Federal University Rio Grande do Norte
Department of Theoretical and Experimental Physics

Phenomenology of the Type-II Seesaw Mechanism

Yoxara Sánchez Villamizar

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Farinaldo Queiroz

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Phenomenology of the Type-II Seesaw Mechanism

Yoxara Sánchez Villamizar

Supervisor: Prof. Dr. Farinaldo Queiroz

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Examining Committee:
Prof. Dr. Farinaldo Queiroz - Supervisor (International Institute of Physics-UFRN)
Prof. Dr. Carlos Antônio de Sousa Pires (UFPB)
Prof. Dr. Diego Alejandro Cogollo Aponte (UFCG)

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Abstract

In this dissertation the theoretical and experimental aspects of the type-II seesaw model will be reviewed. The type-II seesaw is a mechanism that generates neutrino masses by adding a weak-scalar triplet to the Standard Model spectrum. This model is phenomenologically attractive because it can generate neutrino masses without invoking new physics effects at very high energy scales. We revisit the constraints coming from the ρ-parameter and electroweak precision data. Later we discuss collider signatures of the type II seesaw model focusing on the Large Hadron Collider. We exhibit the viable parameter space of the type II seesaw model and show the complementarity between collider physics and lepton flavor violation. This dissertation represents a first-step to our goal which is to assess the High-Luminosity and High Energy LHC sensitivity to the type II seesaw model in connection to lepton flavor violation and neutrino mass ordering.

Key words: (type-II seesaw mechanism, neutrino mass, doubly-charged scalar, weak-scalar triplet, lepton flavor violation, collider phenomenology).

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Chapter 1

Introduction

Neutrino is a fundamental particle, which is a very small piece in the universe structure. Its history goes back to 1896, when Becquerel (Nobel Prize in Physics in 1903) discovered the spontaneous radioactivity effect in uranium decay [8]. This would violate the energy conservation principle [9], but this could not be explained with the classical theories at that time. For this reason, different investigations were opened with the purpose of explaining this principle, as for instance: between 1897 and 1913 the electron was discovered; the atomic nucleus was evidenced; Bohr created the atomic spectrum quantum theory, etc. In 1914, Chadwick [10] for the first time observed that the beta spectrum was continuous*.The confirmation that the beta spectrum was continuous was carried out in 1927 by Ellis and Wooster [11]. Consequently, the proposal of the neutrino was put forward by W. Pauli by means of a letter in 1930†, where he will refer to his new particle as, the neutron:

> *“. . . I have hit upon a desperate remedy to save the . . . energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 . . . The mass of the neutron must be . . . not larger than 0.01 proton mass. . . . in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant” [12].*

And, in this way, the neutrino was born as a postulate in letter words, with the aim of solving the problem of energy conservation principles.

Later on, two years after Pauli’s proposal, in 1932, J Chadwick discovered a new type of electric chargeless massive particle that he called the neutron (as we know it today) and

*Which is an indirect evidence on the existence of neutral penetrating particles [11].

†This Pauli’s letter can be seen in the reference [12], in which the following greeting is read: "Dear radioactive ladies and gentlemen,"

Heisenberg suggested that nuclei are composed of protons and neutrons. In 1934, Pauli explains the continuous electron spectrum of β decay as proposal for the neutrino [11]

$$n \longrightarrow p + e^- + \bar{\nu}_e \ .$$

In the same year, Fermi took Pauli's idea and developed a field theory for the β decay, which indicates that the neutron decays in three particles: proton (p), neutrino (ν) and electron (e). Fermi proposed an effective Lagrangian to describe the β decay, based on his famous four-fermion interaction Lagrangian [11, 13, 14].

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} J_\nu^\dagger J\nu = \frac{G_F}{\sqrt{2}} \left(\bar{\Psi}_p \, \gamma_\mu \, \Psi_n\right) \left(\bar{\Psi}_e \, \gamma^\mu \, \Psi_\nu\right) \ .$$

With $G_F \simeq 1.166 \times 10^{-5} GeV^{-2}$, which is called Fermi coupling constant and its determination comes from measurements of the exponential decay of the muon. Currently, the decay can be represented by means of a Feynman diagram, as can be seen in figure 1.1. Where the neutron (n) decays into a proton (p), an electron (e^-) and an electronic antineutrino $\bar{\nu}_e$. In which u and d are quarks. It was not until 1956, when the neutrinos

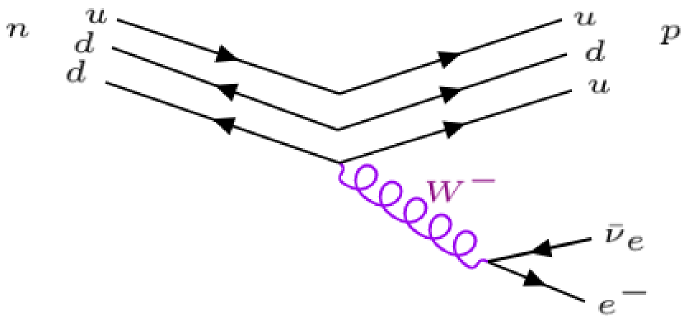

Figure 1.1: Beta decay Feynman diagram. Where W^- is a mediator of the electroweak group and it will be explained in chapter 2

were found by Reines and Cowan by using the known inverse beta decay: $\bar{\nu}_e + p \longrightarrow n + e^+$ (Nobel Prize in Physics 1995). After this, the first idea of neutrino oscillations was considered by B. Pontecorvo in 1957 [15] and the mixing was introduced in the beginning of the 60's by Z. Maki, M. Nakagawa, S. Sakata [16, 17]. In the same year (1957) the idea of weak and electromagnetic interaction unification was suggested by Schwinger as a way to explain both interactions [14]. Later, Sheldon Glashow [18] (who correctly identified the gauge group in 1961), Abdus Salam and Steven Weinberg [19] (who independently introduced the Higgs mechanism to account for the gauge boson masses [14]) contributed to the construction of the electroweak standard model. However, this model was built assuming that the neutrino does not have mass, since there was no proof of this. For this reason, leptons do not mix among families in the SM unlike quarks.

While the standard model was being shaped, the solar and atmospheric neutrinos were observed, by means of various experiments. During the 60's, some experiments gave hints that the neutrinos could have mass. Where the first signal of neutrino oscillation was known by Raymond Davis' experimental result announced in 1968 [20], because the neutrino detectors on the earth had observed a lesser number of neutrinos that the solar neutrinos foretold by theoretical models, which was known as "The Solar Neutrino Problem".

Later on, thanks to new technological advances, physics could make a significant progress, succeeding in determining the existence of three neutrino varieties: the electric one (ν_e), the muonic (ν_μ) and the tautonic ν_τ), every neutrino was associated to a family of charged leptons. Also, Takaaki Kajita and Arthur B. McDonald[‡] headed discovery of neutrino oscillations, which show that neutrino masses are not exactly zero, their masses are small and also that they mix each other, constituting the first new experimental evidence for new physics, given that in the standard model, leptons do not mix each other.

Consequently, it is clear that one of the major problems of the electroweak SM is the absence of neutrinos masses, since this clearly contradicts the experimental evidence. As a solution to this issue, there are some models beyond the standard model that nicely explain neutrino masses by adding new energy scales and they are called seesaw mechanism. Currently, there are three well-known seesaw models: type-I, Type-II and type-III, which consist of including new particles. these particles are right-handed neutrinos, a scalar triple [21, 22] and a left-handed fermionic triplet with zero hypercharge, for every one of the seesaw models respectively. In the same way, It is important to mention that these particles are represented by heavy fields that directly include a new energy scale Λ (or new physics scale) and this would correspond to each one of the squared masses of those introduced heavy fields.

In this dissertation, we have focused on the phenomenology on the type-II seesaw model, which is one of the simplest frameworks to incorporate neutrino masses in the SM and it is phenomenologically interesting, because it adds a weak-scalar triplet that can be produced at colliders through interactions with gauge bosons. Besides, a component and very attractive of this complex scalar triplet is the doubly charged-scalar, because is subjects to intensive collider searches. Since it offers a clear signature. For this reason, it is a key ingredient in this neutrino mass model.

The goal of this work is to review the theoretical and experimental aspect of the type-II seesaw model and asses the sensitivity reach of future hadron collider to the presence of a

[‡]both Nobel Prize in Physics 2015

doubly-charged scalar. This dissertation represents a first-step to what we aim to do in the near future which is to connect neutrino physics to lepton flavor violation, collider physics and dark matter which has become a popular avenue in the literature [23–47].

Finally, In this dissertation, not only the SM (chapter 2) will be described, but also the type-II seesaw model (chapter 3) and its theoretical and experimental phenomenology (chapter 4).

Chapter 2

Elementary particles and the Standard Model electroweak

In this section we will review the standard model, which is the base of the type-II seesaw model that we will discuss later. The Standard Model (SM) of the elementary particles describes the electromagnetic, weak and strong interactions in the sphere of Quantum Field Theory. The standard model is described for the internal local symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ where C, L and Y denote color, left-handed fermions and weak hypercharge respectively. The weak hypercharge and the weak isospin T_3 are associated to the electric charge by mean of the Gell-Mann-Nishijima relation: $Q = T_3 + \frac{Y}{2}$ [11]. The gauge group $SU(2)_L \otimes U(1)_Y$ is related to the unification of the electromagnetic and weak interactions in fermions by means of three massive gauge bosons W^+_μ, W^-_μ, Z_μ and another non-massive gauge boson A_μ. The $SU(3)_C$ group implies the strong interaction between the quarks through eight non-massive gluons.

2.1 The elementary particles and the organization of the Standard Model

The elementary particles of the SM are fermions, gauge bosons (mediators of the interactions) and the Higgs boson. The interactions of these particles with the Higgs boson gives mass of them. The SM Lagrangian contains three types of fields: fermionic, vector and scalar fields as we describe below.

2.1.1 Fermionic fields

It is important to highlight that the fermions have spin one-half and obey the Fermi-Dirac statistics. Its dynamics is described by the free Dirac Lagrangian (2.1).

$$\mathcal{L} = \bar{\Psi}(x)(i\partial\!\!\!/ - m)\Psi(x)\,, \tag{2.1}$$

$\Psi(x)$ is a Dirac spinor that represents the fermionic field of particles, where m is the particle mass, $\bar{\Psi}(x) = \Psi^{\dagger}\gamma^{0}$, $\partial\!\!\!/ = \gamma^{\mu}\partial_{\mu}$ and γ^{μ} are gamma matrices.

In this way, the matter we known is composed of a set of elementary Dirac fermions, which are classified as leptons (l) and quarks (q). Some of its characteristics may be seen in the table 2.1. The leptons are: electrons (e), muon (μ), tau (τ), and their correspondent neutrinos (ν_e), (ν_μ), (ν_τ). These leptons have antiparticles which have the same mass, but with opposite charges. In the case of neutrinos, they are charge-less particles, so that their antiparticles could be themselves, which would implies that they are Majorana fermions (this will be explained in the section 3.2). Besides, in the SM, the neutrinos are massless. On the other hand, there are six quarks up (u), down (d), charm (c), strange (s), top (t) and bottom (b), where each one has its antiparticle. These are susceptible to the strong force, and for this reason, they have a color charge, but the leptons are colorless and do not feel this force. As can be seen in the table 2.1, the fermions are organized in three families

Table 2.1: Some characteristics of the elementary fermion: type, family, massa [7], and electric charge (Q)

Type	Name (symbol)	Family	Massa	Q
Lepton	electron (e^-)	first	0.511 MeV	-1
	electron neutrino (ν_e)	first	–	0
	muon (μ)	second	105.7 MeV	-1
	muon neutrino (ν_μ)	second	–	0
	tau (τ)	third	(1776.86 $\pm$ 0.12) MeV	-1
	tau neutrino (ν_τ)	third	—	0
Quark	up (u)	first	$\left(2.16\ {}^{+0.49}_{-0.26}\right)$ MeV	2/3
	down (d)	first	$\left(4.67\ {}^{+0.48}_{-0.17}\right)$ MeV	-1/3
	charm (c)	second	(1.27 $\pm$ 0.02) GeV	2/3
	strange (s)	second	$\left(93\ {}^{+11}_{-5}\right)$ MeV	-1/3
	top (t)	third	(172.9 $\pm$ 0.4) GeV	2/3
	bottom (b)	third	$\left(4.18\ {}^{+0.03}_{-0.02}\right)$ GeV	-1/3

with a mass hierarchy. That is, the fermions of the first family have a smaller mass than that of the second family and the second family are lighter than the third family.

It is important to say, that the fermionic fields may be represented as right-hand (R) and left-hand (L) fermions:

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} , \qquad \psi_L = P_L \psi \, , \qquad \psi_R = P_R \psi \, , \tag{2.2}$$

where P_L and P_R are left and right helicity projectors:

$$P_L \equiv \frac{1}{2}(1 - \gamma_5) \quad , \quad P_R \equiv \frac{1}{2}(1 + \gamma_5) \, . \tag{2.3}$$

These projectors satisfy the following properties [11]:

$$\begin{aligned} P_L + P_R &= 1 \, , \\ P_R \, P_L = P_L \, P_R &= 0 \, , \\ P_L^2 &= P_L \, , \\ P_R^2 &= P_R \, . \end{aligned}$$

Thus, the conjugated spinors can be written with the use of these properties as: $\bar{\psi}_L = \bar{\psi} P_R$ and $\bar{\psi}_R = \bar{\psi} P_L$. Where the left-handed fields transform as doublet of the SU(2) or isospin group, while the right fields as singlet of this group. It is important to mention that some general aspects of the fermionic fields:

- the Dirac mass term (m_D) mixes the components of the right and left handed fermion, (by using the helicity projectors properties)

$$\bar{\psi}\psi = \bar{\psi}(P_L + P_R)\psi = \bar{\psi}(P_L^2 + P_R^2)\psi = \bar{\psi} P_L P_L \psi + \bar{\psi} P_R P_R \psi,$$

$$m_D \bar{\psi}\psi = m_D(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) \, . \tag{2.4}$$

- The electromagnetic vector current does not mix those components,

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_R \gamma^\mu \psi_R + \bar{\psi}_L \gamma^\mu \psi_L \, . \tag{2.5}$$

- Another case for the fermions can be seen in the $(V - A)$ fermionic weak current structure, where only left-handed fermions describe the weak interactions, since,

$$\bar{\psi}_L \gamma^\mu \psi_L = \bar{\psi} P_R \gamma^\mu P_L \psi = \bar{\psi} \gamma^\mu P_L^2 \psi = \bar{\psi} \gamma^\mu P_L \psi,$$

$$\bar{\psi}_L \gamma^\mu \psi_L = \frac{1}{2} \, \bar{\psi} \gamma^\mu (1 - \gamma_5) \psi \, , \tag{2.6}$$

where the properties of the helicity operators were used.

In this way, we may arrange the fermions as follows:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, e_R, \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \rightarrow \quad 1^{\text{st}} \quad \text{family}, \tag{2.7}$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \mu_R, \begin{pmatrix} c \\ s \end{pmatrix}_L, c_R, s_R \rightarrow \quad 2^{\text{nd}} \quad \text{family}, \tag{2.8}$$

$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \tau_R, \begin{pmatrix} t \\ b \end{pmatrix}_L, t_R, b_R \rightarrow \quad 3^{\text{rd}} \quad \text{family}, \tag{2.9}$$

where the left fields are doublets and the right fields are singlets under the group $SU(2)_L \otimes U(1)_Y$. The Isospin T_3 and hypercharge (Y) by the left-handed Fermion in the group SU(2) are displayed in the table 2.2. Considering group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, each quark

Table 2.2: Isospin T_3 and hypercharge (Y) by the left-handed Fermions

Name	T_3	Y	Name	T_3	Y
electron (e^-)	-1/2	-1	electron neutrino (ν_e)	1/2	-1
muon (μ)	-1/2	-1	muon neutrino (ν_μ)	1/2	-1
tau (τ)	-1/2	-1	tau neutrino (ν_τ)	1/2	-1
up (u)	-1/2	1/3	down (d)	1/2	1/3
charm (c)	-1/2	1/3	strange (s)	1/2	1/3
top (t)	-1/2	1/3	bottom (b)	1/2	1/3

is represented by three types of "color charge". So, these are defined as color triplets and can be written as follows:

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \quad , \quad d = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \tag{2.10}$$

and can be written alike for the other quarks.

2.1.2 Vector fields

Vector fields are the mediators of the interactions and rise when we impose local gauge invariance. There are three massive gauge bosons W_μ^+, W_μ^-, Z_μ and another non-massive gauge boson A_μ (photon) that are the mediators in the electroweak force. And there are eight gluons ($G_\mu^a; a = 1...8$) that are the mediators in the stronge-force (these only have color charge). Although the SM does not include gravitational interaction, it is assumed that there is a mediator called graviton, which has not yet been observed. These gauge bosons may be seen in the table 2.3. Furthermore, the gauge bosons have spin one and obey the Bosen-Einstein statistics.

Table 2.3: Mediating particles of the SM electroweak [7]

Interactions	Boson	Mass	Q
Strong	*Gluons* $G^a; a = 1...8$	0	0
Electromagnetic	Photon (γ)	$< 1 * 10^{-18} eV$	0
Weak	$W^{\pm}, Z^0$	$M_W = (80.38 \pm 0.01) GeV$ $M_Z = (91.188 \pm 0.002) GeV$	$\pm 1, 0$
Gravitational	Graviton (?)	$< 6 * 10^{-32} eV$	—

2.1.3 Scalar fields

The scalar field (ϕ) will be later identified as the Higgs boson which has spin zero, hypercharge 1, electric charged zero, the mass is (125.10 ± 0.14) GeV [7] and obeys the Bosen-Einstein statistics. Its dynamics is described by the scalar field Lagrangian (2.11)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \,, \tag{2.11}$$

where $V(\phi)$ is the scalar potential,

$$V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \,, \tag{2.12}$$

with λ real and positive. In the case of the charged self–interacting scalar field Lagrangian, this is:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \,, \tag{2.13}$$

where (ϕ) is the complex scalar field and with a similar potential:

$$V(\phi^* \phi) = \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2 \,. \tag{2.14}$$

The higgs field gives mass to the fermions and the gauge bosons $W^{\pm}_\mu, Z_\mu$ as we will discuss later in the Higgs mechanism. Finally, the SM elementary particles are the fermions in the Eqs. (2.7), (2.8), (2.9), the Higgs boson and the gauge bosons in the table 2.3, except for the graviton which is not part of SM spectrum yet.

2.2 Principle of gauge invariance and symmetries

2.2.1 Symmetry

The symmetries may be classified in two main groups: discrete and continuous, where the continuous symmetries are grouped as temporary space and internal symmetries. On the other hand, it is important to mention that in group theory these symmetries may be represented through a Lie group by means of unitary transformations, which are described by the generators and transformation parameters of the group. In this dissertation, transformations of internal symmetries will be considered and these are classified in global and local. Where the global symmetry transformation parameters do not depend on the temporal space coordinates and, for the local, these parameters depend on these coordinates. The global and local symmetries are descried respectively by:

$$U_{(\alpha)} = \exp\left(i\sum_{a=1}^{n}\alpha_a T^a\right) \simeq 1 - iT^a\alpha^a \tag{2.15}$$

$$U_{(\alpha(x))} = \exp\left(i\sum_{a=1}^{n}\alpha_a(x) T^a\right) \simeq 1 - iT^a\alpha^a(x)\,, \tag{2.16}$$

where α_a are the transformation parameters and T_a the group generators, which in Lie algebra satisfy the Eq. (2.17)

$$[T^a, T^b] = i f^{abc} T^c, \tag{2.17}$$

f^{abc} are called structure constants. Notice that the parameters α_a do not depend on the temporal space coordinates for the global transformations. On the other hand, it is important to mention that, if $f^{abc} = 0$ for any permutation of the indices, then it refers to Abelian algebra and if these are given by the Eqs. (2.18) and all other f^{abc} not related to these by permuting indices are zero, is a Non-Abelian algebra

$$\begin{aligned} f^{123} &= 1\,, \\ f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} &= \frac{1}{2}\,, \\ f^{458} = f^{678} &= \frac{\sqrt{3}}{2}\,. \end{aligned} \tag{2.18}$$

2.2.2 Gauge principle

The SM particles are described by fields and its dynamic by Lagrangians, as it was previously discussed. These Lagrangians are invariant under a global transformation, but these are not invariant under a local transformation. In this latter transformation, other terms appear from the original Lagrangians, which correspond to interactions. In this way, when that happens, the gauge principle is used. The gauge principle consists of leaving an invariant theory a under local transformation one. So the gauge procedure consists of introducing a covariant derivative D_μ and, thus, at the same time introducing a new vector field (A_μ),

$$U(1) : D_\mu \equiv \partial_\mu + ieqA_\mu \,, \tag{2.19}$$

and simultaneously, it is required that A_μ transforms like

$$A_\mu \longrightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha(x) \,. \tag{2.20}$$

Where "e" is a coupling constant and q the group generator operator. The gauge fields introduced by the covariant derivative depend on the symmetry group so that the number of gauge fields equals the number of generators of the symmetry group.

