A note on limits on new interactions from the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio

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We investigate the constraints on pseudoscalar-type $\bar{d}u \to e\nu_e$ interactions from the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio. We point out that for some important cases the usual assumption that the contributions of these interactions to $\pi \to e\nu/\pi \to \mu\nu$ are small relative to the standard model contribution is not justified. We give an exact treatment of the constraints from $\pi \to e\nu/\pi \to \mu\nu$ and investigate the implications for tree-level sources of pseudoscalar-type $\pi \to e\nu_e$ interactions. One of the results is that, contrary to previous belief, the bounds on the masses of some of the leptoquarks which couple to both left-handed and right-handed quarks are not considerably stronger than the bounds on the masses of chiral leptoquarks even for equal strength for the left-handed and right-handed couplings.

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I. INTRODUCTION

The branching ratio

$$R_\pi \equiv \frac{\Gamma(\pi \to e\nu + \pi \to e\nu\gamma)}{\Gamma(\pi \to \mu\nu + \pi \to \mu\nu\gamma)} \tag{1}$$

is a sensitive probe of a variety of physics beyond the standard model [1]. This is made possible by the high level of precision at which R_π can be calculated in the standard model. R_π is especially sensitive to CP-invariant pseudoscalar-type $\pi \to e\nu$ interactions (four-fermion interactions containing $\bar{u}\gamma_5 d$) involving the same neutrino state as the one in the $W^+ \to e^+\nu_e^{(L)}$ amplitude: the contribution of pseudoscalar-type interactions to the $\pi \to e\nu$ amplitude has a large enhancement factor, and moreover, if the interaction is CP invariant and involves $\nu_e^{(L)}$, the amplitude interferes in the rate with the standard model amplitude. Limits on such interactions implied by the experimental value of R_π have been derived in previous works under the assumption that the contribution of pseudoscalar-type interactions is small relative to the contribution from the standard model, and neglecting therefore the term quadratic in the pseudoscalar coupling. In this paper we wish to point out that in some important cases this assumption is not justified [2]. In the next section we give an exact treatment of the constraints from R_π. In Sec. III we analyze the implications for tree-level sources of pseudoscalar-type $\pi \to e\nu_e$ interactions. A summary of our conclusions is given in Sec. IV.

II. LIMITS ON PSEUDOSCALAR-TYPE $\bar{d}u \to e\nu_e$ INTERACTIONS FROM R_π

In the standard model the decays $\pi \to l\nu_l (l = e, \mu)$ are described by the V-A Hamiltonian [3]

$$H = (g^2 U_{ud}/8m_W^2)\bar{l}\gamma_\lambda(1-\gamma_5)\nu_l \bar{u}\gamma^\lambda(1-\gamma_5)d + \text{H.c.}\,, \tag{2}$$

where U_{ud} is the ud element of the Kobayashi-Maskawa matrix. Only the $\bar{u}\gamma_\lambda\gamma_5 d$ part of (2) contributes to $\pi \to l\nu_l$. The decay rate is given by [4]

$$\Gamma(\pi \to l\nu_l) = (m_\pi/4\pi)(1-r_l^2)^2 m_l^2 f_\pi^2 (g^2 U_{ud}/8m_W^2)^2\,, \tag{3}$$

where $r_l = m_l/m_\pi$, and f_π is defined by $\langle 0|\bar{d}\gamma_\lambda\gamma_5 u|\pi^+(p)\rangle = if_\pi p_\lambda$ ($f_\pi \simeq 131$ MeV). The muonic decay mode, which has the large rate, provides the value of f_π. Here we are interested in possible contributions from interactions beyond the standard model. The quantity which is sensitive to new physics is the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$, or the ratio (1), in which the factor $f_\pi U_{ud}$ drops out [5].

Including radiative corrections, the value of R_π in the standard model is [6]

$$(R_\pi)_{\text{SM}} = (1.2352 \pm 0.0005) \times 10^{-4}\,. \tag{4}$$

An experiment at TRIUMF [7], and an experiment at PSI [8] have obtained

$$(R_\pi)_{\text{expt}} = [1.2265 \pm 0.0034(\text{stat}) \pm 0.0044(\text{syst})] \times 10^{-4}\,, \tag{5}$$

and

$$(R_\pi)_{\text{expt}} = [1.2346 \pm 0.0035(\text{stat}) \pm 0.0036(\text{syst})] \times 10^{-4}\,, \tag{6}$$

respectively. Combining (5) and (6) gives [9]

$$(R_\pi)_{\text{expt}} = (1.2303 \pm 0.0036) \times 10^{-4}\,. \tag{7}$$

Let us consider the decays $\pi \to l\nu_l$ in the framework of the interaction

$$H = H^{(e)} + H^{(\mu)}\,, \tag{8}$$

where

$$H^{(e)} = \bar{e}\gamma_\lambda(1-\gamma_5)\nu_e^{(L)}[a_{LL}\bar{u}\gamma^\lambda(1-\gamma_5)d + a_{LR}\bar{u}\gamma^\lambda(1+\gamma_5)d] + \bar{e}\gamma_\lambda(1+\gamma_5)\nu_e^{(R)}[a_{RL}\bar{u}\gamma^\lambda(1-\gamma_5)d + a_{RR}\bar{u}\gamma^\lambda(1+\gamma_5)d]$$
$$+\bar{e}(1-\gamma_5)\nu_e^{(L)}[A_{LL}\bar{u}(1+\gamma_5)d + A_{LR}\bar{u}(1-\gamma_5)d] + \bar{e}(1+\gamma_5)\nu_e^{(R)}[A_{RL}\bar{u}(1+\gamma_5)d + A_{RR}\bar{u}(1-\gamma_5)d]\,, \quad (9)$$

and

$$H^{(\mu)} = \bar{\mu}\gamma_\lambda(1-\gamma_5)\nu_\mu^{(L)}[b_{LL}\bar{u}\gamma^\lambda(1-\gamma_5)d + b_{LR}\bar{u}\gamma^\lambda(1+\gamma_5)d] + \bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu^{(R)}[b_{RL}\bar{u}\gamma^\lambda(1-\gamma_5)d + b_{RR}\bar{u}\gamma^\lambda(1+\gamma_5)d]$$
$$+\bar{\mu}(1-\gamma_5)\nu_\mu^{(L)}[B_{LL}\bar{u}(1+\gamma_5)d + B_{LR}\bar{u}(1-\gamma_5)d] + \bar{\mu}(1+\gamma_5)\nu_\mu^{(R)}[B_{RL}\bar{u}(1+\gamma_5)d + B_{RR}\bar{u}(1-\gamma_5)d]\,. \quad (10)$$

In Eqs. (9) and (10) the first and the second subscript of a_{ik} and A_{ik} denotes the chirality of the neutrino and of the u quark, respectively. The constants a_{LL} and b_{LL} contain the standard model contributions $(a_{LL})_{SM}$ and $(b_{LL})_{SM}$, given by

$$(a_{LL})_{\rm SM} = g^2 U_{ud}/8m_W^2\,,$$
$$(b_{LL})_{\rm SM} = (a_{LL})_{\rm SM}\,. \quad (11)$$

