Two-Higgs Doublet Models

Highlights:

- Basic constraints
- The model
- Spectrum
- Some preciison constraints: $g-2$, $b \to s\gamma$, $\overline{B^0} - B^0$ mixing, and ρ parameter.
- Model III
- $b \to s\gamma$

I. BASIC CONSTRAINTS

The first constraint is the experimental value of

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_{\rm w}} \simeq 1$$

equals to 1 very closely. The structure of the Higgs sector will affect the ρ parameter. Higgs doublets and singlets will satisfy $\rho = 1$ automatically. But it is not true for an arbitrary Higgs representation. The general formula for arbitrary representations is

$$\rho = \frac{\sum_{T,Y} [4T(T+1) - Y^2] |V_{T,y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 |V_{T,Y}|^2}$$

where $V_{T,Y} = \langle \phi_{T,Y} \rangle$ denotes the VEV of each neutral Higgs field, T is the total $SU(2)_L$ isospin and Y is the hypercharge. The constant $c_{T,Y}$ is

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation} \\ \frac{1}{2}, & (T,Y) \in \text{real representation} \end{cases}$$

It is easy to see that for arbitrary $V_{T,Y}$ the condition

$$4T(T+1) - Y^2 = 2Y^2 \qquad \Leftrightarrow \qquad (2T+1)^2 - 3Y^2 = 1$$

can make sure $\rho = 1$.

Consider an example of Higgs triplet of $T = 1, Y = 0$ OR $T = 0, Y = 2$

$$\begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}, \qquad \begin{pmatrix} \phi^{++} \\ \phi^+ \\ \phi^0 \end{pmatrix}$$

Obviously, the triplets do not satisfy $(2T+1)^2 - 3Y^2 = 1$ condition. One can satisfy the $\rho = 1$ within experimental uncertainty by restricting the VEV of the triplet (use the current value from PDG):

$$1.0002\ ^{+0.0007}_{-0.0004} = \frac{8|V_{1,0}|^2 + 2|V_{1/2,1}|^2}{2|V_{1/2,1}|^2}$$

which gives

$$\frac{|V_{1,0}|}{|V_{1/2,1}|} \leq 0.03$$

The second constraint is the flavor-changing neutral current:

$$s \leftrightarrow d, \qquad c \leftrightarrow u$$

A theorem due to Glashow and Weinberg stated that tree-level FCNC mediated by Higgs bosons will be absent if all fermions of a given electric charge couple to no more than one Higgs doublet. There are two natural choices:

- Model I: of 2HDM is that one of the Higgs doublets do not couple to fermions at all;
- Model II: of 2HDM is that the $Y = 1$ doublet couples to the up-type fermions while the $Y = -1$ doublet couples to the down-type fermions and the charged leptons. This is also the basis for the MSSM.

II. THE 2HDM MODEL

There are two complex $Y = 1$ doublets, ϕ_1 and ϕ_2 with the following Higgs potential

$$\begin{aligned} V(\phi_1, \phi_2) &= \lambda_1(\phi_1^\dagger\phi_1 - v_1^2)^2 + \lambda_2(\phi_2^\dagger\phi_2 - v_2^2)^2 + \lambda_3\left[(\phi_1^\dagger\phi_1 - v_1^2) + (\phi_2^\dagger\phi_2 - v_2^2)\right]^2 \\ &+ \lambda_4\left[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right]^2 \\ &+ \lambda_5\left[\Re e(\phi_1^\dagger\phi_2) - v_1 v_2 \cos\xi\right]^2 + \lambda_6\left[\Im m(\phi_1^\dagger\phi_2) - v_1 v_2 \sin\xi\right]^2 \end{aligned} \tag{1}$$

Some comments are in order here.

- All λs are real. This potential is the most general with respect to gauge invariance.
- For a large range of parameters correct pattern of EWSB is guaranteed. The minimum of the potential occurs at

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

which breaks the $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$.

- If $\sin\xi \neq 0$ then CP is violated in the Higgs sector. But if $\lambda_5 = \lambda_6$ the last two terms can be combined into a single one $|\phi_1^\dagger \phi_2 - v_1 v_2 e^{i\xi}|^2$ and the phase can be removed by a redefinition of one of the fields, e.g.,

$$\phi_2 \longrightarrow \phi_2 e^{i\xi}$$

which does not change any other terms in the potential.

- We set $\xi = 0$, there will be CP violation in the Higgs sector.
- Define the ratio of the VEVs

$$\tan\beta = \frac{v_2}{v_1}$$

III. SPECTRUM

There are 8 d.o.f. in two complex doublets. 3 of which will be eaten to become the longitudinal components of the gauge bosons. From the potential in Eq. (1), one can determine the spectrum. We substitute

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

into the potential.

- Charged Higgs: The mass terms of the charged fields are

$$\lambda_4 (\phi_1^- \;\; \phi_2^-) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

It can be diagonalized by

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

After subsituting we obtain

$$\lambda_4 (G^- \;\; H^-) \begin{pmatrix} 0 & 0 \\ 0 & v_1^2 + v_2^2 \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

The charged Higgs mass is

$$m_{H^+}^2 = \lambda_4 (v_1^2 + v_2^2)$$

- Pseudoscalar: Again look for the mass terms for $\Im m \phi_1^0$ and $\Im m \phi_2^0$:

$$\lambda_6 (\phi_1^{0,i} \;\; \phi_2^{0,i}) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^{0,i} \\ \phi_2^{0,i} \end{pmatrix}$$

We rotate them by the same angle as the charged fields:

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^{0,i} \\ \phi_2^{0,i} \end{pmatrix}$$

Then the mass term becomes

$$\frac{\lambda_6}{2} (G^0 \;\; A^0) \begin{pmatrix} 0 & 0 \\ 0 & v_1^2 + v_2^2 \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

The G^0 is the goldstone boson. The pseudoscalar mass is

$$m_A^2 = \lambda_6 (v_1^2 + v_2^2)$$

- Neutral Higgs bosons: We rotate the real part of ϕ_1^0 and ϕ_2^0 as

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^{0,r} - v_1 \\ \phi_2^{0,r} - v_2 \end{pmatrix}$$

where it is assumed $m_{H^0} > m_{h^0}$. The mass matrix was

$$(\phi_1^{0,r} - v_1 \;\; \phi_2^{0,r} - v_2) \begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2 \lambda_5 & (4\lambda_3 + \lambda_5) v_1 v_2 \\ (4\lambda_3 + \lambda_5) v_1 v_2 & 4v_2^2(\lambda_2 + \lambda_3) + v_1^2 \lambda_5 \end{pmatrix} \begin{pmatrix} \phi_1^{0,r} - v_1 \\ \phi_2^{0,r} - v_2 \end{pmatrix}$$

The masses can be obtained as

$$m^2_{H^0,h^0} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(M_{11}-M_{22})^2 + 4M_{12}^2}\right]$$

and the mixing angle is

$$\sin 2\alpha = \frac{2M_{12}}{\sqrt{(M_{11}-M_{22})^2 + 4M_{12}^2}}, \qquad \cos 2\alpha = \frac{M_{11}-M_{22}}{\sqrt{(M_{11}-M_{22})^2 + 4M_{12}^2}},$$

- So totally, we have 5 physical Higgs bosons: 2 charged, 2 CP even, and 1 CP odd.

A. Model I

In model I, all fermions couple to one of the Higgs doublet only, let us assume it is the ϕ_2. Consider

$$\mathcal{L} = -y_u \overline{Q}_L u_R \tilde{\phi}_2 - y_d \overline{Q}_L d_R \phi_2$$

where $\tilde{\phi}_2 = i\tau^2 \phi_2^*$. We can put in the VEV of $\langle\phi_2\rangle = v\sin\beta$. So we have

$$m_u = y_u v \sin\beta, \qquad m_d = y_d v \sin\beta$$

We obtain the Yukawa interactions

$$\begin{aligned}
\mathcal{L} &= -\frac{g m_u}{2 m_w s_\beta}\bar{u}u(\sin\alpha H^0 + \cos\alpha h^0) + \frac{g m_u \cot\beta}{2m_w}\bar{u}i\gamma^5 u A^0 \\
&\quad -\frac{g m_d}{2 m_w s_\beta}\bar{d}d(\sin\alpha H^0 + \cos\alpha h^0) - \frac{g m_d \cot\beta}{2m_w}\bar{d}i\gamma^5 d A^0 \\
&\quad +\frac{g\cot\beta}{\sqrt{2}m_W}\left[\bar{d}(m_u P_R - m_d P_L)u\, H^- + \bar{u}(m_u P_L - m_d P_R)d\, H^+\right] \qquad (2)
\end{aligned}$$

where we wrote $s_\beta \equiv \sin\beta$, $P_L, R = (1 \mp \gamma^5)/2$. Note that in this normalization $m_w = gv/\sqrt{2}$.

