Modelling CS u: control input, y: plant output State variable CS is in state variable form if IO to SS Model 1. Define x s.t. highest order derivative in x

2.1 If LTI, then

*Write x = Ax + Bu = f(x, u) by isolating for components of x *Write y = Cx + Du = h(x, u) by setting measurement output y to component of x2.2 If not LTI, then

*Write $\dot{x} = f(x, u)$ by isolating for components of \dot{x}

Equilibria y_d (steady state) b/c if $y(0)=y_d$ at t=0, then $y(t)=y_d$ $\forall t\geq 0$.

Equilibrium pair Consider the system $\dot{x} = f(x, u)$. The pair (\bar{x}, \bar{u}) is an equilibrium pair if $f(\bar{x}, \bar{u}) = 0$.

 (\bar{x},\bar{u}) is an equilibrium pair if $f(\bar{x},\bar{u})=0$. Equilibrium point \bar{x} is an equilibrium point w/ control $u=\bar{u}$. *If $u=\bar{u}$ and $x(0)=\bar{x}$ then $x(t)=\bar{x}$ $\forall t\geq 0$ (i.e. a system that starts at equilibrium remains at equilibrium). Find Equilibrium Pair/Point 1. Set f(x,u)=02. Solve f(x,u)=0 to find $(x,u)=(\bar{x},\bar{u})$. 3. If specific $u=\bar{u}$, then find $x=\bar{x}$ by solving $f(x,\bar{u})=0$.

Linearization of Nonlinear System Consider system $\dot{x} = f(x, u)$ w/ equ. pair (\bar{x}, \bar{u}) , then error coordinates around equ. pair $\delta x = x - \bar{x}$, $\delta u = u - \bar{u}$, $\delta v = v - h(\bar{x}, \bar{u})$ $\delta \dot{x} = \dot{x} - f(\bar{x}, \bar{u})$ w/ $\delta \dot{x} = A\delta x + B\delta u, \ A = \frac{\partial f(\bar{x}, \bar{u})}{\partial \underline{x}} \in \mathbb{R}^{n_1 \times n_1}, \ B = \frac{\partial f(\bar{x}, \bar{u})}{\partial u} \in \mathbb{R}^{n_1},$ $\delta y = C\delta x + D\delta u, \ C = \frac{\partial \underline{h}}{\partial \underline{x}}(\bar{x}, \bar{u}) \in \mathbb{R}^{1 \times n_1}, \ D = \frac{\partial h(\bar{x}, \bar{u})}{\partial u} \in \mathbb{R}$ *Only valid at equ. pairs.

Linear Approx. Given a diff. fcn. $f:\mathbb{R}\to\mathbb{R}$, its linear approx at \bar{x} is $f_{\lim}=f(\bar{x})+f'(\bar{x})(x-\bar{x})$.

*Remainder Thm: $f(x) = f_{\text{lin}} + r(x)$ where $\lim_{x \to \bar{x}} \frac{r(x)}{x - \bar{x}} = 0$.



*Note: Can provide a good approx. near \bar{x} but not globally.

*Gen. $f: \mathbb{R}^{n_1} \to \mathbb{R}^{n_2}, \ f(x) = f(\bar{x}) + \frac{\partial f}{\partial x}(\bar{x})(x - \bar{x}) + R(x)$

*Jacobian: $\frac{\partial f}{\partial x}(\bar{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(\bar{x}) & \dots & \frac{\partial f}{\partial x_{n_1}}(\bar{x}) \end{bmatrix} \in \mathbb{R}^{n_2 \times n_1}$

Linearization Steps 1. Find equ. pair (\bar{x}, \bar{u}) 2. Derive A, B, C, D and then evaluate at (\bar{x}, \bar{u})

3. Write $\delta \dot{x} = A\delta x + B\delta u$ and $\delta y = C\delta x + D\delta u$

Laplace Transform Given a fcn $f : \mathbb{R}_+ = [0, \infty) \to \mathbb{R}^n$, its Laplace transform is $F(s) = \mathcal{L}\{f(t)\} := \int_0^\infty f(t)e^{-st} dt$, $s \in \mathbb{C}$. $^*\mathcal{L}: f(t) \mapsto F(s)$, $t \in \mathbb{R}_+$ (time dom.) & $s \in \mathbb{C}$ (Laplace dom.). *\$\mathcal{L}: f(t) \to F'(s), t \in \mathbb{R}_+ \text{ (time dom.).} & s \in \mathcal{L}: Laplace dom.).\$ P.W. CTS: A fcn \$f: \mathbb{R}_+ \to \mathbb{R}^n\$ is p.w. cts if on every finite interval of \$\mathbb{R}\$, \$f(t)\$ has at most a finite \$\mathcal{H}\$ of discontinuity points \$(t_i)\$ and the limits \$\lim_{t \to t_i} + f(t)\$, \$\lim_{t \to t_i} - f(t)\$ are finite.



