SOLID MECHANICS:

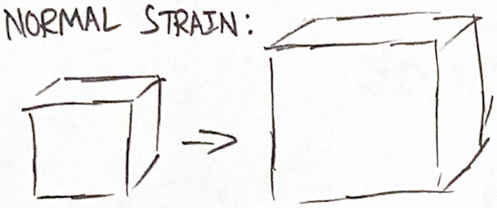
DISPLACEMENT OF DEFORMATION:

POSITION: $x = x_1 e_1 + x_2 e_2 + x_3 e_3 = x_i e_i$

DISPLACEMENT: $u(x,t) = u_1 e_1 + u_2 e_2 + u_3 e_3 = u_i(x,t) e_j$

AT ANY POINT — TIME DEPENDENT.

NORMAL STRAIN:



TENSION/

$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}$ $\varepsilon_{22} = \frac{\partial u_2}{\partial x_2}$ $\varepsilon_{33} = \frac{\partial u_3}{\partial x_3}$ 3COMPONENTS

DISPLACEMENT VECT. VS. STRAIN TENSOR

$$\boxed{\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

SHEAR STRAIN:

3COMPONENTS.

$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$ $\varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$ $\varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

STRAIN TENSOR.
COMPLETELY DESCRIBES DEFORMATION

ENGINEERING SHEAR STRAIN:

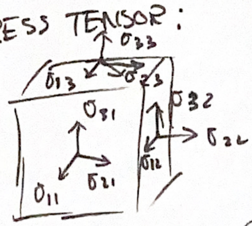
- $\gamma_{12} = 2\varepsilon_{12} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}$ $\gamma_{13} = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}$

$\gamma_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}$

~~DETEMINING~~ STRAIN TENSOR:

E = YOUNG'S MODULUS
ν = POISSON'S RATION
G = SHEAR MODULUS.

STRESS TENSOR:



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

HOOKES LAW: SHEAR

$\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \nu(\sigma_{22} - \sigma_{33})\right]$

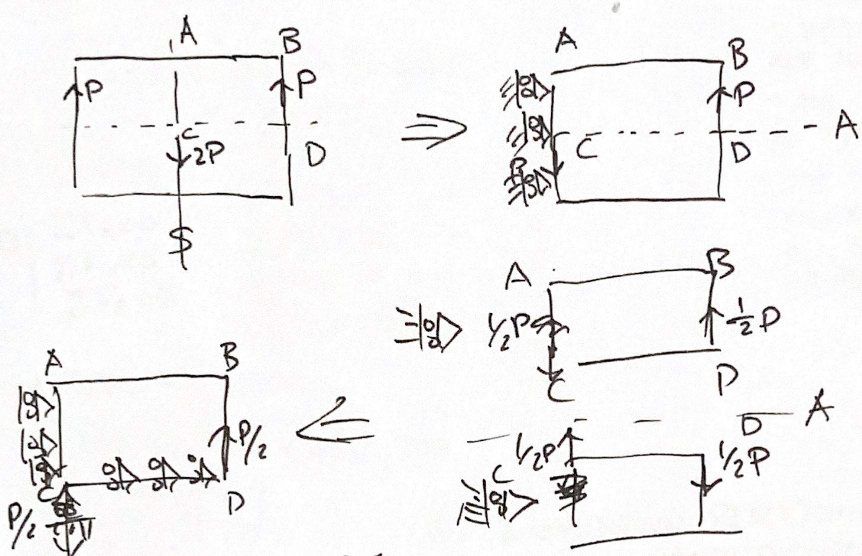
$\varepsilon_{22} = \frac{1}{E}\left[\sigma_{22} - \nu(\sigma_{33} - \sigma_{11})\right]$

$\varepsilon_{33} = \frac{1}{E}\left(\sigma_{33} - \nu(\sigma_{11} - \sigma_{22})\right]$