B. Model II

In model II, up-type fermions couple to ϕ_1 while down-type fermions couple to ϕ_2:

$$\mathcal{L} = -y_u \overline{Q}_L u_R \tilde{\phi}_2 - y_d \overline{Q}_L d_R \phi_1$$

We obtain the Yukawa interactions

$$\begin{aligned}
\mathcal{L} &= -\frac{g m_u}{2 m_w s_\beta}\bar{u}u(\sin\alpha H^0 + \cos\alpha h^0) + \frac{g m_u \cot\beta}{2m_w}\bar{u}i\gamma^5 u A^0 \\
&\quad -\frac{g m_d}{2 m_w c_\beta}\bar{d}d(\cos\alpha H^0 - \sin\alpha h^0) + \frac{g m_d \tan\beta}{2m_w}\bar{d}i\gamma^5 d A^0 \\
&\quad +\frac{g}{\sqrt{2}m_W}\left[\bar{d}(m_u \cot\beta P_R + m_d \tan\beta P_L)u\, H^- + \bar{u}(m_u \cot\beta P_L + m_d \tan\beta P_R)d\, H^+\right] \qquad (3)
\end{aligned}$$

IV. PRECISION CONSTRAINTS AND MODEL II

We take the parameter space of the model as given by a set of six Higgs-sector parameters

$$m_h\ ,\quad m_H\ ,\quad m_A\ ,\quad m_{H^\pm}\ ,\quad \tan\beta\ ,\quad \text{and}\quad \alpha\ .$$

In the general 2HDM, there are enough parameters in the Higgs potential such that all the above 6 parameters can be regarded as free. This is in constrast to the one with SUSY, which only has two parameters in the Higgs sector. The Yukawa couplings of h, H, and A to up- and down-type quarks are given by, with a common factor of $-igm_f/2M_W$,

	$t\bar{t}$	$b\bar{b}$	$\tau^-\tau^+$
h:	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$
H:	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos\alpha/\cos\beta$
A:	$-i\cot\beta\,\gamma_5$	$-i\tan\beta\,\gamma_5$	$-i\tan\beta\,\gamma_5$

while the charged Higgs H^- couples to t and $\bar{b}$ via

$$\bar{b}tH^-\ :\qquad \frac{ig}{2\sqrt{2}M_W}\,[m_t\cot\beta\,(1+\gamma_5)+m_b\tan\beta\,(1-\gamma_5)]\ .$$

Other relevant couplings in our study are those to gauge bosons, as given by,

$$\begin{aligned}
hZZ\ &:\quad ig\ M_Z\frac{\sin(\beta-\alpha)}{\cos\theta_W}\ g^{\mu\nu}\\
HZZ\ &:\quad ig\ M_Z\frac{\cos(\beta-\alpha)}{\cos\ \theta_W}\ g^{\mu\nu}\\
hAZ\ &:\quad g\ \frac{\cos(\beta-\alpha)}{2\cos\theta_W}\ (p-p')^\mu\\
HAZ\ &:\quad -g\ \frac{\sin(\beta-\alpha)}{2\cos\theta_W}\ (p-p')^\mu\\
H^+H^-Z\ &:\quad -ig\ \frac{\cos 2\theta_W}{2\cos\ \theta_W}\ (p-p')^\mu\ ,
\end{aligned}$$

where $p(h,H,H^+)$ and $p'(A,H^-)$ are the 4-momenta going into the vertex.

A. $b\to s\gamma$

The major contribution comes from the charged-Higgs loop of the 2HDM, in addition to the $W-t$ loop in the SM. The detail formulism can be found in Ref. [1]. The effective

Hamiltonian for $B \to X_s\gamma$ at a factorization scale of order $O(m_b)$ is given by

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \Big[\sum_{i=1}^{6} C_i(\mu) Q_i(\mu) + C_{7\gamma}(\mu) Q_{7\gamma}(\mu) + C_{8G}(\mu) Q_{8G}(\mu) \Big] . \tag{4}$$

The operators Q_i can be found in Ref.[1], of which the Q_1 and Q_2 are the current-current operators and $Q_3 - Q_6$ are QCD penguin operators. $Q_{7\gamma}$ and Q_{8G} are, respectively, the magnetic penguin operators specific for $b \to s\,\gamma$ and $b \to s\,g$. The decay rate of $B \to X_s\gamma$ normalized to the experimental semileptonic decay rate is given by

$$\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e\bar{\nu}_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\,\alpha_{\rm em}}{\pi f(m_c/m_b)} |C_{7\gamma}(m_b)|^2 , \tag{5}$$

where $f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$. The Wilson coefficient $C_{7\gamma}(m_b)$ is given by

$$C_{7\gamma}(\mu) = \eta^{\frac{16}{23}} C_{7\gamma}(M_W) + \tfrac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_{8G}(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i} , \tag{6}$$

where $\eta = \alpha_s(M_W)/\alpha_s(\mu)$. The a_i's an d h_i's can be found in Ref. [1]. The coefficients $C_i(M_W)$ at the leading order in 2HDM II are given by

$$\begin{aligned}
C_j(M_W) &= 0 \qquad\qquad (j = 1,3,4,5,6) , && (7)\\
C_2(M_W) &= 1 , && (8)\\
C_{7\gamma}(M_W) &= -\frac{A(x_t)}{2} - \frac{A(y_t)}{6}\cot^2\beta - B(y_t) , && (9)\\
C_{8G}(M_W) &= -\frac{D(x_t)}{2} - \frac{D(y_t)}{6}\cot^2\beta - E(y_t) , && (10)
\end{aligned}$$

where $x_t = m_t^2/M_W^2$, and $y_t = m_t^2/m_{H^\pm}^2$. The Inami-Lim functions are given by

$$\begin{aligned}
A(x) &= x\Big[\frac{8x^2+5x-7}{12(x-1)^3} - \frac{(3x^2-2x)\ln x}{2(x-1)^4}\Big] , && (11)\\
B(y) &= y\Big[\frac{5y-3}{12(y-1)^2} - \frac{(3y-2)\ln y}{6(y-1)^3}\Big] \;; , && (12)\\
D(x) &= x\Big[\frac{x^2-5x-2}{4(x-1)^3} + \frac{3x\ln x}{2(x-1)^4}\Big] , && (13)\\
E(y) &= y\Big[\frac{y-3}{4(y-1)^2} + \frac{\ln y}{2(y-1)^3}\Big] . && (14)
\end{aligned}$$

The most recent experimental data on $b \to s\,\gamma$ rate has been reported, giving

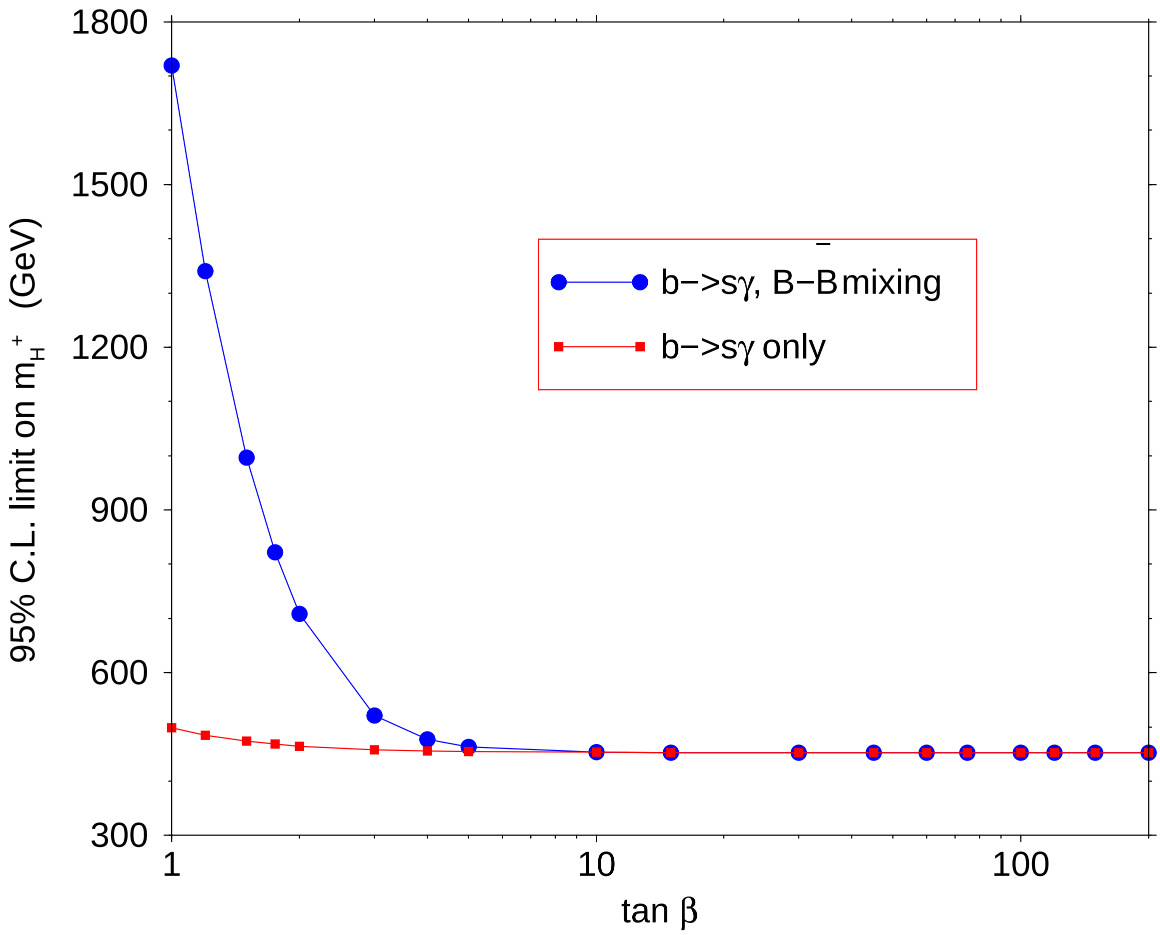
$$B(b \to s\,\gamma)|_{\rm exp} = 3.88 \pm 0.36({\rm stat}) \pm 0.37({\rm sys})^{+0.43}_{-0.28}({\rm theory}) .$$

