

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

$$\langle\psi|\sigma_i^z\sigma_{i+1}^z|\psi\rangle = \dots$$

$$|x\rangle = |q_{n-1} \dots q_1 q_0\rangle \quad (\equiv |\psi\rangle)$$

$$x = \sum_{j=0}^{n-1} q_j 2^j \quad \begin{matrix} \sigma_i^z & \sigma_{i+1}^z \\ q_i & q_{i+1} \end{matrix}$$

$$\Rightarrow \langle\psi|\sigma_i^z\sigma_{i+1}^z|\psi\rangle$$

$$\Rightarrow \langle\psi|\sigma_i^z\sigma_{i+1}^z|\psi\rangle = |\alpha_0|^2 + |\alpha_3|^2 - |\alpha_1|^2 - |\alpha_2|^2$$

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

$$\langle 00|\sigma_i^z\sigma_{i+1}^z|00\rangle = \langle 0|\sigma_0^z|0\rangle \times \langle 0|\sigma_1^z|0\rangle$$

$$\langle\psi|\sigma_0^z\sigma_1^z|\psi\rangle = |\alpha_0|^2 + |\alpha_3|^2 - |\alpha_1|^2 - |\alpha_2|^2$$