^+ - Y_{\alpha\beta}^{H_2} \bar{L}_\alpha H_2 l_\beta^R - Y_{\alpha\beta}^{H_1} \bar{L}_\alpha H_1 l_\beta^R + \text{h.c.}$$

$$V(H_1, H_2, S^+) \supset \mu H_1^T i\sigma_2 H_2 S^-$$

$$\text{The mixing angle } c_\beta s_\beta = \frac{\mu \langle H_1 \rangle}{m_{\eta^+}^2 - m_{H^+}^2}$$

$$\begin{pmatrix} \eta^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} S^+ \\ H_2^+ \end{pmatrix}$$

$$M_\nu = \frac{s_\beta c_\beta}{8\pi^2} \log \left( \frac{m_{\eta^+}^2}{m_{H^+}^2} \right) [Y^S M_l (Y^{H_2})^T - Y^{H_2} M_l (Y^S)^T]$$

$\sim 1 \text{ eV}$

Pinpointed term:  $Y^S M_l (Y^{H_2})^T$

$$|Y^{S,H_2}| \sim \frac{\mathcal{O}(10^{-2}-10^{-3})}{\sqrt{|c_\beta s_\beta|}}$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_0 \end{pmatrix}, \quad H_1 \begin{pmatrix} 0 \\ \frac{H_1 (\equiv H_{SM})}{\sqrt{2}} \end{pmatrix}$$

Higgs basis

Alignment/decoupling limit

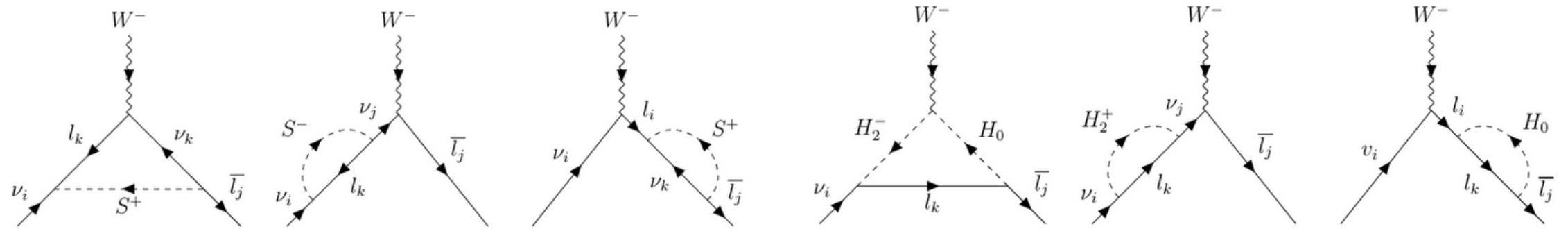
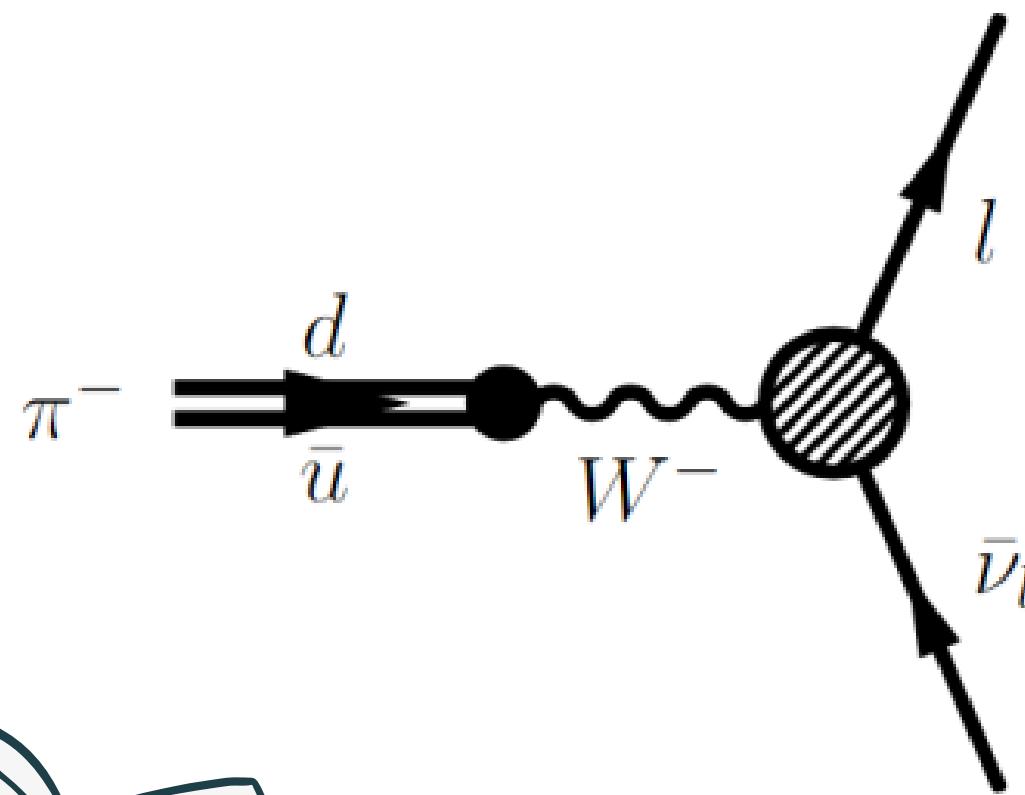


# **Phenomenological constraints**

\*All data taken from PDG

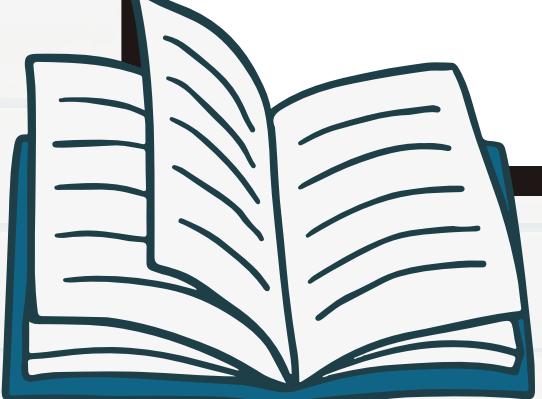
# Universality test in Pion decay

## Pion decay

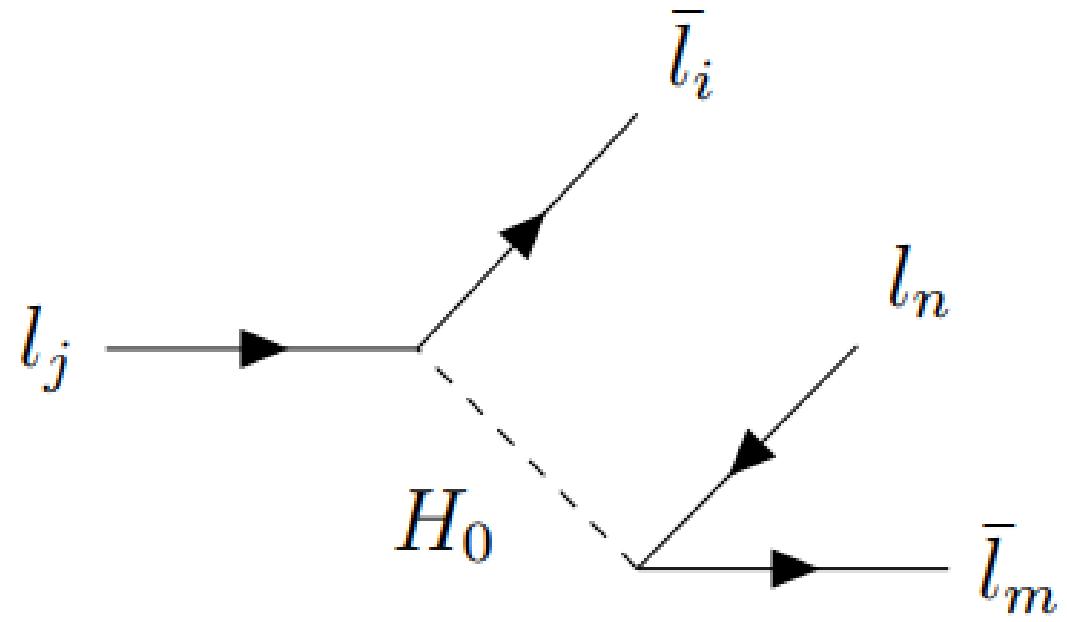


$$\left\{ \begin{array}{l} \left| |Y_{e\tau}^S|^2 - |Y_{\mu\tau}^S|^2 \right| < \mathcal{O}(10^2 - 10^5) \quad m_S \in (100 - 1000) \text{ and } m_{H_2^+} \approx m_{H_0} \\ \left| |Y_{ek}^{H_2}|^2 - |Y_{\mu k}^{H_2}|^2 \right| < \mathcal{O}(10^{-3}) \quad m_{H_2^+} \neq m_{H_0} \end{array} \right.$$

Weak constraint !



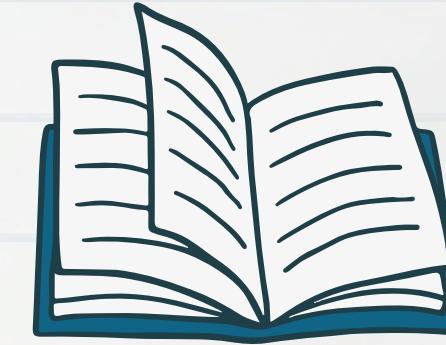
# Trilepton decays L->3l



$$Br(l_i \rightarrow l_j \bar{l}_m l_n) = \frac{Y}{n! 128 G_F^{i2}} Br(l_i \rightarrow l_j \bar{\nu}_m \nu_n)$$

$$Y = \frac{1}{16 m_{H_0}^4} \left( |2Y_{ji}^{H_2} Y_{nm}^{H_2*} - Y_{mi}^{H_2} Y_{nj}^{H_2*}|^2 + |2Y_{ij}^{H_2} Y_{mn}^{H_2*} - Y_{im}^{H_2} Y_{jn}^{H_2*}|^2 + 4 |Y_{ji}^{H_2} Y_{mn}^{H_2}|^2 + 4 |Y_{ij}^{H_2} Y_{nm}^{H_2}|^2 + 3 |Y_{mi}^{H_2} Y_{nj}^{H_2*}|^2 + 3 |Y_{im}^{H_2} Y_{jn}^{H_2*}|^2 + 4 |Y_{mi}^{H_2} Y_{jn}^{H_2}|^2 + 4 |Y_{im}^{H_2} Y_{nj}^{H_2}|^2 \right)$$

# Trilepton decays L->3l



$$\frac{|Y^{H_2}|^2}{m_{H_0}^2} < \mathcal{O}(10^{-9} - 10^{-8}) \text{ GeV}^{-2}$$

