Low-Energy Effective Field Theory for Muon Experiments

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Abstract

This study presents a detailed construction of a low energy Effective Field Theory (EFT) framework that introduces novel charged bosons potentially deemed to be the SM and Dark sector portals. The newly charged mediators have coupling to photon and generally arbitrary couplings to fermions, both matter and dark matter particles, which therefore significantly contribute to hugely vast experimental processes. That adjusts the theoretical prediction of experimental values, particularly lepton decays within the scope of our study. Preliminary to this theoretical development is the verification of the anapole moment's disappearance at the one-loop order, guaranteeing its gauged invariant Lagrangian. By employing muon decay, we impose constraints on the newly introduced couplings. A pivotal aspect of our analysis involves the use of Michel parameters [1, 2] derived from polarized muon decay, which plays a key role in assessing the chirality effects predicted by the theory. Our findings indicate that the proposed couplings are too constrained to explain the muon g-2 anomaly if a universal coupling assumption is made. In addition, the given coupling pattern of UV models can also be applied to offer a better lower bound of detecting charged boson mass than collider searches that surpass and/or benchmarks with other current experimental searches.

Muon experiment

Muon experiments provide some of the most precise measurements in particle physics, essential for probing fundamental phenomena and observables. Key quantities such as the Fermi constant [3], Michel parameters [4], and the anomalous magnetic moment of the muon (g-2) [5] are crucial for testing the Standard Model (SM) and exploring new physics (NP) beyond it. The recent well-known discrepancy in the muon g-2 measurement suggests potential new physics. In this study, we construct a low-energy EFT model that includes a new charged boson, either scalar or vector, taking action in muon's decay, resolves most anomalies in the muon's weak interaction sector, and generalizes to other lepton weak processes. This EFT approach simplifies the parameter space and BSM operators, which predicts abundant phenomena as well as being a grounded benchmark for UV models with similar properties.

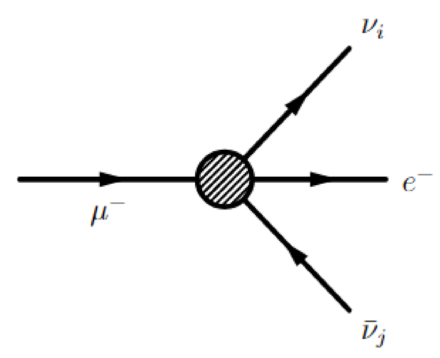

Figure 1: Muon decay process.

Effective Field Theory of new charged boson at low-energy scale

We souped up a generic EFT Lagrangian at the low-energy scale, typically satisfying Lorentz invariance and $U(1)_{\text{EM}}$ group, that is formulated in the following Lagrangian:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{S}} + \mathcal{L}_{\text{Y}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{F-}V^\pm} + \mathcal{L}_{V^\pm} + \mathcal{L}_\gamma, \tag{1}$$

Where each part is defined

$$\mathcal{L}_{\text{S}} = |D_\mu\phi|^2 - m_\phi^2\,|\phi|^2\,, \tag{2}$$
$$\mathcal{L}_{\text{F}} = i\bar\psi \not{D}\psi - m_f^2\bar\psi\psi, \tag{3}$$
$$\mathcal{L}_\gamma = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{4}$$
$$\mathcal{L}_{V^\pm} = -\frac{1}{2}\tilde{V}^+_{\mu\nu}\tilde{V}^{-\mu\nu} + m_V^2V^+_\mu V^{-\mu} + ieQ_VaF^{\mu\nu}V^+_\mu V^-_\nu\,, \tag{5}$$
$$\mathcal{L}_{\text{F-S}} = \bar\psi_i\left(Y^L_{ij}\hat{L} + Y^R_{ij}\hat{R}\right)\psi_j\phi + \text{h.c}, \tag{6}$$
$$\mathcal{L}_{\text{F-}V^\pm} = \bar\psi_i\left(V^L_{ij}\hat{L} + V^R_{ij}\hat{R}\right)\gamma^\mu\psi_jV^-_\mu + \text{h.c} \tag{7}$$

Where each Lagrangian part in the above

- Eq. 2 describes the dynamics of new charged scalar ϕ under $U(1)_{\text{EM}}$ group
- Eq. 3 describes the dynamics of new charged vector $V^\pm$ under $U(1)_{\text{EM}}$ group
- Eq. 6 describes the interaction of newly charged scalar with fermions
- Eq. 7 describes the interaction of newly charged vector with fermions

Importantly, the unknown factor "a" in Eq. 3 is assigned to **1** to preserve gauge invariant at all quantum levels. The couplings $Y^{R,L}_{ij}, V^{R,L}_{ij}$, which are general but simplified complex element matrices such that their left-right relative phases are $k\pi$ for the CP invariant condition, are our main task where using muon experiments to constrain the parameter space.

