Neutrino Mass Generation Models

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1 Neutrino mass

Neutrino has non-zero but very small mass compared to other ferminos: $m_\nu \neq 0 \ll m_f$
There are possibilities to explain this:

- Possibility 0: Neutrinos are Dirac fermions, but $m_\nu \simeq 10^{-12}\, v_{\mathrm{EW}}$
  $\Rightarrow$ Yukawa coupling is unnaturally small: $\lambda_{\mathrm{Yuk},\nu} \simeq 10^{-12}$
- Possibility 1: Seesaw models I, II and III
  - Tree level mechanisms.
- Possibility 2: Radiative mass generation mechanisms
  - Loop level mechanisms.
  - Loop suppression of mass: $\left(\frac{1}{16\pi^2}\right)$ * (number of loops)
  - Product of several small coupling constants

2 Seesaw Mechanisms

2.1 Lepton terms in the Lagrangian

EW gauge: SU(2)$_L$$\otimes$U(1)$_Y$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim 2(-1) \tag{1}$$

$$e_R \sim 1(-2) \tag{2}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim 2(+1) \tag{3}$$

where numbers are SU(2)$_L$ (U(1)$_Y$) quantum numbers, and

$$< \phi >= \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{4}$$

Note that the electric charge is given by

$$Q = I_{3L} + Y/2, \;\; I_{3L} = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} = \frac{1}{2}\sigma_3 \tag{5}$$

so,

$$\begin{aligned} &\nu_L : Q = 1/2 + (-1/2) = 0 \\ &e_L : Q = -1/2 + (-1/2) = -1 \\ &e_R : Q = 0 + (-2/2) = -1 \\ &\phi^+ : Q = 1/2 + (+1/2) = +1 \\ &\phi^0 : Q = -1/2 + (+1/2) = 0 \end{aligned}$$

The Yukawa term in the Lagrangian is

$$\begin{aligned}
&\mathcal{L}_{Yuk} = \lambda_e \bar{l_L}\phi e_R + \text{H.C.} \\
&= \lambda_e(\bar{\nu_L}, \bar{e_L}) \begin{pmatrix} 0 \\ v \end{pmatrix} e_R + \text{H.C.} \\
&= (\lambda_e v)\bar{e_L} e_R + \text{H.C.} \\
&= m_e \bar{e} e
\end{aligned} \tag{6}$$

where m_e is the charged lepton mass. Note that the hyper change of the Yukawa term is

$$Y = +1 + 1 - 2 = 0$$

Now, let's introduce a right handed neutrino:

$$\nu_R \sim 1(0) \tag{7}$$

This introduce additional terms in the Yukawa Lagranginan

$$\begin{aligned}
&\mathcal{L}_{Yuk} = ... + \lambda_\nu \bar{l_L}\tilde{\phi}\nu_R + \text{H.C.} \\
&\Rightarrow m_\nu = \lambda_\nu v
\end{aligned} \tag{8}$$

where

$$\tilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \tag{9}$$

and m_ν is the Dirac neutrino mass (Possibility 0).

2.1.1 Majorana mass term

Mass term couples left-handed fermion to right-handed one. For example, the electron mass term is $(\lambda_e v)\bar{e_L} e_R$. Since,

$$(\nu_R)^c \equiv C\bar{\nu_R}^T = \text{charge conjugate of } \nu_R \tag{10}$$

is left-handed, a possible mass term is

$$\frac{1}{2} M_R \bar{\nu_R} (\nu_R)^c \tag{11}$$

However, this term violates lepton number conservation by 2 units.
Similarly, terms like $\bar{l_L}(l_L)^c$ and $\bar{e_R}(e_R)^c$ violate charge conservation.
This can be seen as follows. Consider a transformation

$$l_{L,R} \to e^{i\theta} l_{L,R} \tag{12}$$

Terms like $\bar{l_L}\phi e_R$ and $\bar{l_L}\tilde{\phi}\nu_R$ are invariant, but

$$\bar{\nu_R}(\nu_R)^c \to e^{-2i\theta} \bar{\nu_R}(\nu_R)^c \tag{13}$$

3 Seesaw Models

As given in Equation (8), possible mass terms for neutrino have the following forms:

$$\bar{\nu_L}\phi\nu_R \quad \text{or} \quad \bar{\nu_{\mathrm{L}}}\nu_{\mathrm{L}}^{\mathrm{c}}\Delta \tag{14}$$