$H_0$

$\bar{l}$

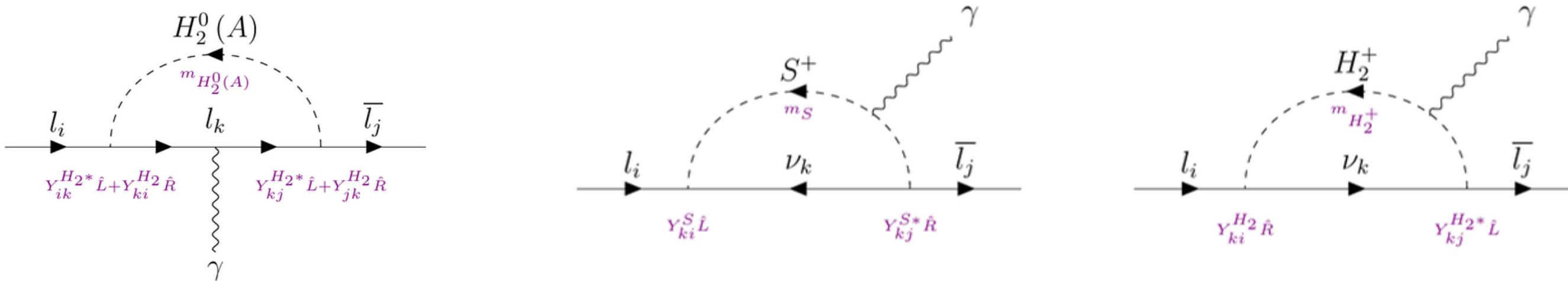
$|Y^{H_2}|^2$   
 $mn$   
 $ij$

$|Y^{H_2}|^2$   
 $mn$   
 $ij$

Process	Exp. data	Coupling	Constraint $\left(\frac{m_{H_0}}{\text{GeV}}\right)^4$
$\mu^- \rightarrow e^- e^+ e^-$	$< 10^{-12}$	$ Y_{ee}^{H_2} ^2  Y_{e\mu}^{H_2}   Y_{\mu e}^{H_2} $	$< 1.45 \times 10^{-21}$
$\tau^- \rightarrow e^- e^+ e^-$	$< 2.7 \times 10^{-8}$	$ Y_{ee}^{H_2} ^2  Y_{e\tau}^{H_2}   Y_{\tau e}^{H_2} $	$< 2.24 \times 10^{-16}$
$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	$< 2.1 \times 10^{-8}$	$ Y_{\mu\mu}^{H_2} ^2  Y_{\tau\mu}^{H_2}   Y_{\mu\tau}^{H_2} $	$< 1.74 \times 10^{-16}$
$\tau^- \rightarrow e^- \mu^+ e^-$	$< 1.5 \times 10^{-8}$	$ Y_{\mu e}^{H_2}   Y_{e\tau}^{H_2}   Y_{e\mu}^{H_2}   Y_{\tau e}^{H_2} $	$< 1.32 \times 10^{-16}$
$\tau^- \rightarrow \mu^- e^+ \mu^-$	$< 1.7 \times 10^{-8}$	$ Y_{e\mu}^{H_2}   Y_{\mu\tau}^{H_2}   Y_{e\mu}^{H_2}   Y_{\tau\mu}^{H_2} $	$< 1.49 \times 10^{-16}$
$\tau^- \rightarrow \mu^- \mu^+ e^-$	$< 2.7 \times 10^{-8}$	$ Y_{\mu e}^{H_2}   Y_{e\mu}^{H_2}   Y_{\tau e}^{H_2}   Y_{e\tau}^{H_2}  +  Y_{\mu\mu}^{H_2} ^2  Y_{\tau e}^{H_2}   Y_{e\tau}^{H_2} $ $- \frac{1}{8}  Y_{\mu\mu}^{H_2}  ( Y_{\mu\tau}^{H_2}   Y_{\mu e}^{H_2}   Y_{e\tau}^{H_2}  +  Y_{\tau\mu}^{H_2}   Y_{e\mu}^{H_2}   Y_{\tau e}^{H_2} )$	$< 1.68 \times 10^{-16}$
$\tau^- \rightarrow e^+ \mu^- e^-$	$< 1.8 \times 10^{-8}$	$ Y_{\mu e}^{H_2}   Y_{e\mu}^{H_2}   Y_{\tau e}^{H_2}   Y_{e\tau}^{H_2}  +  Y_{ee}^{H_2} ^2  Y_{\tau\mu}^{H_2}   Y_{\mu\tau}^{H_2} $ $- \frac{1}{8}  Y_{ee}^{H_2}  ( Y_{\mu\tau}^{H_2}   Y_{e\mu}^{H_2}   Y_{e\tau}^{H_2}  +  Y_{\tau\mu}^{H_2}   Y_{\mu e}^{H_2}   Y_{\tau e}^{H_2} )$	$< 1.11 \times 10^{-16}$

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$

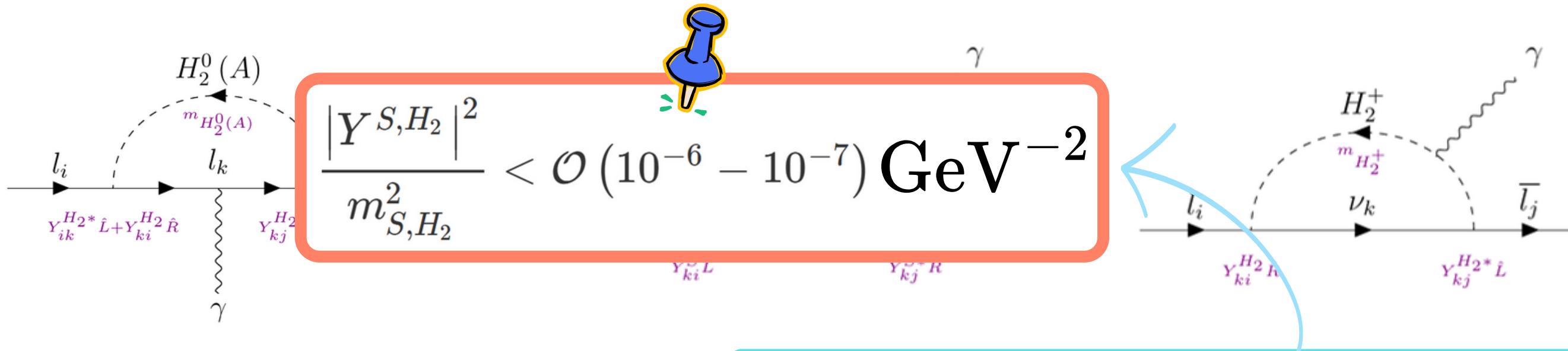


$$\begin{aligned} \left|Y_{ij}^{F_2}\right|^2 &= \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} - \sum_{\alpha=S, H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right], \\ \left|Y_{ij}^{G_2}\right|^2 &= \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S, H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right]. \end{aligned}$$

Process	Exp. data	Coupling	Constraint
$\mu^- \rightarrow e^- \gamma$	$< 5.7 \times 10^{-13}$	$\frac{ Y_{\mu e}^{G_2} ^4}{m_{S, H_2}^4} + \frac{ Y_{\mu e}^{F_2} ^4}{m_{S, H_2}^4}$	$< 2.93 \times 10^{-16}$
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	$\frac{ Y_{\tau e}^{G_2} ^4}{m_{S, H_2}^4} + \frac{ Y_{\tau e}^{F_2} ^4}{m_{S, H_2}^4}$	$< 1.70 \times 10^{-11}$
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	$\frac{ Y_{\tau \mu}^{G_2} ^4}{m_{S, H_2}^4} + \frac{ Y_{\tau \mu}^{F_2} ^4}{m_{S, H_2}^4}$	$< 2.26 \times 10^{-11}$

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$

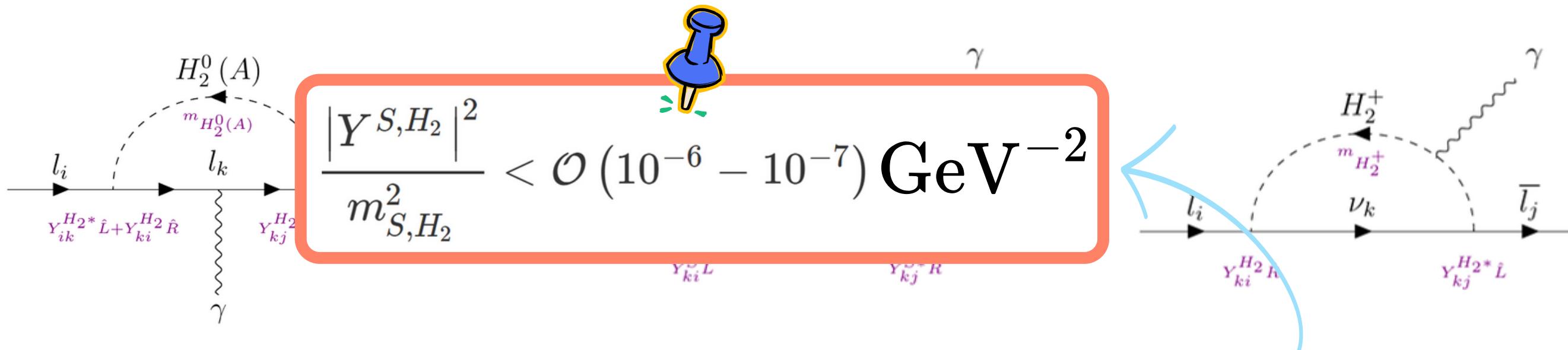


$$\begin{aligned} \frac{|Y_{ij}^{F_2}|^2}{m_{S,H_2}^2} &= \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} - \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right], \\ \frac{|Y_{ij}^{G_2}|^2}{m_{S,H_2}^2} &= \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right]. \end{aligned}$$

Process	Exp. data	Coupling	Constraint
$\mu^- \rightarrow e^- \gamma$	$< 5.7 \times 10^{-13}$	$\frac{ Y_{\mu e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\mu e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.93 \times 10^{-16}$
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	$\frac{ Y_{\tau e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 1.70 \times 10^{-11}$
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	$\frac{ Y_{\tau \mu}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau \mu}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.26 \times 10^{-11}$

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$



$$\frac{|Y_{ij}^{F_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right],$$

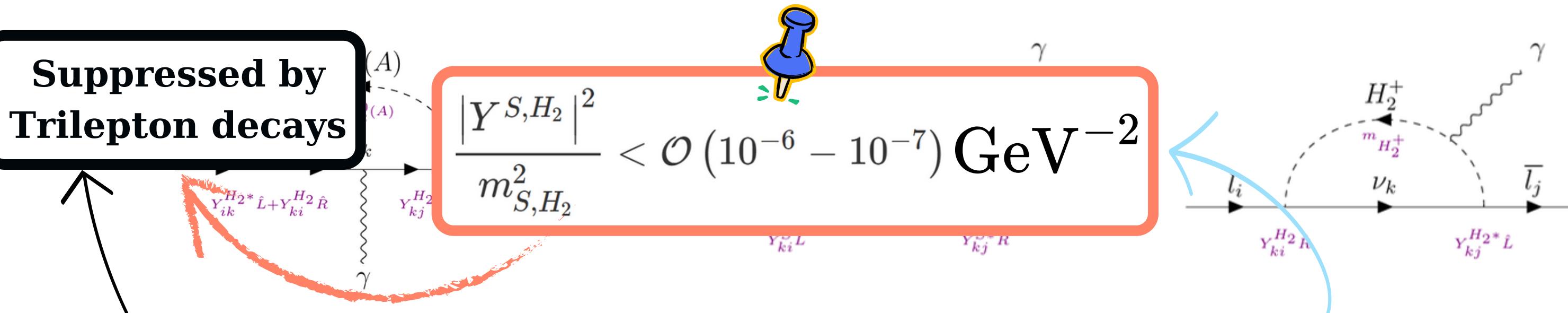
$$\frac{|Y_{ij}^{G_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right].$$

Heavy charged scalar mass

Process	Exp. data	Coupling	Constraint
$\mu^- \rightarrow e^- \gamma$	$< 5.7 \times 10^{-13}$	$\frac{ Y_{\mu e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\mu e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.93 \times 10^{-16}$
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	$\frac{ Y_{\tau e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 1.70 \times 10^{-11}$
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	$\frac{ Y_{\tau \mu}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau \mu}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.26 \times 10^{-11}$

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$



$$\frac{|Y_{ij}^{F_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right],$$