The most updated SM prediction is

$$B(b \to s\,\gamma)|_{\rm SM} = (3.64 \pm 0.31) \times 10^{-4} ,$$

which agrees very well the data. Both the experimental data and the SM prediction have been extrapolated to the total branching ratio. Therefore, there is only a little room for new physics contributions. The constraint on new physics contribution is, explicit ly,

$$\Delta B(b \to s\,\gamma) \equiv B(b \to s\,\gamma)|_{\rm exp} - B(b \to s\,\gamma)|_{\rm SM} = (0.24^{+0.67}_{-0.59}) \times 10^{-4}\ , \tag{15}$$



B. $B^0 - \overline{B^0}$

The quantity that parameterizes the $B^0 - \overline{B^0}$ mixing is

$$x_d \equiv \frac{\Delta m_B}{\Gamma_B} = \frac{G_F^2}{6\pi^2}|V_{td}^*|^2|V_{tb}|^2 f_B^2\, B_B\, m_B \eta_B \tau_B\, M_W^2\, (I_{WW} + I_{WH} + I_{HH})\ , \tag{16}$$

$$\begin{aligned}
I_{WW} &= \frac{x}{4}\Big[1 + \frac{3-9x}{(x-1)^2} + \frac{6x^2\log x}{(x-1)^3}\Big]\ ,\\
I_{WH} &= xy\cot^2\beta\Big[\frac{(4z-1)\log y}{2(1-y)^2(1-z)} - \frac{3\log x}{2(1-x)^2(1-z)} + \frac{x-4}{2(1-x)(1-y)}\Big]\ ,\\
I_{HH} &= \frac{xy\cot^4\beta}{4}\Big[\frac{1+y}{(1-y)^2} + \frac{2y\log y}{(1-y)^3}\Big]\ ,
\end{aligned}$$

with $x = m_t^2/M_W^2$, $y = m_t^2/m_{H^\pm}^2$, $z = M_W^2/m_{H^\pm}^2$, and the running top mass $m_t = m_t(m_t) = 166 \pm 5$ GeV. The experimental value is

$$x_d = 0.755 \pm 0.015 \, . \tag{17}$$

We use the following input parameters $|V_{tb}V_{td}^*| = 0.0079 \pm 0.0015$, $f_B^2 B_B = (198 \pm 30 \text{ GeV})^2(1.30 \pm 0.12)$, $m_B = 5279.3 \pm 0.7$ MeV, $\eta_B = 0.55$, and $\tau_B = 1.542 \pm 0.016$ ps. Note that the value of $|V_{tb}V_{td}^*|$ is in fact determined by the measurement of x_d.

C. $g-2$

Experimental measurements

- The 1977 CERN measurement:

$$a_\mu^{\rm exp} = 116\ 592\ 300(840) \times 10^{-11} \qquad \text{(CERN77)}$$

- 1998 E821 measurement combined with CERN 77:

$$a_\mu^{\rm exp} = 116\ 592\ 050(460) \times 10^{-11} \qquad \text{(CERN77 + BNL98)}$$

- New 1999 E821 result:

$$a_\mu^{\rm exp} = 116\ 592\ 020(160) \times 10^{-11} \qquad \text{(BNL99)}$$

- Combined all:

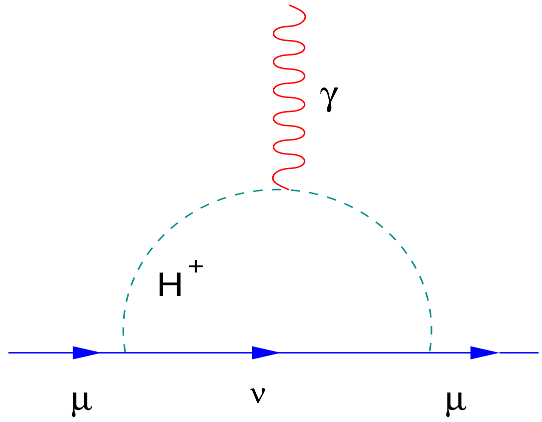
$$a_\mu^{\rm exp} = 116\ 592\ 023(151) \times 10^{-11} \qquad \text{(Worldin2000)}$$

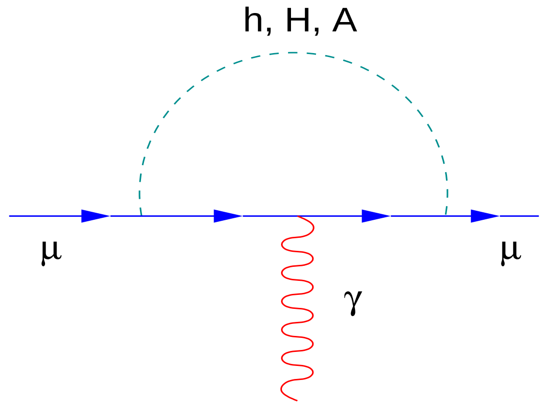
- Compared with $a_\mu^{\rm SM}$, the deviation is

$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426 \pm 165 \times 10^{-11}$$

A 2.6σ deviation, which may indicate new physics. At the present moment, the deviation stands at about 3σ with improved hadronic calculations.

For 2HDM: all higgs bosons contribute to a_μ at one-loop level.





$$\Delta a_\mu^h \simeq \frac{m_\mu^2}{8\pi^2 m_h^2}\left(\frac{g m_\mu}{2m_W}\frac{\sin\alpha}{\cos\beta}\right)^2 \left(-\frac{7}{6} - \ln(m_\mu^2/m_h^2)\right)$$

$$\Delta a_\mu^H \simeq \frac{m_\mu^2}{8\pi^2 m_H^2}\left(\frac{g m_\mu}{2m_W}\frac{\cos\alpha}{\cos\beta}\right)^2 \left(-\frac{7}{6} - \ln(m_\mu^2/m_H^2)\right)$$

$$\Delta a_\mu^A \simeq -\frac{m_\mu^2}{8\pi^2 m_A^2}\left(\frac{g m_\mu}{2m_W}\tan\beta\right)^2 \left(-\frac{11}{6} - \ln(m_\mu^2/m_A^2)\right)$$

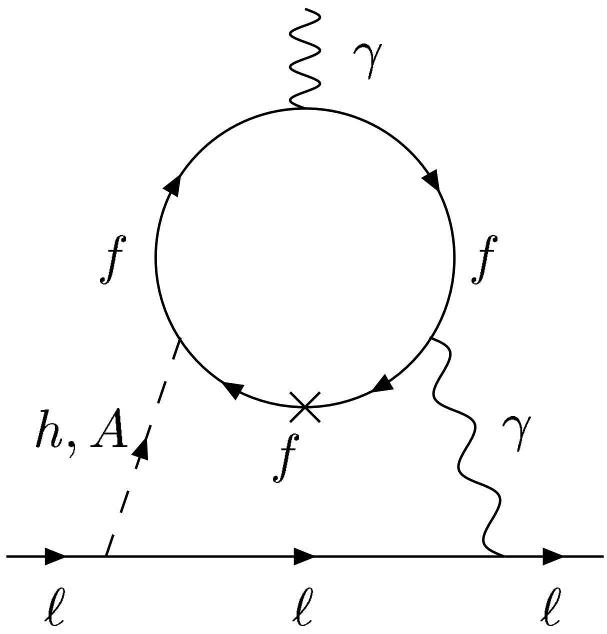
$$\Delta a_\mu^{H^+} \simeq \frac{m_\mu^2}{8\pi^2 m_{H^+}^2}\left(\frac{g m_\mu}{2m_W}\tan\beta\right)^2 \left(-\frac{1}{6} - \frac{1}{12}\frac{m_\mu^2}{m_{H^+}^2}\right)$$

Dominated by small h and A.

$$\Delta a_\mu^h(\mathrm{one-loop})\ \text{ is positive}$$

$$\Delta a_\mu^A(\mathrm{one-loop})\ \text{ is negative}$$

Two-loop Barr-Zee diagrams with heavy fermions.