here, ν_R is a new fermion and Δ is a new scalar.
Since ν_L and ϕ are SU(2) doublets:

$$2 \otimes 2 = 1 \oplus 3$$

means that the new fermion must be either SU(2) singlet or triplet. This is because

$$1 \otimes 1 = 1 \qquad 3 \otimes 3 = 1 \oplus 3 \oplus 5$$

can produce a singlet, but additional doublet

$$1 \otimes 2 = 2 \qquad 3 \otimes 2 = 2 \oplus 4$$

cannot produce a singlet. The type of seesaw models, thus, defined as

- SU(2) singlet ν_R: Type I seesaw model
- SU(2) triplet f_R: Type III seesaw model

Similarly, the new scalar must be either SU(2) single or triplet.

- SU(2) singet scalar violates charge conservation.
- SU(2) triplet Δ: Type II seesaw model

3.1 Type I Seesaw Model

The Yukawa Lagrangian is

$$\begin{aligned} &\mathcal{L}_{Yuk} = \lambda_e \bar{l_L}\phi e_R + \lambda_\nu \bar{l_L}\tilde{\phi}\nu_R + \text{H.C.} \\ &\Rightarrow (\lambda_\nu v)\bar{\nu_L}\nu_R + (\lambda_\nu v)\bar{\nu_R}\nu_L \end{aligned} \tag{15}$$

If we allow lepton number violation, there is nothing to prevent having Majorana mass terms in the Lagrangian. Then

$$\mathcal{L}_{Yuk} = (\lambda_\nu v)\bar{\nu_L}\nu_R + \frac{1}{2}M_R\bar{\nu_R}(\nu_R)^c + (\lambda_\nu v)\bar{\nu_R}\nu_L + \frac{1}{2}M_R\bar{(\nu_R)}^c\nu_R \tag{16}$$

Using

$$\bar{\nu_L}\nu_R = \bar{(\nu_R)}^c(\nu_L)^c \tag{17}$$

The above Lagrangian is

$$\mathcal{L}_{Yuk} = \frac{1}{2}\begin{pmatrix} \bar{\lambda_L} & \bar{(\lambda_R)}^c \end{pmatrix}\begin{pmatrix} 0 & \lambda_\nu v \\ \lambda_\nu v & M_R \end{pmatrix}\begin{pmatrix} (\lambda_L)^c \\ \nu_R \end{pmatrix} + \text{H.C.} \tag{18}$$

where

- ν_L: Left-handed neutrino
- $(\nu_R)^c$: Left-handed anti-neutrino
- $(\nu_L)^c$: Right-handed anti-neutrino
- ν_R: Right-handed neutrino

The mass matrix can be diagonalized to give two eigenvalues:

- Neutrino mass: $-\frac{(\lambda_\nu v)^2}{M_R} \equiv -m_\nu$
- Heavy neutral lepton mass: M_R

So, when M_R gets heavier the neutrino mass m_ν gets lighter like a seesaw. The Yukawa coupling λ_ν does not have to be unnaturally small. If $\lambda_\nu \sim 1$, then $M_R \sim 10^{14}$GeV.
The eigenvectors of the mass matrix are

- $\nu_L - \frac{\lambda_\nu v}{M_R}(\nu_R)^c \simeq \nu_L$: neutrino
- $\nu_R + \frac{\lambda_\nu v}{M_R}(\nu_L)^c \simeq \nu_R$: sterile neutrino

3.2 Type III Seesaw Model

New SU(2) triplet fermion:

$$f_R = \begin{pmatrix} f^+ \\ f^0 \equiv \nu_R \\ f^- \end{pmatrix} \sim 3(0) \tag{19}$$

The charges, $Q = I_{3L} + Y/2$, are

- f^+: $Q = +1 + 0 = +1$
- f^0: $Q = 0 + 0 = 0$
- f^-: $Q = -1 + 0 = -1$

The Yukawa Lagrangian is

$$\mathcal{L}_{Yuk} = \lambda \bar{l_L} \tilde{\phi} f_R + \frac{1}{2} M_f \bar{f_R} (f_R)^c + \text{H.C.} \tag{20}$$

The hyper charge of the first term is

$$Y = (+1) + (-1) + 0 = 0$$

By diagonalizing the mss matrix, the neutrino mass is

$$m_\nu = \frac{(EW)^2}{M_f} \tag{21}$$

3.3 Type II Seesaw Model

New SU(2) triplet scalar

$$\Delta = \begin{pmatrix} \Delta^0 \\ \Delta^- \\ \Delta^{--} \end{pmatrix} \sim 3(-2) \tag{22}$$