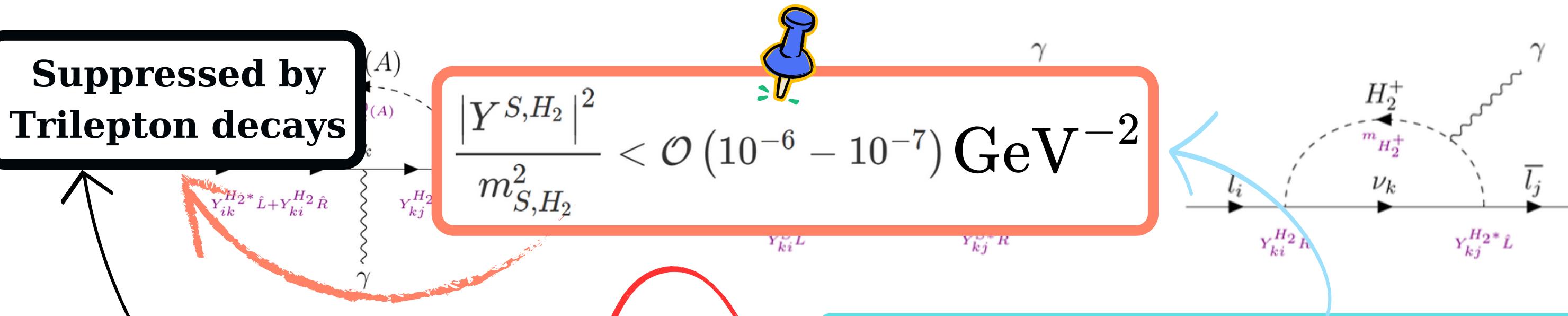
$$\frac{|Y_{ij}^{G_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right].$$

Heavy charged scalar mass

Process	Exp. data	Coupling	Constraint
$\mu^- \rightarrow e^- \gamma$	$< 5.7 \times 10^{-13}$	$\frac{ Y_{\mu e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\mu e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.93 \times 10^{-16}$
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	$\frac{ Y_{\tau e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 1.70 \times 10^{-11}$
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	$\frac{ Y_{\tau \mu}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau \mu}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.26 \times 10^{-11}$

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$



$$\frac{|Y_{ij}^{F_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right],$$

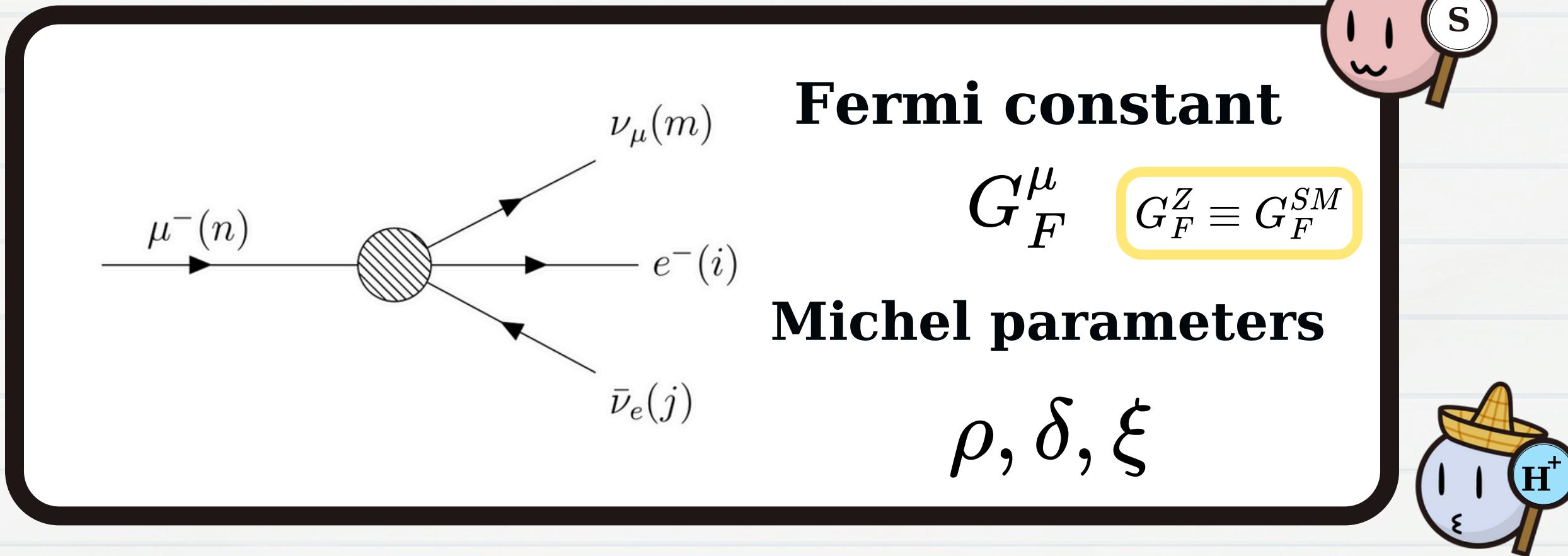
$$\frac{|Y_{ij}^{G_2}|^2}{m_{S,H_2}^2} = \left[ 2 \frac{(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*})}{m_{H_0}^2} + \sum_{\alpha=S,H_2^+} \frac{Y_{ki}^\alpha Y_{kj}^{\alpha*}}{m_\alpha^2} \right]$$

SEE IN MUON DECAY

Heavy charged scalar mass

Process	Exp. data	Coupling	Constraint
$\mu^- \rightarrow e^- \gamma$	$< 5.7 \times 10^{-13}$	$\frac{ Y_{\mu e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\mu e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.93 \times 10^{-16}$
$\tau^- \rightarrow e^- \gamma$	$< 3.3 \times 10^{-8}$	$\frac{ Y_{\tau e}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau e}^{F_2} ^4}{m_{S,H_2}^4}$	$< 1.70 \times 10^{-11}$
$\tau^- \rightarrow \mu^- \gamma$	$< 4.4 \times 10^{-8}$	$\frac{ Y_{\tau \mu}^{G_2} ^4}{m_{S,H_2}^4} + \frac{ Y_{\tau \mu}^{F_2} ^4}{m_{S,H_2}^4}$	$< 2.26 \times 10^{-11}$

# Muon decay



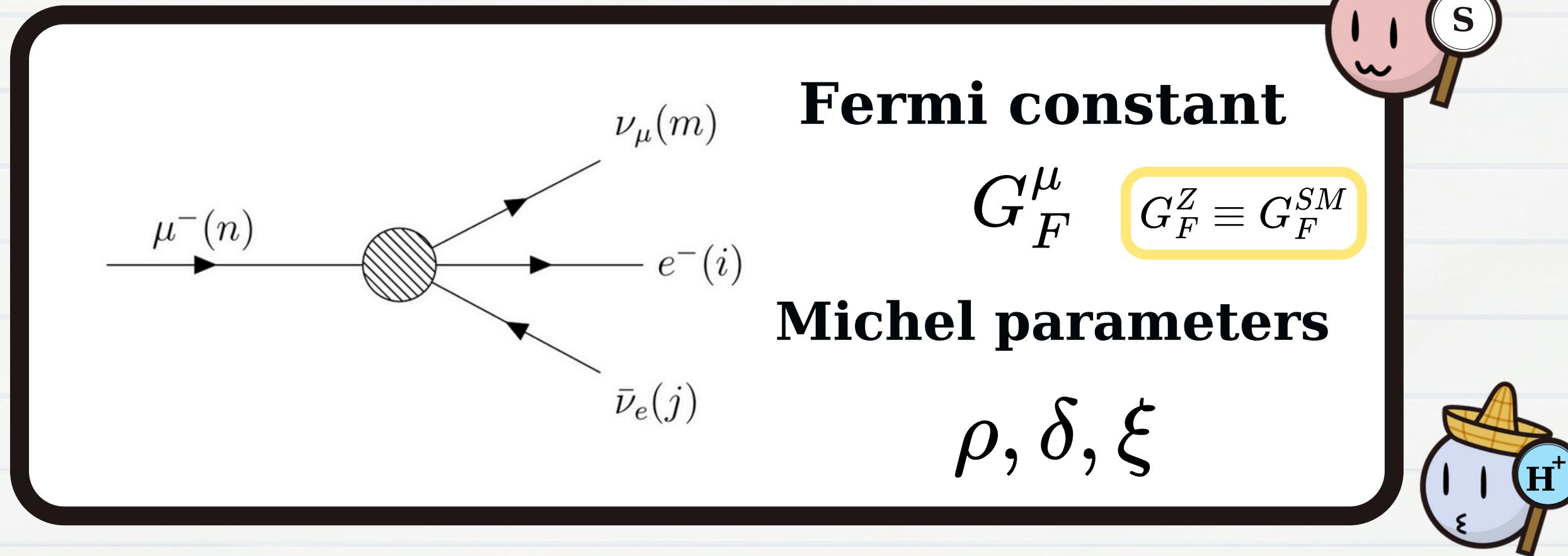
$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1 - x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1 - x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