In the SM, a local transformation is made in the free particle Lagrangians and so that it can generate the terms of interactions of the electroweak and strong forces. The covariant derivatives of some fields in the fundamental representation groups are:

$$SU(3)_C : D_\mu = \partial_\mu + icG^a_\mu \frac{\lambda_a}{2} \,, \tag{2.21}$$

$$SU(2)_L : D_\mu = \partial_\mu + igW^i_\mu \frac{\sigma_i}{2} \,, \tag{2.22}$$

$$U(1)_Y : D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu \,, \tag{2.23}$$

$$U(1)_{electromagnetic} : D_\mu = \partial_\mu - ieQA_\mu \,, \tag{2.24}$$

where *c, g,g'* and e are coupling constants, G^a_μ, W^i_μ, B_μ and A_μ are the gauge fields and $\lambda_a/2$, $\sigma_i/2$, $Y/2$ and Q are the each group generators, being σ_i and λ_a the Pauli and Gell-Mann matrices respectively.

In the case of the SM electroweak group:

$$SU(2)_L \otimes U(1)_Y : D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu + igW^i_\mu \frac{\sigma_i}{2} \,, \tag{2.25}$$

in which Y takes the value of the hypercharge of each particle, for example, for the Higgs boson $Y = 1$

$$D_\mu \phi = \left(\partial_\mu - ig'\frac{\mathbb{I}_{2x2}}{2}B_\mu - igW^i_\mu \frac{\sigma_i}{2}\right)\phi \,, \tag{2.26}$$

where $\mathbb{I}_{2x2}$ is the identity matrix. The kinetic term for the gauge fields is,

$$\mathcal{L}_{gauge} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ . \tag{2.27}$$

In this case the negative sign is added in order to obtain a positive energy density for the electromagnetic field and the term $\frac{1}{4}$ is chosen so that the Euler-Lagrange equations describe the Maxwell equations of electromagnetism correctly with the conventional normalization of the electric charge e, where $F_{\mu\nu}$ is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^b_\mu + g f_{abc} A^b_\mu A^c_\nu \ . \tag{2.28}$$

The gauge field $F_{\mu\nu}$ will be denoted as $G_{\mu\nu}$ for $SU(3)_C$, $W_{\mu\nu}$ for $SU(2)_L$ and $B_{\mu\nu}$ for $U(1)_Y$, which are strength tensors definided as

$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g\, f^{abc} G^b_\mu G^c_\nu \ , \tag{2.29}$$

$$W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g \epsilon^{ijk} W^j_\mu W^k_\nu \ , \tag{2.30}$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \ , \tag{2.31}$$

where ϵ^{ijk} comes from the commutation relation between the generators.

2.3 Spontaneous Symmetry Breaking (SSB).

The Spontaneous Symmetry Breaking occurs when a system defined by a Lagrangian symmetric under the gauge group falls into a vacuum state (ground state) that is not symmetric or it is not invariant. The SSB is necessary to later analyze the Higgs mechanism that is one of the pillars of the SM. We will follow closely [11].

2.3.1 SSB of a Discrete symmetry

In field theory, the SSB may be analyzed with a scalar self–interacting real field Lagrangian. The whole Lagrangian (Eq. ̊lag) is invariant under the discrete transformation (Eq. (2.32))

$$\phi \rightarrow -\phi \ . \tag{2.32}$$

The vacuum ($<\phi>$) can be obtained from the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\left[(\partial_0\phi)^2 + (\nabla\phi)^2\right] + V(\phi) \ ,$$

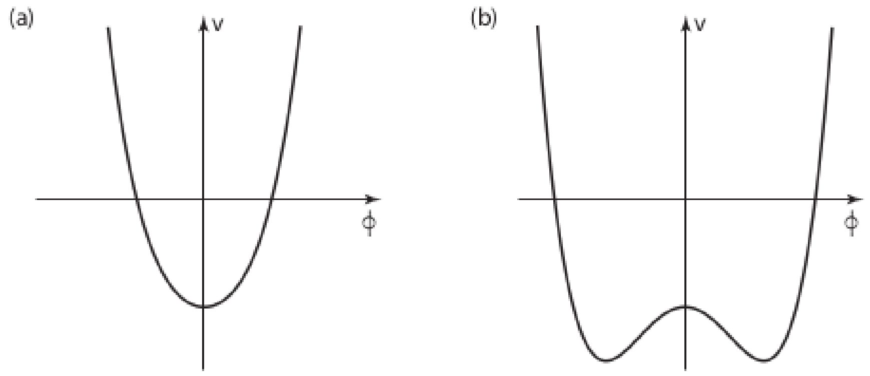

Figure 2.1: (a) $V(\phi)$ for $\mu^2 \geqslant 0$ with a unique energy minimum (vacuum) at $\phi = 0$. (b) $V(\phi)$ for $\mu^2 < 0$ with two minimum energy (degenerate vacuum), where the symmetry is spontaneously broken [1].

where $V(\phi)$ corresponds to the Eq. (2.12) with $<\phi>=$ constant, the minimum energy does not depend on x and t, it requires that the potential to be minimized and consequently the Hamiltonian, in this way,

$$\partial_0\phi = \nabla\phi = 0\,,$$
$$\frac{d\mathcal{H}}{d\phi} = \frac{dV}{d\phi} = 0\,,$$

since, λ must be positive to guarantee a minimum energy (vacuum) and then the energy minimum depends on μ parameter.

$$\phi(\mu^2 + \lambda\phi^2) = 0 \rightarrow \begin{cases} \phi = 0 & \text{for} \quad \mu^2 \geqslant 0 \\ \phi^{\pm} = \pm\sqrt{-\mu^2/\lambda} & \text{for} \quad \mu^2 < 0\,. \end{cases} \tag{2.33}$$

From the Eq. (2.33) can be seen that for $\mu^2 \geqslant 0$, there is just one vacuum and this is invariant under transformation in the Eq. (2.32) (see Figure 2.1 (a)). However, for $\mu^2 < 0$, there are two vacuum states corresponding to $\phi^{\pm}$, in this case, the vacuum is not invariant under transformation in the Eq. (2.32). Then, the symmetry is spontaneously broken (see Figure 2.1 (b)). To sum up, the vacuum of this theory is not invariant under assumed the discrete transformation and this is called spontaneous symmetry breaking. However, we really want the minimum of this theory to be zero, so the expected value of the vacuum of ϕ must be zero. For this, a new field is defined (ϕ') when we consider perturbations in the fundamental state

$$\phi \longrightarrow \phi' = \phi - <\phi>\,,$$

where the state in the vacuum of the old field is chosen as

$$<\phi>= \phi^+ = \sqrt{-\mu^2/\lambda} = \mathrm{v}\,,$$

thus,

$$\phi' \equiv \phi - \mathrm{v}\,, \tag{2.34}$$

in this way, it can be obtained $< \phi' >= 0$, which implies that it is an adequate theory for perturbations in the ground stated. Finally, substituting the Eq. (2.34) in the Klein-Gordon Lagrangian (Eq. (2.11)), it can be obtained the scalar field Lagrangian that is described by ϕ',

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi'\partial^\mu\phi' - \frac{1}{2}\left(\sqrt{-2\mu^2}\right)^2 \phi'^{\,2} - \lambda\,\mathrm{v}\,\phi'^{\,3} - \frac{1}{4}\lambda\phi'^{\,4}\,,$$

where the real and positive mass is $M_{\phi'} = \sqrt{-2\mu^2}$ and this last Lagrangian is not symmetric under $\phi \rightarrow -\phi'$, because it have a $\phi'^{\,3}$ term. [11].

2.3.2 SSB of a Continuous symmetry and Nambu-Goldstone bosons

In this case, to study a continuous symmetry is spontaneously broken, we may analyze the the charged self–interacting scalar field Lagrangian Eq. (2.13) with the potential of the Eq. (2.14), for a (ϕ) complex scalar field written in terms of two real fields as:

$$\phi = \frac{(\phi_1 + i\phi_2)}{\sqrt{2}}\,,$$

thus, the Lagrangian Eq. (2.13) is

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2\right) - V(\phi_1,\phi_2)\,, \tag{2.35}$$

which is invariant under $SO(2)$ rotations [11]. With

$$V(\phi_1,\phi_2) = \frac{\mu^2\phi_1^2}{2} + \frac{\mu^2\phi_2^2}{2} + \lambda^2(\phi_1^2 + \phi_2^2)^2 \tag{2.36}$$

This Lagrangian is invariant under the global transformation in the U(1) group:

$$U_{(\theta)} = \exp\left(-i\theta\right) \tag{2.37}$$

so,

$$\phi \rightarrow U_{(\theta)}\phi = \exp\left(-i\theta\right)\phi \tag{2.38}$$

The vacuum $< \phi >$ can be obtained with the same arguments from the previous subsection 2.3.1. Then, the minimum energy does not depend on x and t, it requires the minimal potential ($V(\phi_1,\phi_2)$). In this way, the minimum depends only of the μ parameter, because

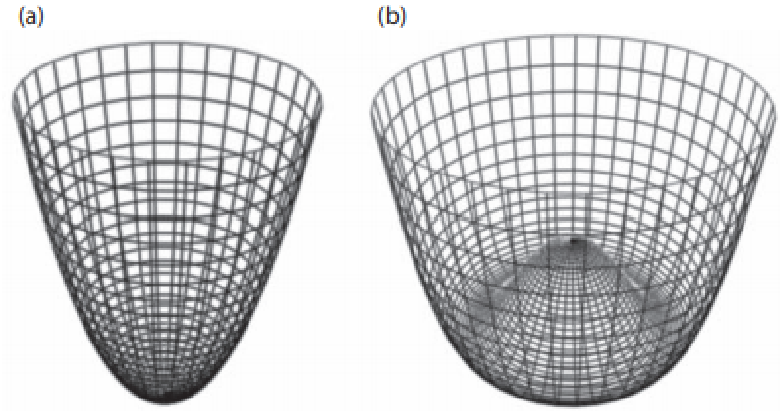

Figure 2.2: (a)$V(\phi_1, \phi_2)$ with a unique energy minimum at $\phi_1 = \phi_2 = 0$. (b) $V(\phi_1, \phi_2)$ with a degenerate vacuum at $\phi_1^2 + \phi_2^2 \neq 0$. [1].

λ is positive. Thus, two cases can be considered, $\mu^2 > 0$ and $\mu^2 < 0$. For $\mu^2 > 0$ the vacuum is not broken and the unique minimum corresponding to the vacuum state occurs at $\phi_1 = \phi_2 = 0$, as can be illustrated in the potential of the figure 2.2 a).

For the case $\mu^2 < 0$ there are a continuum of distinct vacuum states located at $< |\phi|^2 >$ (Eq. (2.39)) [11], degenerated in energy. This degeneracy is a consequence of the SO(2) symmetry, because the Eq. (2.35) is invariant under $SO(2)$ rotations. It can be seen in the potential graph (see figure 2.2 b)), which is a structure called a Mexican hat.

$$< |\phi|^2 >= \frac{(< \phi_1 >^2 + < \phi_2 >^2)}{2} = \frac{-\mu^2}{2\lambda} \equiv \frac{\mathrm{v}^2}{2} \,. \tag{2.39}$$

Moreover, the SO(2) symmetry is a spontaneous broken when at selecting one state as the particular vacuum for the case $\mu^2 < 0$. In this way, let us choose the physical vacuum state with the configuration $\phi_1 = \mathrm{v}$ and $\phi_2 = 0$, where we may do the new fields appropriate for small perturbations. Thus, these fields can be defined as,

$$\begin{aligned} \phi_1' &= \phi_1 - \mathrm{v} \,, \\ \phi_2' &= \phi_2 \,. \end{aligned}$$

Now, for small oscillations the Lagrangian (Eq. (2.35)) becomes as,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_1' \partial^\mu \phi_1' - \frac{1}{2}(-2\mu^2)\phi_1'^2 + \frac{1}{2}\partial_\mu \phi_2' \partial^\mu \phi_2' + \text{ interaction terms} \,,$$

in terms of these new fields. So, there are two particles in the spectrum, where the field ϕ_1' has mass $\sqrt{2}\mu$ while the field ϕ_2' remains without mass, because there is no quadratic factor associated with this. Thus this massless field are called to Nambu-Goldstone bosons or Goldstone bosons, which are described by Goldstone's theorem.

Goldstone's theorem can be formulated as follows [1]:

> *"one massless spin-zero particle will occur for each broken generator of the original symmetry group"*

On the other hand, it is important to know that the mass matrix for the massless spin-zero field, in tree approximation can be considered as:

$$M_{ij}^2 = \left. \frac{\partial^2 V(\phi_1', \phi_2')}{\partial \phi_i' \partial \phi_j'} \right|_{\phi'=\phi_0'} . \tag{2.40}$$

In this case, for this mass matrix ϕ_2' corresponds to the zero eigenvalue and ϕ_1' it is positive.

2.3.3 The Higgs Mechanism

The Higgs mechanism has an additional advantage that would be the gauge boson that becomes massive. Because, this mechanism is based on the Spontaneous Breaking of a gauge symmetry and the gauge principle. In this way, a contribution will arise by means of the massless gauge fields and the Nambu–Goldstone bosons that appear in the spontaneous symmetry breaking process. [1]. In this section, we will consider the Higgs mechanism to an U(1) abelian local symmetry gauge group.

The Abelian Higgs Mechanism

The case of Abelian Higgs Mechanism, it can be considered the charged self–interacting scalar field Lagrangian (Eq. (2.13)) with the potential (Eq. (2.14)), and we will use a local transformation,

$$\phi \longrightarrow \exp\left(i\, q\, \alpha(x)\right) \phi \,, \tag{2.41}$$

where it is remembered that the vector field has no mass, due to the gauge invariance. Due to this, Lagrangian is not invariant under this transformation and following the principle of gauge, a covariant derivative D_μ (Eq. (2.19)) and the gauge boson (A_μ, Eq. (2.20)) must be introduced (with e = 1) in such a way that it can be left invariant. In this way the Lagrangian will be

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)(D^\mu \phi)^\dagger - V(\phi^* \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} . \tag{2.42}$$

As discussed in the previous section for the case of SSB in a continuous symmetry, for $\mu^2 > 0$ the vacuum $< |\phi|^2 >$ is 0, but for $\mu^2 < 0$, the spontaneous symmetry breaking occurs with the vacuum $< |\phi|^2 >$ given by (Eq. (2.39)). Choosing a convenient configuration of the vacuum state of the new fields called η and $\mathcal{Z}$, in such a way that they are appropriate for small perturbations, *i.e.* [11]

$$\phi = \exp\left(i\frac{\mathcal{Z}}{\mathrm{v}}\right)\frac{(\eta+\mathrm{v})}{\sqrt{2}} \simeq \frac{1}{\sqrt{2}}(\eta+\mathrm{v}+i\mathcal{Z}) = \phi' + \frac{\mathrm{v}}{\sqrt{2}}\ . \tag{2.43}$$

In this case, $\exp\left(i\frac{\mathcal{Z}}{\mathrm{v}}\right)$ is known as the phase corresponding to the goldstone boson. Assuming that only the real part is dislocated, the imaginary part is not. Then, $\phi' = \phi - \mathrm{v}$. Therefore, substituting ϕ (Eq. (2.43)) in the Lagrangian (Eq. (2.13)),

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}(-2\mu^2)\eta^{\,2} + \frac{1}{2}\partial_\mu\mathcal{Z}\partial^\mu\mathcal{Z} + \text{ interact.}\\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu\partial^\mu\mathcal{Z}\ .\end{aligned} \tag{2.44}$$

This Lagrangian presents three particles, a scalar field η with mass $M_\eta = \sqrt{-2\mu^2}$, a massless scalar boson $\mathcal{Z}$ that correspond at Goldstone boson and a massive vector boson A_μ, with mass $M_A = q\mathrm{v}$ [11]. Nevertheless, the last term in the Eq. (2.44) is a mixing term, because it mixes the propagators of A_μ and $\mathcal{Z}$ particles and it is a problem, since it does not describe a well-defined mass.

In order to eliminate the mixing term in the Lagrangian, the gauge parameter can be chosen in the gauge transformation (Eq. (2.41)) as [11]

$$\alpha(x) = -\frac{1}{q\mathrm{v}}\mathcal{Z}(x)\ , \tag{2.45}$$

which is called *unitary gauge*, then

$$\phi = \exp\left(i\,q\,\alpha(x)\right)\phi = \exp\left(iq\left(-\frac{\mathcal{Z}}{q\mathrm{v}}\right)\right)\phi\ . \tag{2.46}$$

Thus, ϕ (Eq. (2.43)) is given as,

$$\phi = \exp\left(-i\frac{\mathcal{Z}}{\mathrm{v}}\right)\exp\left(i\frac{\mathcal{Z}}{\mathrm{v}}\right)\frac{(\eta+\mathrm{v})}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\eta+\mathrm{v})\ .$$

Now, we can see the Goldstone boson disappears (the $\mathcal{Z}$ field disappeared). The same result is obtain in the limit $\frac{\mathcal{Z}}{\mathrm{v}} << 1$. Thus, the Lagrangian becomes,

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}(-2\mu^2)\eta^{\,2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2\mathrm{v}^2}{2}A'_\mu A^{\mu\,\prime}\\ & + \frac{1}{2}q^2(\eta+2\mathrm{v})\,\eta A'_\mu A^{\mu\,\prime} - \frac{\lambda}{4}\eta^{\,3}(\eta+4\mathrm{v})\ .\end{aligned} \tag{2.47}$$

Thus, it is important to observed the degrees of freedom by comparing the initial Lagrangian (Eq. (2.13)) with the latter obtained (Eq. (2.47)). In this way, in the Eq. (2.13) there are two degrees of freedom for $\phi^{(*)}$ charged scalar and two for A_μ massless vector, but in the final Lagrangian Eq. (2.47) there are three degrees of freedom for A'_μ massive vector and one for η neutral scalar [11]. In both cases, there are 4 degrees of freedom in total, which are distributed, but preserved. The extra degree of freedom acquired by the massive gauge boson comes from the disappearance of one of the degrees of freedom of the scalar field.

From the aforementioned, it is important to say, that the most interesting result in this mechanism was the disappearance of $\mathcal{Z}$ field and the appearance of a mass term for A_μ field. This can be interpreted in the following way: by using the redefinition of the field and the gauge invariance, a massless field (Goldstone's boson) can be generated, which is absorbed generating a mass term for field A_μ.

2.4 Standard Model Lagrangian

In this section we will review the key aspects of the SM following [11].

2.4.1 Introducing the electroweak gauge group

Starting points concerning the SM are local gauge symmetries. Having in mind the β-decay which has a V-A structure, Eq. (2.6), we need to incorporate in the SM the weak leptonic charged current in the form,

$$J^+_\mu = \bar{\ell}\gamma_\mu(1-\gamma_5)\nu = 2\,\bar{\ell}_L\gamma_\mu\nu_L\,, \tag{2.48}$$

where ℓ corresponds e, μ and τ flavors since leptons are arranged in families.

As the left-handed fermions (ψ_L) transform as isospin doublets $T = 1/2$ (see T_3 in the table 2.2), whereas the right-handed ones (ψ_R) as weak isospin singlets with $T = 0$, these fermions are arranged as follows,

$$\psi_{L\ell} \equiv \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} P_L\,\nu \\ P_L\,\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}\,, \tag{2.49}$$

$$\psi_{R\ell} \equiv P_R\,\ell = \ell_R\,, \tag{2.50}$$

where, we assume that the neutrinos are massless, there is no such right-handed neutrino component.