The charges, $Q = I_{3L} + Y/2$, are

- Δ^0: $Q = +1 + (-1) = 0$
- Δ^-: $Q = 0 + (-1) = -1$
- Δ^{--}: $Q = -1 + (-1) = -2$

The Yukawa Lagrangian is

$$\mathcal{L}_{Yuk} = \frac{1}{2}\lambda_\Delta \bar{l_L}(l_L)^c\Delta + \text{H.C.} \tag{23}$$

$$= \frac{1}{2}\lambda_\Delta \bar{\nu_L}(\nu_L)^c\Delta^0 + \frac{1}{2}\lambda_\Delta \bar{e_L}(\nu_L)^c\Delta^- + \frac{1}{2}\lambda_\Delta \bar{e_L}(e_L)^c\Delta^{--} + ... \tag{24}$$

$$= \frac{1}{2}\lambda_\Delta \bar{\nu_L}(\nu_L)^c < \Delta^0 > + \text{interacation terms} + ... \tag{25}$$

The mass is

$$m_L = \lambda_\Delta < \Delta^0 > \; \ll \; < \phi^0 > \tag{26}$$

This can be achieved by a seesaw mechanism.
Potential is

$$V = -\mu_\phi^2\phi^\dagger\phi + \lambda_\phi(\phi^+\phi)^2 + \mu_\Delta^2\Delta^\dagger\Delta + \lambda_\Delta(\Delta^\dagger\Delta)^2 + \lambda_{\phi\Delta}\phi^\dagger\phi\Delta^\dagger\Delta + (a\phi\phi\Delta + \text{H.C.}) \tag{27}$$

The $\phi\phi\Delta$ term

- Weak hypercharge: $(-1) + (-1) + 2 = 0$
- Lepton number: $0 + 0 + (-2) = -2$; violates lepton number by 2

When EW symmetry is broken

$$V(\phi \rightarrow < \phi >) = \mu_\Delta^2\Delta^\dagger\Delta + \lambda_\Delta(\Delta^\dagger\Delta)^2 + (\lambda_{\phi\Delta} < \phi >^2)\Delta^\dagger\Delta + (a < \phi >^2 \Delta + \text{H.C.}) \tag{28}$$

This potential has a minimum at

$$< \Delta^0 > \sim \frac{a < \phi >^2}{\mu_\Delta^2} \tag{29}$$

If the mass of Δ

$$m_\Delta \sim \mu_\Delta^2 \tag{30}$$

is large, the $< \Delta^0 >$ can be small so that $\lambda_\Delta < \Delta^0 > \; \ll \; < \phi^0 >$ is possible for the coupling with $\lambda_\Delta \sim 1$

3.4 Additional SU(2) singlet scalar?

Consider a Yukawa term

$$\bar{l_L}(l_L)^c\Delta \tag{31}$$

Since, the total hyper charge needs to be zero

$$(+1) + (+1) + Y_\Delta = 0 \Rightarrow Y_\Delta = -2 \tag{32}$$

Also total charge needs to be zero

$$(-1) + (+1) + Q_\Delta = 0 \Rightarrow Q_\Delta = 0 \tag{33}$$

However

$$Q = I_{3L} + Y/2 = 0 + (-1) = -1 \tag{34}$$

So, SU(2) singlet does not work.

4 Radiative generation of neutrino mass

Small neutrino masses can be generated with loop diagrams without right-handed neutrino or Majorana terms. Since, the loops make the neutrino mass smaller, we don't need seesaw mechanism.

- Zee model: one loop; excluded by experiments.
- Zee-Babu model: two loop; Two new scalar h^+ and k^{++}, violates lepton number conservation.
- Coloured Zee-Babu model: two loop; A new diquark and a new leptoquark.

This is a small part of possible models.

