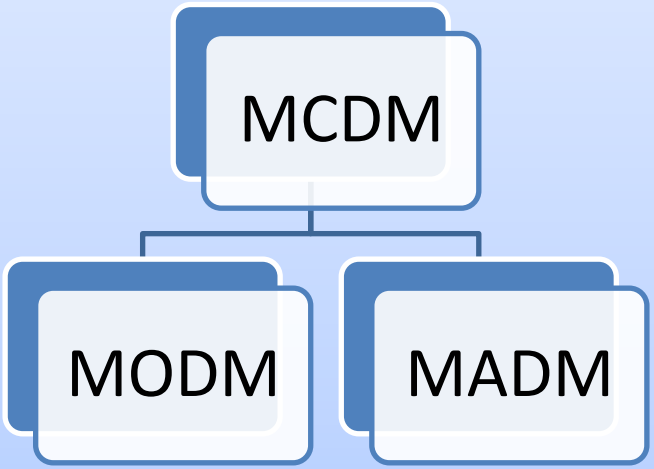
Multi Criteria Decision Making (MCDM)

Lê Ngọc Quỳnh Lam, PhD.

MCDM

- MCDM deals with decision problems under the presence of a number of decision criteria



  - MODM: Multi-Objective Decision Making
  - MADM: Multi-Attribute Decision Making

Alternatives

- Represent the different **choices of action** available to the decision maker.
- Usually, the set of alternatives is assumed to be finite
- They are supposed to be **screened, prioritized** and eventually **ranked.**

Multiple attributes

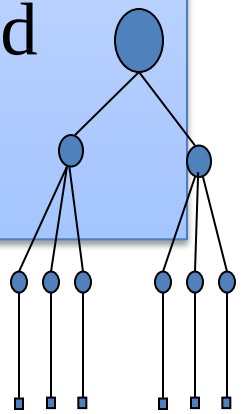
- Each MADM problem is associated with multiple attributes.
- Attributes are also referred to as *"goals" or "decision criteria"*.
- Attributes represent the different dimensions from which the alternatives can be viewed

Examples

Small Group:	• Take the next 5 minutes to discuss in your group about attributes of a car
Class	• Discuss findings

Multiple attributes

- In cases in which the number of attributes is large (e.g., more than a few dozens), attributes may be arranged in a **hierarchical manner**.
  - Some attributes may be **major** attributes.
  - Each major attribute may be associated with several **sub-attributes**.
  - Similarly, each sub-attribute may be associated with several **sub-sub-attributes** and so on.



Examples

Small Group:	• Now, take the next 5 minutes to arrange all findings from previous activity in a hierarchy manner
Class	• Discuss new findings

Conflict among attributes

- Since different attributes represent different dimensions of the alternatives, they may conflict with each other.

  Eg. Cost vs. benefit

Problems in doing trade-off

- Criteria have different units
- Some are quantitative, some are qualitative
- For some, **smaller is better** (resource use), for others, **bigger is better** (quality issues)

How to picture diverse criteria together so we can decide between alternatives?

Examples

- Car buying
- Job selection
- Human resource planning
- Computer buying

Classification of MCDM Methods

- Based on data:
  - Deterministic
  - Stochastic
  - Fuzzy
- Based on number of decision makers:
  - Single
  - Group

single decision maker deterministic MADM methods

Single decision maker deterministic MADM methods

- WSM: Weighted Sum Model
- AHP: Analytical Hierarchy Process

Weighted Sum Model (WSM)

- **Step 1**: Define all factors or attributes
- **Step 2**: Assign weight to each factor
- **Step 3**: List all non-dominated alternatives
- **Step 4**: Evaluate each alternative based on each factor using scale of (0, 1); (1, 10); or (1, 100)
- **Step 5**: Calculate weighted sum for each alternative
- **Step 6**: Compare and select the best alternative

Example

A just graduated student consider for a job

- **Step 1: Three important factors:**
  - Salary
  - Promotion
  - Company location

Ex.

- Step 2: Assign weight
  - Salary 0.5
  - Promotion 0.3
  - Company location 0.2

Ex.

- Step 3: Alternatives
  - Company A, B and C
- Step 4: Evaluate

Factor	A	B	C
Salary	0.8	0.4	0.7
Promotion	0.3	0.9	0.4
Location	0.6	0.6	0.2

Ex.

- Step 5: Calculate weight sum for each alternative

Factor	Weight	A	B	C
Salary	0.5	0.8	0.4	0.7
Promotion	0.3	0.3	0.9	0.4
Location	0.2	0.6	0.6	0.2
Weighted sum		**0.61**	**0.59**	**0.56**

EX.