Thus

$$a_{LL} = (a_{LL})_{\rm SM} + a'_{LL}\,,$$
$$b_{LL} = (b_{LL})_{\rm SM} + b'_{LL}\,, \quad (12)$$

where a'_{LL} and b'_{LL} represent contributions from new $V-A$ interactions. In Eqs. (9) and (10) we have made the assumption that in all the new interaction terms the electron (muon) couples only to the neutrino states $\nu_e^{(L)}$ and $\nu_e^{(R)}$ ($\nu_\mu^{(L)}$ and $\nu_\mu^{(R)}$), where $\nu_l^{(L)}$ $(l = e, \mu)$ is the neutrino state in the $W^+ \to e^+\nu_l^{(L)}$ amplitude and $\nu_l^{(R)}$ $(l = e, \mu)$ is a right-handed singlet state [10]. The states $\nu_l^{(L)}$ and $\nu_l^{(R)}$ are in general linear combinations of the left-handed and the right-handed components of the neutrino mass-eigenstates ν_i:

$$\nu_l^{(L)} = \sum_i U_{li}\nu_{iL} \quad (l = e, \mu)\,,$$
$$\nu_l^{(R)} = \sum_i V_{li}\nu_{iR} \quad (l = e, \mu)\,, \quad (13)$$

where $\nu_{iL} = \frac{1}{2}(1-\gamma_5)\nu_i$, $\nu_{iR} = \frac{1}{2}(1+\gamma_5)\nu_i$, and U_{ei} and V_{ei} are (in a basis where the charged leptons are diagonal) elements of the neutrino mixing matrix.

Apart from our assumptions regarding the neutrino states, the part of the Hamiltonians (9) and (10) containing $\bar{u}\gamma_\lambda\gamma_5 d$ (axial-vector-type interaction) and $\bar{u}\gamma_5 d$ (pseudoscalar-type interaction) is the most general nonderivative four-fermion interaction that can contribute to $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$. Assuming further that the neutrinos are either too heavy to be produced in $\pi \to l\nu_l$ or light enough that the effect of their masses on the decay probability can be neglected, the $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$ decay rates due to (9) and (10) can be obtained from Eq. (3) by replacing $(g^2U_{ud}/8m_W^2)^2$ with the quantities [11]

$$Q_e = (g^2U_{ud}/8m_W^2)^2 u_e(|1 + \eta'_{LL} - \eta_{LR} + \omega_e\eta_{LP}|^2$$
$$+\tilde{v}_e|\eta_{RL} - \eta_{RR} + \omega_e\eta_{RP}|^2) \quad (14)$$

and

$$Q_\mu = (g^2U_{ud}/8m_W^2)^2 u_\mu(|1 + \beta'_{LL} - \beta_{LR} + \omega_\mu\beta_{LP}|^2$$
$$+\tilde{v}_\mu|\beta_{RL} - \beta_{RR} + \omega_\mu\beta_{RP}|^2)\,, \quad (15)$$

respectively. In Eq. (14),

$$\eta'_{LL} = a'_{LL}/(a_{LL})_{\rm SM}\,,$$
$$\eta_{ik} = a_{ik}/(a_{LL})_{\rm SM} \quad (ik = LR, RL, RR)\,,$$
$$\eta_{iP} = (A_{iL} - A_{iR})/(a_{LL})_{\rm SM} \quad (i = L, R)\,, \quad (16)$$

and, in Eq. (15),

$$\beta'_{LL} = b'_{LL}/(b_{LL})_{\rm SM}\,,$$
$$\beta_{ik} = b_{ik}/(b_{LL})_{\rm SM} \quad (ik = LR, RL, RR)\,,$$
$$\beta_{iP} = (B_{iL} - B_{iR})/(b_{LL})_{\rm SM} \quad (i = L, R)\,. \quad (17)$$

The quantities $u_l(l = e, \mu)$ and $\tilde{v}_l(l = e, \mu)$ are given by

$$u_l = {\sum_i}' |U_{li}|^2\,, \quad (18)$$

and

$$\tilde{v}_l = v_l/u_l\,, \quad (19)$$

where

$$v_l = {\sum_i}' |V_{li}|^2\,. \quad (20)$$

The prime on the summation in Eqs. (18) and (20) indicates that the sum extends only over the neutrinos that are light enough to be produced in $\pi \to l\nu_l$. The quantities $\omega_l(l = e, \mu)$ are [12]

$$\omega_l = \frac{m_\pi}{m_l}\frac{m_\pi}{m_u + m_d} \quad (l = e, \mu)\,. \quad (21)$$

The ratio R_π is given by

$$R_\pi = (R_\pi)_{\rm SM}(Q_e/Q_\mu)\,. \quad (22)$$

As seen from Eqs. (14), (15), and (22), R_π is not sensitive to new axial-vector-type interactions for which $b'_{LL} = a'_{LL}$, $b_{ik} = a_{ik}$, and to pseudoscalar-type interactions for which $A_{ik}/B_{ik} = m_e/m_\mu$ [13]. In this paper we shall not be dealing with models of this kind. The constraints from $(R_\pi)_{\rm expt}$ could still be weakened by accidental cancellations between the electronic and the muonic terms. In the following, while keeping this possibility in mind, we shall ignore it in our considerations.

From the experimental result (7) and Eq. (22) one obtains $(Q_e/Q_\mu)^{1/2} = 0.9980 \pm 0.0015$ so that

$$0.9951 < (Q_e/Q_\mu)^{1/2} < 1.0009 \quad (95\% \text{ C.L.}) . \tag{23}$$

The ratio R_π is very sensitive to pseudoscalar-type $\bar{d}u \to e\nu_e$ interactions, which are enhanced in R_π by the factor $\omega_e(\simeq 3.3 \times 10^3)$. Let us consider the special case when from the new couplings only η_{LP} is nonzero, and $u_e/u_\mu = 1$. Also, we shall neglect the contribution of the $\text{Im}\eta_{LP}$ term (regarding this see the discussion further on). A limit on general $\text{Re}\eta_{LP}$ type couplings has been deduced in Ref. [14]. This was done under the assumption that the pseudoscalar contribution is small relative to the contribution from the standard model (i.e., that $\omega_e\text{Re}\eta_{LP}$ is small relative to unity). In such a case $(Q_e/Q_\mu) = (1 + \omega_e\text{Re}\eta_{LP})^2 \simeq 1 + 2\omega_e\text{Re}\eta_{LP}$, and from Eqs. (7) and (22) one obtains [15]

$$-1.5 \times 10^{-6} < \text{Re}\eta_{LP} < 2.8 \times 10^{-7} \quad (95\% \text{ C.L.}) . \tag{24}$$

However, especially in view of the large enhancement factor ω_e, whether the assumption of the smallness of $\omega_e\text{Re}\eta_{LP}$ is permissible depends on what are the available constraints on the $\text{Re}\eta_{LP}$ interaction. Without this assumption we have from Eqs. (14) and (23)

$$0.9951 < |1 + \omega_e\text{Re}\eta_{LP}| < 1.0009 (95\% \text{ C.L.}) , \tag{25}$$

which allows $\text{Re}\eta_{LP}$ to be either in the range

$$-1.5 \times 10^{-6} < \text{Re}\eta_{LP} < 2.8 \times 10^{-7} \tag{26a}$$

(corresponding to $\omega_e\text{Re}\eta_{LP}$ in the neighborhood of zero), or in the range

$$-6.14 \times 10^{-4} < \text{Re}\eta_{LP} < -6.12 \times 10^{-4} \tag{26b}$$

[corresponding to $\omega_e\text{Re}\eta_{LP}$ in the neighborhood of (-2)]. The range (26a) is at the level of accuracy shown the same as (24), but the range (26b) allows $|\text{Re}\eta_{LP}|$ to be as large as 6×10^{-4}.