Michel parameters

The point-like interaction Fig. 2 in muon decay is governed by the 4-Fermi theory that is matched to the EFT couplings by the Fierz transformation. If the massless of the final states is supposed, then the differential decay rate of the muon is in the form:

$$d\Gamma = \frac{m_\mu^5G_F^2}{3*2^6*\pi^4}\left\{3(1-x) + 2\rho\left(\frac{4}{3}x-1\right) - \xi\cos\theta\left[(1-x) + 2\delta\left(\frac{4}{3}-1\right)\right]\right\}x^2dx, \tag{8}$$

where ρ, ξ, δ are Michel parameters [1] measured from experiment, which are defined [2].

References

[1] L. Michel. Interaction between four half spin particles and the decay of the μ meson. *Proc. Phys. Soc. A*, 63:514–531, 1950.

[2] Toichiro Kinoshita and Alberto Sirlin. Polarization of electrons in muon decay with general parity-nonconserving interactions. *Phys. Rev.*, 108:844–850, Nov 1957.

[3] Andreas Crivellin, Martin Hoferichter, and Claudio Andrea Manzari. Fermi constant from muon decay versus electroweak fits and cabibbo-kobayashi-maskawa unitarity. *Physical Review Letters*, 127(7), August 2021.

[4] Toichiro Kinoshita and Alberto Sirlin. Muon decay with parity nonconserving interactions and radiative corrections in the two-component theory. *Phys. Rev.*, 107:593–599, Jul 1957.

[5] etc. Bennett, G. W. Final report of the e821 muon anomalous magnetic moment measurement at bnl. *Physical Review D*, 73(7), April 2006.

Anomalous magnetic moment of the muon $(g-2)_\mu$

Figure 2: $(g-2)_\mu$ contributions from new particle.

$$\Gamma^\mu(q^2=0) = \gamma^\mu F_1 + \gamma^\mu\gamma_5G_1 + i\sigma^{\mu\nu}q_\nu F_2 + i\sigma^{\mu\nu}q_\nu\gamma_5G_2, \tag{9}$$

The matrix element for the process $l_j \to l_i\gamma$ in Fig. 2 emitting a real photon $\mathcal{M}^\mu = \bar{u}_i(p)\Gamma^\mu u_j(p+q)$, the interaction vertex Γ^μ can be formally decomposed into form factors in Eq 9.
Where F_1, G_1 associated to γ^μ are Dirac and Anapole form factors, gauge invariance property enforces $G_1 = 0$ though. While F_2, G_2 associated to $\sigma^{\mu\nu}q_\nu$ are Magnetic Dipole (MDM) and Electric Dipole Moment (EDM) respectively, got the expression in the expressions are under the approximation of the heavy charged boson

$$F_2^\phi = \frac{-e}{3.2^7m_\phi^2\pi^2}\left[(m_i+m_j)(Q_\phi-2Q_k)\left(Y_L^{*ki}Y_L^{kj} + Y_R^{*ki}Y_R^{kj}\right)\right.$$
$$\left. +6m_k\left(Q_\phi + 3Q_k + 2Q_k\log\left[\frac{m_k^2}{m_\phi^2}\right]\right)\left(Y_L^{*ki}Y_R^{kj} + Y_R^{*ki}Y_L^{kj}\right)\right] \tag{10}$$

and

$$F_2^V = \frac{e}{3.2^7m_V^2\pi^2}\left[(m_i+m_j)(Q_V-8Q_k+9aQ_V)\left(V_L^{*ki}V_L^{kj} + V_R^{*ki}V_R^{kj}\right)\right.$$
$$\left. -24m_kQ_V\left(V_L^{*ki}V_R^{kj} + V_R^{*ki}V_L^{kj}\right)\right]. \tag{11}$$

Numerical study in singly charge boson $Q_{V,\phi} = 1$

(1) Universal coupling

Couplings (V,Y) satisfy muon decay experiment constraints defined $(V,Y)L \equiv \frac{|(V,Y)^L|^2/m_V^2}{G_F}$, $(V,Y)R \equiv \frac{|(V,Y)^R|^2/m_V^2}{G_F}$. The blue is represented by G_F and the orange one is represented by $|\xi|$ constraints.