$$\Delta a_\mu^h = -\frac{\alpha^2}{4\pi^2 \sin^2\theta_W} \frac{m_\mu^2 \lambda_\mu}{M_W^2} \sum_{f=t,b,\tau} N_c^f Q_f^2 \lambda_f f\left(\frac{m_f^2}{m_h^2}\right) ,$$

$$\Delta a_\mu^A = \frac{\alpha^2}{4\pi^2 \sin^2\theta_W} \frac{m_\mu^2 A_\mu}{M_W^2} \sum_{f=t,b,\tau} N_c^f Q_f^2 A_f g\left(\frac{m_f^2}{m_A^2}\right)$$

The couplings λ_f and A_f are

$$\begin{array}{llll} h \ \ (\lambda_f): & \frac{\cos\alpha}{\sin\beta} & -\frac{\sin\alpha}{\cos\beta} & -\frac{\sin\alpha}{\cos\beta} \\ H \ \ (\lambda_f): & \frac{\sin\alpha}{\sin\beta} & \frac{\cos\alpha}{\cos\beta} & \frac{\cos\alpha}{\cos\beta} \\ A \ \ (A_f): & \cot\beta & \tan\beta & \tan\beta \end{array}$$

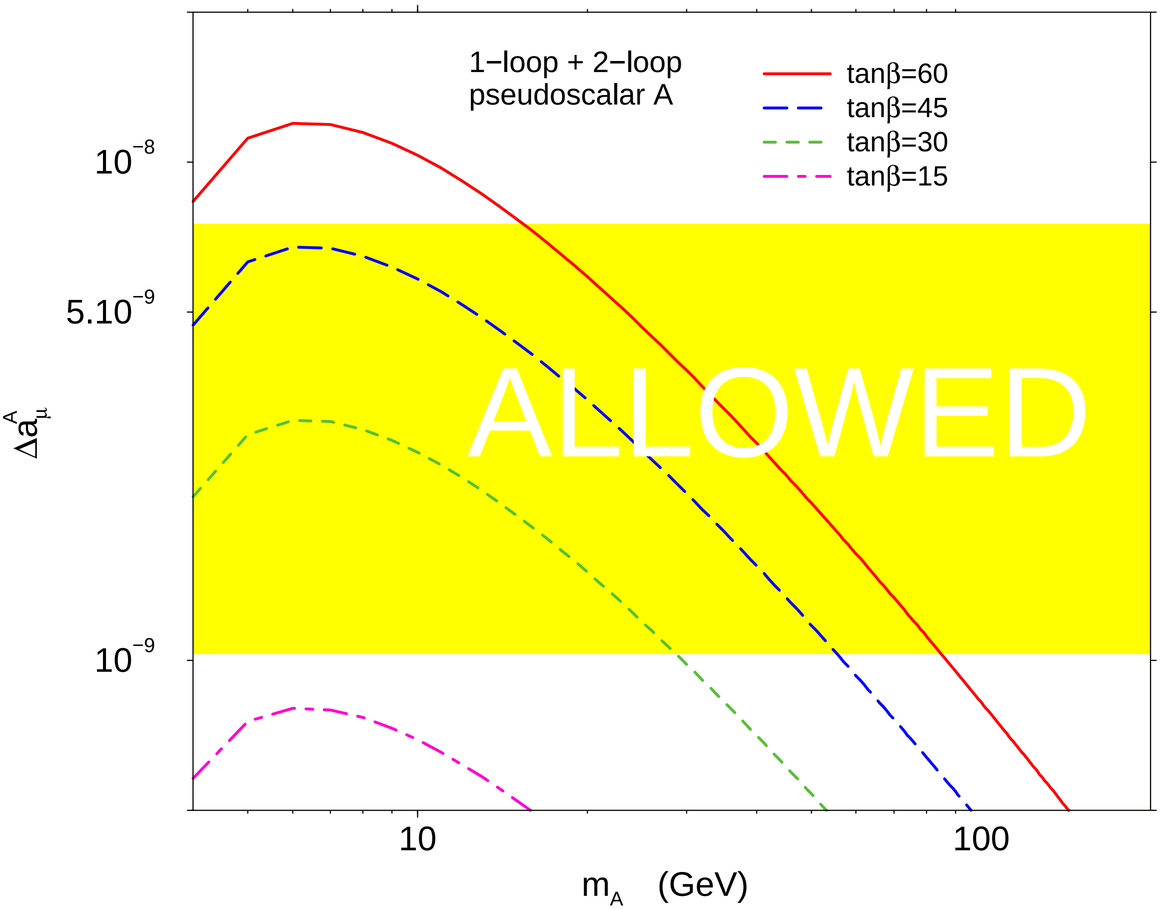
$$f(x) \to x(\frac{3}{2} + \frac{\pi^2}{6} + -\frac{1}{2}(\ln(x)+1)^2) , \quad g(x) \to \frac{\pi^2}{3} + \ln^2(x)$$

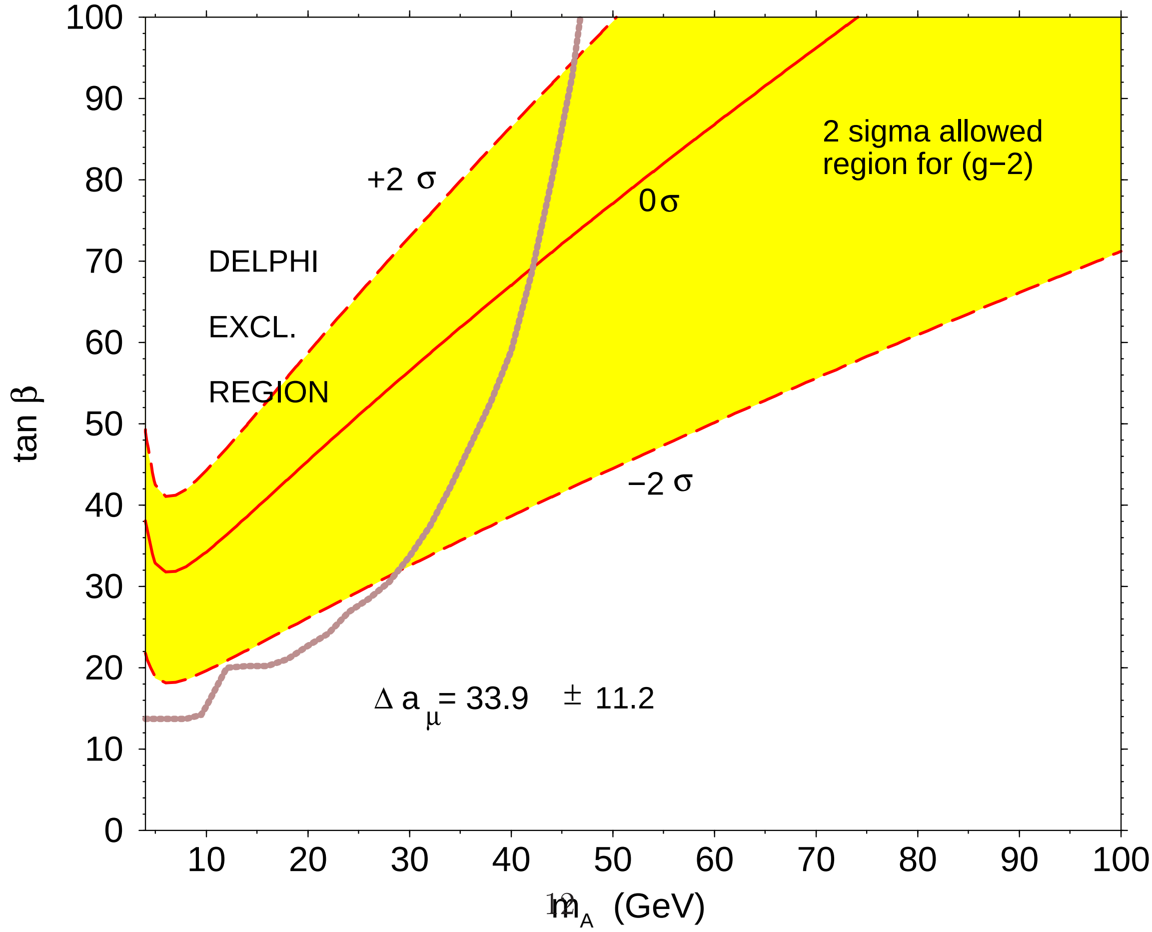
- Dominated by τ and b loops.

$$\Delta a_\mu^h(\mathrm{two-loop}) \ \ \text{is} \ \ \text{negative}$$

$$\Delta a_\mu^A(\mathrm{two-loop}) \ \ \text{is} \ \ \text{positive}$$

Since the deviation is positive, we want to make A^0 light and the h^0 heavy such that the overall contribution is positive and large enough.