That said, we can write the charged weak current in terms of leptonic isospin currents

$$J^i_\mu = \bar{\psi}_{L\ell}\,\gamma_\mu\;\; T^i\;\psi_{L\ell}\;\; = \bar{\psi}_{L\ell}\,\gamma_\mu\,\frac{\sigma^i}{2}\,\psi_{L\ell}\,,$$

were the isospin T^i is described by the Pauli matrices σ^i ($i = 1,2,3$), since $T^i = \frac{\sigma^i}{2}$. With this information we can write down the charged weak current of $SU(2)$ as,

$$\begin{aligned}
J^1_\mu &= \frac{1}{2}(\bar{\nu}_L\;\;\bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}\left(\bar{\ell}_L\gamma_\mu\nu_L + \bar{\nu}_L\gamma_\mu\ell_L\right)\,,\\
J^2_\mu &= \frac{1}{2}(\bar{\nu}_L\;\;\bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{i}{2}\left(\bar{\ell}_L\gamma_\mu\nu_L - \bar{\nu}_L\gamma_\mu\ell_L\right)\,,\\
J^3_\mu &= \frac{1}{2}(\bar{\nu}_L\;\;\bar{\ell}_L)\,\gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}\left(\bar{\nu}_L\gamma_\mu\nu_L - \bar{\ell}_L\gamma_\mu\ell_L\right)\,.
\end{aligned}$$

Notice that we may combine the J^1_μ and J^2_μ currents to and define the charged current as,

$$J^+_\mu = 2\;\left(J^1_\mu - iJ^2_\mu\right)\;.$$

Concerning the current associated with the weak hypercharge, J^Y_μ, we can find it using the J^3_μ current involving both left-handed and right-handed fields as follows,

$$J^Y_\mu \equiv \left(Y_{\psi_{L\ell}}\bar{\psi}_{L\ell}\,\gamma_\mu\,\psi_{L\ell} + Y_{\psi_{R\ell}}\,\bar{\psi}_{R\ell}\,\gamma_\mu\,\psi_{R\ell}\right) = -\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{\ell}_L\gamma_\mu\ell_L + 2\,\bar{\ell}_R\gamma_\mu\ell_R\right)\,, \quad (2.51)$$

where we have defined $Y_{\psi_{L\ell}} = -1$ and $Y_{\psi_{R\ell}} = -2$. In other words, left-handed doublets have hypercharge equal to -1 while right-handed singlets hypercharge equal to -2. Therefore, the hypercharge differentiates singlets from doublets fields.

Also, in the same way, we can write the electromagnetic current as

$$J^{\text{em}}_\mu = -\,\bar{\ell}\,\gamma_\mu\,\ell = -\left(\bar{\ell}_L\gamma_\mu\ell_L + \bar{\ell}_R\gamma_\mu\ell_R\right) = J^3_\mu + \frac{1}{2}J^Y_\mu\,, \quad (2.52)$$

which involves the Gell-Mann–Nishijima relation,

$$Q = T_3 + \frac{1}{2}Y\,. \quad (2.53)$$

Where T_3 is isospin operator, Y is hypercharged operator and the conservation of the electric charge is guaranteed by the definition of the hypercharged, satisfying the group algebra $SU(2) \otimes U(1)$ group:

$$[T^i, T^j] = i\, \epsilon^{ijk} T^k \qquad \text{and} \qquad [T^i, Y] = 0\,,$$

guaranteeing that $SU(2)$ commutes with $U(1)$.

Since we can obtain a charged and neutral current using the $SU(2)$ and $U(1)$ groups they were selected to describe the electromagnetic and weak interactions in nature. Knowing the local gauge groups of the SM, one of the first things one needs to obtain are the neutral and charged currents using the formalism of gauge invariance as we discuss below.

2.4.2 Leptonic Kinetic Lagrangian

In this section, we will derive the charged and neutral currents via the introduction of vector fields using gauge invariance. As it was mentioned earlier in the section 2.2.2, the gauge bosons of $SU(2)_L$ are W^1_μ, W^2_μ, W^3_μ and for the $U(1)_Y$ the gauge boson is B_μ. Therefore, the gauge fields of the selected group are these four fields. The kinetic terms of these fields are similar to the one presented in the Eq. (2.27), where we change $F_{\mu\nu}$ for each boson respectively. The fundamental representation for the matter field is chosen. So, it is necessary to analyze the interactions between fermions and gauge bosons via the Dirac Lagrangian for massless fields,

$$\begin{aligned}
\mathcal{L}_{\text{leptons}} &= \bar{\psi}_{R\ell}\, i\, \partial\!\!\!/\, \psi_{R\ell} + \bar{\psi}_{L\ell}\, i\, \partial\!\!\!/\, \psi_{L\ell} \\
&= \bar{\ell}_R\, i\, \partial\!\!\!/\, \ell_R + \bar{\ell}_L\, i\, \partial\!\!\!/\, \ell_L + \bar{\nu}_L\, i\, \partial\!\!\!/\, \nu_L \\
&= \bar{\ell}\, i\, \partial\!\!\!/\, \ell + \bar{\nu}\, i\, \partial\!\!\!/\, \nu\,,
\end{aligned} \tag{2.54}$$

where this Lagrangian Eq. (2.54) is invariant under a global transformation, but not under a local one. As we want to introduce interactions between the gauge fields and the fermions we need to require a local gauge invariance. Therefore, the next step is to introduce the covariant derivative (D_μ) and write the most general Lagrangian invariant under the gauge group as follows,

$$D_{\mu(\psi_{L\ell})} = \partial_\mu + i\, \frac{g}{2}\, \tau^i\, W^i_\mu + i\, \frac{g'}{2}\, Y B_\mu\,, \tag{2.55}$$

$$D_{\mu(\psi_{R\ell})} = \partial_\mu + i\, \frac{g'}{2}\, Y\, B_\mu\,, \tag{2.56}$$

where $D_{\mu(\psi_{L\ell})}$ is associated to left-handed fermions and $D_{\mu(\psi_{R\ell})}$ to the right-handed ones, where g and g' are the coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups.

Substituting Eq. (2.55) and Eq. (2.56) in Eq.(2.54) we get,

$$\begin{aligned}
\mathcal{L}_\ell \quad \longrightarrow \quad & \mathcal{L}_{\text{leptons}} + \bar{\psi}_{L\ell}\, i\gamma^\mu \left(i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2} Y B_\mu \right) \psi_{L\ell} \\
& + \bar{\psi}_{R\ell}\, i\gamma^\mu \left(i\frac{g'}{2} Y B_\mu \right) \psi_{R\ell} \; .
\end{aligned} \tag{2.57}$$

The gauge fields are in the flavor basis and it is necessary to analyze and diagonalize them to obtain the physical ones.

Charged Lagrangian

Extending the left-handed term of (Eq. (2.57)) we find,

$$\begin{aligned}
\mathcal{L}_\ell^L \quad = \quad & -g\, \bar{\psi}_{L\ell}\, \gamma^\mu \left(\frac{\tau^1}{2} W_\mu^1 + \frac{\tau^2}{2} W_\mu^2 \right) \psi_{L\ell} \\
& - g\, \bar{\psi}_{L\ell}\, \gamma^\mu\, \frac{\tau^3}{2}\, \psi_{L\ell}\, W_\mu^3 \; - \; \frac{g'}{2} Y\, \bar{\psi}_{L\ell}\, \gamma^\mu\, \psi_{L\ell}\, B_\mu .
\end{aligned} \tag{2.58}$$

Notice that the first term of Eq.(2.58) mixes a neutral fermion with a charged one. Therefore, charge conservation requires the presence of a charged gauge boson defined as:

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \; . \tag{2.59}$$

With Eq. (2.59) we can rewrite the first of (2.58) as,

$$\mathcal{L}_\ell^{L(\pm)} = -\frac{g}{2}\, \bar{\psi}_{L\ell}\, \gamma^\mu \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix} \psi_{L\ell} \; ,$$

which leads to:

$$\mathcal{L}_\ell^{L(\pm)} = -\frac{g}{2\sqrt{2}} \left[\bar{\nu}\gamma^\mu (1-\gamma_5)\ell\; W_\mu^+ + \bar{\ell}\gamma^\mu(1-\gamma_5)\nu\; W_\mu^- \right] \; . \tag{2.60}$$

This result has the vector-axial form $(V - A)$ of the weak charged current which reproduces the Fermi theory where at low-energy we match $g/(2\sqrt{2}) = G_W$, where G_W is the coupling constant in the Fermi theory with,

$$G_W^2 = \frac{M_W^2 G_F}{\sqrt{2}} \; .$$

Where G_F is the Fermi coupling constant. Thus,

$$\frac{g}{2\sqrt{2}} = \left(\frac{M_W^2 G_F}{\sqrt{2}}\right)^{1/2} . \tag{2.61}$$

Notice that the g coupling constant is related to the mass of the W boson and the Fermi constant. This is important because the until now arbitrary gauge coupling, g is now related to observables, G_F and M_W. G_F is extracted from the neutron lifetime and M_W has been measured by CERN. We shall now move to the neutral current.

Neutral Lagrangian

Picking up the right term of Eq. (2.57), the second and third terms of the Eq. (2.58), the neutral Lagrangian can be written as:

$$\begin{aligned}\mathcal{L}_\ell^{(\mathrm{L+R})(0)} &= -\, g\, \bar{\psi}_{L\ell} \left(\gamma^\mu \frac{\tau^3}{2}\right) \psi_{L\ell}\, W_\mu^3 \\ &\quad - \frac{g'}{2} \left(\bar{\psi}_{L\ell}\, \gamma^\mu\, Y\, \psi_{L\ell} \;+\; \bar{\psi}_{R\ell}\, \gamma^\mu\, Y\, \psi_{R\ell}\right) B_\mu \,,\end{aligned} \tag{2.62}$$

which can be simplified as,

$$\mathcal{L}_\ell^{(\mathrm{L+R})(0)} = -g\, J_3^\mu\, W_\mu^3 - \frac{g'}{2}\, J_Y^\mu\, B_\mu \,, \tag{2.63}$$

where we used J_3^μ and J_Y^μ defined in Eq. (2.51). This neutral current does not involve the physical fields as we are going to show in the next section. This has to do with the spontaneous symmetry breaking mechanism which as a result leads to a mixing between W_3 and B fields that will give rise to the Z boson and the photon, and consequently generate the electromagnetic and neutral weak current where [11],

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \quad \text{and} \quad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \,,$$

in this case, θ_W is the Weinberg angle, also known as the weak angle. In the diagonalization procedure we find the relations between coupling constants g and g',

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \tag{2.64}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{2.65}$$

Substituting Eq.(2.64) and Eq.(2.65) into Eq.(2.63), we find:

$$\begin{aligned}\mathcal{L}_\ell^{(\mathsf{L+R})(0)} &= -(g\sin\theta_W J_3^\mu + \frac{1}{2}g'\cos\theta_W J_Y^\mu)A_\mu \\ &\quad +(-g\cos\theta_W J_3^\mu + \frac{1}{2}g'\sin\theta_W J_Y^\mu)Z_\mu, \end{aligned} \tag{2.66}$$

which leads to,

$$\begin{aligned}\mathcal{L}_\ell^{(\mathsf{L+R})(0)} &= -g\sin\theta_W\ (\bar{\ell}\gamma^\mu\ell)\ A_\mu \\ &\quad -\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu(g_V^i - g_A^i\gamma_5)\psi_i Z_\mu\ . \end{aligned} \tag{2.67}$$

In Eq.(2.67) we observe a vectorial current, which should be matched with the electromagnetic current, thus $e = g\sin\theta_W = g'\cos\theta_W$. With this relation the electromagnetic current is reproduced, where the photon field, A, couples to fermions proportionally to their electric charge. Moreover, the gauge couplings g and g' are now known. This is important because the covariant derivative initially invoked undetermined gauge couplings, but they are now related to the observables such as the W mass and electric charge. This brings predictability to the model. Furthermore, the SM introduces the vector and axial couplings in the neutral current. In this way, the Z boson does not couple only to left-handed fermions as the W boson which has purely a V-A structure as required by the β decay where parity is maximally violated. After some math one can find,

$$g_V^i \equiv T_3^i - 2Q_i\sin^2\theta_W\ , \tag{2.68}$$

$$g_A^i \equiv T_3^i\ . \tag{2.69}$$

In summary,

$$\mathcal{L}_{\text{leptons}} = \bar{\ell}\ i\ \partial\!\!\!/\ \ell + \bar{\nu}\ i\ \partial\!\!\!/\ \nu\ + \mathcal{L}_\ell^{(\mathsf{L+R})(0)} + \mathcal{L}_\ell^{\mathsf{L}(\pm)}\ , \tag{2.70}$$

where the the kinetic Lagrangian that describes the leptons and gauge bosons is given by:

$$\mathcal{L}_{\text{kinetic}} = \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{gauge}}\ , \tag{2.71}$$

$$\text{with}\quad \mathcal{L}_{\text{gauge}} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\ . \tag{2.72}$$

The weak interactions are of short-range nature. Therefore, it involves massive gauge fields. We have not addressed thus far how these gauge bosons, $W^\pm$ and Z gain mass. To do so, we need to invoke the Higgs mechanism (to be discussed in the next section).

2.4.3 The Higgs Mechanism and massive gauge boson in the SM

We have already introduced the spontaneous symmetry breaking mechanism. We will now implement it within the $SU(2)_L \otimes U(1)_Y$ gauge group. We start adding a $SU(2)$ doublet, Φ, with hypercharge equal to unit with,

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} . \tag{2.73}$$

This definition of ϕ ensures that the scalar field Lagrangian (Eq. 2.13) is invariant in front of the group $SU(2) \otimes U(1)$, by the transformations. We use the gauge principle to introduce the covariant derivative D_μ alike,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i\, g \frac{\tau^i}{2} W_\mu^i + i\, \frac{g'}{2} Y B_\mu \, ,$$

and we can get the vacuum from the Hamiltonian,

$$\mathcal{H} = \frac{1}{2} \left[(\partial_0 \Phi)^2 + (\nabla \Phi)^2 \right] + V(\Phi) \, .$$

Using the same arguments presented before on the section 2.3.1, to find the vacuum expectation value of ϕ we impose,

$$\frac{d\mathcal{H}}{d\Phi} = \frac{dV}{d\Phi} = 0 \, ,$$

We will not review here again the spontaneous symmetry breaking mechanism which occurs for $\mu^2 < 0$. The important information is that we can choose the *vacuum expectation value* (VEV) of the Higgs field without loss of generality as,

$$< \Phi >_0 = \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} ,$$

where, the neutral component of Φ acquires a VEV, v, where,

$$\mathrm{v} = \sqrt{-\frac{\mu^2}{2\lambda}} \tag{2.74}$$

with μ and λ referring to the bilinear and quartic couplings in the scalar potential.

As it already was mentioned above where v corresponds to the minimum of the potential and instead of the energy. Having in mind that each gauge boson is associated with a generator of the gauge group, each generator that is broken in the spontaneous symmetry breaking mechanism leads to a massive gauge field. Therefore, in order to reproduce the electromagnetism with a massless photon we need the electric charge operator to preserve the vacuum. To realize that we first need to notice that,

$$\Phi \longrightarrow \Phi' = e^{i\alpha Q} < \Phi >_0 \simeq \ (1 + i\ \alpha\ Q) < \Phi >_0 = < \Phi >_0 \ .$$

Therefore, we need that $Q < \Phi >_0 = 0$. With this information that Gell-Mann–Nishijima relation (Eq. (2.53)) surfaced where,

$$\begin{aligned} Q < \Phi >_0 &= \left(T_3 + \frac{1}{2}Y\right) < \Phi >_0 \\ &= \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} = 0 \ . \end{aligned}$$

Hence, keeping $Q = T_3 + \frac{1}{2}Y$ with $Y = 1$ for the Higgs boson the spontaneous symmetry breaking mechanism leads to a charge conservation with,

$$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{\mathrm{em}} \ ,$$

where the photon generator of the $U(1)_{\mathrm{em}}$ group remains massless. However, the others gauge bosons will acquire mass, as they correspond to the broken generators T^1,T^2 and $T^3 - \frac{Y}{2}$ [11]. For this, the Higgs doublet will be parametrized by taking into account the Eq. (2.43) in section 2.3.3,

$$\Phi \equiv \exp\left(i\frac{\tau^i}{2}\frac{\chi_i}{\mathrm{v}}\right)\begin{pmatrix} 0 \\ (\mathrm{v} + H)/\sqrt{2} \end{pmatrix}$$

By continuing with what has been done in section (2.3.3), we now choose a parametrization for the $SU(2)_L$ gauge transformation as in the Eq. (2.45) (unitary gauge),

$$\alpha_i = \frac{\chi_i}{\mathrm{v}} \ .$$

The unitary gauge is the gauge in which the goldstone bosons disapear in the theory. It is easier to work with the unitary gauge because there will be no need to include the goldstone bosons in the theory, since they their presence is related to the the preservation of unitarity in the theory. In summary, we will work in the unitary gauge throughout where,

$$\Phi \longrightarrow \Phi' = \exp\left(-i\frac{\tau^i}{2}\frac{\chi_i}{\mathrm{v}}\right)\Phi = \frac{(\mathrm{v} + H)}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{2.75}$$

Thus, the scalar Lagrangian can be written as,

$$\begin{aligned}\mathcal{L}^{SM}_{\text{scalar}} &= \left|\left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + i\frac{g'}{2}YB_\mu\right)\frac{(\text{v}+H)}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\right|^2 \\ &\quad -\mu^2\frac{(\text{v}+H)^2}{2} - \lambda\frac{(\text{v}+H)^4}{4}\,,\end{aligned} \tag{2.76}$$

where $W^i = W^1, W^2, W^3$. Using the Eq. (2.59), we may see that the first term of Eq. (2.76) is,

$$\left|\begin{pmatrix}0\\ \frac{\partial_\mu H}{\sqrt{2}}\end{pmatrix} + \frac{(\text{v}+H)}{\sqrt{2}}\left[\frac{ig}{2}\begin{pmatrix}W^3_\mu & \sqrt{2}W^+_\mu\\ \sqrt{2}W^-_\mu & -W^3_\mu\end{pmatrix} + \frac{ig'}{2}\begin{pmatrix}B_\mu & 0\\ 0 & B_\mu\end{pmatrix}\right]\begin{pmatrix}0\\1\end{pmatrix}\right|^2,$$

leading to,

$$\left|\begin{pmatrix}0\\ \frac{\partial_\mu H}{\sqrt{2}}\end{pmatrix} + \frac{(\text{v}+H)}{2\sqrt{2}}\left[\begin{pmatrix}igW^3_\mu + ig'B_\mu & ig\sqrt{2}W^+_\mu\\ ig\sqrt{2}W^-_\mu & -igW^3_\mu + ig'B_\mu\end{pmatrix}\right]\begin{pmatrix}0\\1\end{pmatrix}\right|^2.$$

In this way, we have,

$$\left|\begin{pmatrix}0\\ \frac{\partial_\mu H}{\sqrt{2}}\end{pmatrix} + \frac{(\text{v}+H)}{2\sqrt{2}}\left[\begin{pmatrix}ig\sqrt{2}W^+_\mu\\ -igW^3_\mu + ig'B_\mu\end{pmatrix}\right]\right|^2.$$

Using the vector fields defined below,

$$W^3_\mu = sen\theta_W A_\mu + cos\theta_W Z_\mu \quad \text{and} \quad B_\mu = cos\theta_W A_\mu - sen\theta_W Z_\mu$$

, we may obtain the following,

$$-igW^3_\mu + ig'B_\mu = -ig\left[sen\theta_W A_\mu + cos\theta_W Z_\mu\right] + ig'\left[cos\theta_W A_\mu - sen\theta_W Z_\mu\right].$$

With the relations between coupling constants g and g' in the $SU(2)_L \otimes U(1)_Y$ group, $cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$ and $sen\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$, we get,

$$-igW^3_\mu + ig'B_\mu = \frac{i}{\sqrt{g^2+g'^2}}\left[-gg'A_\mu - g^2Z_\mu + gg'A_\mu - g'^2Z_\mu\right],$$

Thus,

$$-igW^3_\mu + ig'B_\mu = -iZ_\mu\sqrt{g^2+g'^2} = -\frac{igZ_\mu}{c_W},$$

where $c_W \equiv \cos\theta_W$. The first term of Eq. (2.76) remains as,

$$\left| \begin{pmatrix} 0 \\ \frac{\partial_\mu H}{\sqrt{2}} \end{pmatrix} + i\frac{g}{2}(\mathrm{v}+H) \begin{pmatrix} W^+_\mu \\ (-1/\sqrt{2}c_W)Z_\mu \end{pmatrix} \right|^2 .$$

The quadratic terms in the vector fields are,

$$\frac{g^2}{4}(\mathrm{v}+H)^2 \left(W^+_\mu W^{-\,\mu} + \frac{1}{2c_W^2} Z_\mu Z^\mu \right) , \tag{2.77}$$

where the terms with v^2 constitute mass terms for the $W^\pm$ and Z bosons,

$$\frac{g^2\mathrm{v}^2}{4} W^+_\mu W^{-\,\mu} + \frac{g^2\mathrm{v}^2}{8\cos^2\theta_W} Z_\mu Z^\mu .$$

When comparing these last terms with *Proca* equation we get,

$$M_W^2 W^+_\mu W^{-\,\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu .$$

Thereby, the masses for the massive gauge bosons in the SM electroweak can be identified as follows,

$$M_W = \frac{g\mathrm{v}}{2} , \tag{2.78}$$

$$M_Z = \frac{g\mathrm{v}}{2c_W} = \frac{M_W}{c_W} . \tag{2.79}$$

As it can be seen, no quadratic terms appear in Eq. (2.77) for the gauge field A_μ. Hence, the photon remains massless and the global symmetry associated with is kept unbroken, $U(1)_{\text{em}}$. In this way, the Higgs mechanism generate masses for the $W^\pm$ and Z bosons via the spontaneous symmetry breaking of the $SU(2) \otimes U(1)$. Using Eq. (2.61) and Eq. (2.78) we find,

$$\mathrm{v} = \left(\sqrt{2}G_F\right)^{1/2} \simeq 246GeV . \tag{2.80}$$

Consequently, with the value of v, the value of the masses of the gauge bosons W and Z can be determined by assuming a experimental value for $s_W^2 \equiv \sin^2\theta_W \sim 0.22$ [11]. Thereby, $M_W \sim 80$ GeV and $M_Z \sim 90$ GeV. In an analogous way, by observing the second term of the Eq. (2.76) and by comparing them with terms of usual masses, we can identify the mass for the Higgs boson as,

$$M_H = \sqrt{-2\mu^2} , \tag{2.81}$$

where $\mu^2 < 0$.