- Step 6: Decision making

  Select company A

Activity

Small group

- Take the next 5 minutes to discuss all advantages and disadvantages of this method

Class

- Class discussion

The Analytic Hierarchy Process (AHP)

- Founded by **Saaty** in 1980.
- It is a popular and widely used method for MCDM.
- Allows the use of qualitative, as well as quantitative criteria in evaluation.
- Wide range of applications exists:
  - Selecting a car for purchasing
  - Deciding upon a place to visit for vacation
  - Deciding upon an MBA program after graduation.

AHP-General Idea

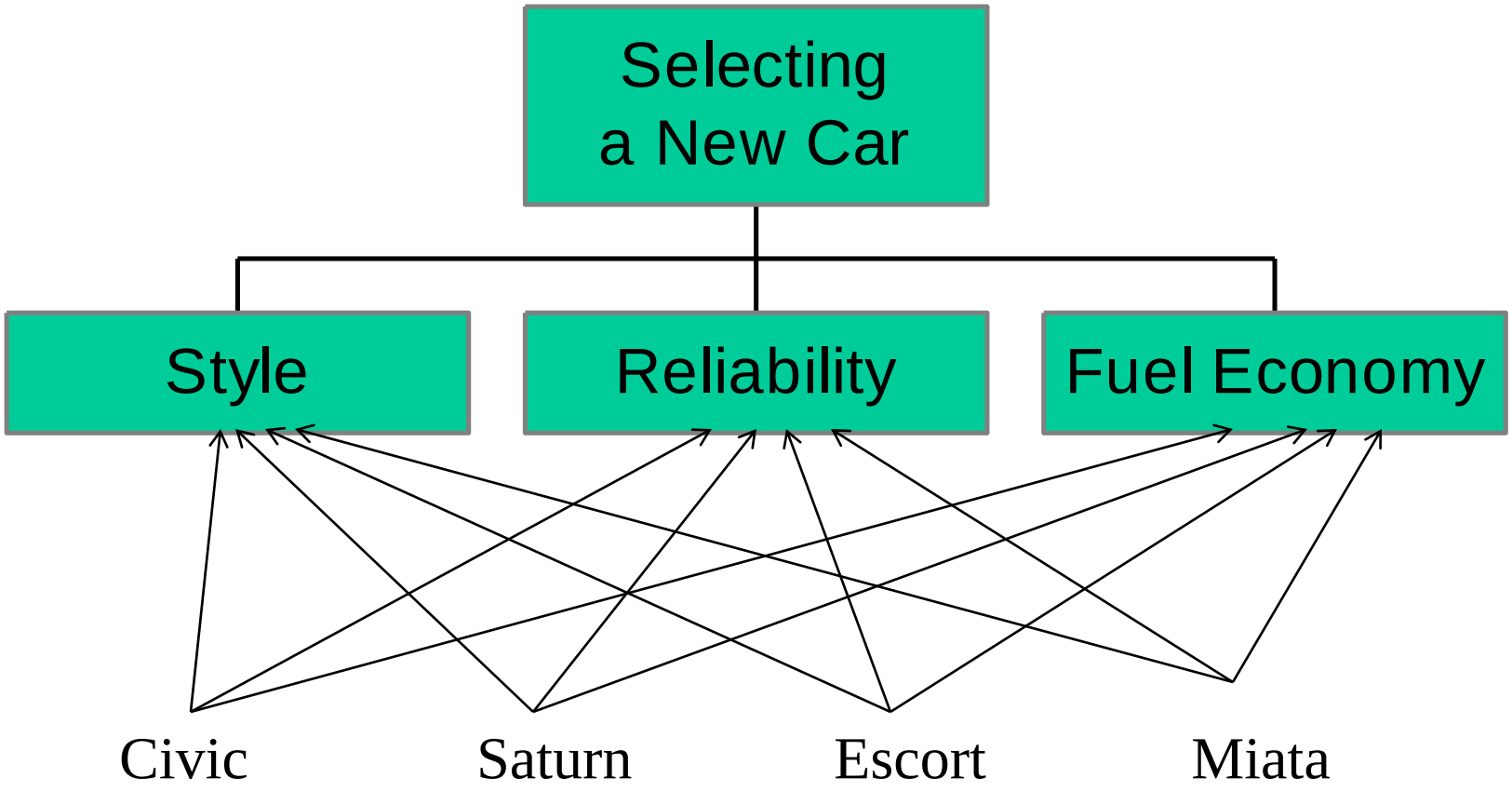
- Develop an hierarchy of decision criteria and define the alternative courses of actions.

- AHP algorithm is basically composed of two steps:
  1. Determine the relative weights of the decision criteria
  2. Determine the relative rankings (priorities) of alternatives

Example: Car Selection

- Objective
  - Selecting a car
- Criteria
  - Style, Reliability, Fuel-economy
- Alternatives
  - Civic Coupe, Saturn Coupe, Ford Escort, Mazda Miata

Hierarchy tree



Alternative courses of action

Ranking of Criteria and Alternatives

- **Pairwise comparisons** are made with the grades ranging from 1-9.
- A basic, but very reasonable assumption for comparing alternatives:

  If attribute A is absolutely more important than attribute B and is rated at 9, then B must be absolutely less important than A and is graded as 1/9.
- These pairwise comparisons are carried out for all factors to be considered, usually not more than 7, and the matrix is completed.

