

DISPLACEMENT INTERPOLATION FUNCTION

- USE DISPLACEMENT AT NODE TO INTERPOLATE / APPROXIMATE THE DISPLACEMENT WITHIN THE FIELD.

N SHAPE FUNCTION MATRIX

$$u = N \tilde{u}^e$$

\tilde{u}^e NODAL DISPLACEMENT MATRIX OF THE ELEMENT.

u DISPLACEMENT VECTOR AT ANY POINT WITHIN ELEMENT.

\rightarrow SHAPE FUNC. 3 NODE TRI. ELE.:

$$u(x, y) = a + bx + cy$$

$$\left. \begin{array}{l} u(x_1, y_1) = a + bx_1 + cy_1 = u_1 \\ u(x_2, y_2) = a + bx_2 + cy_2 = u_2 \end{array} \right\}$$

$$\left. \begin{array}{l} u(x_3, y_3) = a + bx_3 + cy_3 = u_3 \end{array} \right\}$$

UNKNOWNNS:

$$a = \frac{1}{2A} [u_1(x_2y_3 - x_3y_2) + u_2(x_3y_1 - x_1y_3) + u_3(x_1y_2 - x_2y_1)]$$

$$b = \frac{1}{2A} [u_1(y_2 - y_3) + u_2(y_3 - y_1) + u_3(y_1 - y_2)]$$

$$c = \frac{1}{2A} [u_1(x_3 - x_2) + u_2(x_1 - x_3) + u_3(x_2 - x_1)]$$

$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = \frac{1}{2} [(x_3 - x_2)y_1 + (x_1 - x_3)y_2 + (x_2 - x_1)y_3]$$

AREA OF TRIANGLE

INTERPOLATION FUNC.:

$$\left. \begin{array}{l} N_1(x, y) = \frac{1}{2A} [(x_2y_3 - x_3y_2) + (y_2 - y_3)x + (x_3 - x_2)y] \\ N_2(x, y) = \frac{1}{2A} [(x_3y_1 - x_1y_3) + (y_3 - y_1)x + (x_1 - x_3)y] \\ N_3(x, y) = \frac{1}{2A} [(u_1y_2 - x_2y_1) + (y_1 - y_2)x + (x_2 - x_1)y] \end{array} \right\}$$

NO DES

FULL DISPLACEMENT FUNC.:

$$u(x, y) = N_1(x, y)u_1 + N_2(x, y)u_2 + N_3(x, y)u_3$$

$$\tau(x, y) = N_1(x, y)\tau_1 + N_2(x, y)\tau_2 + N_3(x, y)\tau_3$$

SHAPE