Another important point for the description of the SM is the ρ-parameter, which is a dimensionless parameter,

$$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z^2} \,, \tag{2.82}$$

ρ-parameter is equal to unit at tree level, in the SM, and represents a successful prediction of the SM, being a good test for the isospin structure of the Higgs sector [11]. The ρ parameter is important to probe the presence of extended higgs sectors such as the one we will address later in this work.

2.4.4 Lepton Masses

Until now the charged fermions are massless and the quarks do not mix with leptons, since they have barionic and leptonic numbers respectively, and these numbers are conserved in the SM. Having in mind the gauge groups of the SM, the only renormalizable way to give mass to the SM leptons is via the Yukawa Lagrangian that reads,

$$\mathcal{L}^{\ell}_{\text{yuk}} \;=\; -G_\ell \left[\bar{\psi}_{R\ell}\,\left(\Phi^\dagger\,\psi_{L\ell}\right) + \left(\bar{\psi}_{L\ell}\,\Phi\right)\,\psi_{R\ell}\right] \,, \tag{2.83}$$

with G_ℓ the Yukawa constant and the Higgs field as,

$$\Phi = \begin{pmatrix} 0 \\ \frac{(\text{v}+H)}{\sqrt{2}} \end{pmatrix}, \quad \text{then,} \quad \mathcal{L}^{\ell}_{\text{yuk}} = -G_\ell \frac{(\text{v}+H)}{\sqrt{2}} \left[\bar{\ell}_R\,(0 \quad 1)\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + (\bar{\nu}_L \quad \bar{\ell}_L)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\ell_R\right] \,,$$

$$\mathcal{L}^{\ell}_{\text{yuk}} \;=\; -\frac{G_\ell\,\text{v}}{\sqrt{2}}\,\bar{\ell}\,\ell - \frac{G_\ell}{\sqrt{2}}\,\bar{\ell}\,\ell\,H \,. \tag{2.84}$$

where in the first terms of Eq. (2.84), we may see that the term constitutes the mass term for the charged leptons with,

$$M_\ell = \frac{G_\ell\,\text{v}}{\sqrt{2}} \,. \tag{2.85}$$

Besides, from the second term of Eq. (2.84) we get,

$$\frac{M_\ell}{v}\sqrt{2} = G_\ell \,.$$

Therefore, the Higgs boson couples to fermions proportionally to their masses.

2.4.5 Kinetic Lagrangian of the quarks

Before 1970's, the phenomenology of the weak and electromagnetic interactions of the quarks discovered at that time was not fully known. This could be observed due to the hadronic neutron current,

$$J_\mu^H(0) = \bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}'\gamma_\mu(1-\gamma_5)d' \ . \tag{2.86}$$

It was only described by the quarks of the first generation,

$$\psi_{Lq} = \begin{pmatrix} u_L \\ d_{L\theta} \end{pmatrix} . \tag{2.87}$$

The hadronic neutral current comes as a consequence of Cabibbo's theory described by the quarks u and d 'as a dublet under SU(2) where,

$$\begin{pmatrix} u_L \\ d'_L \end{pmatrix} = \begin{pmatrix} u_L \\ d_{L\theta} \end{pmatrix} . \tag{2.88}$$

In this case the d quark is seen as a superposition of d and s quarks, and the mixing is controlled by the Cabibbo angle,

$$d' = d_\theta \ = d\cos\theta_C + s\sin\theta_C \ ,$$

where the quark s of the 2^{nd} family is also included as a singlet under SU(2), because these were the known quarks. This is known as the theory of Cabibbo. When the Cabibbo mechanism was firstly proposed, the charm quark had not been discovered as an immediate consequence of this theory. That said, the hadronic neutral current results in terms of the quarks u and d' is,

$$\begin{aligned} J_\mu^H(0) &= \bar{u}\gamma_\mu(1-\gamma_5)u + \cos^2\theta_C \ \bar{d}\gamma_\mu(1-\gamma_5)d + \sin^2\theta_C \ \bar{s}\gamma_\mu(1-\gamma_5)s \\ &+ \cos\theta_C\sin\theta_C \left[\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d\right] , \end{aligned}$$

where it can be seen that the Cabibbo mechanism describes neutral current transitions, specifically the latter term that implies Flavor Changing Neutral Currents (FCNC). In the electroweak theory of the SM the neutral current does not mix the families, only the charged currents, hence, one can suggest some theories to suppress the last term of the above equation. For example, a possibility it would rise, if we had replaced the quark s by $s' = s_\theta = s\cos\theta_C - d\sin\theta_C$ (which is orthogonal to d'), which implies a combination of the orthogonal charge quarks to Eq. (2.87) with Q = -1/3 [1]. However, this still leads to a mixing between generations in the neutral current.

However, FCNC processes are rare, as can be observed in the branching ratio (BR) of kaon decays via charged current [11, 48],

$$BR\left(K^+_{u\bar{s}} \to W^+ \to \mu^+ \nu\right) \cong \; 63.5\% \,,$$

$$BR\left(K^+_{u\bar{s}} \to W^+ \to \pi^0 e^+ \nu_e\right) \cong \; (4.98 \pm 0.07) x 10^{-2} \,,$$

whereas kaons decaying involving FCNC are quite suppressed [11],

$$BR\left(K^+_{u\bar{s}} \to \pi^+_{u\bar{d}} \nu \bar{\nu}\right) \cong \; 4.2 x 10^{-10} \,,$$

$$BR\left(K^L_{d\bar{s}} \to \mu^+ \mu^-\right) \cong \; 7.2 x 10^{-9} \,.$$

These processes are displayed in figure 2.3.

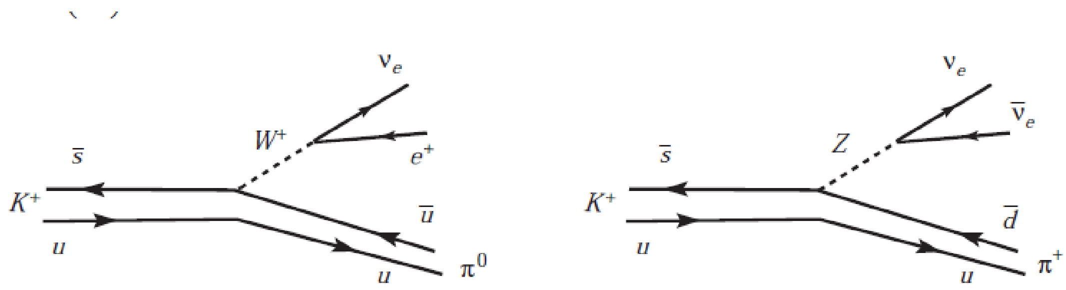

Figure 2.3: Strangeness changing charged and neutral current decays. [2].

Thus, in order to solve this problem, Glashow, Iliopoulos, and Maian proposed the GIM mechanism to suppress flavor changing neutral currents. For this, the GIM mechanism required the existence of a fourth quark, in this case the c quark (charm). It would be no longer a singlet under group SU(2), it would be a doublet under SU(2) instead. Therefore, the hadronic of the SM would be the quarks doublets $\binom{u_L}{d'_L} = \binom{u_L}{d_{L\theta}}$ and $\binom{c_L}{s'_L} = \binom{c_L}{s_{L\theta}}$ and the respective right-hand singlets u_R, d_R, c_R and s_R. Moreover, the mechanism requires that s' to be orthogonal to d' with,

$$d_\theta \; = d \cos\theta_C + s \sin\theta_C \quad \text{and} \quad s_\theta \; = s \cos\theta_C - d \sin\theta_C \,,$$

which can be seen as a rotation basis,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} .$$

In this way, two additional terms appear to $J_\mu^H(0)$,

$$\begin{aligned}\bar{c}\gamma_\mu(1-\gamma_5)c + \bar{s}'\gamma_\mu(1-\gamma_5)s' &= \bar{c}\gamma_\mu(1-\gamma_5)c + \cos^2\theta_C\bar{s}\gamma_\mu(1-\gamma_5)s \\ &+ sin^2\theta_C\bar{d}\gamma_\mu(1-\gamma_5)d \\ &+ C_{\theta_C}S_{\theta_C}[\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d]\,,\end{aligned}$$

where $\cos\theta_C = C_{\theta_C}$ and $\sin\theta_C = S_{\theta_C}$. In summary, the resulting neutral current is,

$$J_\mu^H(0) = \bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}\gamma_\mu(1-\gamma_5)d + \bar{c}\gamma_\mu(1-\gamma_5)c + \bar{s}\gamma_\mu(1-\gamma_5)s\,. \tag{2.89}$$

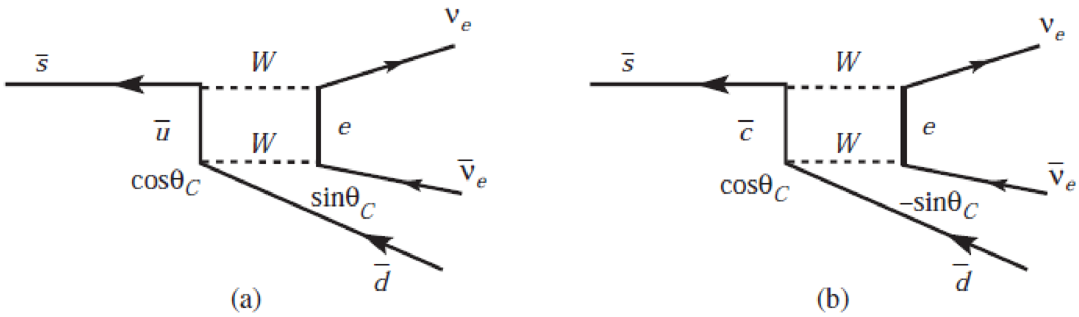

Figure 2.4: Second-order diagrams of the quarks level for the decay $k^+ \to \pi^+ + \nu_e + \bar{\nu}_e$ a) The diagram of the relevant second order (the corresponding calculated rate is much greater than the experimental value.) If a fourth quark exists diagram appears b) [2]

Hence, via the GIM mechanism no source of FCNC appears. The same problem could be studied by considering higher order contributions (beyond tree-level diagrams). For example, when considering the second order (as seen in figure 2.4), where the lower vertices are $\pm\cos\theta_C\sin\theta_C$, the two contributions could be cancelled, but the sum of the 2 diagrams is not zero, this cancellation would be perfect if the masses of u y c quarks were the same. However, the cancellation is small enough to be perfectly compatible with the observations [2].

In this way, the GIM mechanism is successful, being able to predict the existence of the c quark, but also its mass from the measurement of the meson K decay.

Similarly to leptons quarks are describeD by the free massless Dirac Lagrangian,

$$\begin{aligned}\mathcal{L}_{\text{quarks}} &= \bar{\psi}_{Lu}\, i\,\partial\!\!\!/\,\psi_{Lu} + \bar{\psi}_{Lc}\, i\,\partial\!\!\!/\,\psi_{Lc} \\ &+ \bar{\psi}_{Ru}\, i\,\partial\!\!\!/\,\psi_{Ru} + \cdots + \bar{\psi}_{Rc}\, i\,\partial\!\!\!/\,\psi_{Rc}\,.\end{aligned} \tag{2.90}$$

with hyperchages equal to,

$$Y_{\psi_{Lq}} = \frac{1}{3} \quad , \qquad Y_{\psi_{Ru}} = \frac{4}{3} \quad , \qquad Y_{\psi_{Rd}} = -\frac{2}{3} \; .$$

Such fractional values for the hyperchages are obtained using the Gell-Mann–Nishijima relation (Eq. (2.53)) and the electric charge of the up-type quark ($+\frac{2}{3}$) and the down-type ($-\frac{1}{3}$) quarks.

Moreover, using the covariante derivative of the fermions one can find the charged and neutral currents as follows,

$$\mathcal{L}_q^{(\pm)} = \frac{g}{2\sqrt{2}} \left[\bar{u}\gamma^\mu(1-\gamma_5)d' + \bar{c}\gamma^\mu(1-\gamma_5)s'\right] W_\mu^+ + \text{h.c.} \, , \tag{2.91}$$

$$\mathcal{L}_q^{(0)} = -\frac{g}{2c_W} \sum_{\psi_q = u, \cdots, c} \bar{\psi}_q \gamma^\mu (g_V^q - g_A^q \gamma_5) \psi_q \; Z_\mu \; . \tag{2.92}$$

As before, the the vector (g_V) and axial (g_A) couplings for the quarks given by Eq. (2.68) and Eq. (2.69), for $i = q$ [11].

2.4.6 Quarks Masses

The procedure to obtain the mass terms of the quarks is similar to that of the leptons, where the Yukawa Lagrangian should be used. However, for quarks it is a bit more complex because both quarks in the SU(2) doublet are massive.The Yukawa Lagrangian for three generations of quarks can be written as [11],

$$\mathcal{L}_{\text{yuk}}^q = -\sum_{i,j=1}^{3} \left[G_{ij}^U \; \bar{\psi}_{R_{U_i}} \left(\tilde{\Phi}^\dagger \psi_{Lj}\right) + G_{ij}^D \; \bar{\psi}_{R_{D_i}} \left(\Phi^\dagger \psi_{Lj}\right)\right] + \text{h.c.} \, , \tag{2.93}$$

where we have up ($U_i = u$, c, and t) and down ($D_i = d$, s, and b) quarks. Also we need a $Y = -1$ Higgs doublet. Defining the conjugate doublet Higgs as,

$$\tilde{\Phi} = i \; \sigma_2 \; \Phi^* = \begin{pmatrix} \phi^{0^*} \\ -\phi^- \end{pmatrix} = \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} , \tag{2.94}$$

For simplificity, we will concentrate on the first generation of quarks only, but our procedure is easily applied to the others. That said, we can write the Yukawa Lagrangian as follows,

$$\mathcal{L}_{\text{yuk}}^q = -G_d \left(\bar{\psi}_{Lq}\Phi d_R - \bar{d}_R \Phi^\dagger \psi_{qL}\right) - G_u \left(\bar{\psi}_{Lq}\tilde{\Phi} u_R - \bar{u}_R \tilde{\Phi}^\dagger \psi_{qL}\right) \; , \tag{2.95}$$

where,

$$\psi_{qL} = \begin{pmatrix} u \\ d \end{pmatrix}_L .$$

Thus,

$$\mathcal{L}^q_{\text{yuk}} = -G_d(\bar{u}_L \quad \bar{d}_L) \begin{pmatrix} 0 \\ \frac{\text{v}+H}{\sqrt{2}} \end{pmatrix} d_R - G_u(\bar{u}_L \quad \bar{d}_L) \begin{pmatrix} \frac{\text{v}+H}{\sqrt{2}} \\ 0 \end{pmatrix} u_R ,$$

$$\mathcal{L}^q_{\text{yuk}} = -\frac{G_d}{\sqrt{2}} \bar{d}_L d_R(\text{v} + H) - \frac{G_u}{\sqrt{2}} \bar{u}_L u_R(\text{v} + H) ,$$

where the terms with v constitute the mass terms for the charged quarks as,

$$M_d = \frac{G_d \text{ v}}{\sqrt{2}} \quad \text{and} \quad M_u = \frac{G_u \text{ v}}{\sqrt{2}} . \tag{2.96}$$

Note that again the Higgs couples to quarks proportionally to their masses since,

$$\frac{M_d}{\text{v}}\sqrt{2} = G_d \quad \text{and} \quad \frac{M_u}{\text{v}}\sqrt{2} = G_u ,$$

where G_d and G_u are the introduce Yukawa constants. Hence, the spontaneous symmetry breaking mechanism, where the Higgs is the key player, is responsible for generating fermion masses, with Higgs interacting with them proportionally to their masses.

2.4.7 Standard Model Lagrangian

Finally, we can collect all the information exposed in the previous sections to write the Lagrangian of the SM. Then we can say that the Lagrangian for the SM is,

$$\mathcal{L}^{SM} = \mathcal{L}^{SM}_{\text{scalar}} + \mathcal{L}^{\text{SM}}_{yuk} + \mathcal{L}_{\text{gauge}} + \bar{\psi}(x)(i\gamma^\mu D_\mu)\psi(x) , \tag{2.97}$$

Besides these terms we could have included the Gauge Fixing terms, is we were not working on the unitary gauge as well as the Faddeev-Popov terms which are important to cancel nonphysical contributions (time like and longitudinal) involving the gauge bosons [49]. In this dissertation, these terms will not be analyzed, since they are not necessary for this investigation. Another sector that we have not reviewed in this work was Quantum Chronodynamics which elegantly describes the interacting between quarks via gluon exchange as well as color confinement and the self-gluon interactions. We emphasize that these aspects are not relevant to our reasoning and for this reason they dropped from our discussion.

We have thus far reviewed the main ingredients of the SM and shown that neutrinos are massless. In the next chapter we will show how to successfully generate neutrino masses.

Chapter 3

Type-II seesaw mechanism

The SM has successfully passed all precision tests. Therefore it stands as the best description of the electroweak and strong interactions in nature. Although, it has setbacks. The SM does not account for neutrino masses, dark matter [50,51], matter-antimatter asymmetry [52], etc. Therefore we should go beyond the SM to explain the phenomena observed in nature. Historically speaking, right-handed neutrinos are absent in the SM, and for this reason they do not have Dirac mass as the other fermions. Nevertheless, nowadays it is known that neutrinos are massive via the observation of neutrino oscillations. An easy explanation for this is in the existence of right-handed neutrinos. Assuming the existence of right-handed neutrinos, these should be hypothetical masses of Majorana that are supposed to be very heavy, they are out of reach of colliders. Another popular alternative to explain neutrino masse is the so called seesaw type II. In this mechanism, a scalar triplet is added to the SM spectrum where neutrinos simply have majorana masses. Such scalar triplet is entitled to a reach phenomenology concerning collider physics and lepton flavor violation. Motivated by the such rich phenomenology aspect, in this dissertation we are going to review it. Before doing so, we are going to to review some interesting aspects concerning neutrino physics.

3.1 Neutrino oscillation

Neutrinos are classified in terms of both flavor (e, ν, τ) and mass eigenstates $(1, 2, 3)$, but these neutrinos cannot be determined at the same time in experiments. In this way, for instance, the electron neutrino is the mixing state of neutrino 1, neutrino 2 and neutrino 3. This is called the neutrino mixing, where the flavor eigenstates and mass eigenstates are connected as follows,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{3.1}$$

The matrix U_{PMNS} is often called Pontecorvo-Maki-Nakagawa-Sakata mixing matrix * and it is usually parametrized in terms of the three mixing angles θ_{12}, θ_{23}, θ_{13}, and one Dirac CP phase denoted as δ and two Majorana (α_1, α_2) phases,

$$U_{PMNS} = U_{Dirac} \times \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) , \tag{3.2}$$

with

$$U_{Dirac} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$. The phase factors α_1 and α_2 are physically meaningful only if neutrinos are Majorana particles. These phases play no role in neutrino oscillations [53,54]. Moreover, the angles θ_{ij} can be taken without loss of generality to lie in the first quadrant, $\theta_{ij} \in [0, \pi/2]$, and the phase δ_{CP} in $[0, 2\pi]$ [55,56]. Values of δ_{CP} different from 0 and π imply CP violation† in neutrino oscillations in vacuum [57,58].