The range of values of $\text{Re}\eta_{LP}$ allowed by $(R_\pi)_{\text{expt}}$ could be wider than the sum of the ranges (26a) and (26b) if there is an additional new contribution to R_π such that Q_e/Q_μ has the form

$$Q_e/Q_\mu = (1 + \omega_e\text{Re}\eta_{LP})^2 + y^2 . \tag{27}$$

The presence of the positive contribution y^2 would weaken the effect of the lower bound in (23) on $\text{Re}\eta_{LP}$, allowing a wider range of values. If y^2 can have any value in the interval $0 \le y^2 \lesssim 1$, the values of $\text{Re}\eta_{LP}$ allowed by $(R_\pi)_{\text{expt}}$ will span the whole interval

$$-6.2 \times 10^{-4} < \text{Re}\eta_{LP} < 2.8 \times 10^{-7} . \tag{28}$$

A natural candidate for y would be $\text{Im}\eta_{LP}$. This contribution is CP violating and therefore it has no interference in the rate with the standard model contribution. But for CP-violating pseudoscalar-type couplings a stringent limit follows from the experimental limit on the parity- and time-reversal-violating tensor-type electron-nucleon interaction [16]. The present experimental bound on the electric dipole moment of the ^{199}Hg atom [17] sets a bound on the latter interaction, which implies $|\text{Im}\eta_{LP}| \lesssim 4 \times 10^{-5}$. The y^2 contribution could however come from an η_{RP} interaction [see Eq. (14)]. Such a term may be present in the Hamiltonian if the extension of the standard model containing η_{LP} includes a light right-handed neutrino. Another possibility, which does not require the presence of a right-handed neutrino, would be the presence of an interaction of the form $\bar{e}(1-\gamma_5)\nu_\tau^{(L)}\bar{u}\gamma_5 d$ (where $\nu_\tau^{(L)}$ is the neutrino state in the $W^+ \to \tau^+\nu_\tau^{(L)}$ amplitude), since this interaction is not constrained yet significantly [10].

So far we have assumed that $u_e = u_\mu = 1$. This would be the case, for example, if all the neutrinos are light. In general $u_e \neq 1$, $u_\mu \neq 1$, and $u_e/u_\mu \neq 1$. For $u_e/u_\mu \neq 1$ we have

$$(Q_e/Q_\mu)^{1/2} = (u_e/u_\mu)^{1/2}|1 + \omega_e\text{Re}\eta_{LP}| . \tag{29}$$

A lower bound on the quantity $(u_e u_\mu)^{1/2}$ follows from a comparison of the predicted and the experimental value of the mass m_W of the W. Assuming that only the standard model interaction contributes to muon decay, the predicted value $(m_W)_p$ of m_W for $u_e u_\mu \neq 1$ is given by

$$(m_W)_p = \left(\frac{\pi\alpha}{\sqrt{2}G_F}\right)^{1/2} [\sin^2\theta_W(1-\Delta r)]^{-1/2}(u_e u_\mu)^{1/4} , \tag{30}$$

where Δr represents radiative corrections [19]. Using in Eq. (30) $\sin^2\theta_W = 0.2247 \pm 0.0019$ and $\Delta r = 0.040 \pm 0.004$ [18], and identifying $(m_W)_p$ with the experimental value $(m_W)_{\text{expt}} = (80.22 \pm 0.26)$ GeV [18], we obtain $(u_e u_\mu)^{1/4} > 0.99$ (95% C.L.), and therefore

$$(u_e u_\mu)^{1/2} > 0.98 \quad (95\% \text{ C.L.}) . \tag{31}$$

Since $u_e \le 1$, $u_\mu \le 1$, the bound (31) implies

$$0.98 < (u_e/u_\mu)^{1/2} < 1.02 . \tag{32}$$

Allowing $(u_e/u_\mu)^{1/2}$ to take any value in the interval (32) changes the ranges (26a) and (26b) to

$$-7.6 \times 10^{-6} < \text{Re}\eta_{LP} < 6.4 \times 10^{-6} \tag{33a}$$

and

$$-6.20 \times 10^{-4} < \text{Re}\eta_{LP} < -6.06 \times 10^{-4} , \tag{33b}$$

respectively. For $u_e/u_\mu \neq 1$ we can therefore have (in the presence of the y^2 term)

$$-6.2 \times 10^{-4} < \text{Re}\eta_{LP} < 6.4 \times 10^{-6} \quad (95\% \text{ C.L.}) . \tag{34}$$

We note yet that the bounds on $(u_e/u_\mu)^{1/2}$ would be somewhat weaker than (32) if also muon decay would involve a pseudoscalar interaction, or any other new interaction which has no interference in the rate with the standard model contribution. In such a case $(u_e u_\mu)^{1/4}$ in Eq. (30) would be replaced by $(u_e u_\mu)^{1/4}(1 + k^2)^{1/4}$, where the positive constant k^2 represents the contribution of the new interaction, and the bound (31) would become $(u_e u_\mu)^{1/2} > 0.98/(1 + k^2)^{1/2}$ [20].

III. IMPLICATIONS FOR TREE-LEVEL SOURCES OF PSEUDOSCALAR INTERACTIONS

Pseudoscalar-type four-fermion interactions contributing to R_π can arise at the tree level from the exchange of charged Higgs bosons, in supersymmetric models with R-parity violation from the exchange of sfermions, and from the exchange of leptoquarks.

A. Charged Higgs bosons

Charged Higgs bosons [21] are present in many extensions of the standard model, and also in the standard $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ model if the Higgs sector contains (for example) two Higgs doublets. In models which use discrete symmetries to eliminate flavor-changing neutral currents (to allow light Higgs bosons), the couplings of the charged Higgs bosons from multiplets which contribute to the fermion mass matrices are proportional to the masses of the fermion fields in the currents. The interactions from the exchange of such Higgs bosons are too weak to be observable in $\pi \to l\nu_l$ (although some enhancement could come from ratios of Higgs vacuum expectation values). Also, as mentioned earlier, if the ratios of the coupling constants for $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$ are proportional to m_e/m_μ, R_π is not sensitive to them. Nevertheless, the ratio R_π is of interest for charged Higgs bosons, since the pattern and size of the Higgs couplings could be different.