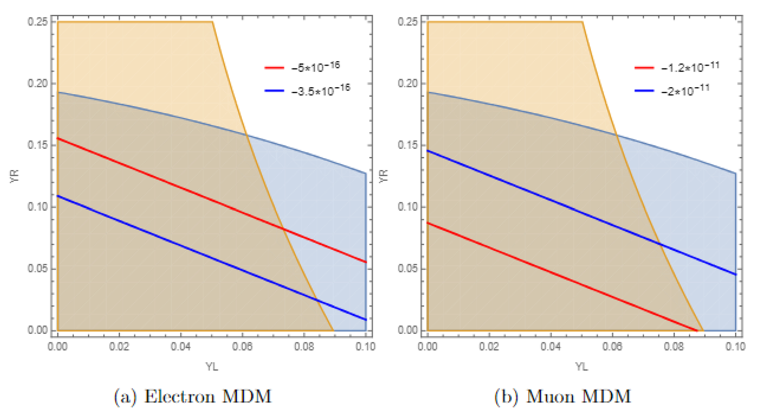

Figure 3: Muon MDM values from the new scalar.

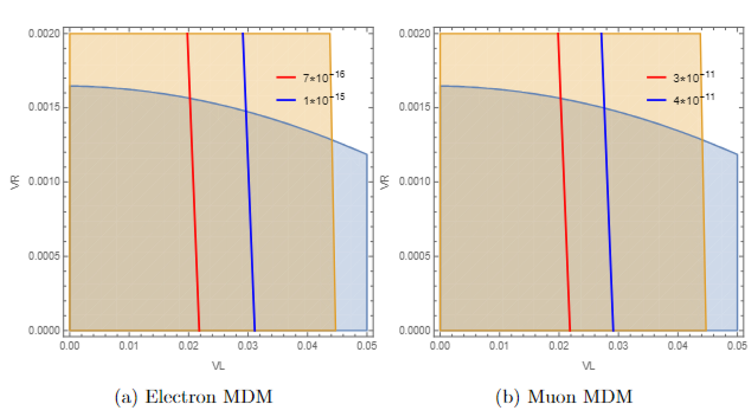

Figure 4: Muon MDM values from the new vector.

We can see that the MDM in the scalar case is highly constrained by the Fermi constant extracted from total Z decay, meanwhile, in the vector case, the MDM is inclined to be strongly constrained by the Michel parameter ξ. The allowed region generates merely a smidgen of MDM values, which is not enough to explain the anomaly

$$\Delta a_\mu = (2.74 \pm 0.73)\times 10^{-9} \tag{12}$$
$$\Delta a_e = (-88 \pm 36)\times 10^{-14}. \tag{13}$$

(2) Non-universal coupling

In the non-universality case, $\frac{|Y^R_{\mu\nu_\mu}|^2}{m_\phi^2} = \frac{|Y^L_{\mu\nu_\mu}|^2}{m_\phi^2} = 6\times10^{-10}$ is tuned to a very small value so as to get the safe regime in electron couplings space satisfying the electron MDM anomaly.