Consequently, with the convention given above for the θ_{ij} angles and for the δ_{CP} phase, there are two non-equivalent orderings for the neutrino masses,

$$\text{Neutrino masses} \longrightarrow \begin{cases} m_1 < m_2 < m_3 & , \ \text{Normal ordering (NO)} \\ m_3 < m_1 < m_2 & , \ \text{Inverted ordering (IO)} \end{cases} .$$

It is important to mention that, as for a time $t = 0$ the neutrinos that are self-interaction states are described as a linear combination of the mass eigenstates as follows,

$$|\nu_i(0)\rangle = a_i |\nu_1\rangle + b_i |\nu_2\rangle + c_i |\nu_3\rangle \quad \text{i} = \text{e}, \mu\,\tau\ . \tag{3.3}$$

In the case of neutrino freely propagating in the vacuum, the interaction eigenstates are described as,

$$|\nu_i(t)\rangle = a_i e^{-iE_1 t} |\nu_1\rangle + b_i e^{-iE_2 t} |\nu_2\rangle + c_i e^{-iE_3 t} |\nu_3\rangle , \quad \text{i} = \text{e}, \mu\,\tau\ , \tag{3.4}$$

*which is a well-defined $3x3$ unitary matrix

†The CP violation is explain in the appendix Eq. (E)

with

$$E_i^2 = p^2 + m_i^2 \ ,$$

where p is the momentum, E_i is the energy of the mass-eigenstate m_i and t is the time from the start of the propagation. However, each mass-eigenstate is different [13]. Therefore, it is necessary to calculate the neutrino interaction oscillation probability to know the probability that any of these neutrinos can be detected. For instance, when calculating the probability $\mathcal{P}$ of detecting a neutrino ν_i at time t this is given by,

$$\mathcal{P}_{(\nu_i,t)} = \left|\left\langle \nu_i | \nu_j(t) \right\rangle\right|^2 \ ,$$

which will give us the neutrino oscillation probability ν_j to the neutrino ν_i [13].

We can obtain in the relativistic limit ($p >> m_i$),

$$E_i \approx p + \frac{m_i^2}{2p} \ .$$

So the probability will depend on the phase $E_i - E_j = \frac{m_i^2 - m_j^2}{2p} \equiv \frac{\Delta m_{ij}^2}{2p}$ that is responsible for oscillation.

In this case, the neutrinos oscillation experiments could help to determine the size of Δm_{ij}^2. With the information concerning the mass hierarchy given about we conclude that the value of Δm_{13}^2 sets the hierarchy,

$$\Delta m_{13}^2 \rightarrow \begin{cases} \text{if it is negative} & \text{Normal hierarchy (NH)} \\ \text{if it is positive} & \text{Inverted hierarchy (IH)} \end{cases} \ .$$

3.2 Mass for neutrinos

We may study neutrinos masses by means of Dirac and Majorana mass terms. With respect to the *Dirac mass*, (see Eq. (2.4)), this term includes right and left states, but for *Majorana mass* is different: $m_M \Psi^T C \psi$ can be written in terms of one helicity state. This happens when neutrinos are their antiparticles. It is important to point out that only chargeless particle can have a Majorana nature, otherwise electric charge would be violated, knowing that being Majorana means $\psi^c = \psi$, where C is the charge conjugation operator. As we discussed a possible way to generate neutrino masses would happen via the addition of right neutrinos ν_R in the SM. By adding the right-neutrino mass, we write a Majorana mass term as follows,

$$M_R \nu_R^T C \nu_R = M_R \bar{\nu_R}^c \nu_R$$

where

$$C\Psi C = -i\gamma^2\Psi^* \ .$$

In this way, if we have one right neutrino for each family, they wil be singlets under the $SU(3) \otimes SU(2) \otimes U(1)$ group. Then, we could give a mass term for the neutrino with the Yukawa Lagrangian, but the neutrino states of both the left and right hands are not independent mass states of other families. So, when considering a single family, for example: $m_D \bar{\nu_L} \nu_R + M_R \bar{\nu^c}_R \nu_R$, the neutrino mass matrix looks like,

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} .$$

Consequently, their eigenvalues are given as:

$$m_{\pm} = \frac{M_R}{2} \pm \frac{M_R}{2}\sqrt{1 + \frac{m_D^2}{M_R^2}} \ .$$

As it is well known, M_R is not related to the SM or to the VEV of the Higgs scalar dublet, so this mass could acquire any value. The Majorana mass terms explicitly violates lepton number and for this M_R is typically associated to a high energy scale. Thus one usually imposes, $M_R >> m_D$ and the mass of the neutrino would be $m_\nu \sim m_D^2/M_R$ for a light mass state or $m_\nu \sim M_R$ for a heavy mass state. See (see appendix A) for futher details.

Now that we have reviewed the type I seesaw mechanism which predicts right-handed neutrinos, we will discuss the type II seesaw mechanism which is the goal of this dissertation.

3.3 Type-II seesaw mechanism

The seesaw mechanism type II introduces one new scalar triplet (Δ),

$$\Delta = \begin{pmatrix} \Delta_1, & \Delta_2, & \Delta_3 \end{pmatrix} . \tag{3.5}$$

The underlying physics behind this mechanism is lepton number violation similarly to the type I seesaw. Albeit, this mechanism does not need to invoke high energy scales and for this reason becomes quite attractive. We will review in a pedagogic way theoretical aspects of the model and discuss interesting phenomenological signatures. We start with the introduction of such scalar triplet.

3.3.1 Construction of the scalar triplet

The scalar is a triplet of SU(2). Thus, Pauli matrices will be used to represent this scalar triplet in the $SU(2)_L$ group as follows,

$$\Delta = \frac{\sigma^i}{\sqrt{2}}\Delta_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix}. \tag{3.6}$$

With i= 1, 2 , 3. As we know, the Yukawa Lagrangian describes the interactions between the scalar and the left-handed lepton doublet with $\psi_{Li} \equiv (\nu, \ell)$, to generate the neutrino mass. In this model, by putting the scalar triplet in the Yukawa Lagrangian usually used in this model ($\mathcal{L}^{II}_{yuk}$), we obtain the following,

$$\mathcal{L}^{II}_{yuk} \supset -(Y_\Delta)_{\alpha\beta}\, \psi^T_{L\alpha} C\, i\sigma_2\Delta\, \psi_{L\beta} + \text{h.c.} = -(Y_\Delta)_{\alpha\beta}\, \psi^{\bar{C}}_{L\alpha}\widetilde{\Delta}\psi_{L\beta} + \text{h.c.}\,, \tag{3.7}$$

where $\psi^T_{L\alpha}C\widetilde{\Delta}\psi_{L\beta}$ is called Weinberg operator, C is charge conjugation matrix with respect to the Lorentz group, $\alpha, \beta = e, \nu, \tau$ are flavor indices, ψ^c denotes right-handed antilepton field, h.c. are Hermitian conjugate terms and the scalar field Δ is rotated through the Pauli matrix σ^2 as $\widetilde{\Delta} = i\sigma_2\Delta$. Thus,

$$\widetilde{\Delta} = i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix},$$

$$\widetilde{\Delta} = \frac{1}{\sqrt{2}}\begin{pmatrix} \Delta_1 + i\Delta_2 & -\Delta_3 \\ -\Delta_3 & -(\Delta_1 - i\Delta_2) \end{pmatrix}. \tag{3.8}$$

Then, the Yukawa Lagrangian is given as,

$$\mathcal{L}^{II}_{yuk} \supset -(G_\nu)_{ij}\begin{pmatrix} \bar{\nu^c}_L & \bar{e^c}_L \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} \Delta_1 + i\Delta_2 & -\Delta_3 \\ -\Delta_3 & -(\Delta_1 - i\Delta_2) \end{pmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}. \tag{3.9}$$

In this way,

$$\begin{aligned}\mathcal{L}^{II}_{yuk} \supset -G_\nu &\left[\bar{\nu^c}_L\frac{\Delta_1 + i\Delta_2}{\sqrt{2}}\nu_L + \bar{e^c}_L\frac{-\Delta_3}{\sqrt{2}}\nu_L\right] \\ -G_\nu &\left[\bar{\nu^c}_L\frac{-\Delta_3}{\sqrt{2}}e_L + \bar{e^c}_L\frac{-(\Delta_1 - i\Delta_2)}{\sqrt{2}}e_L\right],\end{aligned} \tag{3.10}$$

from the obtained in the Yukawa Lagrangian, it can be seen that the scalar field should have a hypercharge equal to 2 ($Y_\Delta = 2$) and, at the same time the electric charge components of the scalar field are defined as follows,

$$\Delta_3 = \Delta^+ \,,$$
$$\frac{\Delta_1 + i\Delta_2}{\sqrt{2}} = \Delta^0 \,,$$
$$\frac{\Delta_1 - i\Delta_2}{\sqrt{2}} = \Delta^{++} \,.$$

In this way, the Yukawa Lagrangian for this mechanism is given as,

$$\mathcal{L}^{II}_{yuk} = (Y_\Delta)_{\alpha\beta}\, \bar{\ell^C}_\alpha \ell_\beta \Delta^{++} + \sqrt{2}\,(Y_\Delta)_{\alpha\beta}\, \bar{\nu^C}_\alpha \ell_\beta \Delta^{+} - (Y_\Delta)_{\alpha\beta}\, \bar{\nu^C}_\alpha \nu_\beta \Delta^{0} + \text{h.c.}\,, \tag{3.11}$$

where the neutrinos get a mass when Δ gets a vacuum expectation value (vev) and $\widetilde{\Delta}$ will be given by,

$$\widetilde{\Delta} = \begin{pmatrix} \Delta^0 & -\Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & -\Delta^{++} \end{pmatrix} . \tag{3.12}$$

Then we, can write the scalar field Δ as,

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} , \tag{3.13}$$

where Δ become $\Delta' = U\Delta U^\dagger$, with U which is a transformation or unitary matrix and we can see that this type of transformation keeps the Weinberg operator invariant under the SM gauge group, with $i\sigma^2 U = U^* i\sigma^2$.

3.3.2 Introducing the sector scalar

Now, with the scalar field built in the electroweak sector of the SM, we can write the scalar Lagrangian as follows,

$$\mathcal{L}^{II}_{\text{scalar}} = (D_\mu\phi)^\dagger (D^\mu\phi) + \text{Tr}\left[(D_\mu\Delta)^\dagger (D^\mu\Delta)\right] - \mathcal{V}(\Phi,\Delta) + \mathcal{L}_{\text{gauge}} \,.0$$

Where the covariant derivative of the triplet scalar field $D_\mu\Delta$ is given as,

$$D_\mu\Delta \;=\; \partial_\mu\Delta - i\frac{g}{2}[\sigma^a W^a_\mu, \Delta] - ig' B_\mu \Delta \qquad\qquad (a = 1,2,3)\,, \tag{3.14}$$

with g and g' that are the weak coupling constants and σ^a are the Pauli matrices, respectively. Where $D_\mu\Delta$ is the object that transform, which guarantee the local invariance. As can be seen before, the Lagrangian scalar contains two kinetic terms. One of these is a trace of the matrix, because it should give a number to add to the others energy terms.

The scalar potential can be written as,

$$\mathcal{V}(\Phi,\Delta) \ = V(\Phi^\dagger,\Phi) + V(\Delta^\dagger,\Delta) + V(\Phi^\dagger,\Phi,\Delta^\dagger,\Delta)\,,$$

where $V(\Phi^\dagger\Phi)$ correspond to the Higgs scalar potential, $V(\Delta^\dagger,\Delta)$ to the Δ scalar field potential and $V(\Phi^\dagger,\Phi,\Delta^\dagger,\Delta)$ to the interaction potential between the two field scalars. Where the total scalar potential is defined as ‡ [59],

$$\begin{aligned}\mathcal{V}(\Phi,\Delta) \ &= -m_\Phi^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + M_\Delta^2\mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 \\ &+ \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right) + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) \\ &+ \lambda_5\Phi^\dagger[\Delta^\dagger,\Delta]\Phi + \left(\mu\Phi^T i\sigma_2\Delta^\dagger\Phi + \mathrm{h.c.}\right)\,, \end{aligned} \tag{3.15}$$

where the coupling μ is a parameter that accounts for the lepton flavor violation [60], with mass dimension and the coupling constants λ_i $(i = 1,2,4,5)$ can be chosen to be real through a phase redefinition of the field Δ [59]. To ensure Standard Model electroweak gauge group spontaneous symmetry breaking $(SU(2)_L \times U(1)_Y$ to $U(1)_{\mathrm{em}})$ λ should be positive. In contrast, we can see that the last term of $\mathcal{V}(\Phi,\Delta)$ remains invariant under the gauge group with the transformation of the Δ scalar field.

3.3.3 Minimum condition for obtaining the masses for the doublet and triplet scalar

Similar to what has been done in section 2.4.3 in the SM, through the Higgs Mechanism we may parameterize the scalar fields in the following way,

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ (\mathrm{v}_\Delta + \delta + i\eta)/\sqrt{2} & -\Delta^+/\sqrt{2}\end{pmatrix}\,, \tag{3.16}$$

$$\Phi = \begin{pmatrix} \phi^+ \\ (\mathrm{v} + \phi + i\chi)/\sqrt{2}\end{pmatrix}\,. \tag{3.17}$$

‡The scalar potential have constraints, which can be seen in the appendix B

The vacuum expectation value for the two scalar fields ϕ and Δ must be acquired for the neutral components of these fields; therefore the charged components are zero,

$$\langle\Delta\rangle^0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ \mathrm{v}_\Delta & 0 \end{pmatrix} \tag{3.18}$$

$$\langle\Phi\rangle^0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \mathrm{v} \end{pmatrix} , \tag{3.19}$$

that is to say, $\langle\phi\rangle^0 = \mathrm{v}/\sqrt{2}$ and $\langle\Delta\rangle^0 = \mathrm{v}_\Delta/\sqrt{2}$.

Now, to find the m_Φ^2 and M_Δ^2 parameters of the Eq. (3.15), where the minimum condition is applied (Eqs. (3.20)) ,

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial\phi}|_{\phi=0,\delta=0,\eta=0,\chi=0} = 0 \quad \text{and} \quad \frac{\partial\langle\mathcal{V}\rangle}{\partial\delta}|_{\phi=0,\delta=0,\eta=0,\chi=0} = 0 \, . \tag{3.20}$$

In this case, $\langle\mathcal{V}\rangle$ is the potential evaluated in the vacuum expectation fields. Where, the charged components are zero and only the neutral components are parameterized in the vacuum,

$$(\mathrm{v} + \phi + \mathrm{i}\chi)/\sqrt{2} = \phi^0, \qquad (\mathrm{v} + \phi + \mathrm{i}\chi)^*/\sqrt{2} = \phi^{0*} \, ,$$

$$(\mathrm{v}_\Delta + \delta + \mathrm{i}\eta)^/\sqrt{2} = \Delta^0 \quad \text{and} \quad (\mathrm{v}_\Delta + \delta + \mathrm{i}\eta)^*/\sqrt{2} = \Delta^{0*} \, .$$

Thus, the m_Φ^2 and M_Δ^2 parameter solutions are obtained directly.

- For m_Φ^2

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial\phi}|_{\phi=0,\delta=0,\eta=0,\chi=0} = -m_\Phi^2\mathrm{v} + \lambda\frac{\mathrm{v}^3}{2} + (\lambda_4 - \lambda_5)\mathrm{v}_\Delta^2\frac{\mathrm{v}}{2} - \sqrt{2}\mu\mathrm{v}\mathrm{v}_\Delta \, . \tag{3.21}$$

Thus,

$$m_\Phi^2 = \frac{1}{2}\lambda\mathrm{v}^2 - \sqrt{2}\mu\mathrm{v}_\Delta + \frac{1}{2}(\lambda_4 - \lambda_5)\mathrm{v}_\Delta^2 \, . \tag{3.22}$$

- For M_Δ^2

$$\frac{\partial\langle\mathcal{V}\rangle}{\partial\delta}|_{\phi=0,\delta=0,\eta=0,\chi=0} = M_\Delta^2\mathrm{v}_\Delta + \lambda_1\frac{\mathrm{v}_\Delta^3}{2} + (\lambda_4 - \lambda_5)\mathrm{v}_\Delta\frac{\mathrm{v}^2}{2} - \mu\frac{\mathrm{v}^2}{\sqrt{2}} \, . \tag{3.23}$$

In this way,

$$M_\Delta^2 = \frac{1}{\sqrt{2}}\frac{\mu\mathrm{v}^2}{\mathrm{v}_\Delta} - \frac{1}{2}(\lambda_4 - \lambda_5)\mathrm{v}^2 - \frac{1}{2}\lambda_1\mathrm{v}_\Delta^2 \, . \tag{3.24}$$

These expression are the masses of the scalar fields. Know that the scalar triplet now contributes to the Higgs mass we conclude that v_Δ should be sufficiently small. Moreover, notice that M_Δ grows with $1/v_\Delta$. We emphasize these aspects because they will dictate the phenomenology ahead. In what follows we will address other features of the type II seesaw.

3.3.4 The masses for the massive gauge bosons W and Z on the Type-II seesaw Model

The mass of bosons W and Z, Higgs, and scalar triplet will be found taking into account Eq. (2.43) in section 2.3.3, using the Higgs Mechanism. Which consists of expanding the neutral scalar fields Φ^0 and δ^0 around their VEV. The scalar triplet Δ contributes to the gauge boson masses through its vacuum expectation value v_Δ. From now on we simply take $\langle\Phi\rangle^0 = \mathrm{v}/\sqrt{2}$ and $\langle\Delta\rangle^0 = \mathrm{v}_\Delta/\sqrt{2}$ in the kinetic Lagrangian of the scalars,

$$\mathrm{Tr}\left[\left(D_\mu\langle\Delta\rangle^0\right)^\dagger\left(D_\mu\langle\Delta\rangle^0\right)\right]+\left(D_\mu\langle\phi\rangle^0\right)^\dagger\left(D_\mu\langle\phi\rangle^0\right)\,, \tag{3.25}$$

which is given as,

$$\mathrm{Tr}\left[\left|\partial_\mu\langle\Delta\rangle^0 - i\frac{g}{2}[\sigma^a W^a_\mu,\langle\Delta\rangle^0] - ig'B_\mu\langle\Delta\rangle^0)\right|^2\right]+\left|\left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + i\frac{g'}{2}YB_\mu\right)\langle\phi\rangle^0\right|^2\,,$$

where the second term was studied the section 2.4.3, which through the Higgs mechanism yields,

$$M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\,.$$

The values of M_W^2 and M_Z^2 were given by the Eqs. (2.78) and (2.79). Now, the first term in the Eq. (3.25) will be given as,

$$\mathrm{Tr}\left[\left|\partial_\mu\begin{pmatrix}0 & 0\\ \mathrm{v}_\Delta/\sqrt{2} & 0\end{pmatrix} - i\frac{g}{2}\left[\sigma^a W^a_\mu,\begin{pmatrix}0 & 0\\ \mathrm{v}_\Delta/\sqrt{2} & 0\end{pmatrix}\right] - ig'B_\mu\begin{pmatrix}0 & 0\\ \mathrm{v}_\Delta/\sqrt{2} & 0\end{pmatrix}\right|^2\right]\,. \tag{3.26}$$

where we obtain

$$\left[\sigma^a W^a_\mu,\langle\Delta\rangle\right] = \left[\begin{pmatrix}W^3_\mu & W1^1_\mu - iW^2_\mu\\ W1^1_\mu + iW^2_\mu & -W^3_\mu\end{pmatrix},\begin{pmatrix}0 & 0\\ \mathrm{v}_\Delta/\sqrt{2} & 0\end{pmatrix}\right]\,,$$

using $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$ we can get,

$$\left[\sigma^a W_\mu^a, \langle\Delta\rangle\right] = \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ \mathrm{v}_\Delta/\sqrt{2} & 0 \end{pmatrix}$$

$$- \begin{pmatrix} 0 & 0 \\ \mathrm{v}_\Delta/\sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix}$$

$$= \begin{pmatrix} W_\mu^+ \mathrm{v}_\Delta & 0 \\ -2W_\mu^3 \mathrm{v}_\Delta/\sqrt{2} & -W_\mu^+ \mathrm{v}_\Delta \end{pmatrix} .$$