$\varepsilon_{12} = \frac{\sigma_{12}}{2G}$ $G = \frac{E}{2(1+\nu)}$

$\varepsilon_{13} = \frac{\sigma_{13}}{2G}$

$\varepsilon_{23} = \frac{\sigma_{23}}{2G}$



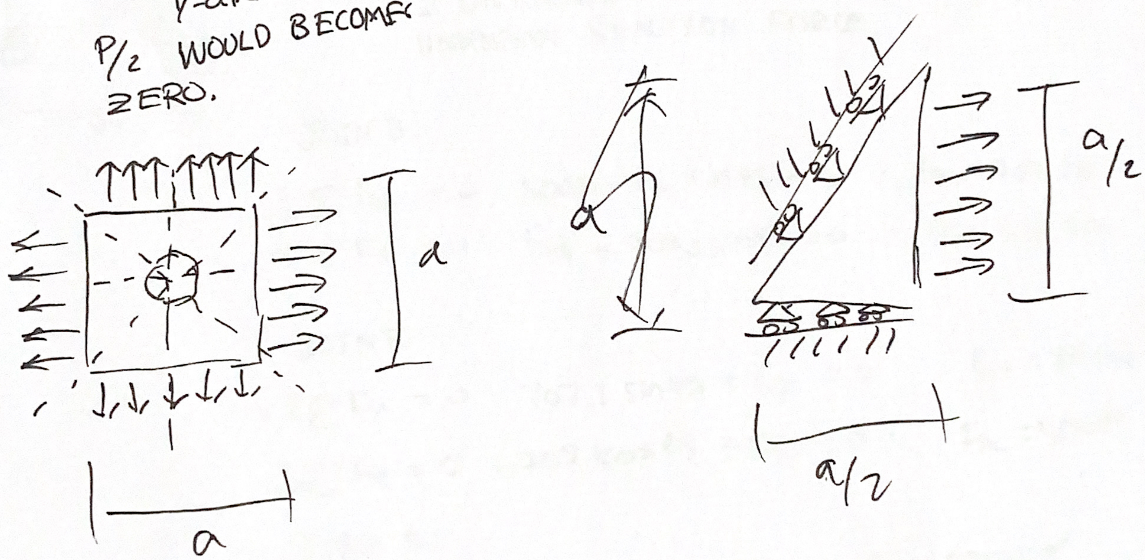
MUST NOT CONSTRAIN

C U_2 BECAUSE
y-dir

P/2 WOULD BECOME

ZERO.

1/8 SYMMETRY?



TRUSS AND BEAM ELEMENT FEA

STATIC EQUILIBRIUM OF BODY:

2D | $\cancel{\Sigma F = 0}$ $\cancel{\Sigma M = 0}$ $\quad \Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$

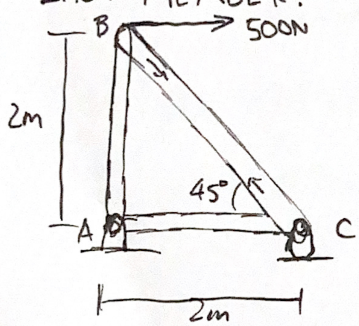
3D | $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ $\quad \Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$

ASSUMPTIONS

- WEIGHT OF THE MEMBERS ARE NEGLECTED.
- ALL LOADING ARE APPLIED AT THE JOINTS.
- THE MEMBERS ARE JOINED TOGETHER BY SMOOTH PINS
- **ALL THE MEMBERS ONLY SUPPORT AXIAL LOADS.**

METHOD OF JOINTS:

- DETERMINE THE FORCE IN EACH MEMBER.



A: 2 UNKNOWN REACTION FORCE AND 2 UNKOWN MEMBER FORCES.

B: 2 UNKNOWN MEMBER FORCES.

C: 2 UNKNOWN MEMBER FORCES AND AN UNKNOWN REACTION FORCE.