3/4

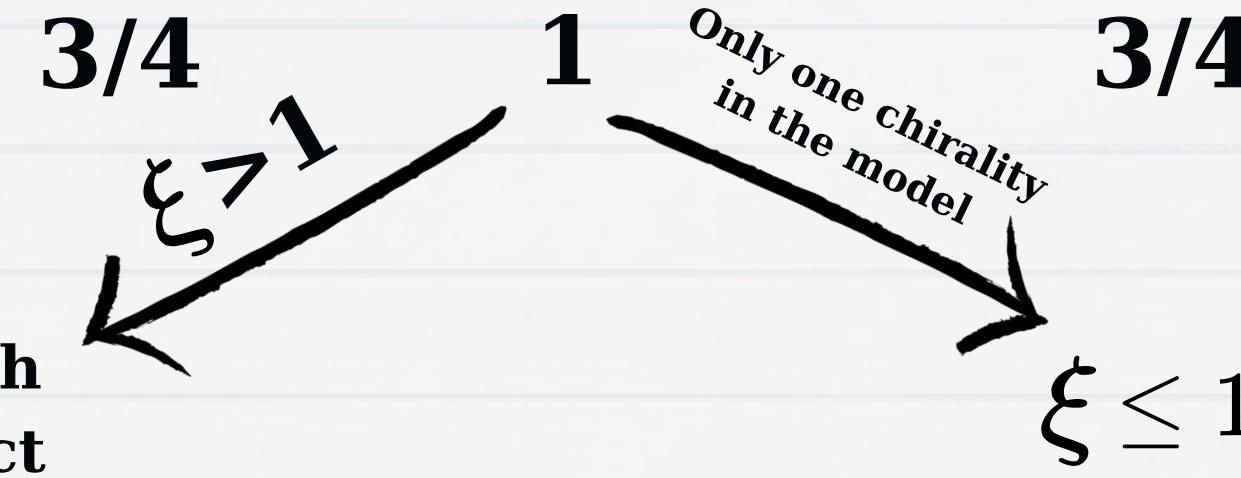
1

3/4

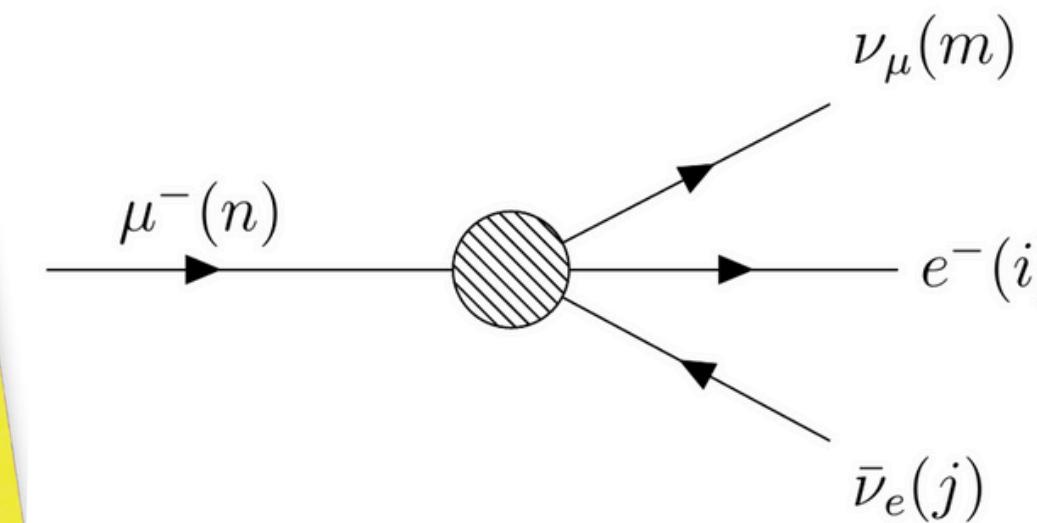
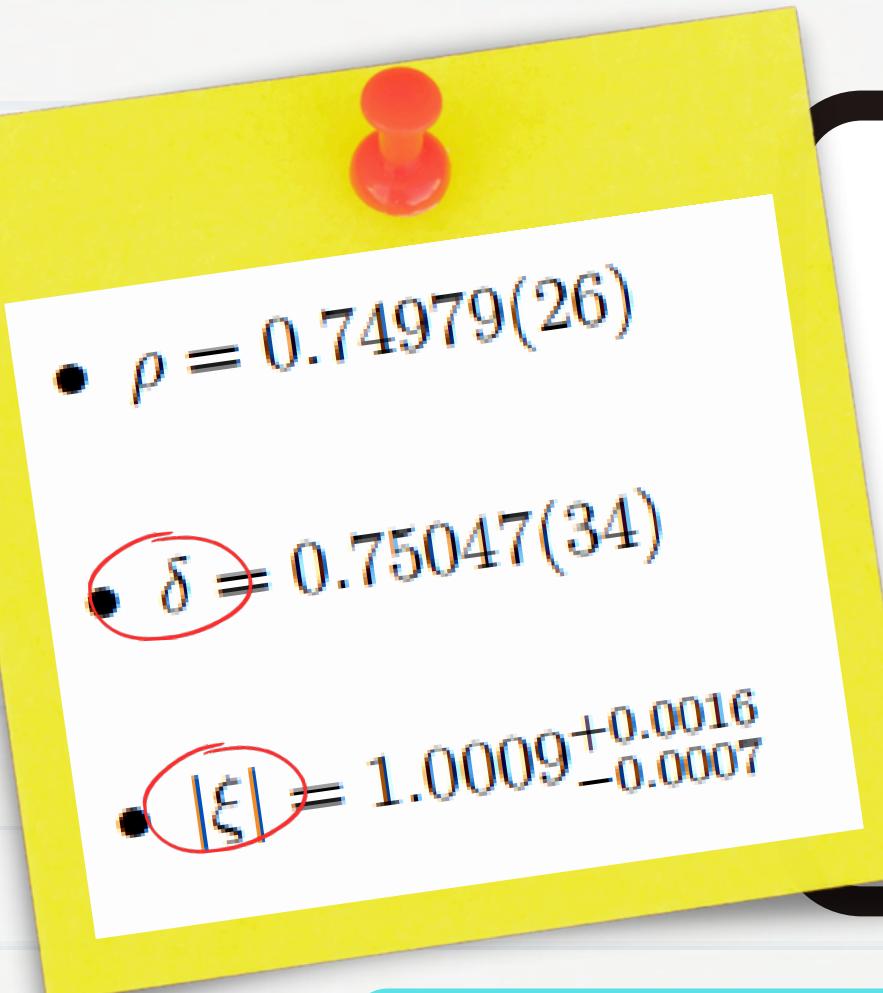
# Muon decay



$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$



# Muon decay



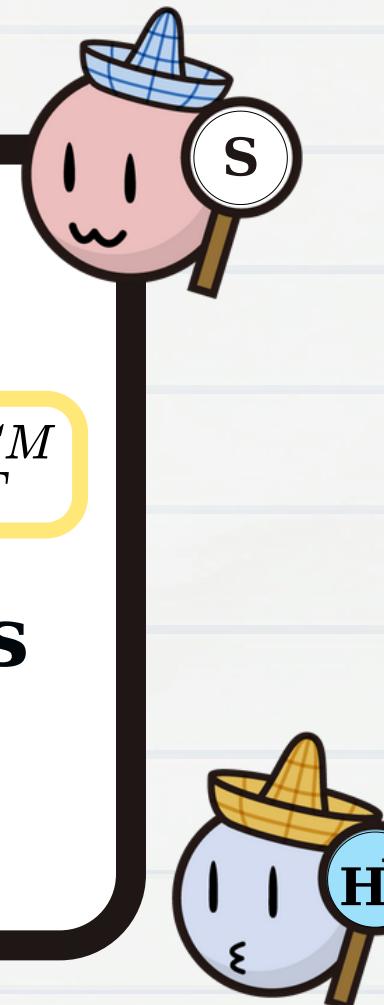
**Fermi constant**

$$G_F^\mu$$

$$G_F^Z \equiv G_F^{SM}$$

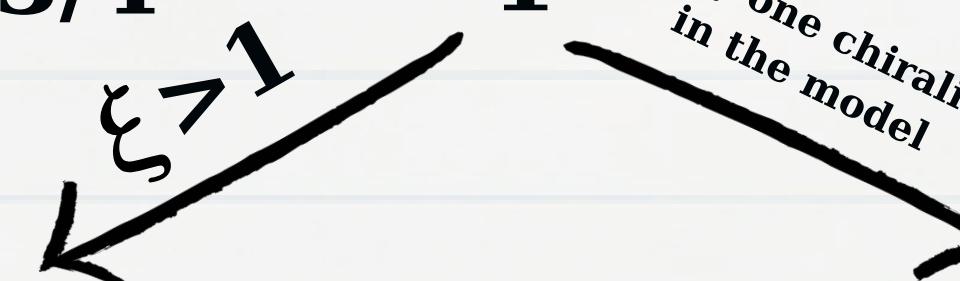
**Michel parameters**

$$\rho, \delta, \xi$$



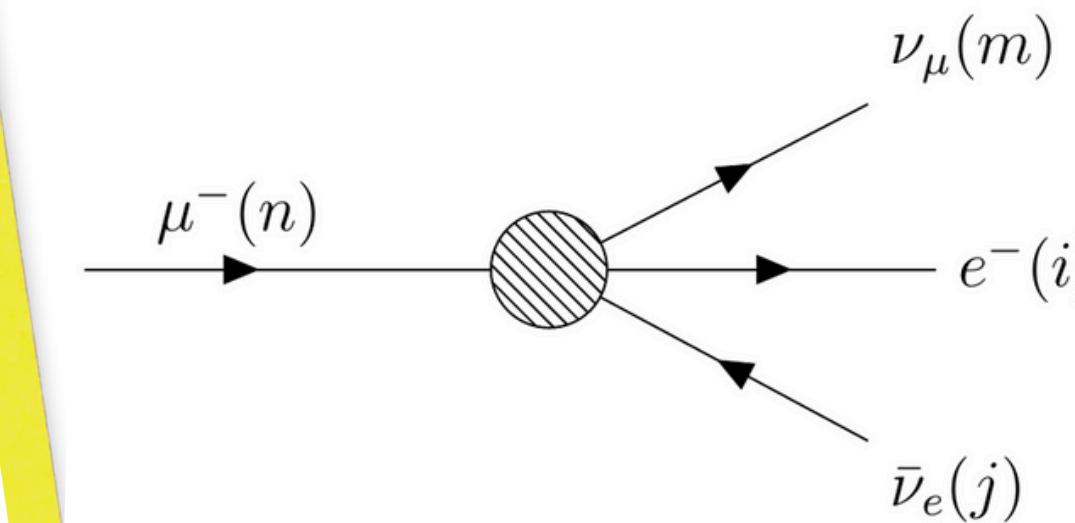
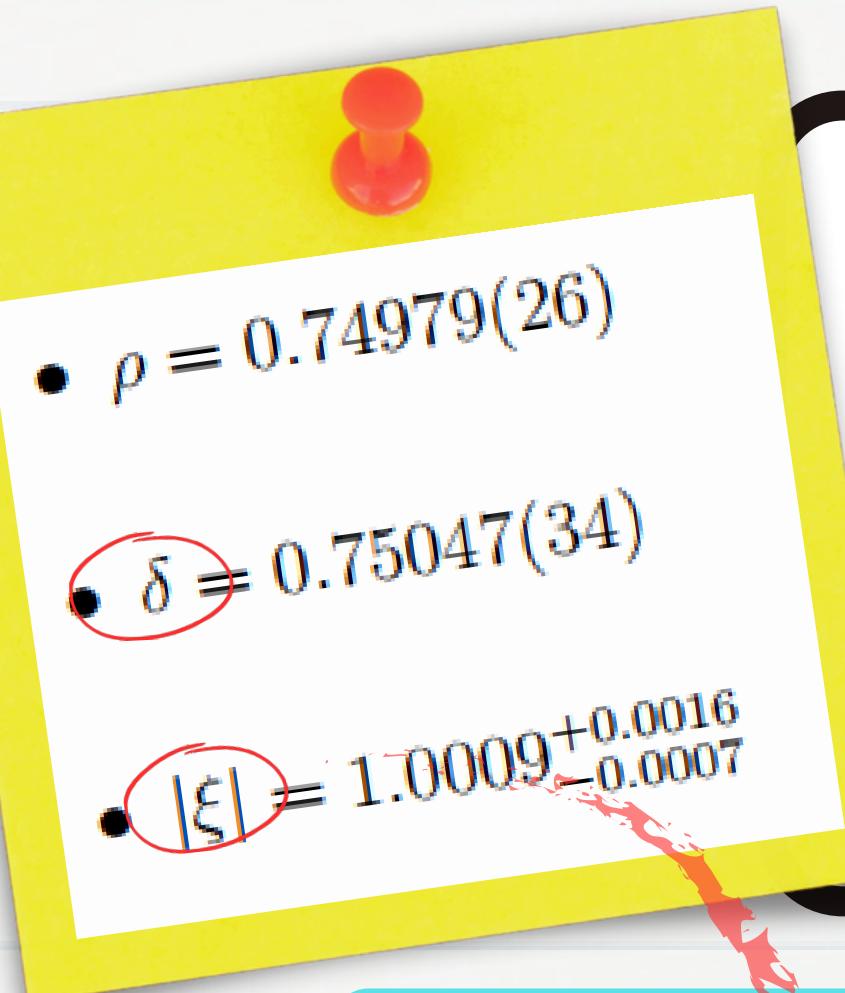
$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