Exp. Order A function $f: \mathbb{R}_+ \to \mathbb{R}^n$ is of exp. order if \exists

Exp. Order A function $f: \mathbb{R}_+ \to \mathbb{R}$ is of exp. order if a constants $K, \rho, T > 0$ s.t. $\|f(t)\| \le Ke^{\rho t}, \forall t \ge T$. Existence of LT Thm If f(t) is p.w. cts and of exp. order w/constants $K, \rho, T > 0$, then $F(\cdot)$ exists and is defined $\forall s \in D := \{s \in \mathbb{C} : \operatorname{Re}(s) > \rho\}$ and $F(\cdot)$ is analytic on D. *Analytic fon iff differentiable fcn. *D: Region of convergence (ROC), open half plane.

Table of Common Laplace Transforms: $f(t) \mid F(s)$ $\mathbf{1}(t) \mapsto \frac{1}{s} \quad t\mathbf{1}(t) \mapsto \frac{1}{s^2} \quad t^k \mathbf{1}(t) \mapsto \frac{k!}{s^{k+1}} \quad e^{at} \mathbf{1}(t) \mapsto \frac{1}{s-a}$

 $t^n e^{at} \ \mathbf{1}(t) \mapsto \frac{n!}{(s-a)^{n+1}} \quad \sin(at) \ \mathbf{1}(t) \mapsto \frac{a}{s^2+a^2}$ $\cos(at)\,\mathbf{1}(t)\mapsto\frac{s}{s^2+a^2}-\frac{1}{2\omega^3}\left[\sin(\omega t)-\omega t\cos(\omega t)\right]\mathbf{1}(t)\mapsto\frac{s}{(s^2+a^2)^2}$

Prop. of Laplace Transform Linearity: $\mathcal{L}\{cf(t)+g(t)\}=c\mathcal{L}\{f(t)\}+\mathcal{L}\{g(t)\},c\sim \text{constant}.$ Differentiation: If the Laplace transform of f'(t) exists, then $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0^{-})$

If the Laplace transform of $f^{(n)}(t) := \frac{d^n f}{dt^n}(t)$ exists, then $\mathcal{L}\{f^{(n)}(t)\} = s^n \mathcal{L}\{f(t)\} - \sum_{i=1}^n s^{n-i} f^{(i-1)}(0^-).$

Integration: $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}\mathcal{L}\left\{f(t)\right\}.$

Convolution: Let $(f*g)(t) := \int_0^t f(\tau)g(t-\tau)d\tau = \int_0^t f(t-\tau)g(\tau)d\tau$, then $\mathcal{L}\{(f*g)(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$. Time Delay: $\mathcal{L}\{f(t-T)\mathbf{1}(t-T)\} = e^{-Ts}\mathcal{L}\{f(t)\}, T \geq 0$.

Multiplication by t: $\mathcal{L}\{tf(t)\} = -\frac{d}{ds}[\mathcal{L}\{f(t)\}].$

Shift in s: $\mathcal{L}\lbrace e^{at}f(t)\rbrace = \mathcal{L}\lbrace f(t)\rbrace \Big|_{s\to s-a} = F(s-a)$, where $F(s) = \mathcal{L}\{f(t)\} \& a \text{ const.}$

Trig. Id. $\frac{1}{2}\sin(2t) = \sin(t)\cos(t)$, $\sin(a-b) = \sin(a)\cos(b)$ $\cos(a)\sin(b), \cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$ Complete the Square: $ax^2 + bx + c = a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c$

Inverse Laplace Transform Given F(s), its inverse LT is f(t) =

Cauchy's Residue THM If F(s) is analytic (complex diff.) everywhere except at isolated poles $\{p_1,\ldots,p_N\}$, then $\mathcal{L}^{-1}\{F(s)\} = \sum_{i=1}^N \operatorname{Res}\left[F(s)e^{st}, s = p_i\right]\mathbf{1}(t),$

*Res $[F(s)e^{st}, s = p_i]$: Residue of $F(s)e^{st}$ at $s = p_i$.

Residue Computation Let G(s) be a complex analytic fcn w/ a pole at s = p, r be the multiplicity of the pole p. Then $Res[G(s), s = p] = \frac{1}{(r-1)!} \lim_{s \to p} \frac{d^{r-1}}{ds^{r-1}} [G(s)(s-p)^r]$.

Inv. LT Partial Frac: 1. Factorize F(s) into partial fractions. 2. Find coefficients and use LT table to find inverse LT.

*Complete the square.

*Inv. LT Residue: 1. Find poles of F(s) and their residues.

*Coughv's Residue THM to find inverse LT.

*cos(x) = $\frac{e^{jx} + e^{-jx}}{2}$, $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$.