The constraints [other than from $(R_\pi)_{\rm expt}$] on pseudoscalar-type interactions which apply in any model are from direct production experiments and from β decay. The bounds on charged Higgs boson masses and couplings from limits on direct production still allow the strength of the pseudoscalar-type interactions (and of the scalar-type interactions, i.e., the interactions involving $\bar{u}d$) to be comparable to the strength of the weak interaction [19]. In nuclear β decay pseudoscalar-type interactions do not contribute in the nonrelativistic limit for the nucleons. The limits on the strength of scalar-type interactions from β decay are not better than $\sim 10^{-2}G_F$ [22]. Consequently, even for equal effective coupling constants for the scalar- and pseudoscalar-type interactions, the quantities $|\omega_e \mathrm{Re}\eta_{LP}|$ and $|\omega_e \eta_{RP}\sqrt{\tilde{v}_e}|$ in Eq. (14) are allowed to be larger than unity. It follows that in models where η_{RP} is present, and where there are no further significant constraints on $\eta_{RP}\sqrt{\tilde{v}_e}$ beyond those which are unavoidable, the bounds on $\mathrm{Re}\eta_{LP}$ from $(R_\pi)_{\rm expt}$ are given by Eq. (34) [or, if $u_e/u_\mu = 1$, by Eq. (28)]. Otherwise the bounds on $\mathrm{Re}\eta_{LP}$ from $(R_\pi)_{\rm expt}$ are given by Eqs. (33a) and (33b) [or by Eqs. (26a) and (26b)].

B. Supersymmetric models with R-parity violation

In supersymmetric models where the superpotential is allowed to contain in addition to the usual Yukawa interaction renormalizable lepton-number violating couplings R parity is not conserved, and therefore contributions arise in processes with the usual particles from the exchange of single sfermions [23]. In such models there are contributions to the $\pi \to l\nu$ interactions mediated by sleptons and squarks. In the supersymmetric standard model the lepton-number-violating part W_1 of the superpotential is given by [23]

$$W_1 = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \mathrm{H.c.}\,, \tag{35}$$

where L, Q, E^c, and D^c are the chiral superfields containing, respectively, the left-handed lepton and quark doublets, the charge conjugate of the right-handed charged lepton, and the charge conjugate of the charge $-1/3$ quark; the i, j, k are generation indices. Inspection shows that the couplings (35) generate A_{LL}-type [see Eq. (9)] $\pi \to e\nu_e$ interactions mediated by slepton exchange, proportional to $\lambda_{121}\lambda'^*_{211}$ and $\lambda_{131}\lambda'^*_{311}$. The corresponding η_{LP} is given by

$$\eta_{LP} = -\left(\frac{\lambda_{121}\lambda'^*_{211}}{4m^2_{\tilde{\mu}_L}} + \frac{\lambda_{131}\lambda'^*_{311}}{4m^2_{\tilde{\tau}_L}}\right)(g^2 U_{ud}/8m_W^2)^{-1}\,, \tag{36}$$

where $m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_L}$ are the masses of the $\tilde{\mu}_L$ and $\tilde{\tau}_L$. Analogous terms (proportional to $\lambda_{212}\lambda'^*_{111}$ and $\lambda_{232}\lambda'^*_{311}$) contribute to $\pi \to \mu\nu_\mu$.

The empirical bounds on the coupling constants λ_{ijk} and λ'_{ijk} have been analyzed in Refs. [24,25]. For comparable sfermion masses the results imply limits of the order of 10^{-2} for the terms in η_{LP} [Eq. (36)]. Thus these bounds allow $|\omega_e \mathrm{Re}\eta_{LP}|$ to be larger than unity. The bounds on $\mathrm{Re}\eta_{LP}$ from $(R_\pi)_{\rm expt}$, which provide new constraints on the R-parity-violating supersymmetric standard model, are given by Eqs. (33a) and (33b) [or, if $u_e/u_\mu = 1$, by (26a) and (26b)]. Since the model does not contain a right-handed neutrino, the only candidates for the y^2 term in Eq. (27) could be interactions contributing to $\pi \to e\nu_x$ with $x \neq e$. Such interactions arise if λ'_{111} is also nonzero. These are also of the A_{LL} type, with $\nu_e^{(L)}$ replaced by $\nu_x^{(L)}$, and with $\eta_{LP} = \eta_{LP}^{(x)}(x = \mu, \tau)$, where

$$\eta_{LP}^{(\mu)} = (-\lambda_{121}\lambda'^*_{111}/4m^2_{\tilde{e}_L})(g^2 U_{ud}/8m_W^2)^{-1} \tag{37}$$

and

$$\eta_{LP}^{(\tau)} = (-\lambda_{131}\lambda'^*_{111}/4m^2_{\tilde{e}_L})(g^2 U_{ud}/8m_W^2)^{-1}\,. \tag{38}$$

In the presence of these interactions y^2 is given by

$$y^2 = \omega_e^2(|\eta_{LP}^{(\mu)}|^2 + |\eta_{LP}^{(\tau)}|^2)\,. \tag{39}$$

The $\eta_{LP}^{(\mu)}$ term cannot affect the bounds on $\mathrm{Re}\eta_{LP}$, since $\lambda_{121}\lambda'^*_{111}$ is severely constrained by experimental results on $\mu^- \to e^-$ conversion in nuclei (see Ref. [24]). Thus if only the first term in (36) is nonvanishing, the bounds from $(R_\pi)_{\rm expt}$ on $\mathrm{Re}\eta_{LP}$ will remain those in Eqs. (33a) and (33b) [or (26a) and (26b)]. However, for $\mathrm{Re}\eta_{LP}$ with the second term in (36) the bounds in Eq. (34) [or in Eq. (28)] apply, since $\eta_{LP}^{(\tau)}$ is not constrained significantly.

We note yet that for $\lambda'_{111} \neq 0$ the decay $\pi \to e\nu_e$ receives also a leptoquark contribution, proportional to $|\lambda'_{111}|^2$, mediated by $\tilde{d}_R$. The presence of this interaction was noted in Ref. [26], and its contribution to $\pi \to e\nu_e$

was considered in Ref. [24]. The $\tilde{d}_R$ here is a special case of an S_1 leptoquark (to be discussed further on), coupled only to the left-handed quarks. The corresponding $\pi \to e\nu_e$ interaction is of the a'_{LL} type, with

$$\eta'_{LL} = (|\lambda'_{111}|^2/8m^2_{\tilde{d}_R})(g^2U_{ud}/8m^2_W)^{-1} . \tag{40}$$

The constraints from $(R_\pi)_{\rm expt}$ apply therefore to $\eta'_{LL} + \omega_e {\rm Re}\eta_{LP}$, rather than to ${\rm Re}\eta_{LP}$. For large $\omega_e {\rm Re}\eta_{LP}$ the η'_{LL} term can be neglected, since the experimental result on parity violation in cesium [27] requires $|\eta'_{LL}| < 1.6 \times 10^{-2}$ (95% C.L.) [28]. Analogous contributions to $\pi \to e\nu_e$ are present from $\tilde{s}_R$ and $\tilde{b}_R$ exchange, proportional to $|\lambda'_{112}|^2$ and $|\lambda'_{113}|^2$, respectively, but $\lambda'_{112} \neq 0$ or $\lambda'_{113} \neq 0$ does not induce with the λ_{131} coupling $\pi \to e\nu_\tau$ decay.