$$3/4 \quad 1 \quad 3/4$$



Involving both chirality effect

# Muon decay



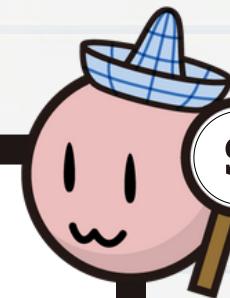
**Fermi constant**

$$G_F^\mu$$

$$G_F^Z \equiv G_F^{SM}$$

**Michel parameters**

$$\rho, \delta, \xi$$



$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

3/4

1

3/4

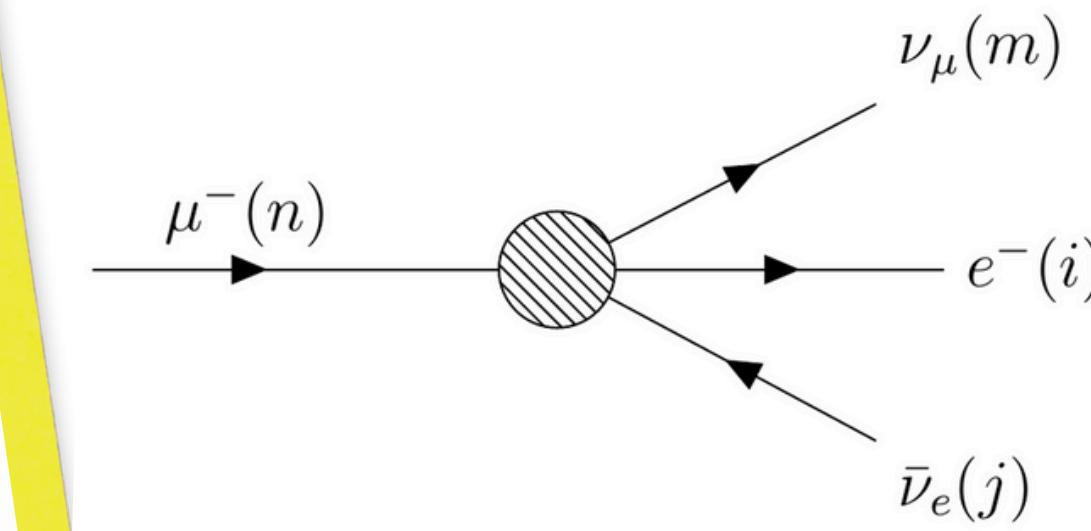
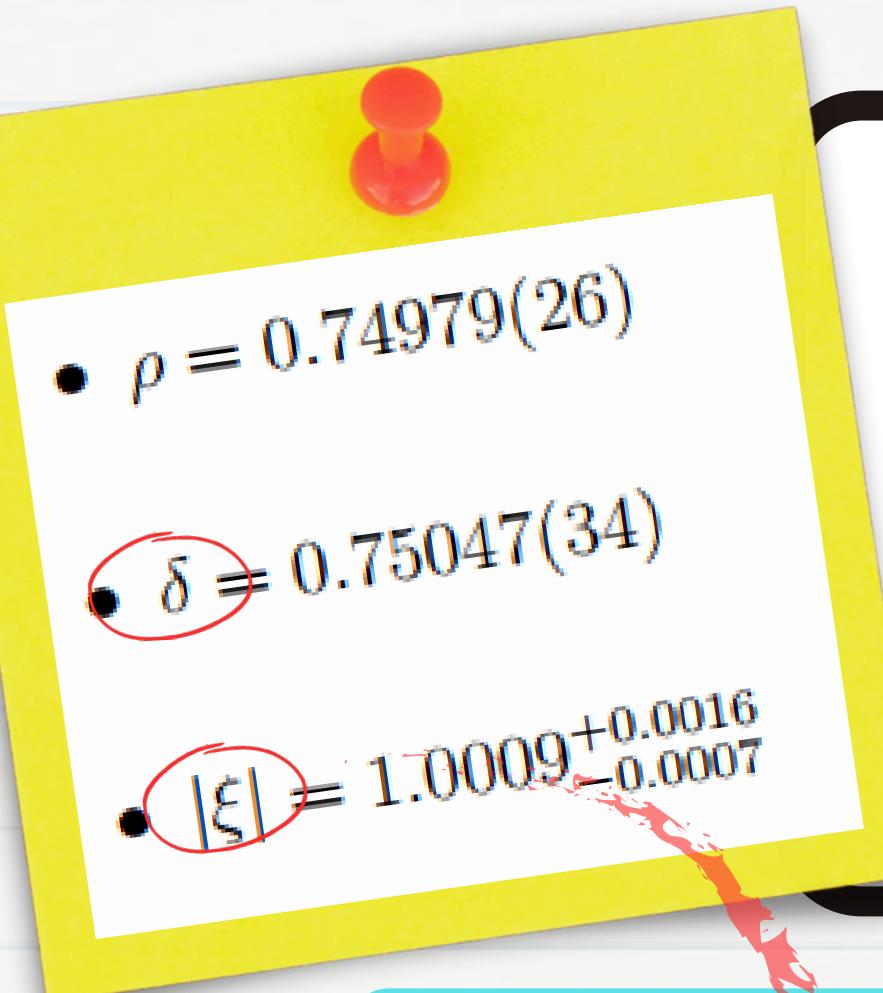
Involving both chirality effect

$\xi > 1$

Only one chirality in the model

$\xi \leq 1$

# Muon decay



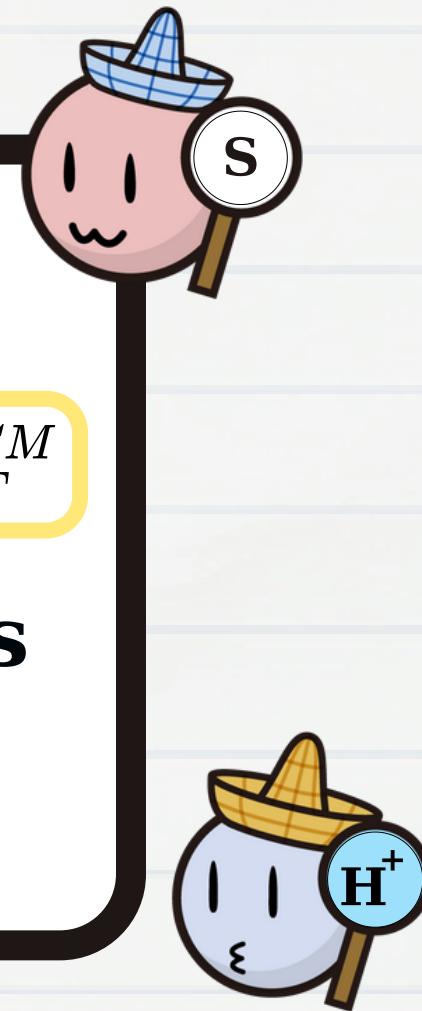
**Fermi constant**

$$G_F^\mu$$

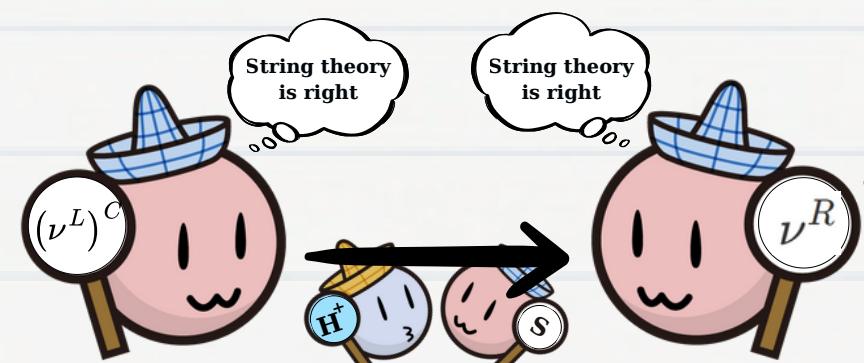
$$G_F^Z \equiv G_F^{SM}$$

**Michel parameters**

$$\rho, \delta, \xi$$



$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$



String theory  
is right

String theory  
is right

Involving both  
chirality effect

3/4

$\xi > 1$

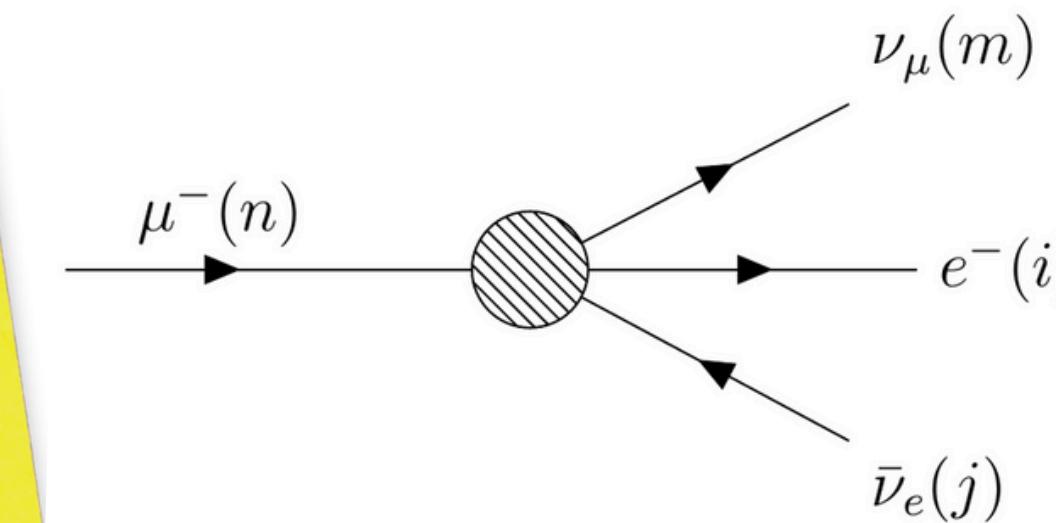
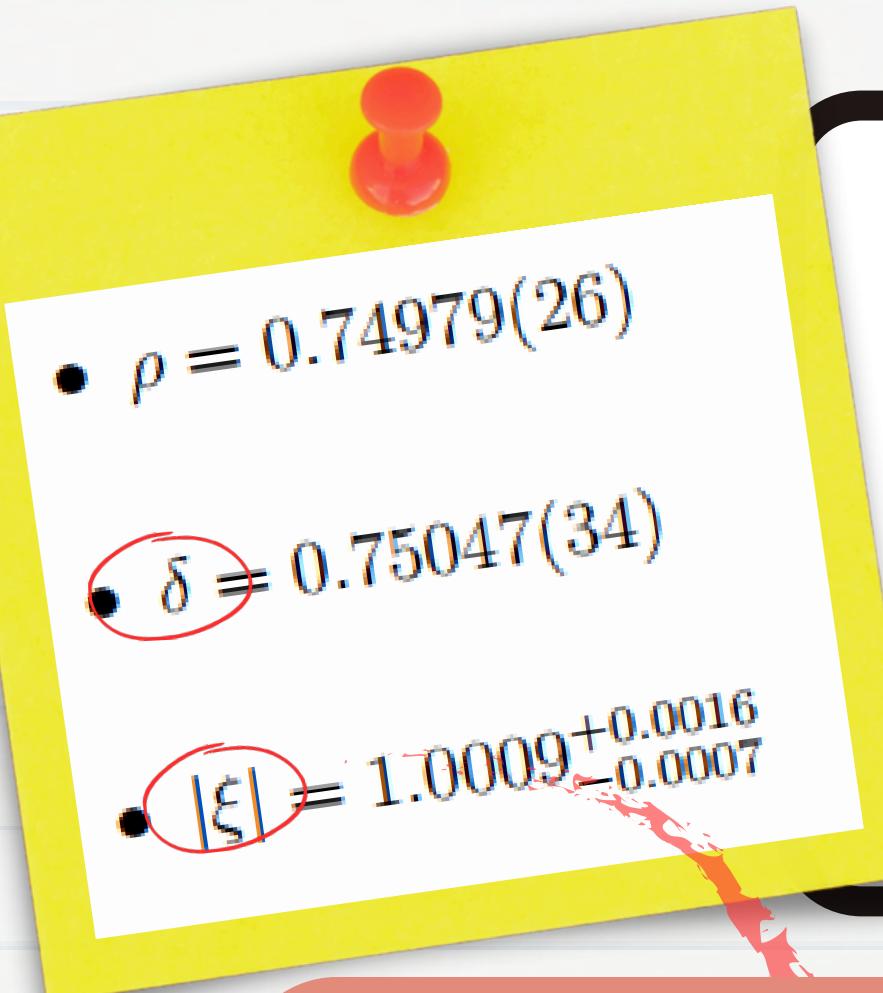
1

Only one chirality  
in the model

3/4

$\xi \leq 1$

# Muon decay



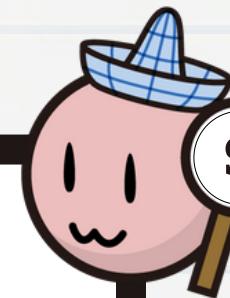
**Fermi constant**

$$G_F^\mu$$

$$G_F^Z \equiv G_F^{SM}$$

**Michel parameters**

$$\rho, \delta, \xi$$



A favorable reason  
for the Zee model !