Thus, the Eq. (3.26) may be written as,

$$\mathrm{Tr}\left[\left|-i\mathrm{v}_\Delta \begin{pmatrix} gW_\mu^+/2 & 0 \\ (-gW_\mu^3 + g'B_\mu)/\sqrt{2} & -gW_\mu^+/2 \end{pmatrix}\right|^2\right] , \qquad (3.27)$$

with the bosonic fields defined above in the section of the SM, we may obtain Eq. (3.27) in terms of the W and Z as follows,

$$-gW_\mu^3 + g'B_\mu = -g\left(sen\theta_W A_\mu + cos\theta_W Z_\mu\right) + g'\left(cos\theta_W A_\mu - sen\theta_W Z_\mu\right)$$

remembering that $cos\theta_W = g/\sqrt{g^2 + g'^2}$ and $sen\theta_W = g'/\sqrt{g^2 + g'^2}$. Then

$$-gW_\mu^3 + g'B_\mu = \frac{Z_\mu}{\sqrt{g^2 + g'^2}}\left(-g^2 - g'^2\right) = -g\frac{Z_\mu}{c_W} ,$$

where $c_W = cos\theta_W$. Hence, the Eq. (3.27) is given as,

$$\mathrm{Tr}\left[\left|-ig\mathrm{v}_\Delta \begin{pmatrix} W_\mu^+/2 & 0 \\ -Z_\mu/\sqrt{2}c_W & -W_\mu^+/2 \end{pmatrix}\right|^2\right] ,$$

which implies that the Eq. (3.27) can be written as follows,

$$\mathrm{Tr}\left[\left[-ig\mathrm{v}_\Delta\begin{pmatrix} W_\mu^+/2 & 0 \\ -Z_\mu/\sqrt{2}c_W & -W_\mu^+/2\end{pmatrix}\right]^\dagger\left[-ig\mathrm{v}_\Delta\begin{pmatrix} W_\mu^+/2 & 0 \\ -Z_\mu/\sqrt{2}c_W & -W_\mu^+/2\end{pmatrix}\right]\right]$$

$$= \mathrm{Tr}\left[-i^2g^2\mathrm{v}_\Delta^2\begin{pmatrix} W_\mu^-/2 & -Z_\mu^*/\sqrt{2}c_W \\ 0 & -W_\mu^-/2\end{pmatrix}\begin{pmatrix} W_\mu^+/2 & 0 \\ -Z_\mu/\sqrt{2}c_W & -W_\mu^+/2\end{pmatrix}\right]$$

$$= \mathrm{Tr}\left[g^2\mathrm{v}_\Delta^2\begin{pmatrix} W_\mu^-W_\mu^+/4 + Z_\mu^*Z_\mu/2c_W^2 & W_\mu^+Z_\mu^*/2\sqrt{2}c_W \\ W_\mu^-Z_\mu/2\sqrt{2}c_W & W_\mu^-W_\mu^+/4\end{pmatrix}\right]$$

$$= g^2\mathrm{v}_\Delta^2\left(\frac{W_\mu^-W_\mu^+}{2} + \frac{Z_\mu^*Z_\mu}{2c_W^2}\right) .$$

Just as in the SM, considering the structure $M_W^2W_\mu^+W^{-\mu} + \frac{1}{2}M_Z^2Z_\mu Z^\mu$, and by taking the values for the second term masses in the Eq. (3.25) too we get,

$$M_W^2 = \frac{g^2}{4}\left(\mathrm{v}^2 + 2\mathrm{v}_\Delta^2\right), \tag{3.28}$$

$$\text{and} \quad M_Z^2 = \frac{g^2}{4\cos^2\theta_W}\left(\mathrm{v}^2 + 4\mathrm{v}_\Delta^2\right) . \tag{3.29}$$

then, in this model, the electroweak VEV can be inferred as,

$$\mathrm{v}^2 + 2\mathrm{v}_\Delta^2 = (\sqrt{2}G_F)^{-1/2} = (246\ \mathrm{GeV})^2 ,$$

to relate it to the value obtained in the SM. Hovewer, the masses differ from those obtained in the SM and for this reason we need to study the parameter ρ. In this case, in the tree level ρ is not trivially satisfied because

$$\rho = \frac{M_W^2}{M_Z^2cos^2\theta_W} = \frac{\mathrm{v}^2 + 2\mathrm{v}_\Delta^2 + 2\mathrm{v}_\Delta^2 - 2\mathrm{v}_\Delta^2}{\mathrm{v}^2 + 4v_\Delta^2} = 1 - \frac{2\mathrm{v}_\Delta^2}{\mathrm{v}^2 + 4\mathrm{v}_\Delta^2}, \tag{3.30}$$

Therefore, $\mathrm{v}_\Delta << \mathrm{v}$, and, this should be so to keep $\rho = 1$, since this is a known observable. A recent global fit to electroweak precision data yielded the value, [61],

$$\rho = 1 + (3.6 \pm 1.9)10^{-4} .$$

On the other hand, from the Eq. (3.24), with $v_\Delta << v$ and $\lambda_4 \simeq \lambda_5$, the relation for v_Δ is given as,

$$v_\Delta \simeq \frac{\mu v^2}{\sqrt{2} M_\Delta^2} \tag{3.31}$$

3.3.5 Masses for the physical mass eigenstates that appear on the Type-II seesaw Model

Expanding the neutral scalar fields Φ^0 and δ^0 around their VEVs (see Eqs. (3.17) and (3.16)) and evaluating the scalar potential $\mathcal{V}(\Phi, \Delta)$ seven physical massive eigenstates remain, denoted here as: two doubly electrically charged scalars $H^{\pm\pm} \equiv \Delta^{\pm\pm}$ and two singly electrically charged $H^\pm$ (see Eqs. (3.32) and (3.33)); three neutral fields h, H^0 and A^0 (a pseudo-scalar). in the process three massless Goldstone arise denoted as $G^\pm$ and G^0 and are responsible for giving mass to the three gauge electroweak bosons $W^\pm$ and Z. The doubly-charged and singly charged scalar masses are given as follows [62],

$$m_{H^{\pm\pm}}^2 \simeq M_\Delta^2 + \frac{1}{2}(\lambda_4 + \lambda_5)v^2 , \tag{3.32}$$

$$\text{and} \quad m_{H^\pm}^2 \simeq M_\Delta^2 + \frac{1}{2}\lambda_4 v^2 , \tag{3.33}$$

where the vacuum expectation value for the scalar triplet was considered to be small $v_\Delta \ll v$. The squared mass of the doubly-charged scalar could be obtained directly from the potential evaluated in the parameterized scalars[§] (because its mass would be well defined), but the other masses involve mixtures [63]. To obtain the $m_{H^\pm}^2$, it is necessary to rotate the scalar field $H^\pm$ to the physical bases, where the rotation angle is called β and it can be defined as [62–64],

$$\tan\beta = \frac{\sqrt{2}v_\Delta}{v} .$$

In the case of A^0 scalar, it is rotated to the respective physical bases and the rotation angle is called β' and may be denoted as [62–64],

$$tan\beta' = \frac{2v_\Delta}{v} .$$

[§]The parameterized potential can be seen in the appendix C and for we can obtain the squared masses could be used the definition the mass matrix which is in the Eq. (2.40)

And the h and H scalar fields are rotated by a angle α to the respective physical bases. This angle is defined as [64],

$$\tan 2\alpha \;=\; \frac{4\mathrm{v}_\Delta}{\mathrm{v}} \frac{M_\Delta^2 + \frac{1}{2}\lambda_1 \mathrm{v}_\Delta^2}{M_\Delta^2 + \frac{1}{2}(\lambda_4 - \lambda_5 - 2\lambda)\mathrm{v}^2 + \frac{3}{2}\lambda_1 \mathrm{v}_\Delta^2} \,. \tag{3.34}$$

3.3.6 Neutrino sector

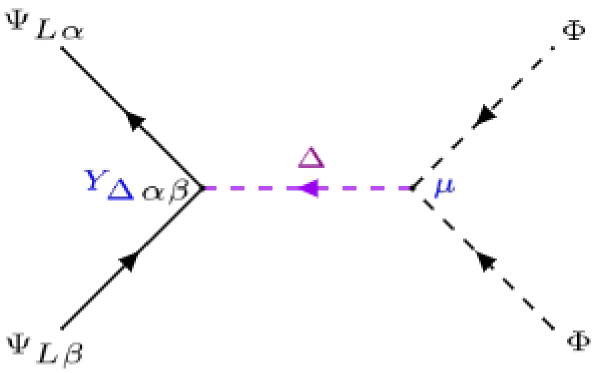

Figure 3.1: Diagram of the type II seesaw model, where μ and $Y_{\alpha\beta}$ are the coupling constants

In this way, the Majorana-type neutrino mass matrix from (Eq. (3.11)) and with the VEV of the Δ, we may find,

$$(M_\nu)_{\alpha\beta} = \sqrt{2}\mathrm{v}_\Delta (Y_\Delta)_{\alpha\beta} \,. \tag{3.35}$$

In this case, the neutino mass matrix can be diagonalized by means of oscillation parameters, using the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix, U_{PMNS},

$$M_\nu = U^*_{PMNS} \quad diag(m_1, m_2, m_3) \quad U^\dagger_{PMNS} \,.$$

In the figure 3.1, we see the Feynman diagram of the type-II seesaw mechanism, where the coupling constants (μ and $Y_{\alpha\beta}$) from the potential and Yukawa Lagrangian respectively are appreciated. Otherwise, in the limit $M_\Delta^2 \gg \mathrm{v}^2$ or $\lambda_4 \simeq \lambda_5$, and from the Eq. (3.31), the Eq. (3.35) becomes,

$$M_\nu \;\simeq\; \frac{\mu \mathrm{v}^2}{M_\Delta^2} Y_\Delta \,, \tag{3.36}$$

where it can be observed that the mass of the neutrino would be small if the mass of the scalar triplet is very large. Since the neutrino masses follow from the combination of three scales: μ, v and M_Δ. Notice that we can successfully explain the smallness of the neutrino masses by tuning the M_Δ and μ and get $m_\nu \sim 0.1$ eV.

Finally, we can write the type-II seesaw mechanism Lagrangian in the following way,

$$\mathcal{L}^{II} = \mathcal{L}^{II}_{\text{scalar}} + \mathcal{L}^{II}_{yuk} + \mathcal{L}^{SM}_{yuk} + \bar{\psi}(x)(i\gamma^{\mu}D_{\mu})\psi(x)\,,$$

where it is shown that this Lagrangian is an extension of the standard model.

We have shown that neutrino masses can be successfully generated within the type II seesaw mechanism that adds to the SM spectrum a scalar triplet. Up to some coupling constants the phenomenology of this model is ruled by M_{Δ} and μ which are the parameters linked to lepton number violation. In the next chapter we will discuss some phenomenological aspects.

Chapter 4

Phenomenology on the Type-II seesaw mechanism

In order to assess the feasibility of this model we need to compare its predictions with experimental data [65–67]. As a matter of fact, the muon anomalous electromagnetic moment $(g-2)^*$ and the lepton flavor violation † observables today push the new physics to low and high energy scales respectively [68–71]. In this way, a point of attention could be to study the phenomenology on these two aspects. However, the fields Δ^{++} and Δ^{+} on the type-II seesaw cannot explain $g-2$ because they give a negative contribution [60]. In accordance with Stefan Antusch et. al [6], in principle, the observed value of muon anomalous electromagnetic moment for some range of the triplet mass and $v_\Delta \lesssim 10^{-10}$ GeV, is already a region excluded by the lepton flavor violation experiments (this can be seen later in the figure 4.9).‡

4.1 Constraints on the ρ-parameter

As we wrote previously, the ρ-parameter must be equal to 1. Then, $v_\Delta << v$ and to meet this condition, we should impose $v_\Delta \leq 2.3$ GeV [65].

*These can be seen in the appendix (F)

†Lepton flavor violating processes are forbidden in the SM due to the assumption of massless neutrinos. In this way, the observation of lepton flavor violation imply new physics beyond the SM.

‡This limit is only written because of the presence in the reference.

4.2 Constraints from electroweak precision data

As v_Δ contributes to the electroweak symmetry breaking at tree-level, as it was mentioned in the previous chapter, the ρ-parameter is constrained by precision electroweak observables. Moreover, the strongest constraint from the LHC comes from electroweak precision test, owing to the the oblique T-parameter (which is an observable beyond the SM)[§], as it imposes strict limit on the mass splitting between the doubly and singly charged scalars, which should be [62],

$$|\Delta m| = |m_{H^{++}} - m_{H^+}| \lesssim 40 GeV$$

In this way, comparing the $m^2_{H^{++}}$ and $m^2_{H^+}$ masses (in the Eqs. (3.32) and (3.33)) we can see that this limit involves the parameter λ_5, since from these equations, the λ_5-coupling can be seen as:

$$m^2_{H^{++}} - m^2_{H^+} = \frac{1}{2}\lambda_5 v^2 , \tag{4.1}$$

which implies that λ_5 should much smaller than one, say,

$$-0.1 \lesssim \lambda_5 \lesssim 0.1 . \tag{4.2}$$

On the other hand, the oblique T-parameter provides an indirect probe of physics beyond the SM for theories with $SU(2) \otimes U(1)$ gauge content [73]. For this reason, this restriction is important. Figure 4.1 shows the regions excluded in the LHC for direct searches of the H^{++} decay into leptons (assuming 100% branching fractions), where Bonilla et al.,[¶] consider the limit on the mass splitting between the doubly and singly charged scalar $\lesssim 50 GeV$, with the Vacuum stability and Perturbativity constraints of the type-II seesaw model as they are explicit in the appendix B and with $m_{H^+} > 100 GeV$. These features are summarized in Fig.4.1 adapted from [3].

4.3 Collider phenomenology

For the type-II seesaw model, the heavy scalars can be (singly and doubly–charged members of the triplet scalar) searched for at accelerators such as LEP (e^+e^-) as well as at hadron colliders such as the LHC.

§check the appendix D and the reference [72]

¶this can be reviewed in the reference [3]

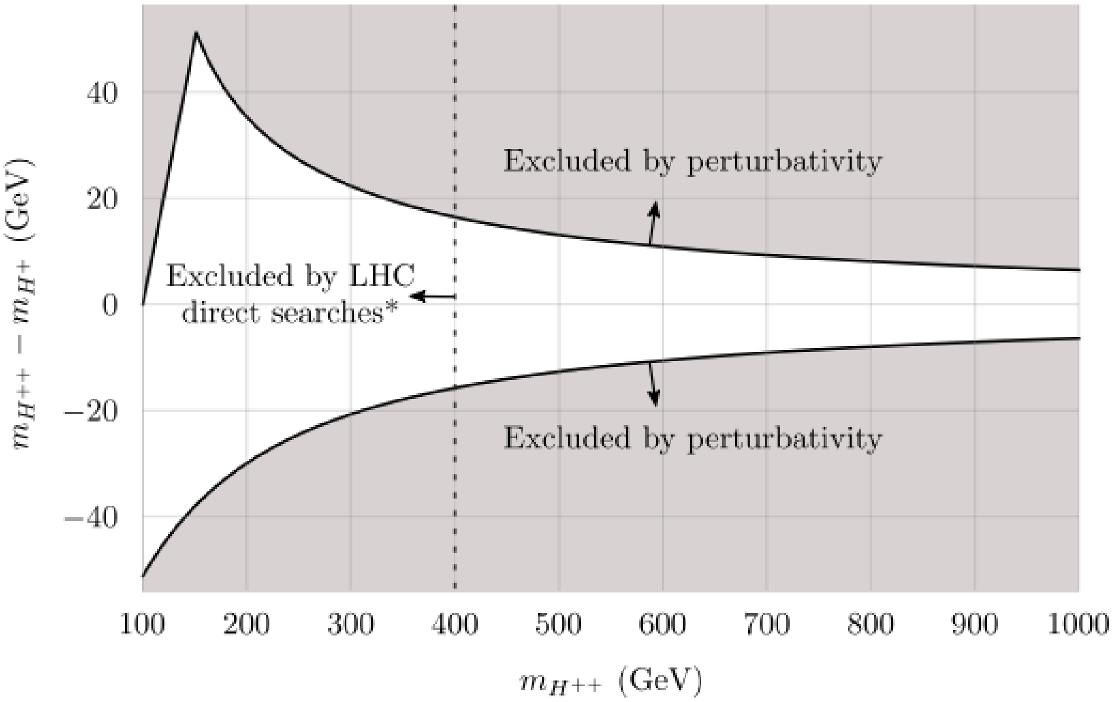

Figure 4.1: Constrains in the mass splitting of the triplet component [3]

4.3.1 Single-charged scalar

To begin with, at the LEP II the direct search on the singly-charged scalar belong to the $e^+e^- \longrightarrow H^-H^+$-decay, which can be seen in the figure 4.2.

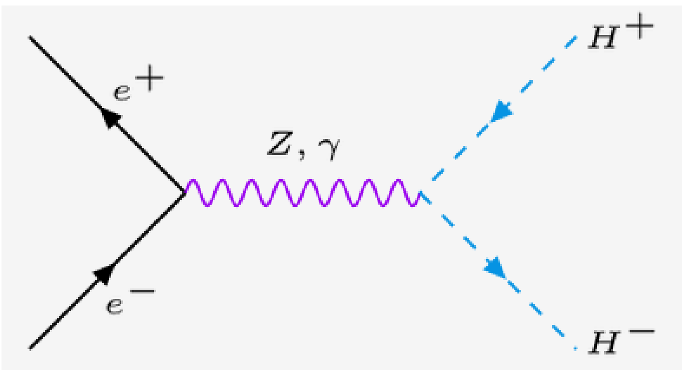

Figure 4.2: $e^+e^- \longrightarrow H^-H^+$-decay diagram

Using this channel they conservatively found $m_{H^+} > 80\ GeV/c^2$ [74]. The charged higgs decays 100% of the time into $\tau\nu$ then the bound ramps up to 94 GeV. This limits are related to the center-of-mass energy of LEP II which was 209 GeV [74].

On the other hand, ATLAS and CMS are interested in the production channels that depend on the tbH^+ coupling, which is suppressed by $tan^2\beta = \text{v}_\Delta^2/\text{v}^2 \lesssim \mathcal{O}(10^{-4})$ [63], for this reason, it is not considered in the study in the type-II seesaw model. The tbH^+ coupling can be seen in the figure 4.3. Moreover, the discovery of a charged higgs boson via this channel involving challenging aspects because it is based on multi-jets studies which are subject to a large QCD background. Latter fact, makes the doubly charged scalar searches be more promising.

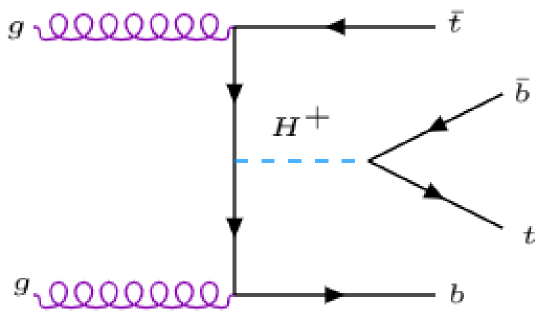

Figure 4.3: Diagram for the production of a heavy charged Higgs boson in association with a top quark and a bottom quark tbH^+ $(mH^+ > mt + mb)$ [4]

4.3.2 Neutral scalars

The gluon fusion production channels $(gg \to \Phi)$ followed by the decay of Φ into b-quarks is typically the target at the LHC (see Fig.4.4), where $\Phi = H$ and A [4]. Although, these production channels are suppressed by $sen^2\alpha$ and/or $sen^2\beta'$, rendering the bounds weak. [63]

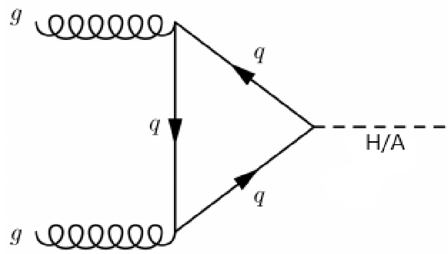

Figure 4.4: Lowest-order Feynman diagrams for gluon–gluon fusion

4.3.3 Doubly charged scalar

The main LHC search for the type II seesaw model is based on the pair production of doubly charged scalars via Z and photon exchange. The associate production that occurs with W mediated processes are also important but not dominant (see Fig. 4.6). It is commonly assumed in these searches that the doubly charged scalar decay exclusively either in charged lepton pairs ($H^{\pm\pm} \to \ell^{\pm}_{\alpha}\ell^{\pm}_{\beta}$) or into gauge bosons ($H^{\pm\pm} \to W^{\pm}W^{\pm}$), which becomes relevant when $m_{H^{++}} \geqslant 160GeV$. If the $m_{H^{++}} < 160$ GeV, the doubly charged scalar may decay into off-shell W bosons that will produce soft jets potentially not energetic enough to pass the cuts imposed by the collaboration as discussed in [63]. If v_{Δ} is sufficiently large, say $v_{\Delta} = 0.1$ GeV LHC searches for doubly charged scalar decaying to WW pairs are more restrictive as can be seen in the figure 4.5. Notice that the multilepton channels yields weaker limits. If $v_{\Delta} \ll 0.1$, then the dilepton decay dominates and the multilepton channel is the most important one.

Generally speaking, the lower mass bound found on the doubly charged scalar mass depend on the branching ratio assumed for the decay into charged leptons as displayed in table 4.1.