JOINT B:

$\xrightarrow{+} \Sigma F_x = 0 \quad 500N - F_{BC} \sin 45 = 0 \quad \therefore F_{BC} = 707.1N$

$+\uparrow \Sigma F_y = 0 \quad F_{AB} - 707.1 \sin 45 = 0 \quad \therefore F_{AB} = 500N$

JOINT C:

$+\uparrow \Sigma F_y = 0 \quad 707.1 \sin 45 + C_y = 0 \quad \therefore C_y = 500N$

$\xrightarrow{+} \Sigma F_x = 0 \quad -707.1 \cos 45 + F_{AC} = 0 \quad \therefore F_{AC} = 500N$

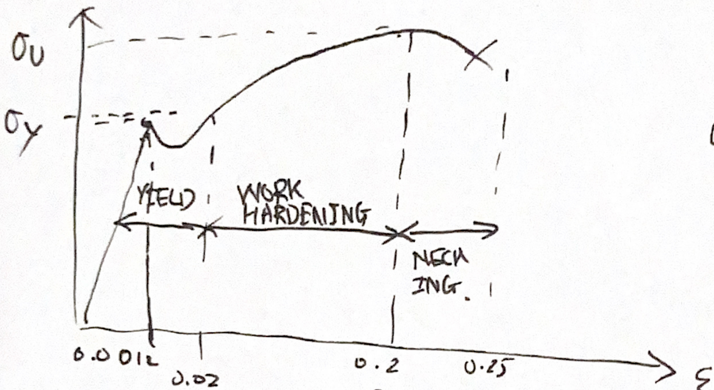
JOIN A:

$+\uparrow \Sigma F_y = 0 \quad A_y - 500N = 0 \quad \therefore A_y = 500N$ ✓

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x - F_{AC} = 0 \quad \therefore A_x = 500N$ ✓

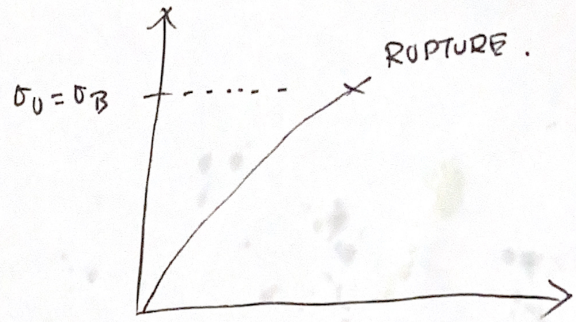
ELASTIC FAILURE:

ELASIC MATERIALS.



LOW CARBON STEEL

BRITTLE MATERIALS:

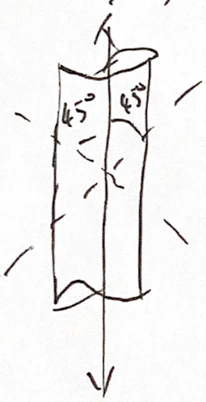


TRESCA CRITERION:

$\tau_{max} < \tau_y$ SAFE

$\tau_{max} \geq \tau_y$ FAILURE

ANY COMBO OF LOAD WILL FAIL WHEN MAX SHEAR STREGHTH EXCEEDS SHEAR STRENGH OF MATERIAL.



FAILURE OFTEN OCCUR 45° TO LOAD AXIS, CORRESPONDES TO THE PLANE OF MAX SHEAR STRESS.

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

σ_{max} – maximum normal principle stress

σ_{min} – min. normal principle stress

$$\Rightarrow \tau_{max} = \tau_y = \frac{1}{2}(\sigma_{max} - \sigma_{min})$$

$$= \frac{1}{2}(\sigma_y - 0) = \frac{1}{2}\sigma_y$$

$$FOS = \frac{\sigma_y}{\sigma_{max} - \sigma_{min}}$$

BECOZ. UNIAXIAL, SO ONLY 1 PRINCIPAL STRESS.

VON MISES CRITERION:

MAX. DISTORTION EN. / VOLUME < DISTORTION EN. /VOLUME TO CAUSE YIELDING.

$$U_D < (U_D)_y$$

$$\sigma_{EQ} = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

PRINCIPLE STRESS

$$\sigma_{EQ} = \left[\frac{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2}{2} + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2)\right]^{1/2}$$

$\sigma_{EQ} \geq \sigma_y$ FAILURE (YIELDING)

$\sigma_{EQ} < \sigma_y$ SAFE (ELASTIC)

$$FOS = \frac{\sigma_y}{\sigma_{eq.}}$$