4.1 Zee-Babu model

This model introduces two new scalar

$$h^+ \sim 1(+2) \qquad k^- = (h^+)^* \sim 1(-2) \tag{35}$$

and

$$k^{++} \sim 1(+4) \qquad k^{--} \sim 1(-4) \tag{36}$$

These couple to l_L and e_R

$$\bar{l_L}(l_L)^c h^- \qquad \bar{e_R}(e_R)^c k^{--} \tag{37}$$

So, decays such as

$$h^- \to e_L^- \nu_L \qquad k^{--} \to e_R^- e_R^-$$

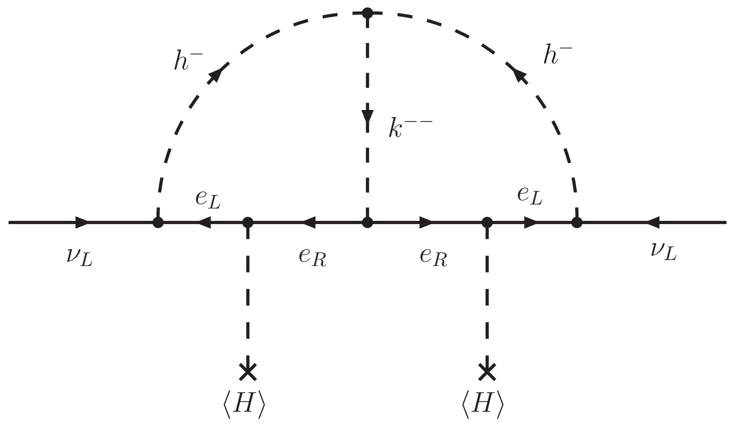

Figure 1: Two loop diagram to generate neutrino mass.

are possible. Then, naturally, the lepton numbers of h^- and k^{--} are both $L = 2$. The model also introduces lepton number violating terms

$$V = ... + \mu k^{--} h^+ h^+ + \text{H.C.} \tag{38}$$

in a potential. This allows a decay like

$$k^{--} \to h^- h^-$$

The left hand side is $L = 2$, but the right hand side is $L = 2 + 2 = 4$. These lepton number violating terms allows to generate neutrino mass with two loops as given in Figure 1.

5 Left-Right Symmetry Model

The Standard Model: $SU(3) \times SU(2)_L \times U(1)_Y$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2)(-1) \quad \nu_R \sim (1,1)(0) \quad r_R \sim (1,1)(-2) \tag{39}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)(1/3) \quad u_R \sim (3,1)(4/3) \quad d_R \sim (3,1)(-2/3) \tag{40}$$

and Higgs

$$\phi \sim (1,2)(1) \quad \widetilde{\phi} = i\tau_2 \phi^* \tag{41}$$

The Left-Right Symmetry Model (LRSM): $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
This symmetry breaks into Standard Model symmetry $SU(3) \times SU(2)_L \times U(1)_Y$ when Higgs field gets a vacuum value v_R. The Y is given by

$$Y = 2I_{3R} + (B - L) \tag{42}$$

This symmetry further breaks into $SU(3) \times U(1)_Q$, when Higgs field gets a vacuum value v_L. The Q is given by

$$Q = I_{3L} + \frac{Y}{2} = I_{3L} + I_{3R} + \frac{B-L}{2} \tag{43}$$

The $v_R \gg v_L$ and W_R and Z' are heavy.

5.1 Field Contents

The Left-Right Symmetry Model (LRSM): $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2,1)(-1) \quad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2,1)(1/3) \tag{44}$$

$$l_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (1,1,2)(-1) \quad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (3,1,2)(1/3) \tag{45}$$

and

$$\begin{aligned} \nu_R: &\quad Y = 2I_{3R} + (B-L) = 2 \times 1/2 + (-1) = 0 \\ e_R: &\quad Y = 2 \times (-1/2) + (-1) = -2 \\ u_R: &\quad Y = 1 + 1/3 = 4/3 \\ d_R: &\quad Y = -1 + 1/3 = -2/3 \end{aligned}$$

The Lagrangian is given by exchanging left-handed and right-handed fields

$$l_L \leftrightarrow l_R \quad q_L \leftrightarrow q_R \quad W_L^\mu \leftrightarrow W_R^\mu \quad Z \leftrightarrow Z'$$

Discrete LR symmetry: $\Delta_L \leftrightarrow \Delta_R$
Parity invariant: $P : \vec{x} \to \vec{x}' = -\vec{x},\ t \to t' = t$

$$\begin{aligned} P : \psi(\vec{x}, t) \to \psi'(\vec{x}', t') = \gamma^0 \psi(\vec{x}, t) \\ \Longrightarrow \psi_L \to \gamma^0 \psi_R \end{aligned}$$