1 -9 Scale

Intensity of Importance	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2, 4, 6, 8	For compromises between the above
Reciprocals of above	In comparing elements i and j - if i is 3 compared to j - then j is 1/3 compared to i
Rationals	Force consistency Measured values available

Ranking of criteria

	Style	Reliability	Fuel Economy
Style	**1**	**1/2**	**3**
Reliability	**2**	**1**	**4**
Fuel Economy	**1/3**	**1/4**	**1**

Ranking of priorities

- Consider [$Ax = \lambda_{max}X$] where
  - A is the comparison matrix of size n×n, for n criteria, also called the priority matrix.
  - x is the Eigenvector of size n×1, also called the **priority vector**.
  - λ_{max} is the Eigenvalue, $\lambda_{max} \in \Re > n$.
- To find the ranking of priorities, namely the Eigen Vector X:

  1) Normalize the column entries by **dividing each entry by the sum** of the column.

  2) Take the overall row **averages.**

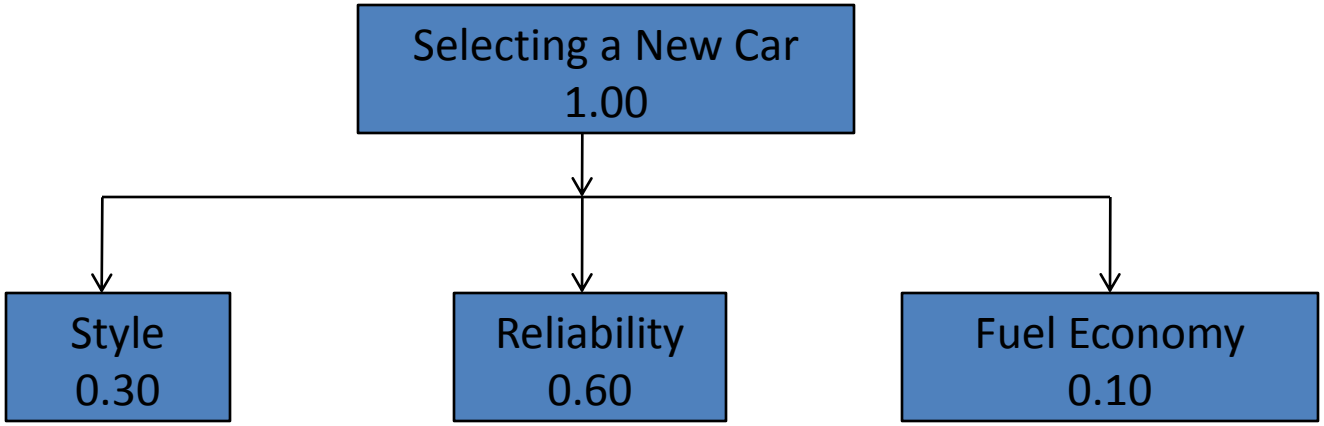
$$A = \begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.33 & 0.25 & 1.0 \end{bmatrix} \xrightarrow{\text{Normalized Column Sums}} \begin{bmatrix} 0.30 & 0.29 & 0.38 \\ 0.60 & 0.57 & 0.50 \\ 0.10 & 0.14 & 0.13 \end{bmatrix} \xrightarrow{\text{Row averages}} X = \begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}$$

Priority vector

Column sums 3.33 1.75 8.00 1.00 1.00 1.00

Criteria weights

- Style .30
- Reliability .60
- Fuel Economy .10



Checking for Consistency

- The next stage is to calculate a Consistency Ratio (CR) to measure how consistent the judgments have been relative to large samples of purely random judgments.
- AHP evaluations are based on the assumption that the decision maker is rational, i.e., if A is preferred to B and B is preferred to C, then A is preferred to C.
- If the CR is greater than 0.1 the judgments are untrustworthy because they are too close for comfort to randomness and the exercise is valueless or must be repeated.

Calculation of Consistency Ratio

- The next stage is to calculate λ_{max} so as to lead to the Consistency Index and the Consistency Ratio.
- Consider [$Ax = \lambda_{max}\, x$] where x is the Eigenvector.

$$\underset{A}{\begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.333 & 0.25 & 1.0 \end{bmatrix}} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}} = \underset{Ax}{\begin{bmatrix} 0.90 \\ 1.60 \\ 0.35 \end{bmatrix}} = \lambda_{max} \underset{x}{\begin{bmatrix} 0.30 \\ 0.60 \\ 0.10 \end{bmatrix}}$$

λmax=average{0.90/0.30, 1.60/0.6, 0.35/0.10}=3.06

Consistency index , CI is found by

CI=(λmax-n)/(n-1)=(3.06-3)/(3-1)= 0.03

Consistency Ratio

- The final step is to **calculate the Consistency Ratio, CR** by using the table below, derived from Saaty's book.
- The upper row is the order of the random matrix, and the lower row is the corresponding index of consistency for random judgments (Random index).