C. Leptoquarks

Leptoquarks are bosons which couple to quark-lepton pairs [29]. Currently there is special interest in leptoquarks which couple to the first fermion family, since for these sensitive direct searches became possible at the electron-positron collider HERA at DESY [26]. In this connection the leptoquarks of interest are those which are not severely constrained by indirect bounds on the masses and couplings. The requirements usually made for this (see Refs. [31,32]) are that the leptoquarks should not couple to diquarks (to avoid fast proton decay), that their couplings to fermions should be as diagonal as possible, i.e., that they should couple only to a single leptonic generation and to a single quark generation, and fermion mixing should be absent for the right-handed quarks and the charged leptons (to circumvent to the extent possible constraints from flavor-changing neutral current processes), and finally, that they should be chiral, i.e., that they should couple either to left- or right-handed quarks, but not to both (to avoid the presence of pseudoscalar-type couplings contributing to $\pi \to e\nu_e$). Here we shall adopt the first two assumptions, but consider also nonchiral leptoquarks.

Assuming that the leptoquark-fermion couplings are dimensionless, the spin of the leptoquarks can only be zero or one. From the possible spin-zero leptoquark states [characterized by definite $SU(2)_L\times U(1)$ quantum numbers and a definite fermion number] the ones which contribute to $\pi \to e\nu_e$ are (in the rotation of Ref. [31]) the Q ($\equiv$ electric charge) $= \frac{1}{3}$ states S_1 and $(S_3)_0$ and the $Q = \frac{2}{3}$ states $(R_2)_-$ and $(\tilde{R}_2)_+$, where the second subscript $(0, \pm)$ represents the value of $T_z (T_z = 0, \pm\frac{1}{2})$ [33].

The couplings of these leptoquarks to the first standard model fermion family extended by a right-handed neutrino are given by [33]

$$\mathcal{L}_{S_1} = \left[\frac{1}{2}g_{1L}(\bar{u}^{c'}(1-\gamma_5)e - \bar{d}^{c'}(1-\gamma_5)\nu_e^{(L)}) + \frac{1}{2}g_{1R}\bar{u}^c(1+\gamma_5)e + \frac{1}{2}g_{1R}^{(\nu)}\bar{d}^c(1+\gamma_5)\nu_e^{(R)}\right] S_1 + \text{H.c.}, \tag{41}$$

$$\mathcal{L}_{(S_3)_0} = -\frac{1}{2}g_{3L}[\bar{u}^{c'}(1-\gamma_5)e + \bar{d}^{c'}(1-\gamma_5)\nu_e^{(L)}](S_3)_0 + \text{H.c.}, \tag{42}$$

$$\mathcal{L}_{(R_2)_-} = \left[\frac{1}{2}h_{2L}\bar{u}(1-\gamma_5)\nu_e^{(L)} - \frac{1}{2}h_{2R}\bar{d}'(1+\gamma_5)e\right](R_2)_- + \text{H.c.}, \tag{43}$$

$$\mathcal{L}_{(\tilde{R}_2)_+} = -\left[\frac{1}{2}\tilde{h}_{2L}\bar{d}(1-\gamma_5)e + \frac{1}{2}\tilde{h}_{2R}\bar{u}'(1+\gamma_5)\nu_e^{(R)}\right](\tilde{R}_2)_+ + \text{H.c.} \tag{44}$$

In Eqs. (41)–(44) the primes on the left-handed quark fields indicate that they are weak eigenstates rather than mass eigenstates; the neutrino states $\nu_e^{(L)}$ and $\nu_e^{(R)}$ are the same as in Eq. (9).

From the above leptoquark states the chiral leptoquarks involving $\nu_e^{(L)}$ are the $(S_3)_0$ and, if $g_{1R} = g_{1R}^{(\nu)} = 0$, the S_1 (which we shall denote as S_{1L} in such a case). The $\pi \to e\nu_e$ interactions generated by the exchange of S_{1L} and $(S_3)_0$ have a pure V-A structure, with

$$\eta'_{LL} = \eta_{LL}^{(1)'} \equiv (|g_{1L}|^2/8m_1^2)(g^2U_{ud}/8m_W^2)^{-1} \tag{45}$$

and

$$\eta'_{LL} = \eta_{LL}^{(3)'} \equiv (-|g_{3L}|^2/8m_3^2)(g^2U_{ud}/8m_W^2)^{-1}, \tag{46}$$

respectively, where m_1 and m_3 are the masses of the S_{1L} and the $(S_3)_0$. The contributions of S_{1L} and $(S_3)_0$ to $\pi \to e\nu_e$ have been considered in Refs. [32,34] assuming that $\eta_{LL}^{(1)'}$ and $(-\eta_{LL}^{(3)'})$ are small relative to unity. In these cases this assumption is justified, since the experimental result on atomic parity violation in cesium requires $\eta_{LL}^{(1)'} \ll 1$, $(-\eta_{LL}^{(3)'}) \ll 1$ [32,34,35]. Taking into account these limits and allowing $u_e/u_\mu \neq 1$, we obtain from $(R_\pi)_{\rm expt}$ the bounds [36]

$$\eta_{LL}^{(1)'} < 2 \times 10^{-2} \ \ (95\% \text{ C.L.}), \tag{47}$$

$$-\eta_{LL}^{(3)'} < 3 \times 10^{-2} \ \ (95\% \text{ C.L.}), \tag{48}$$

or, equivalently,

$$\frac{m_1}{|g_{1L}|} > 0.9 \text{ TeV} \ \ (95\% \text{ C.L.}), \tag{49}$$

$$\frac{m_3}{|g_{3L}|} > 0.8 \text{ TeV} \ \ (95\% \text{ C.L.}). \tag{50}$$

The other major constraints on $\eta_{LL}^{(1)'}$ and $\eta_{LL}^{(3)'}$ are from atomic parity violation, a combination of bounds from K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing, and from searches for single leptoquark production in e-p collisions. The bounds from atomic parity violation are somewhat stronger than the bounds (47) and (48) from $(R_\pi)_{\rm expt}$($\eta_{LL}^{(1)'} < 1.6 \times 10^{-2}$, $-\eta_{LL}^{(3)'} < 5 \times 10^{-3}$; see Ref. [32]). The bounds provided by the combined bound from K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing [37,32] are stronger than the bounds (47) and (48) from $(R_\pi)_{\rm expt}$ for $m_1 \gtrsim 290$ GeV and $m_3 \gtrsim 86$ GeV, respectively [38]. In the range $m_1, m_3 = 120$–200 GeV the most stringent bounds on $\eta_{LL}^{(1)'}$ and $\eta_{LL}^{(3)'}$ are from the results of searches for single leptoquark production at HERA. These yield $\eta_{LL}^{(1)} < 2 \times 10^{-3}$ to 4×10^{-3}, $(-\eta_{LL}^{(3)'}) < 1 \times 10^{-3}$ to 3×10^{-3} (95% C.L.), with upper limits increasing with increasing leptoquark mass [39].