1−loop + 2−loop
pseudoscalar A
tanβ=60
tanβ=45
tanβ=30
tanβ=15
ALLOWED
10^{-8}
5.10^{-9}
10^{-9}
Δa_μ^A
10
100
m_A (GeV)


2 sigma allowed
region for (g−2)
+2 σ
0σ
−2 σ
DELPHI
EXCL.
REGION
$\Delta a_\mu = 33.9 \pm 11.2$
tan β
m_A (GeV)

D. ρ parameter

The parameter ρ was introduced to measure the relation between the masses of $W^\pm$ and Z^0 bosons. In the SM $\rho \equiv M_W^2/M_Z^2\cos^2\theta_W = 1$ at tree-level. The ρ parameter receives contributions from the SM corrections and from new physics. The deviation from the SM prediction is usually described by the parameter ρ_0 defined by

$$\rho_0 \equiv \frac{M_W^2}{\rho\, M_Z^2 \cos^2\theta_W} \,, \tag{18}$$

where the ρ in the denominator absorbs all the SM corrections, including the corrections from the top quark and the SM Higgs boson. By definition, $\rho_0 = 1$ in the SM. Sources of new physics that contribute to ρ_0 can be written as

$$\rho_0 = 1 + \Delta\rho_0^{\rm new} \;; \tag{19}$$

where $\Delta\rho_0^{\rm new} = \Delta\rho^{\rm 2H\ DM} - \Delta\rho^{\rm SM\text{-}Higgs}$ in our case. Note that since the two-doublet Higgs sector (in the 2HDM) is employed here to replace the SM Higgs, the latter contribution to $\Delta\rho$ has to be subtracted out.

The most recent reported value of ρ_0 is

$$\rho_0 = 1.0004 \pm 0.0006, \quad (\text{with } M_{H_{\rm SM}} \text{ fixed at 115 GeV}) \,. \tag{20}$$

In terms of new physics the constraint becomes:

$$\Delta\rho_0^{\rm new} = 0.0004 \pm 0.0006 \,. \tag{21}$$

In 2HDM $\Delta\rho$ receives contributions from all Higgs bosons given by

$$\begin{aligned}
\Delta\rho^{\rm 2HDM} &= \frac{\alpha_{\rm em}}{4\pi\sin^2\theta_W M_W^2}\Big[F(m_A, m_{H^+}) + \cos^2(\beta-\alpha)\,[F(m_{H^+}, m_h) - F(m_A, m_h)] \\
&\quad + \sin^2(\beta-\alpha)\,[F(m_{H^+}, m_H) - F(m_A, m_H)]\Big] \\
&\quad + \cos^2(\beta-\alpha)\Delta\rho^{\rm SM}(m_H) + \sin^2(\beta-\alpha)\Delta\rho^{\rm SM}(m_h) \,, \qquad (22)\\
F(x,y) &= \frac{1}{8}x^2 + \frac{1}{8}y^2 - \frac{1}{4}\frac{x^2y^2}{x^2-y^2}\log\left(\frac{x^2}{y^2}\right) = F(y,x) \,, \\
\Delta\rho^{\rm SM}(M) &= -\frac{\alpha_{\rm em}}{4\pi\sin^2\theta_W M_W^2}\left[3F(M, M_W) - 3F(M, M_Z) + \frac{1}{2}(M_Z^2 - M_W^2)\right] \,. \qquad (23)
\end{aligned}$$

Remarks:

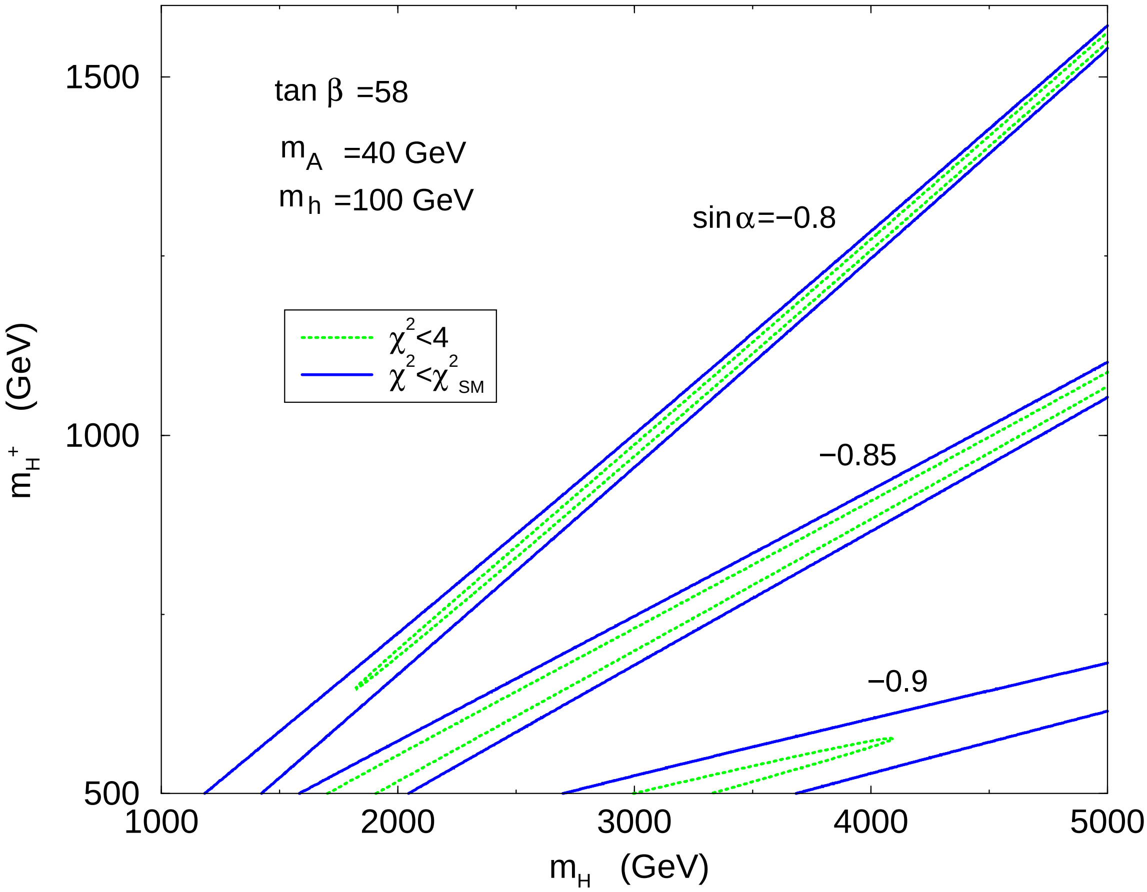
- $\Delta\rho^{\rm SM}(M)$ has a negative value with magnitude increasing with M. The value is about -0.0004 at $M = 115\,$GeV. It has a relatively mild variation, and does not go beyond -0.005 even as M gets to $10\,$TeV.

- Other contributions to $\Delta\rho^{\rm 2HDM}$ in the above formula are very sensitive to the masses involved.

- The $F(x,y)$ function is always positive, vanishes only at $x=y$, and increases with a faster and faster rate with the splitting between x and y.

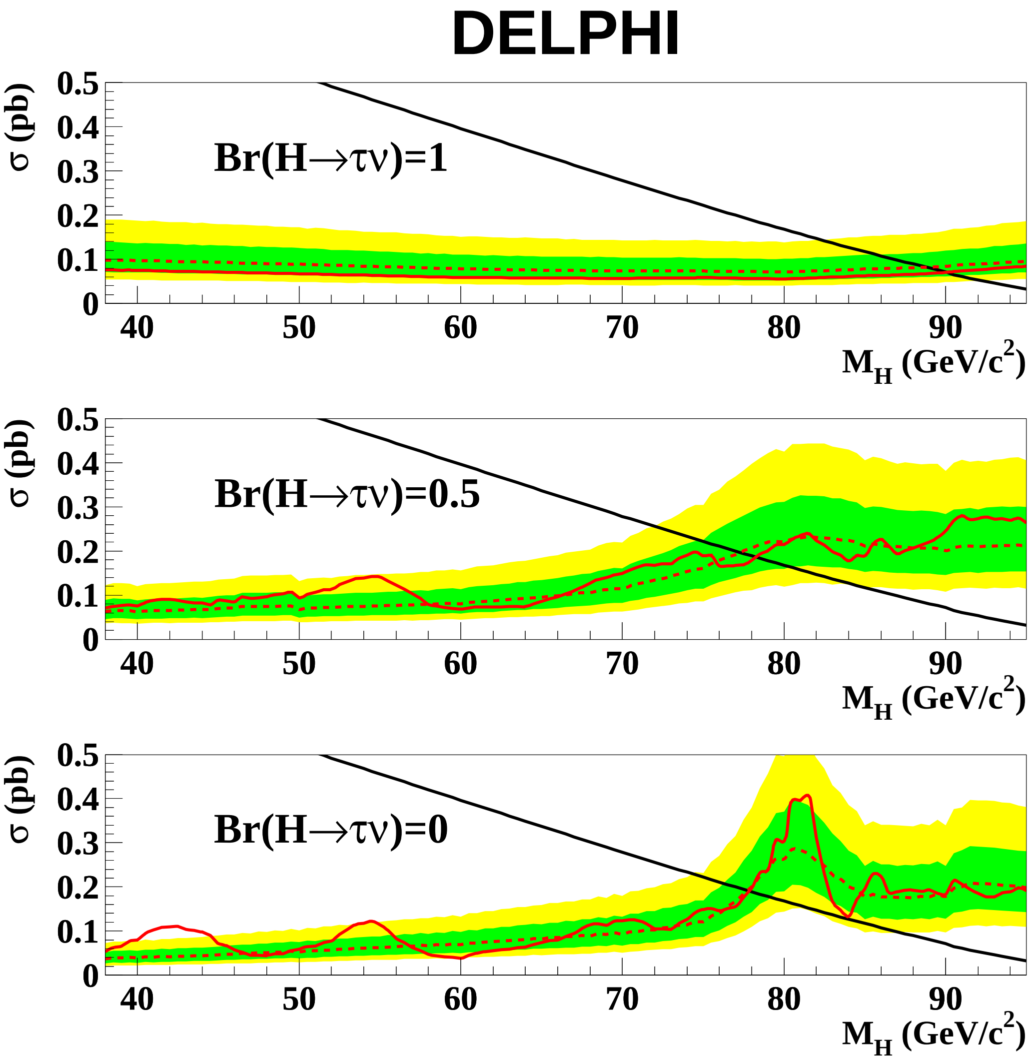
- We want the pseudoscalar A^0 to be the lightest Higgs state to explain the $g-2$ while a quite heavy charged Higgs satisfies the $b\to s\,\gamma$ and $B-\overline{B}$ mixing. But that makes the contribution from the first term [involving $F(m_A, m_{H^+})$] large; indeed of order 0.01.