1	2	3	4	5	6	7
0.00	0.00	0.58	0.90	1.12	1.24	1.32

8	9	10	11	12	13	14	15
1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Consistency Ratio

An inconsistency of **10%** or less implies that the adjustment is small as compared to the actual values of the eigenvector entries.

A CR as high as, say, 90% would mean that the pairwise judgments are just about random and are completely **untrustworthy**! In this case, comparisons should be repeated.

In the above example: CR=CI/0.58=0.03/0.58=0.05

0.05 < 0.1, so the evaluations are consistent!

Ranking alternatives

Style	Civic	Saturn	Escort	Miata
Civic	1	1/4	4	1/6
Saturn	4	1	4	1/4
Escort	1/4	1/4	1	1/5
Miata	6	4	5	1

Priority vector

$$\begin{bmatrix} 0.13 \\ 0.24 \\ 0.07 \\ 0.56 \end{bmatrix}$$

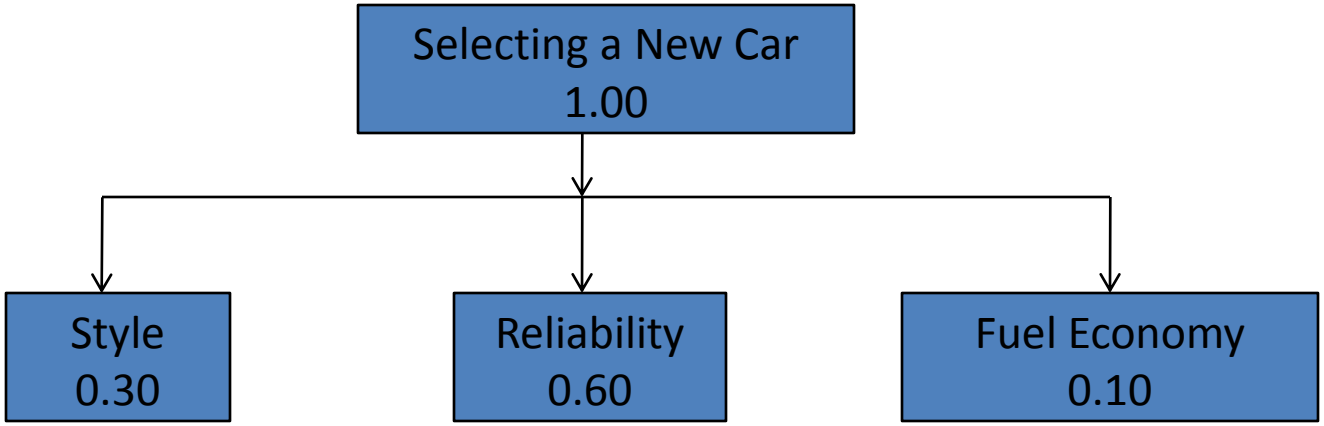
Reliability	Civic	Saturn	Escort	Miata
Civic	1	2	5	1
Saturn	1/2	1	3	2
Escort	1/5	1/3	1	1/4
Miata	1	1/2	4	1