Involving both  
chirality effect

$$\frac{3}{4} \quad \xi > 1$$

$$1 \quad \text{Only one chirality in the model}$$

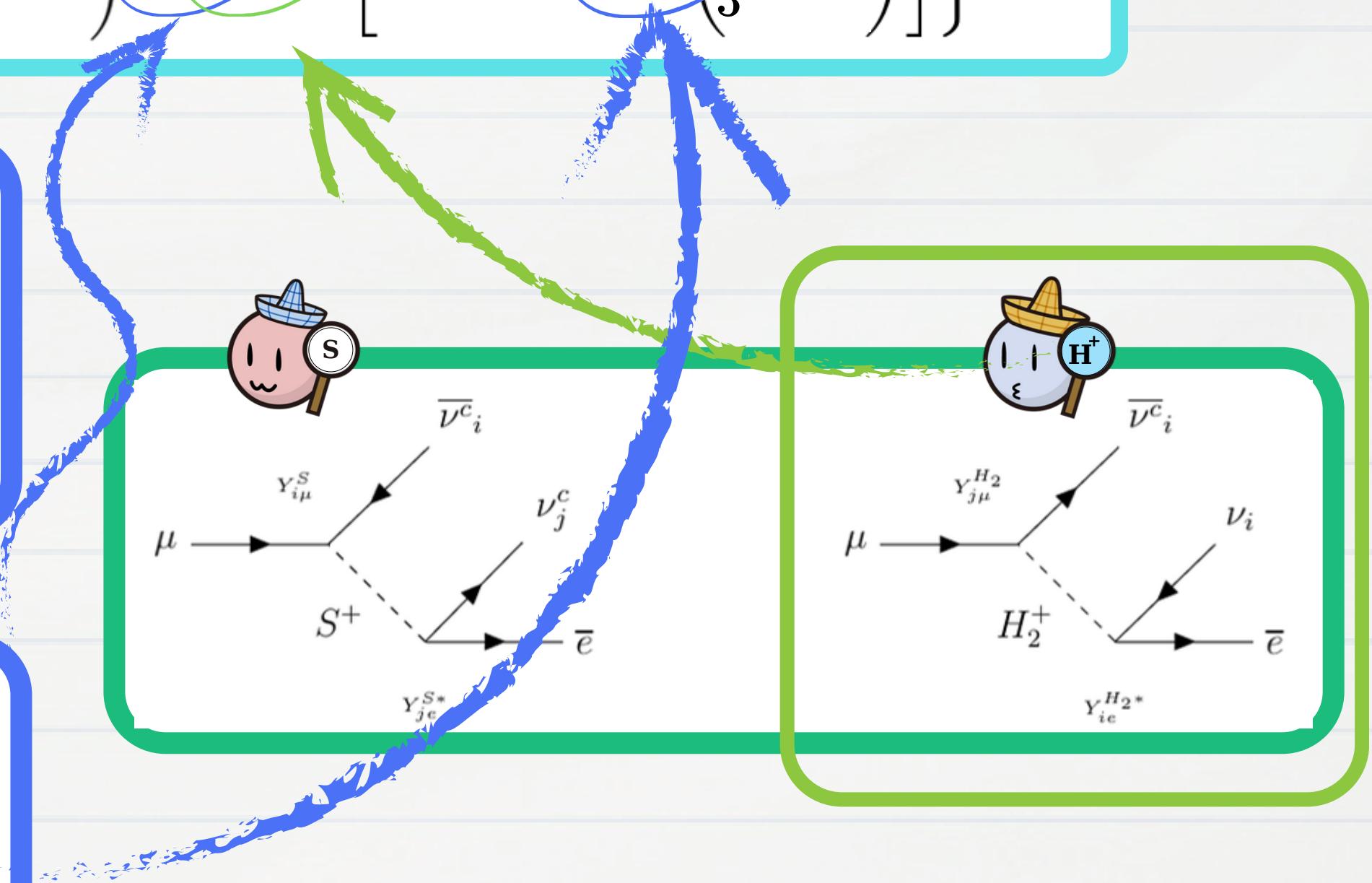
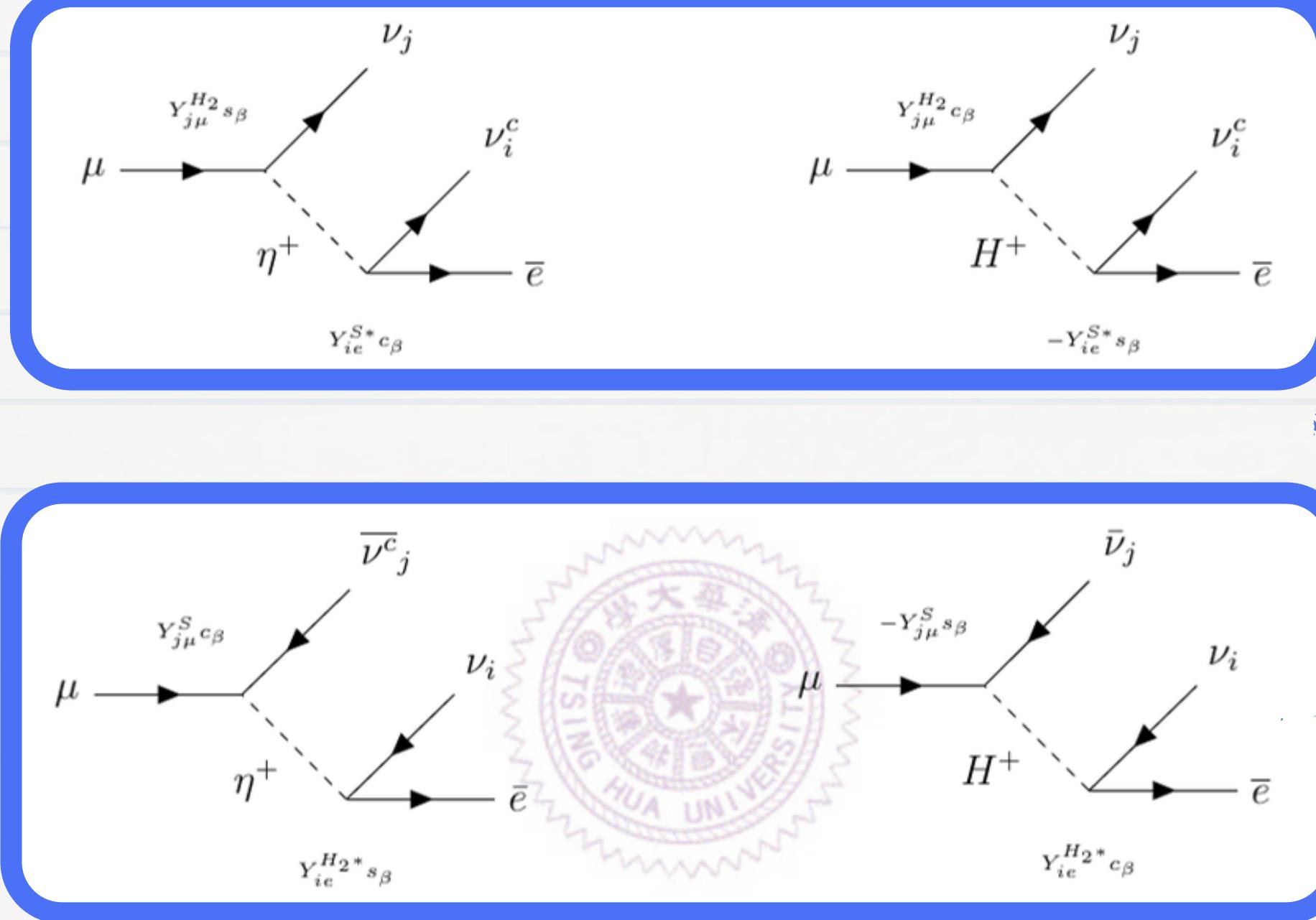
$$\xi \leq 1$$

$$-1) -\xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \} x^2 dx$$

# Zee model in Muon decay



$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$



# Zee model in Muon decay

$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1 - x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1 - x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

$$\begin{aligned} \Delta\rho &= \frac{3}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} \left( Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S + Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} \right) \\ &= (-47 - 5) \times 10^{-5}, \end{aligned}$$

$$\begin{aligned} \Delta G_F^2 &= \frac{G_F^{SM}}{2\sqrt{2}m_S^2} |Y_{e\mu}^S|^2 + \frac{1}{32m_S^4} \sum_{i,j} |Y_{i\mu}^S|^2 |Y_{je}^S|^2 \\ &\quad + \frac{1}{32m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 + \frac{a + 6c}{8} \\ &= (-1.6 - 3.6) \times 10^{-13}. \end{aligned}$$

$$\begin{aligned} \Delta\delta &= \frac{21}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{-3a' + 4b' - 14c'} \operatorname{Re} \left( Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S - Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} \right) \\ &= (1.3 - 8.1) \times 10^{-4}. \end{aligned}$$

$$\begin{aligned} \Delta\xi &= \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} \left( Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} - \frac{3}{2} Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S \right) \\ &\quad - \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \left( |Y_{j\mu}^{H_2}|^2 |Y_{ie}^S|^2 + |Y_{i\mu}^{H_2}|^2 |Y_{je}^S|^2 \right) - \frac{1}{8m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 \\ &= (2 - 25) \times 10^{-4}, \end{aligned}$$

# Zee model in Muon decay

$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1 - x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1 - x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

$$\begin{aligned} \Delta\rho &= \frac{3}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} \left( Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S + Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} \right) \\ &= (-47 - 5) \times 10^{-5}, \end{aligned}$$

$$\begin{aligned} \Delta G_F^2 &= \frac{G_F^{SM}}{2\sqrt{2}m_S^2} |Y_{e\mu}^S|^2 + \frac{1}{32m_S^4} \sum_{i,j} |Y_{i\mu}^S|^2 |Y_{je}^S|^2 \\ &\quad + \frac{1}{32m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 + \frac{a + 6c}{8} \\ &= (-1.6 - 3.6) \times 10^{-13}. \end{aligned}$$

$$\begin{aligned} \Delta\delta &= \frac{21}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{-3a' + 4b' - 14c'} \operatorname{Re} \left( Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S - Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} \right) \\ &= (1.3 - 8.1) \times 10^{-4}. \end{aligned}$$

$$\begin{aligned} \Delta\xi &= \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} \left( Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} - \frac{3}{2} Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S \right) \\ &\quad - \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \left( |Y_{j\mu}^{H_2}|^2 |Y_{ie}^S|^2 + |Y_{i\mu}^{H_2}|^2 |Y_{je}^S|^2 \right) - \frac{1}{8m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 \\ &= (2 - 25) \times 10^{-4}, \end{aligned}$$

  $c_\beta s_\beta \equiv s_{2\beta} \approx \beta > \mathcal{O}(10^{-2})$

# Zee model in Muon decay

$$d\Gamma = \frac{m_\mu^5}{3 * 2^9 * \pi^4} * (a + 4b + 6c) \left\{ 3(1 - x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1 - x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

$$\begin{aligned} \Delta\rho &= \frac{3}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} (Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S + Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2}) \\ &= (-47 - 5) \times 10^{-5}, \end{aligned}$$

$$\begin{aligned} \Delta G_F^2 &= \frac{G_F^{SM}}{2\sqrt{2}m_S^2} |Y_{e\mu}^S|^2 + \frac{1}{32m_S^4} \sum_{i,j} |Y_{i\mu}^S|^2 |Y_{je}^S|^2 \\ &\quad + \frac{1}{32m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 + \frac{a + 6c}{8} \\ &= (-1.6 - 3.6) \times 10^{-13}. \end{aligned}$$

$$\begin{aligned} \Delta\delta &= \frac{21}{16} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{-3a' + 4b' - 14c'} \operatorname{Re} (Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S - Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2}) \\ &= (1.3 - 8.1) \times 10^{-4}. \end{aligned}$$

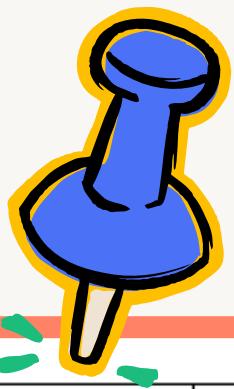
$$\begin{aligned} \Delta\xi &= \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \operatorname{Re} \left( Y_{j\mu}^S Y_{ie}^{H_2*} Y_{i\mu}^{S*} Y_{je}^{H_2} - \frac{3}{2} Y_{j\mu}^{H_2} Y_{ie}^{S*} Y_{i\mu}^{H_2*} Y_{je}^S \right) \\ &\quad - \frac{1}{2} \left( \frac{1}{m_{\eta^+}^2} - \frac{1}{m_{H^+}^2} \right)^2 \frac{c_\beta^2 s_\beta^2}{a + 4b + 6c} \left( |Y_{j\mu}^{H_2}|^2 |Y_{ie}^S|^2 + |Y_{i\mu}^{H_2}|^2 |Y_{je}^S|^2 \right) - \frac{1}{8m_{H_2^+}^4} \sum_{i,j} |Y_{i\mu}^{H_2}|^2 |Y_{je}^{H_2}|^2 \\ &= (2 - 25) \times 10^{-4}, \end{aligned}$$

$$c_\beta s_\beta \equiv s_{2\beta} \approx \beta > \mathcal{O}(10^{-2})$$

$$\frac{|Y_{S,H_2}^S|^2}{m_{S,H_2^+}^2} < (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$$