5.2 Higgs Sector

Higgs bidoublet

$$\Phi \sim (1,2,2)(0) \tag{46}$$

Electric charge

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} = \begin{pmatrix} 1/2 - 1/2 & 1/2 + 1/2 \\ -1/2 - 1/2 & -1/2 + 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{47}$$

Thus

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \tag{48}$$

Also

$$\widetilde{\Phi} = \tau_2 \Phi^* \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \phi_1^{0*} & \phi_2^- \\ \phi_1^+ & \phi_2^{0*} \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} \phi_2^{0*} & -\phi_1^+ \\ -\phi_2^- & \phi_1^{0*} \end{pmatrix} \tag{49}$$

$SU(2)$ transformation:

$$l_L \to u_L l_L \qquad u_L \in SU(2)_L \tag{50}$$

$$l_R \to u_R l_R \qquad u_R \in SU(2)_R \tag{51}$$

For Higgs field:

$$\bar{l_L} \Phi l_R \to \bar{l_L} u_L^\dagger u_L \Phi u_R^\dagger u_R l_R \tag{52}$$

$$\Longrightarrow \Phi \to u_L \Phi u_R^\dagger \tag{53}$$

5.3 Yukawa Term

Yukawa terms in the Lagrangian are

$$\mathcal{L}_{\text{YUK}} = \lambda_1 \bar{l_L} \Phi l_R + \text{H.C.} \tag{54}$$

where

$$\begin{pmatrix} \bar{\nu_L} & \bar{r_R} \end{pmatrix} \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} = \bar{\nu_L} \phi_1^0 \nu_R + \bar{e_L} \phi_1^- \nu_R + \bar{\nu_L} \phi_2^+ r_R + \bar{e_L} \phi_2^0 r_R \tag{55}$$

The first term gives the neutrino mass term and the last gives the charged lepton mass term.

If we add $\widetilde{\Phi}$ terms, for leptons

$$\mathcal{L}_{\text{YUK}} = \lambda_1 \bar{l_L} \Phi l_R + \widetilde{\lambda_1} \bar{l_L} \widetilde{\Phi} l_R + \text{H.C.} \tag{56}$$

Then

$$m_\nu^D = \lambda_1 v_1 + \widetilde{\lambda_1} v_2^* \tag{57}$$

$$m_e = \lambda_1 v_2 + \widetilde{\lambda_1} v_1^* \tag{58}$$

For quarks

$$\mathcal{L}_{\text{YUK}} = \lambda_2 \bar{q_L} \Phi q_R + \widetilde{\lambda_2} \bar{q_L} \widetilde{\Phi} q_R + \text{H.C.} \tag{59}$$

Then

$$m_u = \lambda_2 v_1 + \widetilde{\lambda_2} v_2^* \tag{60}$$

$$m_d = \lambda_2 v_2 + \widetilde{\lambda_2} v_1^* \tag{61}$$

5.3.1 Majorana Neutrino Mass

New scalar triplet (Higgs sector)

$$\Delta_R \sim (1,1,3)(2) \qquad \Delta_L \sim (1,3,1)(2) \tag{62}$$

Their charges are

$$Q_{\Delta_{R,L}} = I_{3L} + I_{3R} + \frac{B-L}{2} = \begin{pmatrix} +1+1 \\ 0+1 \\ -1+1 \end{pmatrix} = \begin{pmatrix} ++ \\ + \\ 0 \end{pmatrix} \tag{63}$$

Thus

$$\Delta_{R,L} = \begin{pmatrix} \Delta^{++}_{R,L} \\ \Delta^{+}_{R,L} \\ \Delta^{0}_{R,L} \end{pmatrix} \tag{64}$$

Mass terms are

$$h_R \overline{(l_R)^c} l_R \Delta_R + h_L \overline{(l_L)^c} l_L \Delta_L \tag{65}$$

Then

$$m_\nu = \begin{pmatrix} h_L < \Delta^0_L > & m^D_\nu \\ (m^D_\nu)^T & h_R < \Delta^0_R > \end{pmatrix} \tag{66}$$

where

$$< \Delta^0_R > \ \gg \ < \Phi > \ \gg \ < \Delta^0_L > \tag{67}$$

The mass $h_L < \Delta^0_L >$, is small.

- $h_L < \Delta^0_L >= 0$: Type I seesaw
- $h_L < \Delta^0_L >\neq 0$: Type II seesaw

The Majorana mass $h_R < \Delta^0_R >$ is large.
The Δ^0_R is responsible for the symmetry breaking: $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$