Transfer Function: Consider a CS in IO form. Assume zero initial conds. $y(0) = \cdots = \frac{d(n-1)}{dt(n-1)}y(0) = 0$ and

 $u(0) = \cdots = \frac{d^{(m-1)}u}{dt^{(m-1)}}(0) = 0$. Then the TF from u to y is

$$G(s) := \frac{y(s)}{U(s)} = \frac{b_m s^m + \dots + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_0}$$
*0 Ini. Conds.: $y_0(s) = G(s) u(s)$

*Ø Ini. Conds.: $y_{\emptyset}(s) = y_0(s) + \frac{\text{poly. based on initial conds.}}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$

TF Steps (IO to TF): 1. Given IO form of CS, assume zero initial conds. 2. Find G(s) by taking LT of IO form and forming Y(s)/U(s). **Careful: Y(s)/U(s) = G(s) not U(s)/Y(s) = G(s). Impulse Response: Given CS modeled by TF G(s), its IR is

*Not Invertible: values of 3 and 2 respond to poles of G(s). Inverse: 1. For $A \in \mathbb{R}^{n \times n}$, find $[\operatorname{cof}(A)]_{(i,j)} = (-1)^{i+j} \det(A_{(i,j)})$ * $A_{(i,j)}$: A w/ row i and col. j removed.

2. Assemble cof(A) and find $det(A) = \sum_{j=1}^{n} a_{ij} [cof(A)]_{(i,j)}$

w/ fixed i or $\det(A) = \sum_{i=1}^{n} a_{ij} [\operatorname{cof}(A)]_{(i,j)}$ w/ fixed j.

3. Find $A^{-1} = \frac{1}{\det(A)}\operatorname{adj}(A) = \frac{1}{\det(A)}[\operatorname{cof}(A)]^T$.

*2 × 2 :
$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

TF (SS to TF): 1. Given SS form, assume zero initial conds. 2. Solve $G(s) = C(sI - A)^{-1}B + D$.

*If $C = \begin{bmatrix} 0 & 1_i & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 1_j & 0 \end{bmatrix}$, then only need ith row

& jth col. of adj(sI - A) s.t. $G(s) = \frac{[\text{adj}(sI - A)](i,j)}{\det(sI - A)} + D.$

*Multiple i, j non-zero entries: Work it out using MM.

TF to SS: Consider $G(s) = \frac{b_m s^m + \dots + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_0}$ where m < n (i.e. G(s) is strictly proper). Then the SS form is

$$*A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} b_0 & \cdots & b_m & | & 0 & \cdots & 0 \end{bmatrix}, D = 0.$$

 $C = \begin{bmatrix} b_0 & \cdots & b_m & | & 0 & \cdots & 0 \end{bmatrix}, D$ *Unique: State space of a TF is not unique.



Block Diagram Types of Blocks

Parallel $y = (G_1(s) + G_2(s))U$

$$V \xrightarrow{\bigcap_{G_1}} \xrightarrow{\circ} Y \qquad \qquad \equiv \qquad V \xrightarrow{\bigcap_{G_1 - G_1}} \longrightarrow Y$$

*SC: Unity Feedback Loop (UFL) if $G_2(s) = 1$.

Manipulations: 1. $y = G(U_1 - U_2) = GU_1 - GU_2$ $\frac{\omega^2}{2} \cdot y_1 = GU \quad y_2 = U \quad | \quad y_1 = GU \quad y_2 = G\frac{1}{G}U$

 $3.\ {\rm From\ feedback\ loop\ to\ UFL}.$

Find TF from Block Diagram: 1. Start from in → out, making simplifications using block diagram rules.

2. Simplify until you get the form $U(s) \to G(s) \to Y(s)$.

Time Response of Elementary Terms: $1(t) \leftarrow \text{pole} @ 0$ $t^{n}\mathbf{1}(t) \leftarrow \text{pole} \ @\ 0 \ \text{w/ mult.} \ n \ |\ e^{at}\mathbf{1}(t) \leftarrow \text{pole} \ @\ a \\ \sin(\omega t + \phi)\mathbf{1}(t) \leftarrow \text{pole} \ @\ \pm j\omega \ |\ \cos(\omega t + \phi)\mathbf{1}(t) \leftarrow \text{pole} \ @\ \pm j\omega$ Real Pole: $y(s) = \frac{1}{s+a}$, real pole at s = -a, then $y(t) = e^{-at} \mathbf{1}(t)$ 1. $a>0 \implies \lim_{t\to\infty} y(t)=0 \mid 2. \ a<0 \implies \lim_{t\to\infty} y(t)=\infty$ 3. $a=0 \implies y(t)=\mathbf{1}(t)$ is constant.