$$\begin{bmatrix} 0.38 \\ 0.29 \\ 0.07 \\ 0.26 \end{bmatrix}$$

Ranking alternatives

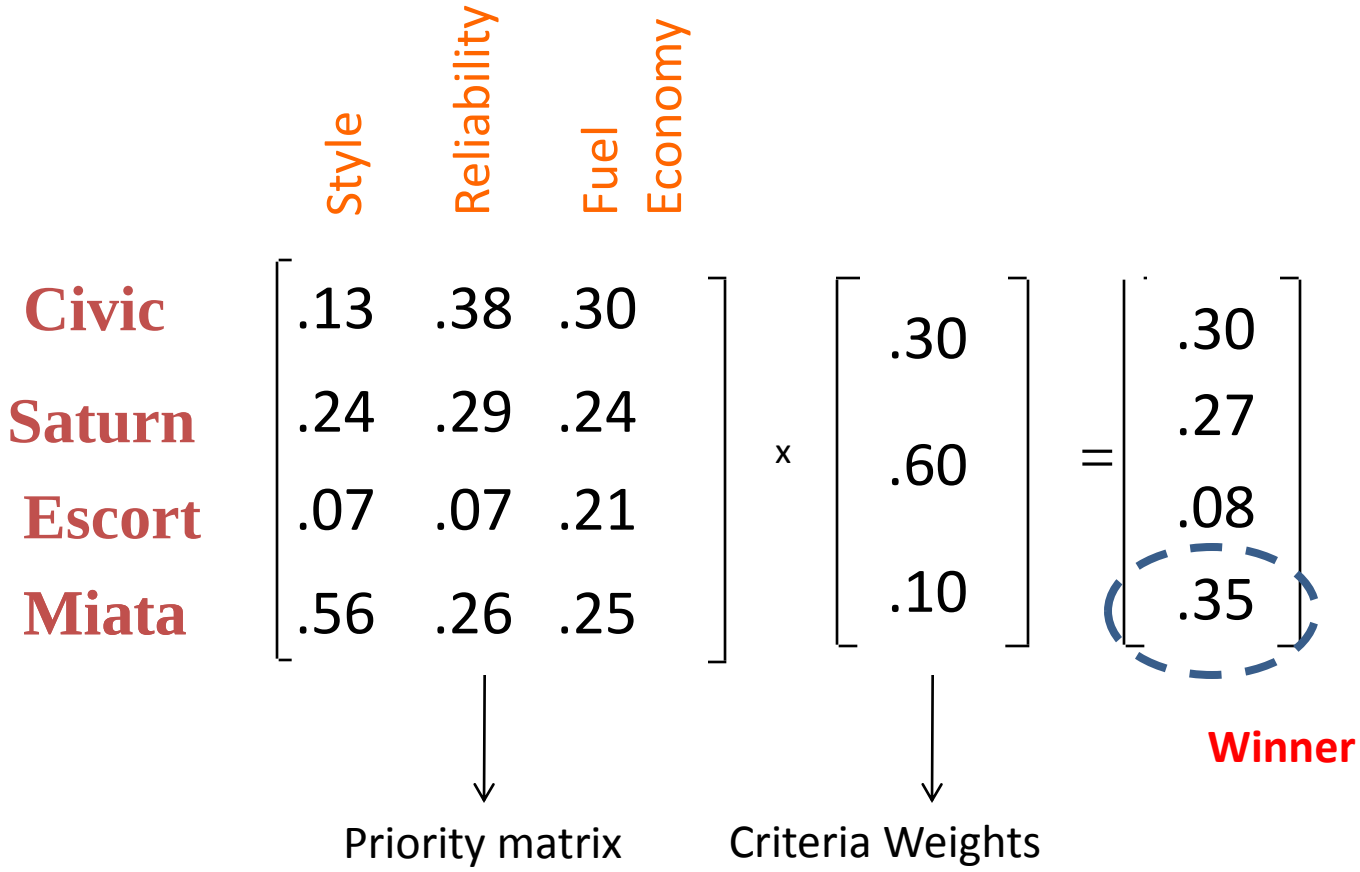
		Miles/gallon	Normalized
Fuel Economy	Civic	34	.30
	Saturn	27	.24
	Escort	24	.21
	Miata	28	.25
		113	1.0

! Since fuel economy is a **quantitative** measure, fuel consumption ratios can be used to determine the relative ranking of alternatives; however this is not obligatory. Pairwise comparisons may still be used in some cases.



Style		Reliability		Fuel Economy	
Civic	0.13	Civic	0.38	Civic	0.30
Saturn	0.24	Saturn	0.29	Saturn	0.24
Escort	0.07	Escort	0.07	Escort	0.21
Miata	0.56	Miata	0.26	Miata	0.25

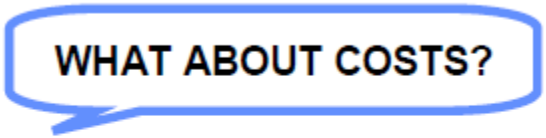
Ranking of alternatives



The Miata is the highest ranked car

In summary AHP provides a logical framework to determine benefits of each alternatives

1. Miata	0.35
2. Civic	0.30
3. Saturn	0.27
4. Escort	0.08



SKEPTIC-GATOR

ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, COSTS SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED

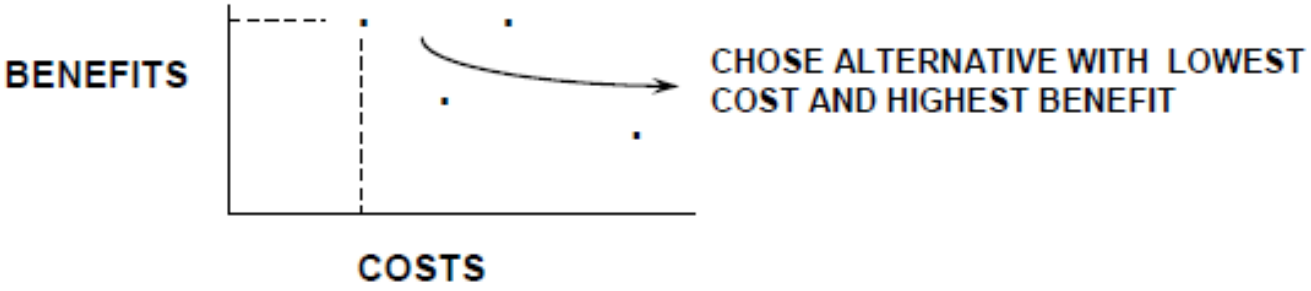
OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON'T CARE ABOUT ITS BENEFITS

DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES

WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:

1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE



2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

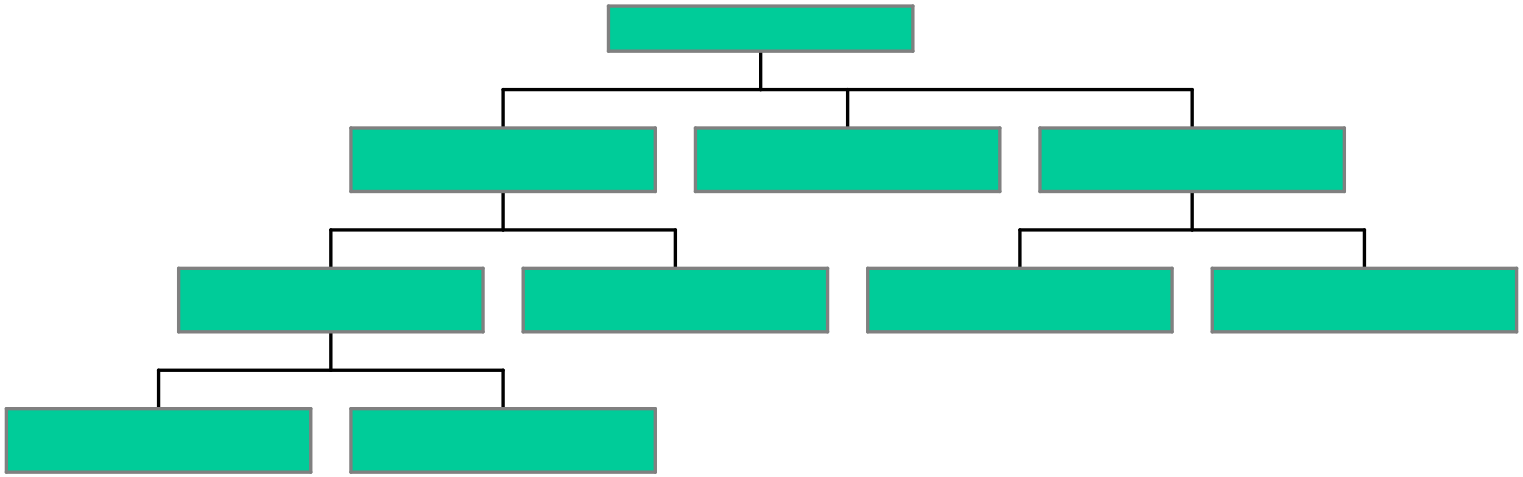
IN OUR EXAMPLE...

No.	Brand	Price (cost) $	Normalized cost	Benefit	B/C
1	Miata	18,000	0.3333	0.35	1.0501
2	Civic	12,000	0.2222	0.30	1.3501
3	Saturn	15,000	0.2778	0.27	0.9719
4	Escort	9,000	0.1667	0.08	0.4799
Total		54,000	1		