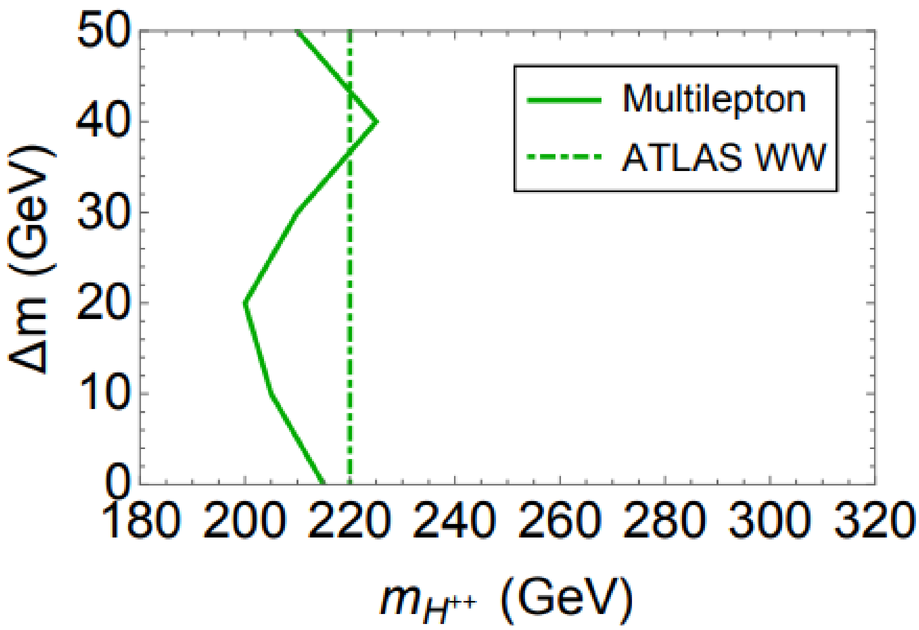

Figure 4.5: The ATLAS same sign diboson bounds ($W^{+}W^{+}$) and the bounds from recasting the CMS multilepton analysis for $v_{\Delta} = 0.1$ GeV and $\Delta m > 0$, here the region on the left of the respective line is excluded, where $\Delta m = m_{H^{++}} - m_{H^{+}}$. Taken from the reference [5]

As these charged scalar are produced on shell, the production cross section, within the narrow width approximation, can be divided into a production cross section of the doubly charged scalars pairs times the branching ratio in the desired final states, in this case, charged leptons. Therefore, one can easily assess the LHC sensitivity to doubly charged scalars for several different decay setups [6, 75].

Channel	$m_{H^{++}}(GeV)$	Branching Ratio
$\ell_\alpha^+ \ell_\beta^+$	$\geqslant 450$ [63]	$Br(H^{++} \to \ell_\alpha^+ \ell_\beta^+) = 10\%$
$\ell_\alpha^+ \ell_\beta^+$	$\geqslant 770$ [63]	$Br(H^{++} \to \ell_\alpha^+ \ell_\beta^+) = 100\%$

Table 4.1: $\ell_\alpha^+ \ell_\beta^+$ channel searched in ATLAS using 36.1 fb^{-1} of data at 13TeV

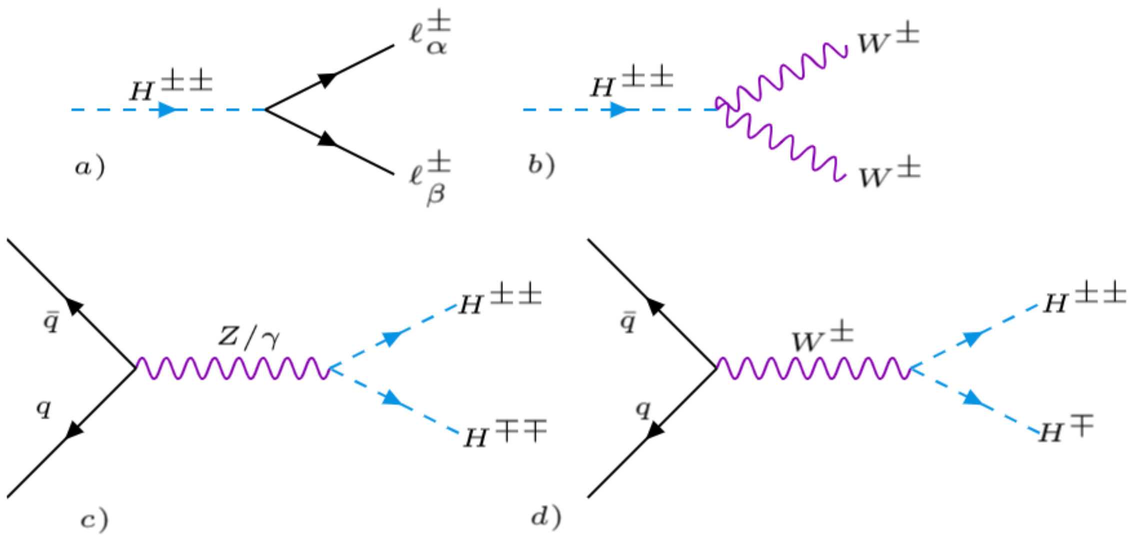

Figure 4.6: Feynman diagrams of the channel production for a) $H^{\pm\pm} \to \ell_\alpha^\pm \ell_\beta^\pm$ and b) $H^{\pm\pm} \to W^\pm W^\pm$), which are the official searches in the LHC. Where c) and d) are the representations of the Drell-Yan process.

4.4 Lepton Flavor Violation (LFV)

The seven physical bosons in this type-II seesaw mechanism contribute to lepton flavor violation processes [62]. Looking $\mathcal{L}_{yuk}^{II}$, we observe that may have flavor-changing interactions that lead to processes such as $\ell_\alpha^- \to \ell_\beta^+ \ell_\theta^- \ell_\sigma^-$ and $\ell_\alpha^- \to \ell_\beta^- \gamma$. The former arises in the tree level and it is mediated by H^{--} and its representation through the Feynman diagrams may be seen in the figure 4.7.

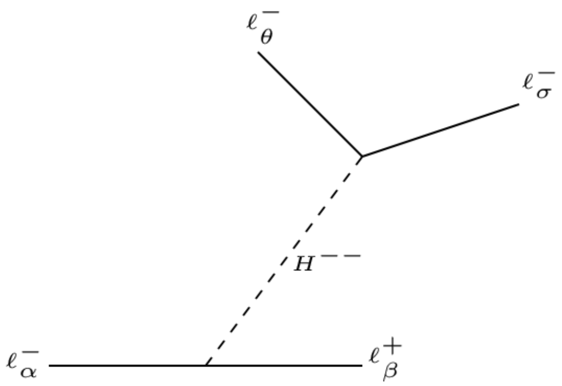

Figure 4.7: Representation of the Feynman diagrams for $\ell_\alpha^- \to \ell_\beta^+ \ell_\theta^- \ell_\sigma^-$ process, which are mediated by H^{--}

Meanwhile, the latter arise on one-loop penguin diagrams. In which, if this second process is mediated by H^-, the photon is emitted only from the single-charged scalar and if it this process is mediated by H^{--}, the photon can be emitted from doubly-charged scalar or from the charged fermion that propagates within the loop. An example of $\ell_\alpha^- \to \ell_\beta^- \gamma$ process can be noticed for the processes $\mu^- \to e^- \gamma$ that may be seen in the diagrams on the figure 4.8.

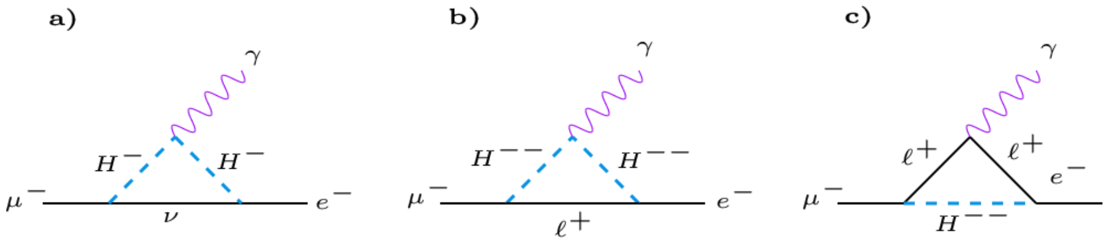

Figure 4.8: Representation pf the Feynman diagrams for $\mu_L^- \to e_L^- \gamma$ process: a) mediated by H^- (the photon is emitted only from H^-), b) mediated by H^{--} (the photon can be emitted from H^{--}) and c) mediated by H^{--} (the photon can be emitted from the charged fermion that propagates within the loop)

The rates for the decays that provided LFV are [63]:

$$\Gamma(\ell_\alpha^- \to \ell_\beta^+ \ell_\theta^- \ell_\sigma^-) = \frac{1}{2(1+\delta_{\theta\sigma})} \frac{m_{\ell_\alpha}^5}{192\pi^3} \left| \frac{Y_{\alpha\beta} Y_{\theta\sigma}}{m_{H^{--}}^2} \right|^2$$

and

$$\Gamma(\ell_\alpha^- \to \ell_\beta^- \gamma) = \frac{m_{\ell_\alpha}^5 \alpha_{em}}{(192\pi^2)^2} |Y^\dagger Y|_{\alpha\beta}^2 \left(\frac{1}{m_{H^-}^2} + \frac{8}{m_{H^{--}}^2} \right)^2$$

here the Kronecker delta accounts for two possible identical final states and α_{em} is the electromagnetic fine-structure constant.

It is easy to see from these decay rates, that they have dependence on the Yukawa coupling constants. Knowing that the yukawa couplings are inversely proportional to the triplet VEV and related to the neutrino masses, once we fix the neutrino masses and v_Δ we have a prediction for these flavor violating decays [6, 67]. Several other lepton flavor violating decays are relevant and we summarize them along with the respective bound in Table 4.2.

Therefore we get,

$$\begin{array}{llll} M_\nu^{22}, M_\nu^{33} \gg M_\nu^{11} & \text{and} & Y^{22}, Y^{33} \gg Y^{11} & \text{for normal hierarchy (NH)} \\ M_\nu^{11} \gg M_\nu^{22}, M_\nu^{33} & \text{and} & Y^{11} \gg Y^{22}, Y^{33} & \text{for inverted hierarchy (IH)} \end{array}$$

In this way, the neutrino mixing and mass relations will be correlated with the decays of the triplet Higgs bosons to charged leptons. This is something we plan to explore in the near future because ATLAS and CMS collaboration make some simplifying assumptions concerning these mixing and yukawa structure. We plan to carry out a more detailed study.

Experimental limit on BR	Constraint on
$Br(\mu \to e\gamma) < 4.2 \times 10^{-13}$	$\lvert(Y_\Delta^\dagger Y_\Delta)_{e\mu}\rvert$
$Br(\mu \to 3e) < 1.0 \times 10^{-12}$	$\lvert(Y_\Delta)_{\mu e}\rvert\lvert(Y_\Delta)_{ee}\rvert$
$Br(\tau \to e\gamma) < 3.3 \times 10^{-8}$	$\lvert(Y_\Delta^\dagger Y_\Delta)_{e\tau}\rvert$
$Br(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$	$\lvert(Y_\Delta^\dagger Y_\Delta)_{\mu\tau}\rvert$
$Br(\tau \to \mu^+\mu^-e^-) < 2.7 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau\mu}\rvert\lvert(Y_\Delta)_{\mu e}\rvert$
$Br(\tau \to e^+e^-e^-) < 2.7 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau e}\rvert\lvert(Y_\Delta)_{ee}\rvert$
$Br(\tau \to e^+\mu^-\mu^-) < 1.7 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau e}\rvert\lvert(Y_\Delta)_{\mu\mu}\rvert$
$Br(\tau \to e^+e^-\mu^-) < 1.8 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau e}\rvert\lvert(Y_\Delta)_{\mu e}\rvert$
$Br(\tau \to \mu^+e^-e^-) < 1.5 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau\mu}\rvert\lvert(Y_\Delta)_{ee}\rvert$
$Br(\tau \to \mu^+\mu^-\mu^-) < 2.1 \times 10^{-8}$	$\lvert(Y_\Delta)_{\tau\mu}\rvert\lvert(Y_\Delta)_{\mu\mu}\rvert$

Table 4.2: Experimental limits on the branching ratios of LFV current processes in the type II seesaw model.

4.5 Parameter space of the type-II seesaw model

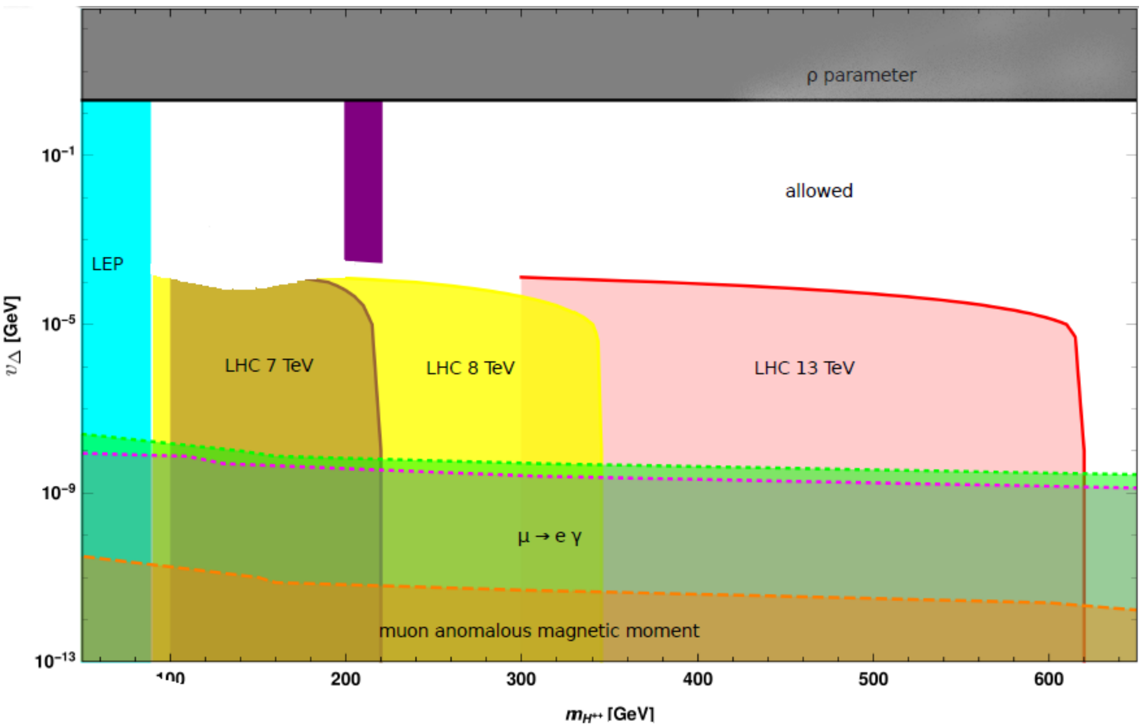

Figure 4.9: Parameter space of the type-II seesaw model, where the allowed and non-allowed regions for the phenomenology in the type-II seesaw model can be seen [6]

In figure 4.9 it is displayed the allowed and excluded regions of the type-II seesaw model. In this way, in this figure, the gray area on top is ruled out by the ρ parameter (which was explained in the section 4.1); the orange region at the bottom is excluded by the experimental measurement for the muon anomalous magnetic moment; the cyan vertical region is excluded by searches at LEP; the magenta area and green area are excluded by the lepton flavor violation process due to the constraints on $\mu \to e\gamma$ (orange region) and constraints on $\mu \to \bar{e}ee$ (green region). It is easy to observe that constraints from lepton flavor violation processes are the most powerful for $m_{H^{++}} > 700$ GeV. Regarding the red, yellow and brown regions, they are excluded by the LHC searches for doubly charged scalars at 7, 8 and 13 TeV respectively (where the lower bound is $m_{H^{\pm\pm}} \gtrsim 620$ GeV at 13 TeV). According to ATLAS Collaboration [76], the searches for di-W bosons are excluded from the mass region where $m_{H^{\pm\pm}}$ lies between 200 and 220 GeV, for $BR(H^{\pm\pm} \to W^{\pm}W^{\pm}) \sim 1$, which is satisfied for $\mathrm{v}_{\Delta} \gtrsim 3 \cdot 10^{-4}$ GeV and it is shown by the purple area in the figure 4.9, which is excluded by LHC. Finally, the white area is allowed.

In conclusion, the type II seesaw model remains a viable alternative to explain neutrino mass while being subject to intensive experimental scrutiny. After presenting all these bounds is quite clear that colliders and lepton flavor violation observables are the most promising avenues to probe the type II seesaw mechanism.

One of our goals in the near future is the derivation of collider bounds having in mind the high-luminosity and high-energy LHC upgrade for several decaying setups. With HE-LHC running at 27 TeV with $15ab^{-1}$ of integrated luminosity we expect to probe doubly charged scalars with masses up to 4.4 TeV. The derivation of lower mass bounds without considering the impact of neutrino mass hierarchy and details concerning the type II seesaw models has been done recently [77]. Our goal is to incorporte those bounds in the context of the type II seesaw in connection to lepton flavor violation.

Chapter 5

Conclusion

We have revisited the theoretical aspects of the type II seesaw model as well as experimental constraints. In particular, we covered electroweak precision, direct production of scalar fields at colliders and lepton flavor violation observables. We stressed the dependence of these constraints with v_Δ and, in turn, with the neutrinos masses. It pointed out the importance of understanding neutrino physics in relation to collider physics and neutrino oscillation experiments, which govern the neutrino mixing which later have an impact on the theoretical predictions of the model.

Lastly we focused on the collider phenomenology of the doubly charged scalar field considering cases where decay into charged leptons and gauge fields are dominant. Thus far, LHC searches for doubly charged scalars where performed under simplifying assumptions concerning the branching ratio of the doubly charged scalar. We plan to perform a more through job concerning the possible decay modes and their connection to neutrino mass hiearchies.

Perspective

As mentioned above, the LHC is continually looking for LFV-charged decays involving the doubly-charged scalar triplet of this model. Knowing that LFV processes are absent in the SM, even in the canonical type I seesaw mechanism, a search for LFV at the LHC is of great importance.

In the near future we plan to devote some time into learning the collider tools and lepton flavor violation codes to investigate this type II seesaw model in greater detail and connect our findings to the neutrino mass hierarchy and lepton flavor violation observables as done in [67] in a different setup.

Moreover, as we have collected convincing evidence for the existence of dark matter over the past decades [78, 79] and we intend to explore the possibility of incorporating dark matter into neutrino mass models [80, 81].

In summary, we plan to test neutrino mass models by exploring the complementary between a large variety of data sets stemming from collider physics to lepton flavor violation.

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Appendix A

Type-I seesaw mechanism

The type-I seesaw mechanism consists of introducing a right-handed neutrino (per family of fermions) into a new renormalizable Yukawa Lagrangian, which is represented by right-handed neutral fields that imply new gauge invariant interactions in Yukawa sector. These right-handed neutrino fields are singlets under the SM group, and they have null hypercharge.

Introducing the right-handed neutrino (ν_R) that is not included in the SM.The leptonic sector is now given by,

$$\bar{\ell}_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \; e_R, \; \nu_R,$$

where $\bar{\ell}_L$ is the leptonic doublet and e_R are singlets of the complete gauge group.