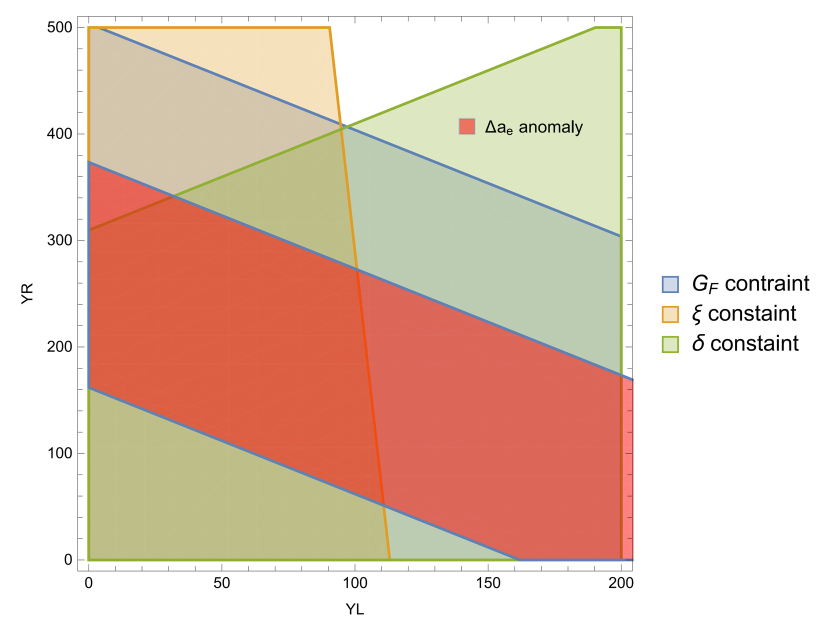

Figure 5: Electron MDM anomaly in the new scalar's non-universality coupling of the constrained region, we defined $YL \equiv \frac{|Y^L_{e\nu_e}|^2/m_\phi^2}{G_F}$, $YR \equiv \frac{|Y^R_{e\nu_e}|^2/m_\phi^2}{G_F}$.

The upper bound is no longer constrained by G_F as the universality case, it is instead from the Michel parameter δ constraint. That emphasizes the importance of Michel parameters in probing and constraining the NP.
For the vector case, then we chose a benchmark point in the muon coupling regime that satisfies the muon MDM anomaly and then see how the electron couplings are constrained.

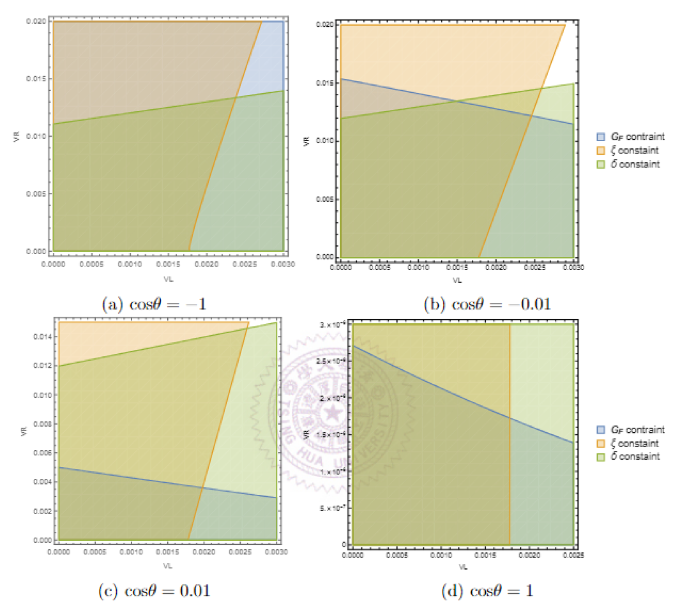

Figure 6: The new vector non-universality coupling's constrained region, we defined $VL \equiv \frac{|V^L_{e\nu_e}|^2/m_V^2}{G_F}$, $VR \equiv \frac{|V^R_{e\nu_e}|^2/m_V^2}{G_F}$. The angle θ is the relative phase between two couplings, $V^R_{e\nu_e}$ and $V^{*R}_{\mu\nu_\mu}$.

(3) Coupling pattern from UV models

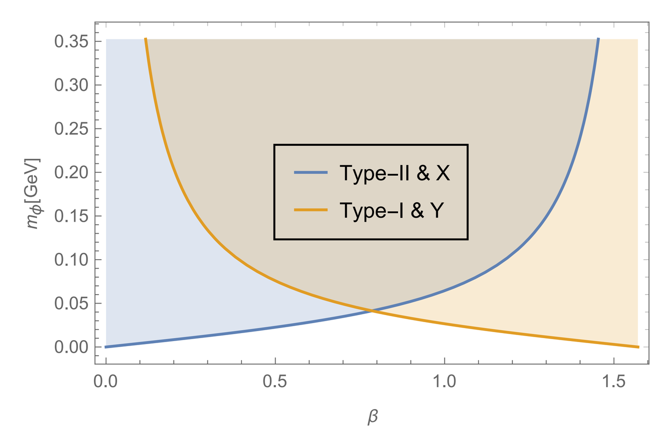

Figure 7: 2HDM mass of charged Higgs in term of mixing angle β

If considering Sequential Standard Model (SSM) models, where the new extended gauge fields share similar properties to that of the SM, from Fig 3, the lower bound of the new gauge boson mass is extracted. The relatively strong **lower limits on the mass scale for the W' boson with only left-handed and only right-handed couplings are 3.3 TeV and 637 GeV**, respectively. These limits are **compared to the 2016 PDG values of 3.7 TeV for left-handed coupling and 715 GeV for right-handed coupling**.