# Conclusion

Exp.	Coupling form	Order of magnitude
Michel Parameters at $1\sigma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$\sim (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
Michel Parameters at $2\sigma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$< (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
$G_F$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$< (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
Neutrino mass	$ Y^{S,H_2} ^2$	$\sim (10^{-6} - 10^{-4})$
$l_i \rightarrow l_j \bar{l}_m l_n$	$\frac{ Y^{H_2} ^2}{m_{H_0}^2}$	$< (10^{-9} - 10^{-8}) \text{ GeV}^{-2}$
$L \rightarrow l \gamma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2}^2}$	$< (10^{-7} - 10^{-6}) \text{ GeV}^{-2}$

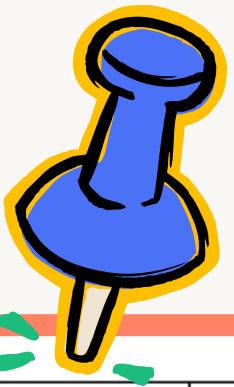


**Trilepton decays  
give strong  
constraints from  
doublet scalar  
sector**

- At  $1\sigma$  the Michel parameter could rule out the feasibility of Zee model
- An upper bound in singlet coupling  $Y^S$  at  $2\sigma$

Yet ! Michel gives constraint for singlet scalar sector, comparable to Lep-to-lep-gamma

# Conclusion



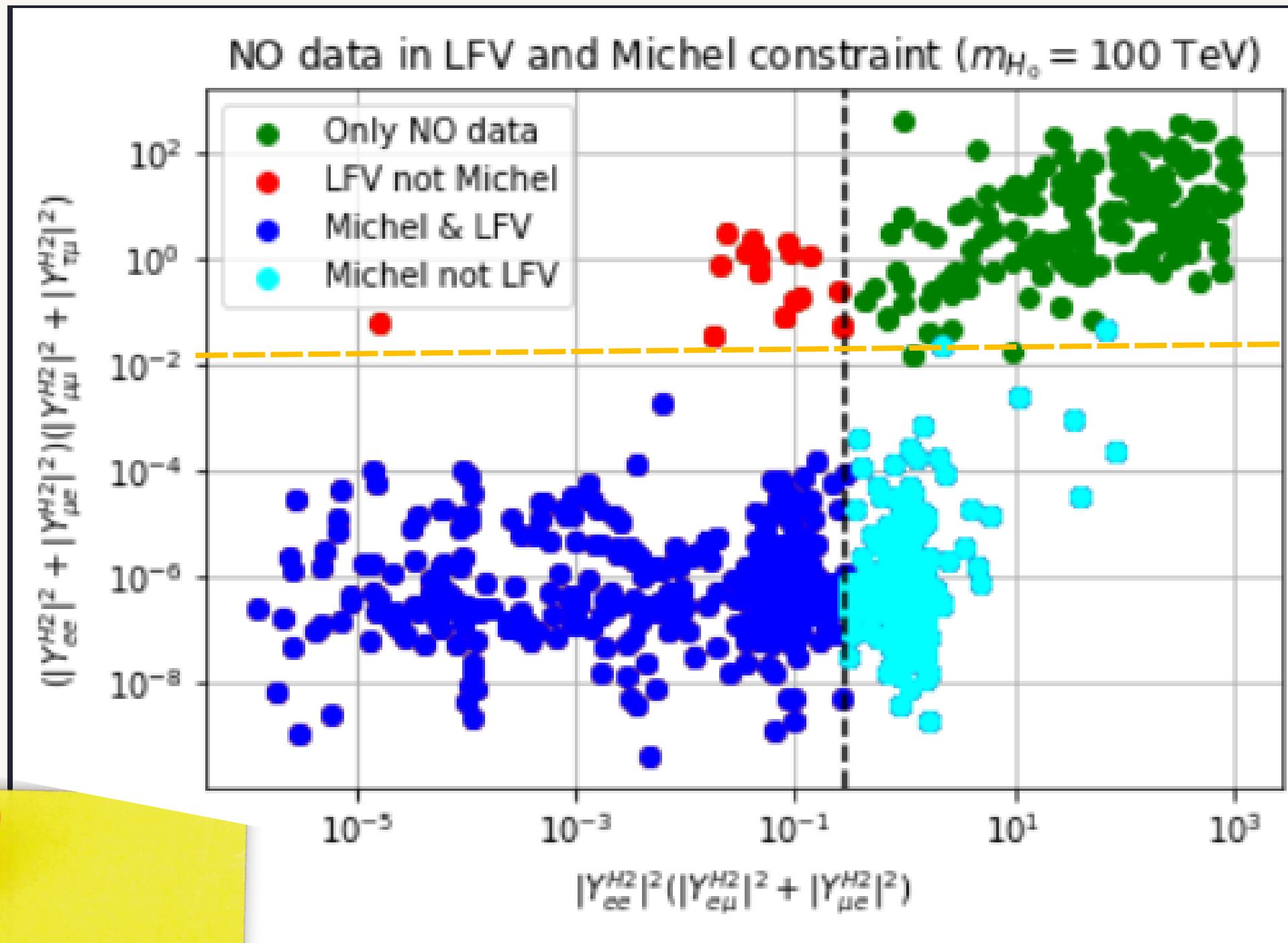
Exp.	Coupling form	Order of magnitude
Michel Parameters at $1\sigma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$\sim (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
Michel Parameters at $2\sigma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$< (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
$G_F$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$< (10^{-6} - 10^{-5}) \text{ GeV}^{-2}$
Neutrino mass	$ Y^{S,H_2} ^2$	$\sim (10^{-6} - 10^{-4})$
$l_i \rightarrow l_j \bar{l}_m l_n$	$\frac{ Y^{H_2} ^2}{m_{H_0}^2}$	$< (10^{-9} - 10^{-8}) \text{ GeV}^{-2}$
$L \rightarrow l \gamma$	$\frac{ Y^{S,H_2} ^2}{m_{S,H_2^+}^2}$	$< (10^{-7} - 10^{-6}) \text{ GeV}^{-2}$

**Trilepton decays give strong constraints from doublet scalar sector**

- At  $1\sigma$  the Michel parameter could rule out the feasibility of Zee model
- An upper bound in singlet coupling  $Y^S$  at  $2\sigma$

Yet ! Michel gives constraint for singlet scalar sector, comparable to Lep-to-lep-gamma

# Outlook



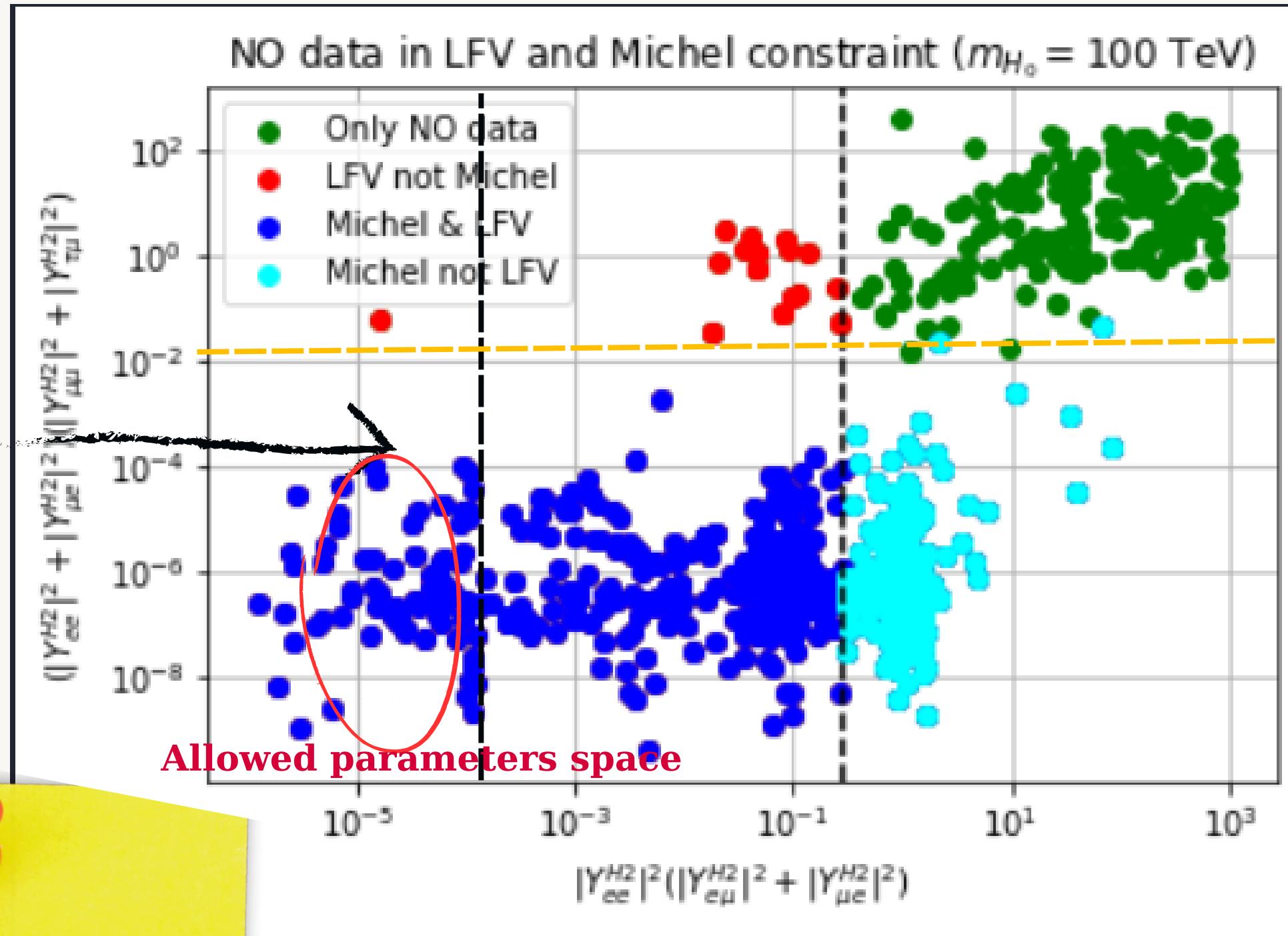
**LFV** = Trilepton decay  
**NO** = Neutrino oscillation  
**Michel** = Michel parameter  $\chi_i$  from muon decay

Reach current experimental limit of scalar mass at  $\mathcal{O}(1)$  TeV to explore the usefulness of the Michel parameter constraint on the Zee model.