In this way, an interaction of Yukawa with the Higgs is included,

$$\mathcal{L}_{yuk} = -(G_\nu)_{ij}\bar{\ell}_{Li}\widetilde{\phi}\nu_{Rj} = -G_\nu \begin{pmatrix} \bar{\nu} & \bar{e} \end{pmatrix}_{Li} i\sigma_2\phi^*\nu_{Rj}.$$

Where G_e are the coupling constants, ϕ is the Higgs doublet that appears in the SM with its expectation value in the vacuum given in the chapter 2, In this way, an Yukawa interaction with the Higgs is included,

$$\mathcal{L}_{yuk} = -(G_\nu)_{ij}\bar{\ell}_{Li}\widetilde{\phi}\nu_{Rj} = -G_\nu \begin{pmatrix} \bar{\nu} & \bar{e} \end{pmatrix}_{Li} i\sigma_2\phi^*\nu_{Rj},$$

where $(G_\nu)_{ij}$ is the Yukawa coupling matrix, ν_R indicates the column matrix of the neutrinos. It is also necessary to write the Majorana mass term for all the current neutrinos,

$$\mathcal{L}_M = -\frac{1}{2}\bar{\nu^C}_{Ri}\left(M_R\right)_{ij}\nu_{Rj} - \frac{1}{2}\nu_{Li}\left(M_L\right)_{ij}\bar{\nu^C}_{Lj}.$$

$(M_R)_{ij}$ is a Majorana mass matrix that relates the different flavors ν_i, where C is a charge conjugation operator and $\frac{1}{2}$ is the factor that was included so that M_R and M_L can be interpreted as the ν masses. Thus, the most general invariant Lagrangian terms of the gauge group ($SU(2)_L \otimes U(1)_Y$) that provide us with mass terms for the neutrinos are given by [82],

$$\mathcal{L}^{\nu} = \mathcal{L}_{yuk} + \mathcal{L}_M,$$

then,

$$\mathcal{L}^{\nu} = -(Y_\upsilon)_{ij}\begin{pmatrix}\bar{\nu} & \bar{e}\end{pmatrix}_{Li} i\sigma_2\phi^*\nu_{Rj} - \frac{1}{2}\bar{\nu^C_{Ri}}\left(M_R\right)_{ij}\nu_{Rj} - \frac{1}{2}\nu_{Li}\left(M_L\right)_{ij}\bar{\nu^C_{Lj}}\,. \tag{A.1}$$

Where $\mathcal{L}_{yuk}$ after electroweak symmetry breaking yields Dirac-type mass terms M_D as,

$$\mathcal{L}^D{}_{yuk} = -\bar{\nu}_{Li}\left(M_D\right)_{ij}\nu_{Rj} = -\frac{1}{2}\bar{\nu}_{Li}\left(M_D\right)_{ij}\nu_{Rj} - \frac{1}{2}\bar{\nu}_{Li}\left(M_D\right)_{ij}\nu_{Rj}\,. \tag{A.2}$$

Applying the following identity,

$$\bar{\nu}_{Li}\left(M_D\right)_{ij}\nu_{Rj} = \bar{\nu^c_{Ri}}\left(M_D\right)_{ji}\nu^C_{Lj},$$

with $(M_D)_{ij} = \mathrm{v}\,(Y_\nu)_{ij}$, because $i\sigma_2\phi^* = \begin{pmatrix}\frac{\mathrm{v}+h}{\sqrt{2}} \\ 0\end{pmatrix}$. So, the full Lagrangian describing the masses in the neutrino sector is given by,

$$\begin{aligned}\mathcal{L}^{\nu} = -\frac{1}{2}\bar{\nu}_{Li}\left(M_D\right)_{ij}\nu_{Rj} &- \frac{1}{2}\bar{\nu^c_{Ri}}\left(M_D\right)_{ji}\nu^C_{Lj} \\ &- \frac{1}{2}\bar{\nu^C_{R}}\left(M_R\right)_{ij}\nu_{Rj} - \frac{1}{2}\nu_{Li}\left(M_L\right)_{ij}\bar{\nu^C_{Lj}} \\ &= -\bar{\psi}_i\mathcal{M}_{ij}\psi_j\,.\end{aligned} \tag{A.3}$$

Writing it in the matricial form,

$$\mathcal{L}^{\nu} = -\frac{1}{2}\begin{pmatrix}\bar{\nu_L} & \bar{\nu^C_R}\end{pmatrix}\begin{pmatrix}M_L & M_D \\ {M_D}^T & M_R\end{pmatrix}\begin{pmatrix}\nu^C_L \\ \nu_R\end{pmatrix}\,. \tag{A.4}$$

For the type-I seesaw mechanism, M_L must be approximately zero [83], given that it cannot derive from Gauge symmetries, because M_R is not related to the SM scale and, basically, it can take any value and for this case it is $M_R >> M_D$.

Note: If $M_R << M_D$, neutrinos would be predominant Dirac particles. For $M_R \cong M_D$, we have a messy combination of Majorana and Dirac. Whereas for $M_R >> M_D$ we would have a predominant Majorana case, where the gauge invariant scale M_R is expected to be above M_W: $M_R > M_W$ [84].

In this way, the Lagrangian for the neutrinos $M_R >> M_D$, in the matricial form is,

$$\mathcal{L}^{\nu} = -\frac{1}{2}\begin{pmatrix} \bar{\nu_L} & \bar{\nu_R^C} \end{pmatrix}\begin{pmatrix} 0 & M_D \\ {M_D}^T & M_R \end{pmatrix}\begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} \bar{\nu_L} & \bar{\nu_R^C} \end{pmatrix}\mathcal{M}_{\nu}\begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} . \tag{A.5}$$

Being M_R, and M_D, matrices NxN for N lepton families. In this case, the matrix $\mathcal{M}_\nu$ has the eigenvalues of the neutrinos mixed at the basis $\begin{pmatrix} \nu_L^C & \nu_R \end{pmatrix}^T$, therefore, the matrix $\mathcal{M}_\nu$ must be diagonalized and your eigenvalues can be obtained by means of the following equation,

$$Det\,|\mathcal{M}_\nu - m\mathbf{1})| = 0,$$

where the eigenvalues are,

$$m_1 = -\frac{|M_D|^2}{M_R} \quad \text{and} \quad m_2 = M_R\ .$$

In which $m_1 m_2 = -|M_D|^2$, so if a mass increases the other one decreases. Being for the seesaw mechanism $M_R >> M_D$, since $m_1 << m_2$. Because the eigenvalues are not positive they do not have physical meaning, where the eigenvalues are now at the basis called physics. That is usually called mass basis and that would be composed of basis vectors $\begin{pmatrix} \nu_L^C & \nu_R \end{pmatrix}^T$. Given that masses do not have a physical meaning since they are negative, the matrix must be diagonalized and this can be obtained by a biunitary transformation [85] $Z^T\mathcal{M}_\nu Z$. With,

$$Z = \begin{pmatrix} 1 & \rho \\ -i\rho & 1 \end{pmatrix}.$$

Resulting the diagonalized matrix as,

$$D_\nu = \begin{pmatrix} \frac{|M_D|^2}{M_R} & 0 \\ 0 & M_R \end{pmatrix} = \begin{pmatrix} m_\nu & 0 \\ 0 & M_R \end{pmatrix} . \tag{A.6}$$

Eventually, m_ν would correspond to the light neutrinos while M_R would correspond to the heavy neutrinos. Where $\frac{|M_D|^2}{M_R} = m_\nu$ is known as type-I seesaw relation and M_R must be above the electroweak scale. Assuming that, from the previous relation $M_R \sim 10^{15} GeV$ and $M_D \sim 10^2 GeV$, then $m_\nu \sim 10^{-2} eV$ is in agreement with the cosmological experimental links. Now, if the (N=3) SM neutrino families are supposed to be considered and the neutrino mass terms become,

$$\mathcal{L}^{\nu} == -(Y_\upsilon)\begin{pmatrix} \bar{\nu} & \bar{e} \end{pmatrix}_L i\sigma_2\phi^* N_R - \frac{1}{2}\bar{N_R^C}\,(M_R)N_R\,C. \tag{A.7}$$

Where N_R corresponds to the heavy right-handed neutrino added in the seesaw mechanism and ν to the light neutrinos (the procedure is the same as the one before, but in a different basis) and the neutrino mass matrix takes clearly the seesaw form. The matricial form of the A.7 is given as,

$$\mathcal{L}^{\nu} = -\frac{1}{2}\begin{pmatrix}\nu_1 & \nu_2 & \nu_3 & N_1 & N_2 & N_3\end{pmatrix}\begin{pmatrix}0 & M_D \\ {M_D}^T & M_R\end{pmatrix}\begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3\end{pmatrix} . \tag{A.8}$$

In this way, the matrices M_R and M_D are 3x3. Where $\mathcal{M}_\nu$, must also be diagonalized, by means of a unitary transformation, as explained above.

$$D_\nu = \begin{pmatrix}\frac{|M_D|^2}{M_R} & 0 \\ 0 & M_R\end{pmatrix} = \begin{pmatrix}m_\nu & 0 \\ 0 & M_R\end{pmatrix} = \begin{pmatrix}m_{light} & 0 \\ 0 & M_{heavy}\end{pmatrix} = m_n . \tag{A.9}$$

So, the masses correspond to

$$Diag(m_n) = (m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{N_1}, M_{N_2}, M_{N_3}),$$

where the matrices for the light and heavy neutrinos with $M_R = M_N$, are given by,

$$m_{light} = -(M_D)^T \frac{1}{M_R} M_D,$$

$$M_{heavy} = M_R.$$

This is the original seesaw formula, currently called Type-I. Being the eigenvectors of these masses as follows,

$$\nu_1 >= a|\nu_e > + b\,|\nu_\mu > + c\,|\nu_\tau > .$$

Where a, b and c are the probabilities to find every neutrino (a+b+c =1). Where ν_e, ν_μ and ν_τ are the neutrino families in the SM. Likewise, the type I seesaw mechanism can explain the neutrino masses ($m_\nu \sim 0.1V$) by means of large Majorana masses of the heavy right-handed neutrino ($M_R \sim 10^{15}$ GeV).

Appendix B

Vacuum stability, Unitarity and Perturbativity of the type-II seesaw model

The vacuum stability has some conditions for the coupling constants λ_i from the scalar potential (3.15), this conditions ensure that the potential of the type II seesaw model is bounded. These constraints are given as [3, 64]:

$$\lambda \geq 0, \tag{B.1a}$$

$$\lambda_1 \geq 0, \tag{B.1b}$$

$$2\lambda_1 + \lambda_2 \geq 0, \tag{B.1c}$$

$$\lambda_4 + \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \tag{B.1d}$$

$$\lambda_4 - \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \tag{B.1e}$$

$$2|\lambda_5|\sqrt{\lambda_1} + \lambda_2\sqrt{\lambda} \geq 0 \quad \text{or} \quad \lambda_4 + \sqrt{\left(\lambda\lambda_2 + 2\lambda_5^2\right)(\lambda_1/\lambda_2 + 1/2)} \geq 0\,. \tag{B.1f}$$

These conditions are sufficient, but are not necessary [62]. Demanding the tree level unitarity to be preserved for different elastic scattering processes the following constraints are written * [62]:

$$\lambda \leq \frac{8}{3}\pi\,, \tag{B.2a}$$

$$\lambda_1 - \lambda_2 \leq 8\pi\,, \tag{B.2b}$$

$$4\lambda_1 + \lambda_2 \leq 8\pi\,, \tag{B.2c}$$

$$2\lambda_1 + 3\lambda_2 \leq 16\pi\,, \tag{B.2d}$$

$$|\lambda_5| \leq \frac{1}{2}\min\left[\sqrt{(\lambda \pm 8\pi)(\lambda_1 - \lambda_2 \pm 8\pi)}\right]\,, \tag{B.2e}$$

$$|\lambda_4| \leq \frac{1}{\sqrt{2}}\sqrt{\left(\lambda - \frac{8}{3}\pi\right)(4\lambda_1 + \lambda_2 - 8\pi)}\,. \tag{B.2f}$$

Where this perturbativity implies that all parameters are smaller than 4π according to A. Arhrib el. at [86].

*These have been studied in the type II seesaw model e.g. in Ref. [86]

Appendix C

Scalar potential parameterized in the type-II seesaw mechanism

By taking the potential of the type-II seesaw model,

$$\begin{aligned}
\mathcal{V}(\Phi,\Delta) \;=\; & -m_\Phi^2\Phi^\dagger\Phi+\frac{\lambda}{2}(\Phi^\dagger\Phi)^2+M_\Delta^2\mathrm{Tr}(\Delta^\dagger\Delta)+\frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 \\
& +\frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2-\mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right)+\lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) \\
& +\lambda_5\Phi^\dagger[\Delta^\dagger,\Delta]\Phi+\left(\mu\Phi^T i\sigma_2\Delta^\dagger\Phi+\mathrm{h.c.}\right)\ . \qquad \text{(C.1)}
\end{aligned}$$

As the potential in the section 3.3.3 must be evaluated with the parameterized scalars (Eqs. (3.17) and (3.16)) and at the same time considering the following,

$$(\mathrm{v}+\phi+\mathrm{i}\chi)/\sqrt{2}=\phi^0, \qquad (\mathrm{v}+\phi+\mathrm{i}\chi)^*/\sqrt{2}=\phi^{0*}\ ,$$

$$(\mathrm{v}_\Delta+\delta+\mathrm{i}\eta)^/\sqrt{2}=\Delta^0 \quad \text{and} \quad (\mathrm{v}_\Delta+\delta+\mathrm{i}\eta)^*/\sqrt{2}=\Delta^{0*}\ .$$

This potential is given as,

$$
\begin{aligned}
\mathcal{V}(\Phi,\Delta) \;=\; & - m_\Phi^2 \begin{pmatrix}\phi^- & \phi^{0*}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} + \frac{\lambda}{2}\left(\begin{pmatrix}\phi^- & \phi^{0*}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\right)^2 \\
& + M_\Delta^2 \mathrm{Tr}\left[\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right] \\
& + \frac{\lambda_1}{2}\left[\mathrm{Tr}\left[\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right]\right]^2 \\
& + \frac{\lambda_2}{2}\left[\mathrm{Tr}\left[\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right]\right]^2 \\
& - \frac{\lambda_2}{2}\mathrm{Tr}\left[\left(\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right)^2\right] \\
& + \lambda_4 \begin{pmatrix}\phi^- & \phi^{0*}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\mathrm{Tr}\left[\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right] \\
& + \lambda_5 \begin{pmatrix}\phi^- & \phi^{0*}\end{pmatrix}\left[\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix} \;,\; \begin{pmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{pmatrix}\right]\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \\
& + \left(\mu \begin{pmatrix}\phi^+ & \phi^0\end{pmatrix} i \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}\Delta^-/\sqrt{2} & \Delta^{0*}\\ \Delta^{--} & -\Delta^-/\sqrt{2}\end{pmatrix}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} + \text{h.c.}\right) \; .
\end{aligned} \tag{C.2}
$$

Thus, the potential may be written as,

$$
\begin{aligned}
\mathcal{V}(\Phi,\Delta) \;=\; & -m_\Phi^2\left(\phi^-\phi^+ + |\phi^0|^2\right) + \frac{\lambda}{2}\left(\phi^-\phi^+ + |\phi^0|^2\right)^2 \\
& + M_\Delta^2\left(\Delta^{--}\Delta^{++} + \Delta^-\Delta^+ + |\Delta^0|^2\right) + \frac{\lambda_1}{2}\left[\left(\Delta^{--}\Delta^{++} + \Delta^-\Delta^+ + |\Delta^0|^2\right)\right]^2 \\
& + \frac{\lambda_2}{2}\left[\left(\Delta^{--}\Delta^{++} + \Delta^-\Delta^+ + |\Delta^0|^2\right)\right]^2 \\
& - \frac{\lambda_2}{2}\left[\left(\frac{\Delta^-\Delta^+}{2} + |\Delta^0|^2\right)\left(\frac{\Delta^+\Delta^-}{2} + |\Delta^0|^2\right)\right] \\
& - \frac{\lambda_2}{2}\left[\left(\frac{\Delta^-\Delta^+}{2} + \Delta^{--}\Delta^{++}\right)\left(\frac{\Delta^+\Delta^-}{2} + \Delta^{++}\Delta^{--}\right)\right] \\
& - \frac{\lambda_2}{4}\left[\left(\Delta^-\Delta^{++} - \Delta^0\Delta^+\right)\left(\Delta^+\Delta^{--} - \Delta^0\Delta^-\right)\right] \\
& - \frac{\lambda_2}{4}\left[\left(\Delta^{--}\Delta^+ - \Delta^-\Delta^0\right)\left(\Delta^{++}\Delta^- - \Delta^+\Delta^0\right)\right] \\
& + \lambda_4\left(\phi^-\phi^+ + |\phi^0|^2\right)\left[\left(\Delta^{--}\Delta^{++} + \Delta^-\Delta^+ + |\Delta^0|^2\right)\right] \\
& + \lambda_5\phi^-\left(\frac{\Delta^-\Delta^+}{2} + |\Delta^0|^2 - \frac{\Delta^+\Delta^-}{2} - \Delta^{++}\Delta^{--}\right)\phi^+ \\
& + \lambda_5\phi^{0*}\left(\frac{\Delta^{--}\Delta^+ - \Delta^-\Delta^0 - \Delta^0\Delta^- + \Delta^+\Delta^{--}}{\sqrt{2}}\right)\phi^+ \\
& + \lambda_5\phi^-\left(\frac{\Delta^-\Delta^{++} - \Delta^0\Delta^+ - \Delta^+\Delta^{0*} + \Delta^{++}\Delta^{--}}{\sqrt{2}}\right)\phi^0 \\
& + \lambda_5\phi^{0*}\left(\frac{\Delta^-\Delta^+}{2} - |\Delta^0|^2 - \frac{\Delta^+\Delta^-}{2} + \Delta^{--}\Delta^{++}\right)\phi^0 \\
& + \mu\left(\phi^+\Delta^{--}\phi^+ - \phi^0\frac{\Delta^-}{\sqrt{2}}\phi^+ - \phi^{0*}\Delta^{0*}\phi^0 - \phi^+\frac{\Delta^-}{\sqrt{2}}\phi^0\right)\;. \qquad \text{(C.3)}
\end{aligned}
$$

Appendix D

T parameter on the type-II seesaw mechanism

We have seen before that the electroweak ρ-parameter for the type-II seesaw model places an upper limit on v_Δ, i.e., $v_\Delta \leq 4.8$ GeV [63], because it is an observable experimentally determined in the tree level $\rho = 1$. However, the new physics beyond the SM uses the Peskin–Takeuchi parameters to constrain models. These parameters are a set of three measurable quantities, called S, T, and U, that parameterize potential new physics contributions to electroweak radiative corrections.

Then, the electroweak oblique parameter T is an observable that combines electroweak precision data to quantify deviation from the SM. This parameter is experimentally determined and may be calculated at one loop in the basis-independent CP-violating using information on the Higgs couplings according to G. Funk [87]. The T parameter in the type-II seesaw model is given by [88],

$$T = \frac{1}{8\pi c_w^2 s_w^2}\left[\eta\left(\frac{m_{H^{++}}^2}{m_Z^2}, \frac{m_{H^+}^2}{m_Z^2}\right) + \eta\left(\frac{m_{H^+}^2}{m_Z^2}, \frac{m_H^2}{m_Z^2}\right)\right], \tag{D.1}$$

where it can be clearly seen that the dependence of the masses of the doubly and singly charged scalar and $\eta(x, y)$ is the following function,

$$WhereEungJinChunet.al[88]providethebestfittedvaluefor\eta(x, y) = x + y - \frac{2xy}{x-y}\ln\frac{x}{y} \quad . \tag{D.2}$$

Where Eung Jin Chun et. al [88] provide the best fitted value for T parameters $T = 0.05 \pm 0.12$.

Appendix E

Symmetry CP

In general, a symmetry in physical suggests that something is conserved. There are three discrete symmetries (transformation) that are usually used in quantum field theory*:

- P-symmetry = spatial reflection symmetry or parity. The parity transformation replaces the system as if it were a mirror, that is, it invests the space or reverses the space coordinates (x, y, z) by (-x, -y-z), but not the time. Then, it should reverse the momentum of a particle without flipping its spin.
- T-symmetry = time reversal, it replaces t by -t. It reverses the momentum of a particle and should also flip the spin.
- C-symmetry is charge conjugation symmetry, it transforms a particle into its antiparticle or vice versa. In this way, it reverses all the internal quantum numbers such as electric charge, hypercharge, strangeness, etc.

In the case of CP-symmetry, it is the parity transformation plus charged conjugation or simply the product of these two symmetries C and P. Failure to comply with any of the symmetries C and P, results in what is called the CP-violation.

*For greater theoretical understanding the reference [89] can be revised.

Appendix F

Muon anomalous electromagnetic moment (g-2)

The muon anomalous magnetic moment is a theory beyond SM.

Where, the muon magnetic moment is defined as follows,

$$\vec{\mu}_\mu = g_\mu \left(\frac{q}{2m_\mu} \right) \vec{S}. \tag{F.1}$$

Where $g_\mu = 2$ is the gyromagnetic ratio; the intrinsic spin is $S = -\frac{1}{2}$ for a particle of mass m and charge $q = \pm e$. The anomalous magnetic moment of a particle is a contribution of effects of quantum mechanics, expressed by Feynman diagrams with loops. In this way, by means of radiative corrections can be introduced an anomalous magnetic moment defined by [90],

$$a_\mu = \frac{1}{2}(g_\mu - 2), \tag{F.2}$$

because, the radiative corrections couple the muon spin to virtual fields.

Besides, the Muon g-2 collaboration [90] measured the anomalous magnetic moment of the muon as follows,

$$a_\mu^{exp} = 11659208.0(6.3) \times 10^{-10}$$

and this result is very precise with the SM predicted value [91],

$$a_\mu^{SM} = 11659183 \times 10^{-10}\ .$$

In the case of the type-II seesaw model, this model modifies the theory prediction at one loop level, since this receives new contributions from $H^{\pm\pm}$ and $H^{\pm}$ fields. According to

Antusch et al., the amplitude is given as [6],

$$\delta a_\mu(H^{\pm\pm}) = -\frac{2\left|(Y_\Delta)_{ij}(Y_\Delta)_{ij}\right|^2 m_\mu^2}{12\pi^2 m_{H^{\pm\pm}}^2}\,,$$

$$\delta a_\mu(H^{\pm}) = -\frac{2\left|(Y_\Delta)_{ij}(Y_\Delta)_{ij}\right|^2 m_\mu^2}{96\pi^2 m_{H^{\pm}}^2}\,.$$

According to Antusch et al. [6] for some range of the triplet mass and $\mathrm{v}_\Delta \lesssim 10^{-10}$ GeV, this modified theory could explain a_μ^{exp}. But, this region is already excluded by the lepton flavor violation experiments.