- We need negative contributions from other terms: $[F(m_{H^+}, m_h) - F(m_A, m_h)]$ and $[F(m_{H^+}, m_H) - F(m_A, m_H)]$.



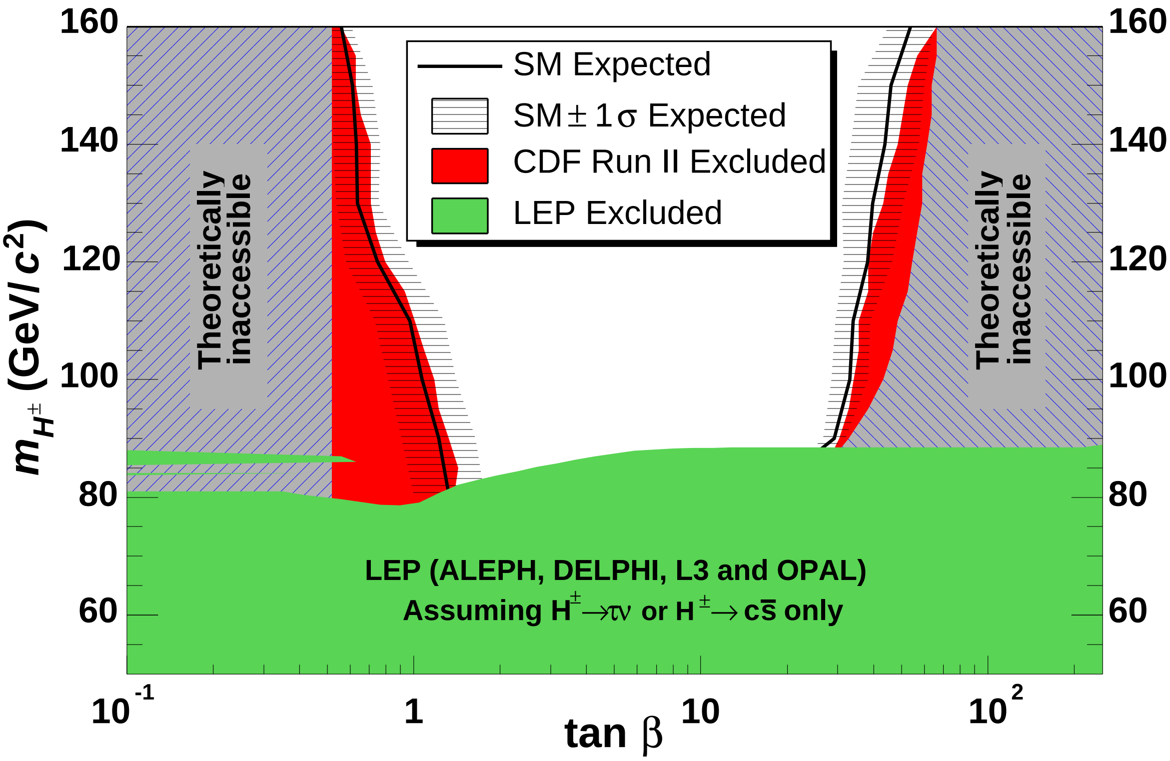
E. Experimental searches on 2HDM

- At LEP: search for $e^+e^- \to H^+H^- \to \tau^+\bar{\nu}_\tau\tau^-\nu_\tau,\ \tau^+\bar{\nu}_\tau\bar{c}s,\ c\bar{s}\bar{c}s$



- Search at the Tevatron The top quark can decay into the charged Higgs

$$t \to bH^+ \to b\tau^+\bar{\nu}_\tau$$



V. MODEL III

Here we introduce a more general 2HDM. without the discrete symmetries as in models I and II. It is often referred as model III. FCNC's in general exist in model III. However, the FCNC's involving the first two generations are highly suppressed from low-energy experiments, and those involving the third generation is not as severely suppressed as the first two generations. It implies that model III should be parameterized in a way to suppress the tree-level FCNC couplings of the first two generations while the tree-level FCNC couplings involving the third generation can be made nonzero as long as they do not violate any existing experimental data, e.g., $B^0 - \overline{B^0}$ mixing.

We simply assume all tree-level FCNC couplings to be negligible.

In a general two-Higgs-doublet model, both the doublets can couple to the up-type and down-type quarks. Without loss of generosity, we work in a basis such that the first doublet generates all the gauge-boson and fermion masses:

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \,, \qquad \langle \phi_2 \rangle = 0 \tag{24}$$

where v is related to the W mass by $M_W = \frac{g}{2} v$. In this basis, the first doublet ϕ_1 is the same as the SM doublet, while all the new Higgs fields come from the second doublet ϕ_2. They are written as

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} G^+ \\ v + \chi_1^0 + i G^0 \end{pmatrix} , \qquad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ \chi_2^0 + i A^0 \end{pmatrix} , \tag{25}$$

where G^0 and $G^\pm$ are the Goldstone bosons that would be eaten away in the Higgs mechanism to become the longitudinal components of the weak gauge bosons. The $H^\pm$ are the physical charged-Higgs bosons and A^0 is the physical CP-odd neutral Higgs boson. The χ_1^0 and χ_2^0 are not physical mass eigenstates but linear combinations of the CP-even neutral Higgs bosons:

$$\chi_1^0 = H^0 \cos\alpha - h^0 \sin\alpha \tag{26}$$

$$\chi_2^0 = H^0 \sin\alpha + h^0 \cos\alpha \ , \tag{27}$$

where α is the mixing angle. In this basis, there is no couplings of $\chi_2^0 ZZ$ and $\chi_2^0 W^+ W^-$.

We can write down the Yukawa Lagrangian for model III as

$$-\mathcal{L}_Y = \eta_{ij}^U \overline{Q_{iL}} \tilde{\phi}_1 U_{jR} + \eta_{ij}^D \overline{Q_{iL}} \phi_1 D_{jR} + \xi_{ij}^U \overline{Q_{iL}} \tilde{\phi}_2 U_{jR} + \xi_{ij}^D \overline{Q_{iL}} \phi_2 D_{jR} \quad + \text{h.c.} \ , \tag{28}$$

where i, j are generation indices, $\tilde{\phi}_{1,2} = i\sigma_2 \phi_{1,2}$, $\eta_{ij}^{U,D}$ and $\xi_{ij}^{U,D}$ are, in general, nondiagonal coupling matrices, and Q_{iL} is the left-handed fermion doublet and U_{jR} and D_{jR} are the right-handed singlets. Note that these Q_{iL}, U_{jR}, and D_{jR} are weak eigenstates, which can be rotated into mass eigenstates. As we have mentioned above, ϕ_1 generates all the fermion masses and, therefore, $\frac{v}{\sqrt{2}} \eta^{U,D}$ will become the up- and down-type quark-mass matrices after a bi-unitary transformation.