Let us allow now all the coupling constants of the S_1 to be nonzero [40]. Neglecting the Imη_{LP} term, which cannot be large enough to play a role in our considerations [41], Q_e/Q_μ is then given by

$$Q_e/Q_\mu = u_e/u_\mu[(1+\eta'_{LL}+\omega_e{\rm Re}\eta_{LP})^2 + \tilde{v}_e|\eta_{RR}-\omega_e\eta_{RP}|^2]\,, \tag{51}$$

where $\eta'_{LL} = \eta_{LL}^{(1)'}$ [see Eq. (45)],

$$\eta_{RR} = -\frac{g_{1R}^* g_{1R}^{(\nu)}}{8m_1^2}(g^2U_{ud}/8m_W^2)^{-1}\,, \tag{52}$$

$$\eta_{LP} = -\frac{g_{1R}^* g_{1L}}{8m_1^2}(g^2U_{ud}/8m_W^2)^{-1}\,, \tag{53}$$

$$\eta_{RP} = \frac{g_{1L}^* g_{1R}^{(\nu)}}{8m_1^2}(g^2U_{ud}/8m_W^2)^{-1}\,. \tag{54}$$

We shall consider the case when $|g_{1L}| = |g_{1R}|$. Then

$$|{\rm Re}\eta_{LP}| = \eta_{LL}^{(1)'} = \frac{|g_{1L}|^2}{8m_1^2}(g^2U_{ud}/8m_W^2)^{-1}\,, \tag{55}$$

$$|\eta_{RP}| = |\eta_{RR}| = \frac{|g_{1L}g_{1R}^{(\nu)}|}{8m_1^2}(g^2U_{ud}/8m_W^2)^{-1}\,. \tag{56}$$

It follows that in (51) the $\eta_{LL}^{(1)'}$ and η_{RR} contributions can be neglected.

The most important constraints on Reη_{LP} and $\eta_{RP}\sqrt{\tilde{v}_e}$ are from β decay and the production experiments at HERA, and for Reη_{LP} also from K^0-$\bar{K}^0$, D^0-$\bar{D}^0$ mixing. β decay and the results on leptoquark production allow still $|\omega_e{\rm Re}\eta_{LP}|$ and $|\omega_e\eta_{RP}\sqrt{\tilde{v}_e}|$ to be larger than unity [42]. Since $|\omega_e\eta_{RP}\sqrt{\tilde{v}_e}|$ can be sufficiently large, the bounds on Reη_{LP} from $(R_\pi)_{\rm expt}$ are given by Eq. (34) [Eq. (28) if $u_e/u_\mu = 1$]. The limit $|{\rm Re}\eta_{LP}| < 6 \times 10^{-4}$ from $(R_\pi)_{\rm expt}$ is equivalent to

$$m_1/\sqrt{|g_{1L}g_{1R}|} = m_1/|g_{1L}| > 5 \text{ TeV}\,. \tag{57}$$

This is to be compared with the bound $m_1/|g_{1L}| > 100$ TeV, which would follow from the bounds (24). For $m_1 > 9.4$ TeV the limit $|m_1/g_{1L}| > \sqrt{2.8m_1}$ TeV$^{1/2}$ [32] from neutral meson mixing is stronger than the limit from $(R_\pi)_{\rm expt}$ [43].

The nonchiral $(\tilde{R}_2)_+$ leptoquark gives only an η_{RP}-type contribution to R_π, with

$$\eta_{RP} = \eta_{RP}^{(2)} \equiv (-\tilde{h}_{2R}\tilde{h}_{2L}^*/8\tilde{m}_2^2)(g^2U_{ud}/8m_W^2)^{-1}\,. \tag{58}$$

$(R_\pi)_{\rm expt}$ sets on this the stringent bound

$$|\eta_{RP}\sqrt{\tilde{v}_e}| < 6 \times 10^{-5} \text{ (95\% C.L.)} \tag{59}$$

($|\eta_{RP}\sqrt{\tilde{v}_e}| < 2 \times 10^{-5}$ if $u_e/u_\mu = 1$). From β decay one has $|\eta_{RP}^{(2)}\sqrt{\tilde{v}_e}| \lesssim 10^{-2}$ [42], and for $|\tilde{h}_{2L}| = |\tilde{h}_{2R}|$ atomic parity violation requires $|\eta_{RP}^{(2)}\sqrt{\tilde{v}_e}| < 4\times10^{-3}$ (95% C.L.). For $(\tilde{R}_2)_+$ the constraint from K^0-$\bar{K}^0$ mixing is absent, and therefore the one from D^0-$\bar{D}^0$ can be evaded [37,32]. The constraints on $(\tilde{R}_2)_+$ from HERA are weaker than for the S_1 and $(S_3)_0$ [30].

The contribution of the nonchiral $(R_2)_-$ to R_π is of the η_{LP} type, with

$$\eta_{LP} = \eta_{LP}^{(2)} \equiv (-h_{2L}h_{2R}^*/8m_2^2)(g^2U_{ud}/8m_W^2)^{-1}\,. \tag{60}$$

The bounds on Re$\eta_{LP}^{(2)}$ from $(R_\pi)_{\rm expt}$ are given by Eqs. (33a) and (33b) [41]. In a model where both R_2 and $\tilde{R}_2$ are present the bounds (34) [or (28)] would apply. For $|h_{2L}| = |h_{2R}|$ the limit from atomic parity violation is $|\eta_{LP}^{(2)}| < 7 \times 10^{-3}$ (95% C.L.) [32]. The limits from β decay [42] and the production experiments at HERA [30] are also weaker than the limit (33b) from $(R_\pi)_{\rm expt}$. For $|h_{2L}| = |h_{2R}|$ the bound on $|{\rm Re}\eta_{LR}^{(2)}|$ from neutral meson mixing is $|{\rm Re}\eta_{LP}^{(2)}| < 5.8$ GeV$/m_2$, which becomes stronger than the bound (33b) for $m_2 > 9.4$ TeV.