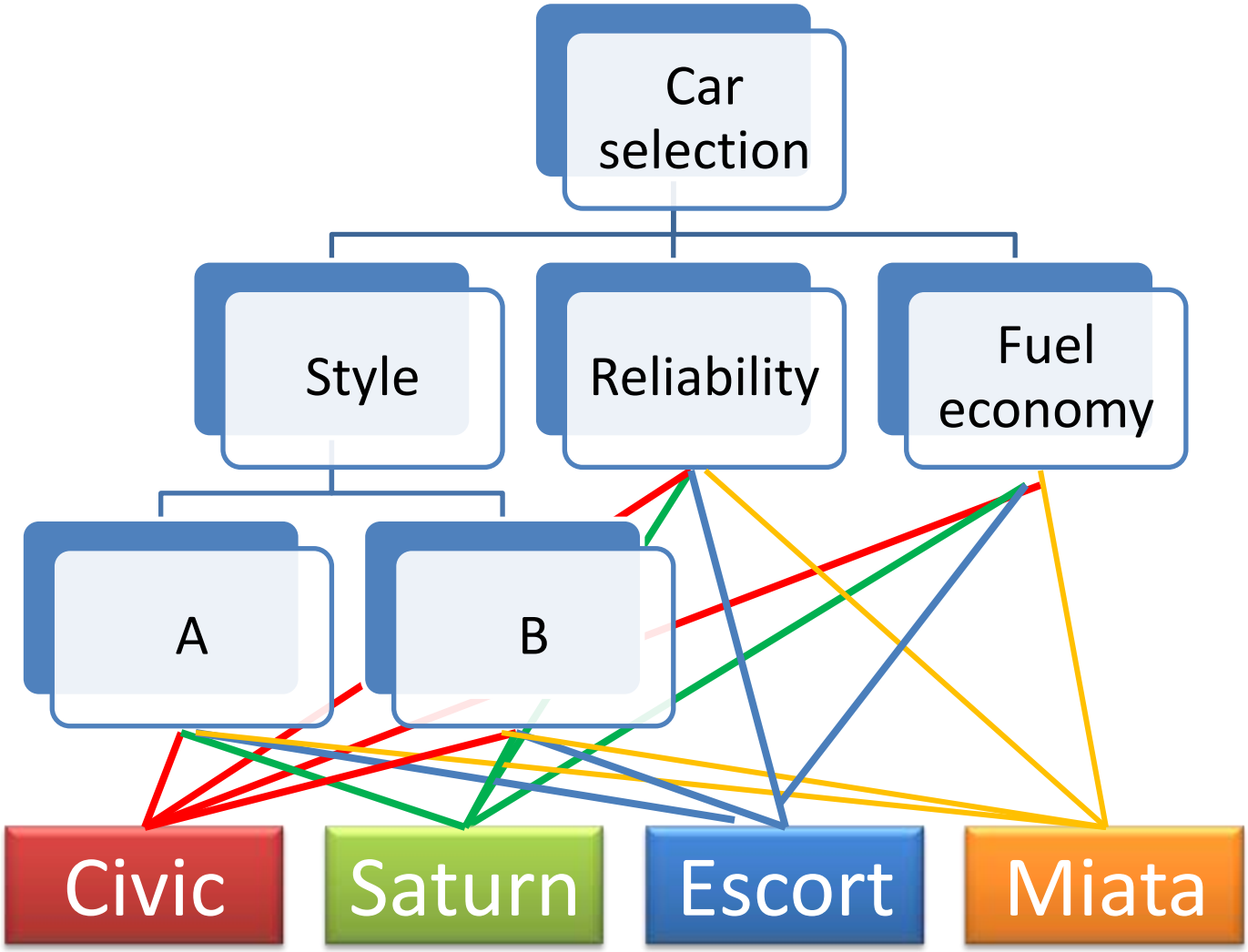
The winner is Civic

Complex decisions

•Many levels of criteria and sub-criteria exists for complex problems.



VÍ DỤ


Car selection
Style
Reliability
Fuel economy
A
B
Civic
Saturn
Escort
Miata

Priority vector for A and B

Style	A	B
A	1	5
B		1

PRIORITY VECTOR

$$\begin{vmatrix} \mathbf{0.83} \\ \mathbf{0.17} \end{vmatrix}$$

Ranking alternatives

Factor A	Civic	Saturn	Escort	Miata
Civic	1	1/5	7	1/6
Saturn	5	1	3	1/4
Escort	1/7	1/3	1	1/5
Miata	6	4	5	1

Priority vector

$$\begin{bmatrix} .1648 \\ .2337 \\ .0646 \\ .5370 \end{bmatrix}$$

Factor B	Civic	Saturn	Escort	Miata
Civic	1	2	5	1
Saturn		1	3	2
Escort			1	1/4
Miata				1

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Ranking alternatives

Factor A	Civic	Saturn	Escort	Miata
Civic	1	1/5		1/6
Saturn	5	1		
Escort	1/7			
Miata	6			

Remember CR ?!

Priority vector

$$\begin{bmatrix} .1648 \\ .2337 \\ .0646 \\ .5370 \end{bmatrix}$$

CR = 0.3626 > 0.1

Ranking alternatives

Factor A

	Civic	Saturn	Escort	Miata
Civic	1	2	5	1/6
Saturn		1	2	1/4
Escort			1	1/7
Miata				1