Just for illustration,  $c_\beta = 0.9$ ,  $m_{H_0} = 100$  TeV,  $m_{H^+} = 210$  and  $m_{\eta^+} = 200$  GeV

# Outlook

$m_{H_0} = 10 \text{ TeV}$



LFV = Trilepton decay

NO = Neutrino oscillation

Michel = Michel parameter  $\chi_i$  from muon decay

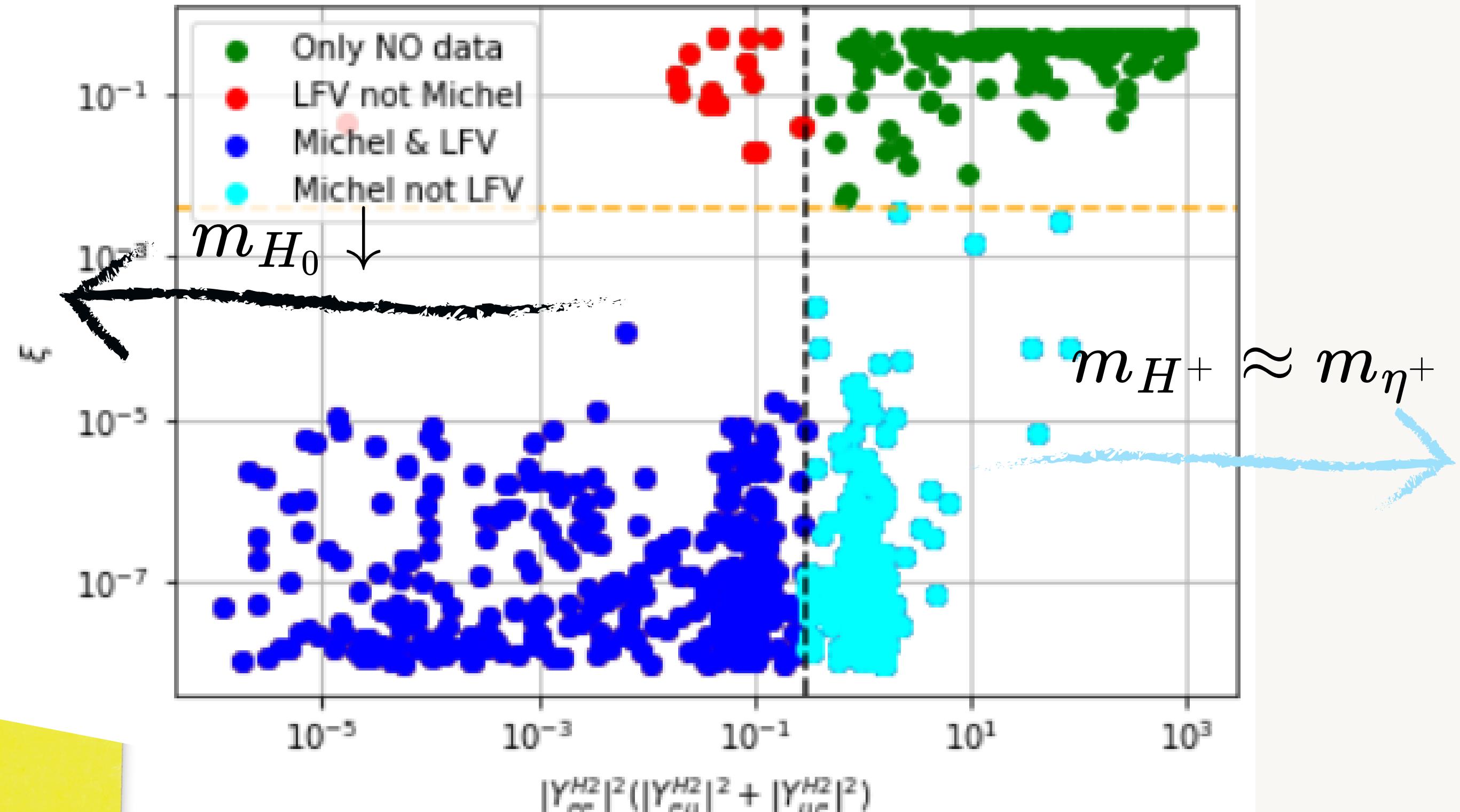
Reach current experimental limit of scalar mass at  $\mathcal{O}(1) \text{ TeV}$  to explore the usefulness of the Michel parameter constraint on the Zee model.

Just for illustration,  $c_\beta = 0.9$ ,  $m_{H_0} = 100 \text{ TeV}$ ,  $m_{H^+} = 210$  and  $m_{\eta^+} = 200 \text{ GeV}$

# Thank's For Listening



## NO data in LFV and Michel constraint ( $m_{H_0} = 100$ TeV)

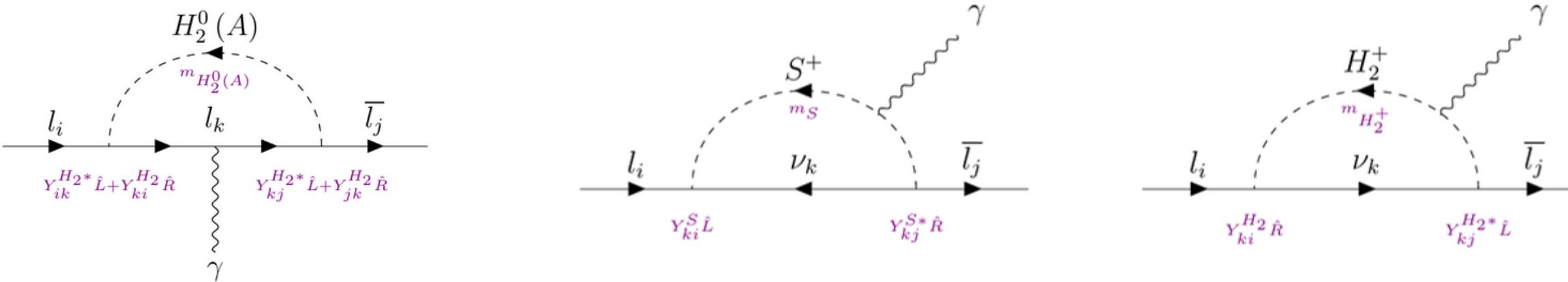


**LFV** = Trilepton decay  
**NO** = Neutrino oscillation  
**Michel** = Michel parameter  $\chi_i$  from muon decay

Just for illustration,  $c_\beta = 0.9$ ,  $m_{H_0} = 100$  TeV,  $m_{H^+} = 210$  and  $m_{\eta^+} = 200$  GeV

# Lep-to-lep-gamma decay $\text{L-L}\gamma$

$$Br(l_i \rightarrow l_j \gamma) = \frac{24\pi^2}{m_i^2 G_F^2} (|F_2|^2 + |G_2|^2) Br(l_i \rightarrow l_j \nu \bar{\nu})$$



$$F_2^{H_2^0(A)} = \frac{e}{3.2^7 \pi^2} \frac{m_i + m_j}{m_{H_2^0(A)}^2} [(Y_{ki}^{H_2} Y_{kj}^{H_2*} + Y_{jk}^{H_2} Y_{ik}^{H_2*}) \\ \mp \frac{3m_k}{m_i + m_j} \left( 3 + 4 \log \frac{m_k}{m_{H_2^0(A)}} \right) (Y_{ki}^{H_2} Y_{jk}^{H_2} + Y_{ik}^{H_2*} Y_{kj}^{H_2*})],$$

$$G_2^{H_2^0(A)} = \frac{e}{3.2^7 \pi^2} \frac{m_i - m_j}{m_{H_2^0(A)}^2} [(Y_{ki}^{H_2} Y_{kj}^{H_2*} - Y_{jk}^{H_2} Y_{ik}^{H_2*}) \\ \pm \frac{3m_k}{m_i - m_j} \left( 3 + 4 \log \frac{m_k}{m_{H_2^0(A)}} \right) (Y_{ki}^{H_2} Y_{jk}^{H_2} - Y_{ik}^{H_2*} Y_{kj}^{H_2*})].$$

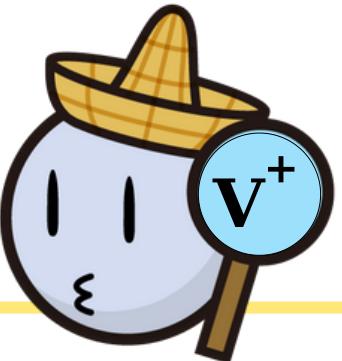
$$F_2^{H_2^+(S^+)} = \frac{-e}{3.2^7 \pi^2} \frac{m_i + m_j}{m_{H_2^+(S)}^2} Y_{ki}^{H_2(S)} Y_{kj}^{H_2(S)*},$$

$$G_2^{H_2^+(S^+)} = \frac{e}{3.2^7 \pi^2} \frac{m_i - m_j}{m_{H_2^+(S)}^2} Y_{ki}^{H_2(S)} Y_{kj}^{H_2(S)*},$$

# Anomalous magnetic moment ( $g-2$ )

$$i\mathcal{M}^\nu = \bar{l} \sigma^{\mu\nu} (F_2 + G_2 \gamma_5) l q_\mu$$

$$a_l = \frac{(g - 2)}{2} = \frac{2m_l F_2}{-eQ_l}$$



$$F_2^V = \frac{e}{3.2^7 m_V^2 \pi^2} \left[ (m_i + m_j) (Q_V - 8Q_k + 9aQ_V) (V_L^{*ki} V_L^{kj} + V_R^{*ki} V_R^{kj}) \right. \\ \left. - 6m_k (5a - 1) Q_V (V_L^{*ki} V_R^{kj} + V_R^{*ki} V_L^{kj}) \right]$$



$$F_2^\phi = \frac{-e}{3.2^7 m_\phi^2 \pi^2} \left[ (m_i + m_j) (Q_\phi - 2Q_k) (Y_L^{*ki} Y_L^{kj} + Y_R^{*ki} Y_R^{kj}) \right. \\ \left. + 6m_k \left( Q_\phi + 3Q_k + 2Q_k \log \left[ \frac{m_k^2}{m_\phi^2} \right] \right) (Y_L^{*ki} Y_R^{kj} + Y_R^{*ki} Y_L^{kj}) \right]$$

# Michel parameters

$$\frac{d\Gamma}{d\Omega} = \frac{m_\mu^5}{3 \cdot 2^9 \cdot \pi^4} (a + 4b + 6c) \left\{ 3(1-x) + 2\rho \left( \frac{4}{3}x - 1 \right) - \xi \cos \theta \left[ (1-x) + 2\delta \left( \frac{4}{3}x - 1 \right) \right] \right\} x^2 dx$$

$$a = \frac{|Y_{ij}^L Y_{nm}^{R*}|^2 + |Y_{ij}^R Y_{nm}^{L*}|^2}{16m_\phi^4}$$

$$a' = \frac{|Y_{ij}^L Y_{nm}^{R*}|^2 - |Y_{ij}^R Y_{nm}^{L*}|^2}{16m_\phi^4}$$

$$b = \frac{|Y_{ij}^L Y_{nm}^{L*}|^2 + |Y_{ij}^R Y_{nm}^{R*}|^2}{16m_\phi^4} + \frac{g^4}{16m_W^4}$$

$$b' = \frac{|Y_{ij}^R Y_{nm}^{R*}|^2 - |Y_{ij}^L Y_{nm}^{L*}|^2}{16m_\phi^4} + \frac{g^4}{16m_W^4}$$

$$c = \frac{|Y_{ij}^L Y_{nm}^{R*}|^2 + |Y_{ij}^R Y_{nm}^{L*}|^2}{32m_\phi^4}$$

$$c' = \frac{|Y_{ij}^R Y_{nm}^{L*}|^2 - |Y_{ij}^L Y_{nm}^{R*}|^2}{32m_\phi^4},$$



$$a = \frac{|V_{ij}^L V_{nm}^{R*}|^2 + |V_{ij}^R V_{nm}^{L*}|^2}{m_V^4}$$

$$a' = \frac{|V_{ij}^L V_{nm}^{R*}|^2 - |V_{ij}^R V_{nm}^{L*}|^2}{m_V^4}$$

$$b = \frac{|V_{ij}^L V_{nm}^{L*}|^2 + |V_{ij}^R V_{nm}^{R*}|^2}{4m_\phi^4} + \frac{g^4}{16m_W^4} + \frac{g^2}{4m_W^2 m_V^2} \text{Re}(V_{ij}^R V_{nm}^{R*})$$

$$b' = \frac{|V_{ij}^R V_{nm}^{R*}|^2 - |V_{ij}^L V_{nm}^{L*}|^2}{4m_\phi^4} + \frac{g^4}{16m_W^4} + \frac{g^2}{4m_W^2 m_V^2} \text{Re}(V_{ij}^R V_{nm}^{R*})$$

$$\rho = \frac{3b + 6c}{a + 4b + 6c}$$

$$\delta = \frac{3b' - 6c'}{-3a' + 4b' - 14c'}$$

$$\xi = \frac{3a' - 4b' + 14c'}{a + 4b + 6c}.$$