So far we have discussed the spin-zero leptoquarks. From the possible spin-one leptoquarks those which contribute to $\pi \to e\nu_e$ are (in the notation of Ref. [31]) the $Q = \frac{2}{3}$ states U_1 and $(U_3)_0$ and the $Q = \frac{1}{3}$ states $(V_2)_-$ and $(\tilde{V}_2)_+$. The couplings of these leptoquarks to the first standard model family extended by $\nu_e^{(R)}$ are [44]

$$\mathcal{L}_{U_1} = \left\{\frac{1}{2}h_{1L}[\bar{u}'\gamma_\mu(1-\gamma_5)\nu_e^{(L)} + \bar{d}'\gamma_\mu(1-\gamma_5)e] + \frac{1}{2}h_{1R}\bar{d}\gamma_\mu(1+\gamma_5)e + \frac{1}{2}h_{1R}^{(\nu)}\bar{u}\gamma_\mu(1+\gamma_5)\nu_e^{(R)}\right\}U_1^\mu + \text{H.c.}\,, \tag{61}$$

$$\mathcal{L}_{(U_3)_0} = \frac{1}{2}h_{3L}[\bar{u}'\gamma_\mu(1-\gamma_5)\nu_e^{(L)} - \bar{d}'\gamma_\mu(1-\gamma_5)e](U_3)_0^\mu + \text{H.c.}\,, \tag{62}$$

$$\mathcal{L}_{(V_2)_-} = \left[\frac{1}{2}g_{2L}\bar{d}^c\gamma_\mu(1-\gamma_5)\nu_e^{(L)} + \frac{1}{2}g_{2R}\bar{u}^{c'}\gamma_\mu(1+\gamma_5)e\right](V_2)_-^\mu + \text{H.c.}\,, \tag{63}$$

$$\mathcal{L}_{(\tilde{V}_2)_+} = -\left[\frac{1}{2}\tilde{g}_{2L}\bar{u}^c\gamma_\mu(1-\gamma_5)e + \frac{1}{2}\tilde{g}_{2R}\bar{d}^{c'}\gamma_\mu(1+\gamma_5)\nu_e^{(R)}\right](\tilde{V}_2)_+^\mu + \text{H.c.} \tag{64}$$

The constraints for spin-one leptoquarks from sources other than $(R_\pi)_{\rm expt}$ do not differ significantly from those for the spin-zero leptoquarks [45].

The ratio Q_e/Q_μ for the U_1 leptoquark is given by Eq. (51) [46] with

$$\eta'_{LL} = (|h_{1L}|^2/4M_1^2)(g^2U_{ud}/8m_W^2)^{-1}, \qquad (65)$$

$$\eta_{RR} = (h_{1R}^* h_{1R}^{(\nu)}/4M_1^2)(g^2U_{ud}/8m_W^2)^{-1}, \qquad (66)$$

$$\eta_{LP} = -(h_{1L}h_{1R}^*/2M_1^2)(g^2U_{ud}/8m_W^2)^{-1}, \qquad (67)$$

$$\eta_{RP} = (h_{1L}^* h_{1R}^{(\nu)}/2M_1^2)(g^2U_{ud}/8m_W^2)^{-1}, \qquad (68)$$

where M_1 is the mass of U_1. For $|h_{1L}| = |h_{1R}|$ one can neglect η'_{LL} and η_{RR}, and the bounds on Reη_{LP} from $(R_\pi)_{\rm expt}$ are given by Eq. (34) [Eq. (28) if $u_e/u_\mu = 1$]. It follows that

$$M_1/\sqrt{|h_{1L}h_{1R}|} = M_1/|h_{1L}| > 10 \text{ TeV}, \qquad (69)$$

rather than $M_1/|h_{1L}| > 200$ TeV, which would be the limit from the bounds (24).

The $\pi \to e\nu_e$ interaction from $(U_3)_0$ has a V-A structure, with

$$\eta'_{LL} = -(|h_{3L}|^2/4M_3^2)(g^2U_{ud}/8m_W^2)^{-1}. \qquad (70)$$

The $(V_2)_-$ and the $(\tilde V_2)_+$ give rise, respectively, to an η_{LP} interaction and an η_{RP} interaction, with

$$\eta_{LP} = -(g_{2L}g_{2R}^*/2M_2^2)(g^2U_{ud}/8m_W^2)^{-1} \qquad (71)$$

and

$$\eta_{RP} = (\tilde g_{2L}^* \tilde g_{2R}/2\tilde M^2)(g^2U_{ud}/8m_W^2)^{-1}. \qquad (72)$$

The bounds from $(R_\pi)_{\rm expt}$ on the η'_{LL} and the Reη_{LP} [46] in Eqs. (70) and (71), and on $|\eta_{RP}\sqrt{\tilde v_e}|$ with η_{RP} from Eq. (72), are the same as the bounds for the same type of couplings generated by the $(S_3)_0$, $(R_2)_-$, and $(\tilde R_2)_+$ [Eq. (48), the limit on the Reη_{LP} in Eq. (60), and Eq. (59)].

IV. CONCLUSIONS

In this paper we have investigated the constraints on pseudoscalar-type $\bar d u \to e\nu_e$ interactions from the $\pi \to e\nu/\pi \to \mu\nu$ branching ratio R_π. We have pointed out that for some important cases the usual assumption that the contributions of these interactions to R_π are small relative to the standard model contribution is not justified. The bounds from $(R_\pi)_{\rm expt}$ [Eq. (7)] on the coupling constant Reη_{LP} [the coupling constant of the CP-invariant pseudoscalar-type interaction involving the left-handed neutrino state $\nu_e^{(L)}$; see Eqs. (9) and (16)] resulting from the exact treatment are given by Eqs. (33a) and (33b), or by Eq. (34). These bounds allow $|\text{Re}\eta_{LP}|$ to be as large as 6×10^{-4}, which is larger by a factor of ~ 400 than the upper limit on $|\text{Re}\eta_{LP}|$ one would obtain with the $|\omega_e \text{Re}\eta_{LP}| \ll 1$ assumption. The bounds (34) apply if in a given model R_π receives also a contribution from a pseudoscalar-type interaction involving the right-handed neutrino state $\nu_e^{(R)}$ [the η_{RP} contribution in Eq. (14)], or a contribution from a $\pi \to e\nu_\tau$ interaction.

The bounds (34) apply to Reη_{LP} from charged Higgs boson exchange in models where there are no significant constraints on $|\eta_{RP}\sqrt{\tilde v_e}|$ beyond those which cannot be avoided, and to Reη_{LP} generated by slepton exchange in the minimal supersymmetric model with R-parity violation if a $\pi \to e\nu_\tau$ interaction is allowed to be present. They apply also to Reη_{LP} from the exchange of some types of nonchiral leptoquarks. An implication is that the bounds on the masses of such nonchiral leptoquarks are not considerably stronger than the bounds on the masses of chiral leptoquarks even for equal strength for the left-handed and the right-handed couplings. An η_{RP} contribution can be present naturally for the S_1 and U_1 leptoquarks. With couplings of electromagnetic strength the lower bounds on the masses of the S_1 and U_1 from $(R_\pi)_{\rm expt}$ are 1.5 TeV and 3 TeV, respectively.