CR = 0.083

Priority vector

$$\begin{bmatrix} .2092 \\ .1230 \\ .0627 \\ .6051 \end{bmatrix}$$

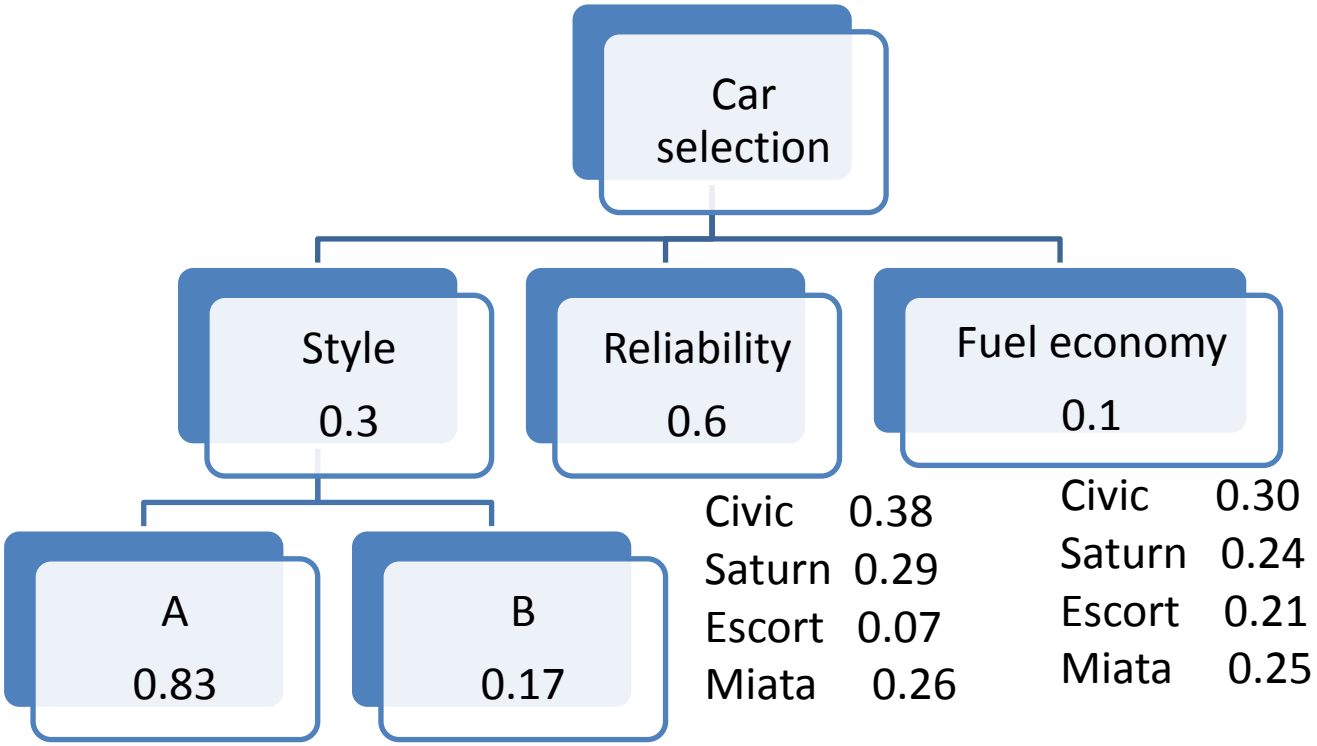
Factor B

	Civic	Saturn	Escort	Miata
Civic	1	2	5	1
Saturn		1	3	2
Escort			1	1/4
Miata				1

CR = 0.079

$$\begin{bmatrix} .3780 \\ .2869 \\ .0742 \\ .2609 \end{bmatrix}$$

Calculate score



Civic score = (0.2092x0.83+0.378x0.17)x 0.3 + 0.38x0.6+0.3x0.1

AHP Software:

Professional commercial software Expert Choice developed by Expert Choice Inc. is available which simplifies the implementation of the AHP's steps and automates many of its computations

- computations
- sensitivity analysis
- graphs, tables

Ex 2: Evaluation of Job Offers

Ex: Peter is offered 4 jobs from Acme Manufacturing (A), Bankers Bank (B), Creative Consulting (C), and Dynamic Decision Making (D). He bases his evaluation on the criteria such as location, salary, job content, and long-term prospects.

Step 1: Decide upon the relative importance of the selection criteria:

	Location	Salary	Content	Long-term
Location	1	1/5	1/3	1/2
Salary	5	1	2	4
Content	3	1/2	1	3
Long-term	2	1/2	1/3	1

Priority Vectors:

1) Normalize the column entries by dividing each entry by the sum of the column.
2) Take the overall row averages

	Location	Salary	Content	Long-term	Average
Location	0.091	0.102	0.091	0.059	0.086
Salary	0.455	0.513	0.545	0.471	0.496
Content	0.273	0.256	0.273	0.353	0.289
Long-term	0.182	0.128	0.091	0.118	0.130
	+				+
	1	1	1	1	1

Example 2: Evaluation of Job Offers

Step 2: Evaluate alternatives w.r.t. each criteria

Location Scores

	A	B	C	D
A	1	1/2	1/3	5
B	2	1	1/2	7
C	3	2	1	9
D	1/5	1/7	1/9	1

Relative Location Scores

	A	B	C	D	Avg.
A	0.161	0.137	0.171	0.227	0.174
B	0.322	0.275	0.257	0.312	0.293
C	0.484	0.549	0.514	0.409	0.489
D	0.032	0.040	0.057	0.045	0.044

Example 2: Calculation of Relative Scores

	Relative Scores for Each Criteria					Relative weights for each criteria		Relative scores for each alternative
	Location	**Salary**	**Content**	**Long-Term**				
A	0.174	0.050	0.210	0.510		0.086		0.164
B	0.293	0.444	0.038	0.012	x	0.496	=	0.256
C	0.489	0.312	0.354	0.290		0.289		0.335
D	0.044	0.194	0.398	0.188		0.130		0.238