After the transformation the Yukawa Lagrangian becomes

$$\begin{aligned}
\mathcal{L}_Y \ = \ & -\overline{U} M_U U - \overline{D} M_D D - \frac{g}{2M_W} (H^0 \cos\alpha - h^0 \sin\alpha) \Big(\overline{U} M_U U + \overline{D} M_D D \Big) \\
& + \frac{ig}{2M_W} G^0 \left(\overline{U} M_U \gamma^5 U - \overline{D} M_D \gamma^5 D \right) \\
& + \frac{g}{\sqrt{2} M_W} G^- \overline{D} V_{\text{CKM}}^\dagger \Big[M_U \tfrac{1}{2}(1+\gamma^5) - M_D \tfrac{1}{2}(1-\gamma^5) \Big] U \\
& - \frac{g}{\sqrt{2} M_W} G^+ \overline{U} V_{\text{CKM}} \Big[M_D \tfrac{1}{2}(1+\gamma^5) - M_U \tfrac{1}{2}(1-\gamma^5) \Big] D \\
& - \frac{H^0 \sin\alpha + h^0 \cos\alpha}{\sqrt{2}} \Big[\overline{U} \Big(\hat{\xi}^U \tfrac{1}{2}(1+\gamma^5) + \hat{\xi}^{U\dagger} \tfrac{1}{2}(1-\gamma^5) \Big) U \\
& + \overline{D} \Big(\hat{\xi}^D \tfrac{1}{2}(1+\gamma^5) + \hat{\xi}^{D\dagger} \tfrac{1}{2}(1-\gamma^5) \Big) D \Big] \\
& + \frac{iA^0}{\sqrt{2}} \Big[\overline{U} \Big(\hat{\xi}^U \tfrac{1}{2}(1+\gamma^5) - \hat{\xi}^{U\dagger} \tfrac{1}{2}(1-\gamma^5) \Big) U - \overline{D} \Big(\hat{\xi}^D \tfrac{1}{2}(1+\gamma^5) - \hat{\xi}^{D\dagger} \tfrac{1}{2}(1-\gamma^5) \Big) D \Big]
\end{aligned}$$

$$
\begin{aligned}
& - \; H^+\overline{U}\Big[V_{\rm CKM}\hat{\xi}^D\tfrac{1}{2}(1+\gamma^5) - \hat{\xi}^{U\dagger}V_{\rm CKM}\tfrac{1}{2}(1-\gamma^5)\Big]D \\
& - \; H^-\overline{D}\Big[\hat{\xi}^{D\dagger}V^\dagger_{\rm CKM}\tfrac{1}{2}(1-\gamma^5) - V^\dagger_{\rm CKM}\hat{\xi}^{U}\tfrac{1}{2}(1+\gamma^5)\Big]U \ ,
\end{aligned} \tag{29}
$$

where U represents the mass eigenstates of u, c, t quarks and D represents the mass eigenstates of d, s, b quarks. The transformations are defined by $M_{U,D} = {\rm diag}(m_{u,d}, m_{c,s}, m_{t,b}) = \frac{v}{\sqrt{2}}(\mathcal{L}_{U,D})^\dagger \eta^{U,D}(\mathcal{R}_{U,D})$, $\hat{\xi}^{U,D} = (\mathcal{L}_{U,D})^\dagger \xi^{U,D}(\mathcal{R}_{U,D})$. The Cabibbo-Kobayashi-Maskawa matrix is $V_{\rm CKM} = (\mathcal{L}_U)^\dagger(\mathcal{L}_D)$.

The FCNC couplings are contained in the matrices $\hat{\xi}^{U,D}$. A simple ansatz for $\hat{\xi}^{U,D}$ would be

$$
\hat{\xi}^{U,D}_{ij} = \lambda_{ij}\frac{g\sqrt{m_i m_j}}{\sqrt{2}M_W} \tag{30}
$$

by which the quark-mass hierarchy ensures that the FCNC within the first two generations are naturally suppressed by the small quark masses, while a larger freedom is allowed for the FCNC involving the third generations.

VI. $b \to s\gamma$

The effective hamiltonian for $B \to X_s\gamma$ at a factorization scale of order $O(m_b)$ is given by

$$
\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}}V^*_{ts}V_{tb}\Big[\sum_{i=1}^{6} C_i(\mu)Q_i(\mu) + C_{7\gamma}(\mu)Q_{7\gamma}(\mu) + C_{8G}(\mu)Q_{8G}(\mu)\Big] \ . \tag{31}
$$

The operators Q_i can be found in Ref.[1], of which the Q_1 and Q_2 are the current-current operators and $Q_3 - Q_6$ are QCD penguin operators. $Q_{7\gamma}$ and Q_{8G} are, respectively, the magnetic penguin operators specific for $b \to s\gamma$ and $b \to sg$. Here we also neglect the mass of the external strange quark compared to the external bottom-quark mass.

The factorization facilitates the separation of the short-distance and long-distance parts, of which the short-distance parts correspond to the Wilson coefficients C_i and are calculable by perturbation while the long-distance parts correspond to the operator matrix elements. The physical quantities should be independent of the factorization scale μ. The natural scale for factorization is of order m_b for the decay $B \to X_s\gamma$. The calculation of the $C_i(\mu)$'s divides into two separate steps. First, at the electroweak scale, say M_W, the full theory is matched onto the effective theory and the coefficients $C_i(M_W)$ at the W-mass scale are extracted in the matching process. In a while, we shall present these coefficients $C_i(M_W)$ in model III. Second, the coefficients $C_i(M_W)$ at the W-mass scale are evolved down to the bottom-mass scale using renormalization group equations.

Since the operators Q_i's are all mixed under renormalization, the renormalization group equations for C_i's are a set of coupled equations:

$$\vec{C}(\mu) = U(\mu, M_W)\vec{C}(M_W) \ , \tag{32}$$

where $U(\mu, M_W)$ is the evolution matrix and $\vec{C}(\mu)$ is the vector consisting of $C_i(\mu)$'s. The calculation of the entries of the evolution matrix U is nontrivial but it has been written down completely in the leading order [1]. The coefficients $C_i(\mu)$ at the scale $O(m_b)$ are given by [1]

$$C_j(\mu) \ = \ \sum_{i=1}^{8} k_{ji}\eta^{a_i} \qquad (j = 1, ..., 6) \ , \tag{33}$$

$$C_{7\gamma}(\mu) \ = \ \eta^{\frac{16}{23}} C_{7\gamma}(M_W) + \tfrac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_{8G}(M_W) + C_2(M_W)\sum_{i=1}^{8} h_i\eta^{a_i} \ , \tag{34}$$

$$C_{8G}(\mu) \ = \ \eta^{\frac{14}{23}} C_{8G}(M_W) + C_2(M_W)\sum_{i=1}^{8} \bar{h}_i\eta^{a_i} \ , \tag{35}$$

with $\eta = \alpha_s(M_W)/\alpha_s(\mu)$. The a_i's, k_{ji}'s, h_i's, and $\bar{h}_i$'s can be found in Ref. [1].

Once we have all the Wilson coefficients at the scale $O(m_b)$ we can then compute the decay rate of $B \to X_s\gamma$. The decay amplitude for $B \to X_s\gamma$ is given by

$$\mathcal{A}(B \to X_s\gamma) = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_{7\gamma}(\mu)\langle Q_{7\gamma}\rangle \ , \tag{36}$$

in which we use the spectator approximation to evaluate the matrix element $\langle Q_{7\gamma}\rangle$ and $m_B \simeq m_b$. The decay rate of $B \to X_s\gamma$ is given by

$$\Gamma(B \to X_s\gamma) = \frac{G_F^2 |V_{ts}^* V_{tb}|^2 \alpha_{\rm em} m_b^5}{32\pi^4} \, |C_{7\gamma}(m_b)|^2 \ , \tag{37}$$

Since this decay rate depends on the fifth power of m_b, a small uncertainty in the choice of m_b will create a large uncertainty in the decay rate, therefore, the decay rate of $B \to X_s\gamma$ is often normalized to the experimental semileptonic decay rate as

$$\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e\bar{\nu}_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \, \frac{6\alpha_{\rm em}}{\pi f(m_c/m_b)} |C_{7\gamma}(m_b)|^2 \ , \tag{38}$$

where $f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$.

The remaining task is the calculation of the Wilson coefficients $C_i(M_W)$ at the W-mass scale. The only contributions at one-loop level come from the charged-Higgs bosons $H^\pm$, the charged Goldstone bosons $G^\pm$, and the SM $W^\pm$ bosons.