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Examples are the couplings involving $\bar{e}(1+\gamma_5)(\nu_e^{(L)})^c$ and $\bar{e}(1-\gamma_5)\nu_\mu^{(L)}$, where $(\nu_e^{(L)})^c = C\bar{\nu}_e^{(L)T}$, and $\nu_\mu^{(L)}$ is the neutrino state in the $W^+ \to \mu^+\nu_\mu^{(L)}$ amplitude. The former coupling is constrained by neutrinoless $\beta\beta$ decay, and the latter by muon-number-violating processes. On the other hand, there are as yet no significant constraints on couplings involving $\bar{e}(1-\gamma_5)\nu_\tau^{(L)}$. We shall encounter such couplings later on in discussing pseudoscalar-type interactions in supersymmetric models with R-parity violation.

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[38] In terms of $\eta_{LL}^{(1)'}$ and $\eta_{LL}^{(3)'}$ the bounds from K^0-$\bar{K}^0$, D^0-$\bar{D}^0$ mixing given in Ref. [32] are $\eta_{LL}^{(1)'} < (5.8\times10^{-2})$ 100 GeV$/m_1$ and $(-\eta_{LL}^{(3)'}) < (2.6\times10^{-2})$ 100 GeV$/m_3$ at 95% C.L.

[39] The limits we quote are from the results of the $H1$ Collaboration (Ahmed *et al.* [30]; I am grateful to Dr. Schleper for sending me the results before publication). Similar results have been obtained by the ZEUS Collaboration (see Derrick *et al.* [30] and Schleper [30]). The $H1$ Collabora-

tion gives limits on $|g_{1L}|$ and $|g_{3L}|$ for $m_{LQ} \sim 80$ GeV to ~ 250 GeV, assuming the absence of other decay modes then into eu and $\nu_e d$. Searches for leptoquark pair production in $\bar{p}p$ collisions have excluded scalar leptoquarks decaying into an electron + quark with branching ratio $\frac{1}{2}$ up to $m_{LQ} = 120$ GeV [D0 Collaboration, S. Abachi *et al.*, Phys. Rev. Lett. **72**, 965 (1994)].

[40] Such S_1 leptoquarks are present for example in $N = 1$ supergravity models with an E_6 gauge symmetry. Their contribution to $\pi \to e\nu_e$ have been considered in Campbell *et al.* [29], and in Kizukuri [29]. The $\pi \to e\nu_e$ amplitude in the latter reference includes the contributions from all the coupling constants, but in the rate only the Reη_{LP} term has been kept. It should be noted that in these models the $g_{1R}^{(\nu)}$ coupling, or the g_{1L} and g_{1R} couplings, have to be suppressed to avoid large Dirac neutrino masses (see Campbell *et al.* [29]). Similar S_1 leptoquarks are present in superstring inspired SU(5) unified theories with non-SU(5) symmetric couplings (Buchmüller and Wyler [29]). In the latter models there are no right-handed neutrinos, and therefore the η_{RP} term is absent.

[41] The limit on Imη_{LP} generated by S_1 exchange is $|\mathrm{Im}\eta_{LP}| \lesssim 10^{-5}$. This follows from the experimental limit on the coupling constant C_T^{eN} of the parity- and time-reversal-violating tensor-type electron-nucleon interaction [17], the calculation of C_T^{eN} in Ref. [16], and from relations between the coupling constants of the scalar-, tensor-, and pseudoscalar-type $\bar{d}u \to e\nu_e$ interactions generated by the S_1 (see Herczeg [22]). The limit on Imη_{LP} from $(R_2)_-$ exchange [see the text around Eq. (60)] is $|\mathrm{Im}\eta_{LP}| \lesssim 5 \times 10^{-6}$, obtained in the same way.

[42] For $|g_{1R}| = |g_{1L}|$ there is no constraint from atomic parity violation (see Ref. [35]). The upper limits on $|\mathrm{Re}\eta_{LP}|$ and $\eta_{RP}|\sqrt{\bar{v}_e}|$ from β decay are $\sim 10^{-2}$, and arise because the pseudoscalar coupling constants are related to the scalar and the tensor ones (see Buchmüller and Wyler [29], Langacker *et al.* [35], and Herczeg [22]). The constraints on S_1 from production experiments should be approximately the same as for S_{1L}. The S_1 coupled to the first family contributes to the $g-2$ of the electron. Although this contribution is enhanced when both g_{1L} and g_{1R} are nonzero, inspection shows that for the light quarks involved here the constraint is not significant yet [see A. Djouadi, T. Köhler, M. Spira, and J. Tutas, Z. Phys. C **46**, 679 (1990); for comparison of experiment and theory see T. Kinoshita and D. R. Yennie, in *Quantum Electrodynamics*, Advanced Series on Directions in High Energy Physics, Vol. 7, edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 1].

[43] It should be noted that for $m_1 > 35.5$ TeV the bound $|m_1/g_{1L}| > \sqrt{2.8 m_1}$ TeV$^{1/2}$ (or equivalently $|\mathrm{Re}\eta_{LP}| = \eta_{LL}^{(1)'} < 5.8$ GeV$/m_1$) can be saturated only for $|g_{1L}| > \sqrt{4\pi}$, where perturbation theory would no longer hold (see Ref. [32]).

[44] The most general $\mathrm{SU}(2)_L \times \mathrm{U}(1) \times \mathrm{SU}(3)_c$ invariant, lepton- and baryon-number-conserving Lagrangian for the couplings of spin-one leptoquarks to a standard model family has been given in Ref. [31]. The Lagrangian contains nine leptoquark states, characterized by definite standard model quantum numbers and a definite fermion number. If a right-handed neutrino is included, there is an additional state $[\tilde{U}_1^{(\nu)}$, with the coupling $\frac{1}{2}\tilde{h}_{1R}^{(\nu)}\bar{d}\gamma^\mu(1+\gamma_5)\nu^{(R)}\tilde{U}_{1\mu}^{(\nu)}]$, and the U_1 and the components of the $\tilde{V}_2$ have an additional coupling.

[45] The constraints on the various spin-one leptoquarks coupled to standard model families have been discussed in Ref. [34]. The limits for the U_1, $(U_3)_0$, $(V_2)_-$, and $(\tilde{V}_2)_+$ from HERA are similar to those for $(R_2)_-$, $(\tilde{R}_2)_+$, S_1, and $(S_3)_0$, respectively [30].

[46] The limit on Imη_{LP} generated by U_1 and $(V_2)_-$ [see the text around Eq. (71)] exchange is $|\mathrm{Im}\eta_{LP}| \lesssim 10^{-4}$. This follows from the experimental limit on the coupling constant C_S^{eN} of the parity- and time-reversal-violating scalar-type electron-nucleon interaction [17] and the calculation of C_S^{eN} in Ref. [16]. The coupling constant of the parity- and time-reversal-violating tensor-type electron-nucleon interaction vanishes due to a relation between the coupling constants of the scalar- and pseudoscalar-type $\bar{d}u \to e\nu_e$ interactions (see Herczeg [22]).