The coefficients $C_i(M_W)$ at the leading order in model III are given by

$$C_j(M_W) \ = \ 0 \qquad (j = 1, 3, 4, 5, 6) \ , \tag{39}$$

$$
\begin{aligned}
C_2(M_W) &= 1 \,, & (40)\\
C_{7\gamma}(M_W) &= -\frac{A(x_t)}{2} - \frac{A(y)}{6}|\lambda_{tt}|^2 + B(y)\lambda_{tt}\lambda_{bb} \,, & (41)\\
C_{8G}(M_W) &= -\frac{D(x_t)}{2} - \frac{D(y)}{6}|\lambda_{tt}|^2 + E(y)\lambda_{tt}\lambda_{bb} \ \,, & (42)
\end{aligned}
$$

where $x_t = m_t^2/M_W^2$, and $y = m_t^2/M_{H^\pm}^2$. The Inami-Lim functions are given by

$$
\begin{aligned}
A(x) &= x\left[\frac{8x^2+5x-7}{12(x-1)^3} - \frac{(3x^2-2x)\ln x}{2(x-1)^4}\right] & (43)\\
B(y) &= y\left[\frac{5y-3}{12(y-1)^2} - \frac{(3y-2)\ln y}{6(y-1)^3}\right] & (44)\\
D(x) &= x\left[\frac{x^2-5x-2}{4(x-1)^3} + \frac{3x\ln x}{2(x-1)^4}\right] & (45)\\
E(y) &= y\left[\frac{y-3}{4(y-1)^2} + \frac{\ln y}{2(y-1)^3}\right] \ . & (46)
\end{aligned}
$$

The SM results for the Wilson coefficients $C_i(M_W)$ for $i = 1, ..., 6$ are the same as in Eqs. (39) and (40), while $C_{7\gamma}(M_W)$ and $C_{8G}(M_W)$ only have the first term as in Eqs. (41) and (42), respectively. Thus, we already have all the necessary pieces to compute the decay rate of $B \to X_s\gamma$.

Before we leave this section we would like to emphasize that the expressions for $C_i(M_W)$ in Eqs. (39) – (42) obtained for model III can be reduced to the results of models I and II by the following substitutions:

$$
\lambda_{tt} \to \cot\beta \quad \text{and} \qquad \lambda_{bb} \to \cot\beta \quad \text{(for model I)}, \tag{47}
$$

and

$$
\lambda_{tt} \to \cot\beta \quad \text{and} \qquad \lambda_{bb} \to -\tan\beta \quad \text{(for model II)}. \tag{48}
$$

We use the following inputs for our calculation: $m_t = 173.8$ GeV, $M_W = 80.388$ GeV, $|V_{ts}^* V_{tb}|^2/|V_{cb}|^2 = 0.95$, $m_c/m_b = 0.3$, and $B(b \to ce^-\bar{\nu}) = 10.45 \pm 0.21\%$, $\alpha_{\rm em}(m_b) \simeq 1/133$, and $\alpha_s(M_Z) = 0.119$ and a 1-loop α_s is employed. The free parameters are then $M_{H^\pm}$, λ_{tt}, and λ_{bb}. Since the term proportional to $\lambda_{tt}\lambda_{bb}$ is, in general, complex we let $\lambda_{tt}\lambda_{bb} = |\lambda_{tt}\lambda_{bb}|e^{i\theta}$. We show the contours of the branching ratio in the plane of θ and $M_{H^\pm}$ for $|\lambda_{tt}\lambda_{bb}| = 3, 1, 0.5$ in Fig.1 (a), (b), and (c), respectively. The contours are symmetric about $\theta = 180^\circ$. The contours are $B = (2, 2.8, 4.5) \times 10^{-4}$, which correspond to 95%CL lower limit, the SM value, and the 95%CL upper limit. The value of $|\lambda_{bb}|$ is set at 50 as preferred in the R_b constraint that will be shown in the next section. The corresponding values of $|\lambda_{tt}|$ are $0.06, 0.02$, and 0.01, which satisfy the constraint from the $B^0 - \overline{B^0}$ mixing, as will also be discussed in the next section. Here the term proportional to $|\lambda_{tt}|^2$ is not crucial because the coefficient of $|\lambda_{tt}|^2$ is small compared with other two terms in Eqs. (41) and (42).

The results of the conventional model II (which can be obtained from our general results by the substitution: $\lambda_{tt} \to \cot\beta$, $\lambda_{bb} \to -\tan\beta$) can be read off from Fig. 1(b) at $\theta = 180^\circ$. The $b \to s\gamma$ data severely constrains $M_{H^\pm} \gtrsim 350$ GeV at 95%CL level, because at $\theta = 180^\circ$ the SM amplitude interferes entirely constructively with the charged Higgs-boson amplitude. It is obvious that at other angles the mass of the charged Higgs-boson mass is less constrained, especially, in the range $\theta = 50^\circ - 90^\circ$ the entire range of charged Higgs-boson mass is allowed by the $b \to s\gamma$ constraint as long as $|\lambda_{tt}\lambda_{bb}| \lesssim 1$. However, when $|\lambda_{tt}\lambda_{bb}|$ is getting larger, say 3, (see Fig. 1(a)) the allowed range of charged Higgs-boson mass becomes narrow. This is because the charged Higgs-boson amplitude becomes too large compared with the SM amplitude. On the other hand, when $|\lambda_{tt}\lambda_{bb}|$ becomes small the allowed range charged Higgs-boson mass is enlarged, as shown in Fig. 1(c). The significance of the phase angle θ is that the constraints previously on $M_{H^\pm}$ and $\tan\beta$ are evolved into θ, $M_{H^\pm}$, λ_{tt}, and λ_{bb}, where we do not need to impose $|\lambda_{tt}| = 1/|\lambda_{bb}|$, as in model II. The previous tight constraint on $M_{H^\pm}$ of model II is now relaxed in model III down to virtually the direct search limit of almost 60 GeV at LEPII.

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[1] G. Buchalla, A. Buras, and M. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

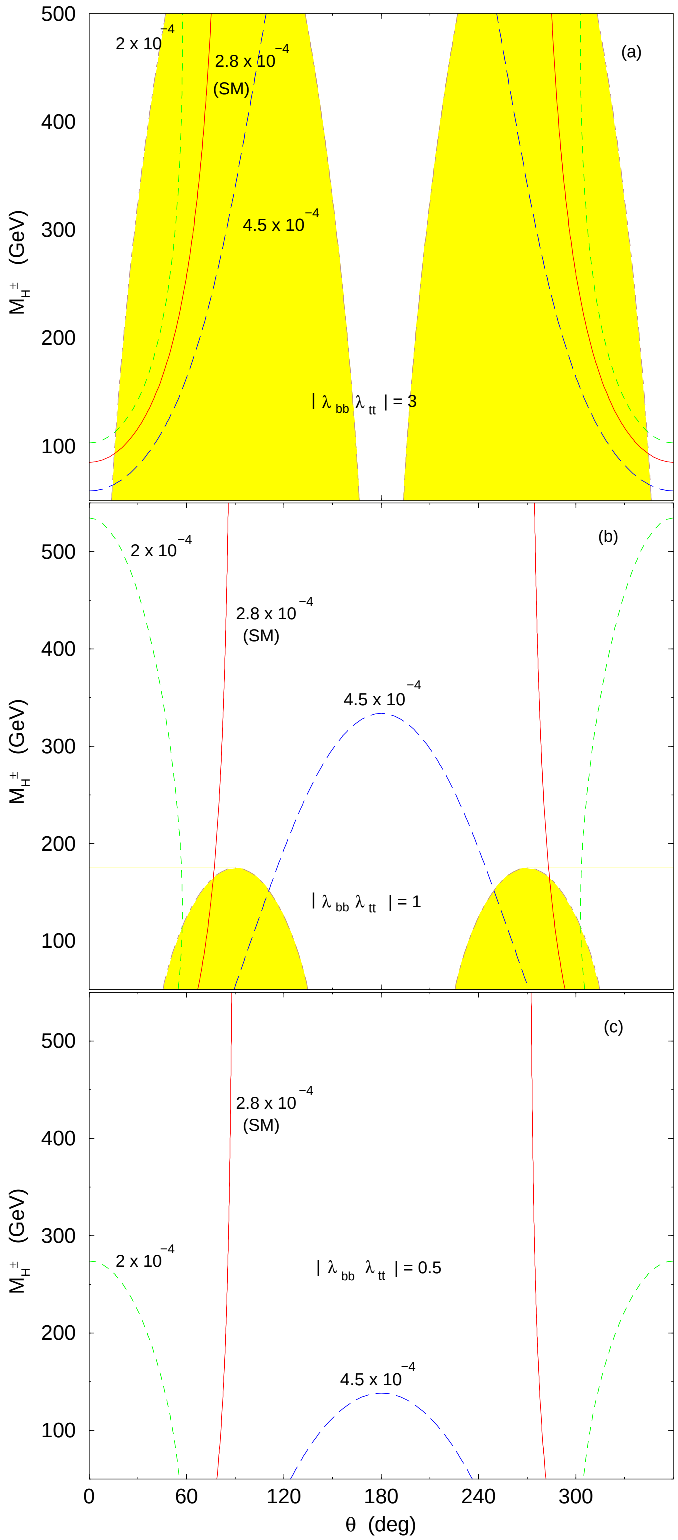

FIG. 1: Contour plot of the branching ratio $b \to s\gamma$ versus $M_H^{\pm}$ and the phase of $\lambda_{tt}\lambda_{bb}$ for various values of $|\lambda_{tt}\lambda_{bb}| = 3, 1, 0.5$. The shaded areas are excluded by the NEDM constraint $|d_n| < 10^{-25}$ e·cm.