Time Constant:
$$\tau = \frac{1}{a}$$
 of the pole $s = -a$ for $a > 0$ Pair of Comp. Conj. Poles:
$$y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \sigma)^2 + \omega_d^2}, \ |\zeta| < 1, \ \text{then}$$

$$y(t) = \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\sigma t} \sin(\omega_d t) \mathbf{1}(t)$$

*Poles: $s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} = -\sigma \pm j \omega_d$

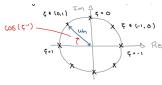
* $\zeta = \frac{\sigma}{\omega_n}$: Damping ratio (or damping coefficient)

 $\sigma^* = \zeta \omega_n$: Decay/growth rate | ω_d : Freq. of oscillation $*\omega_n = \sqrt{\sigma^2 + \omega_d^2} \left[\frac{\text{radians}}{\text{seconds}} \right]$: Undamped natural freq.

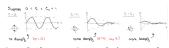
 $*\omega_d = \omega_n \sqrt{1-\zeta^2} \left[\frac{\text{radians}}{\text{seconds}} \right]$: Damped natural freq.

* $|s_{1,2}|^2 = \omega_n^2$: Mag. of poles is ω_n .

 $*\cos^{-1}(\zeta)$: Angle of s_1 on complex plane CW from -ve Re axis



Damping Ratio Effect: $0 < \zeta_1 < \zeta_2 < 1$, then



 $-1 < \zeta_4 < \zeta_3 < 0$, then $\sigma = \zeta \omega_n < 0$, (exp. envelop \uparrow)



Class. of 2nd Order Sys.: y(s) = $\frac{-n}{s^2+2\zeta\omega_n s+\omega_n^2}$, w/ any $|\zeta|$

Loc. of Poles and Behavior:

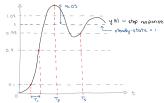


Control Spec. of 2nd Order Sys.: Step Response: Given a TF G(s), its SR is y(t) resulting from applying the input $u(t) = \mathbf{1}(t)$, i.e. $\mathcal{L}^{-1}\left\{G(s)\frac{1}{s}\right\}$.

Control Spec. A control spec. is a criterion specifiying how we

would like a CS to behave.

2nd Order Sys. Metrics: $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ w/ $U(s) = \frac{1}{s}$ *0 < ζ < 1 (i.e. 2 comp. conj. poles w/ Re(pole) < 0).



Rise Time (RT): T_r is the time it takes y(t) to go from 10% to 90% of its steady-state value.

RT: 1. Find $t_1 > 0$ s.t. $y(t_1) = 0.1$, $t_2 > 0$ s.t. $y(t_2) = 0.9$.

3. Compute
$$T_r = t_2 - t_1$$
. $T_r \approx \frac{1.8}{\omega_n}$.

Settling Time (ST): T_s is the time required to reach and stay w/in 2% of the steady-state value. ST: 1. Find when it's first that |y(t) - 1| < 0.02 indefinitely.

4 $T_S \approx -$

Peak Time:
$$T_p$$
 is time req'd to reach the max (peak) value.
Peak Time: 1. Find the first time when $\dot{y}(t) = 0$.

$$* T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}.$$

 $\zeta \omega_n$

 $\% \text{ Overshoot: } \%OS = \frac{\text{[peak value]} - \text{[steady-state value]}}{\text{[steady-state value]}} \times 100\%$ *% $OS = OS \times 100\%$

* OS = exp
$$\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \iff \zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + (\ln(OS))^2}}$$

LT Steps: 1. Write f(t) as a sum and use linearity *Trig. id. may be useful.

2. Use prop. of LT and common LT to find F(s)

Transient Performance Sat.: Given performance spec. $T_r \leq T_r^d$, $T_s \leq T_s^d$, OS \leq OS d , find loc. of poles of G(s). *Admissible region for the poles of G(s) s.t. the step response meets all three spec. is the intersection of the above three regions. Rise Time: $T_r \approx \frac{1.8}{\omega_n} \leq T_r^d \stackrel{\text{app}}{\Longrightarrow} \omega_n \geq \frac{1.8}{T_r^d} \equiv \omega_n^d$



Settling Time: $T_s \approx \frac{4}{\zeta \omega_n} = \frac{4}{\sigma} \leq T_s^d \stackrel{\text{app.}}{\Longleftrightarrow} \sigma \geq \frac{4}{T_s^d} \equiv \sigma^d$



$$\operatorname{OS:} \, \exp \left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \right) \leq \operatorname{OS}^d \, \stackrel{\operatorname{app.}}{\Longleftrightarrow} \, \zeta \geq \frac{-\ln(\operatorname{OS}^d)}{\sqrt{\pi^2 + (\ln(\operatorname{OS}^d))^2}} \equiv \zeta^d$$

Add. Poles & Zeros: The analysis remains approx. correct

under the following assumptions:

1. Any add. poles of G(s) have much more -ve real part (5-10 times) than the real part of the dom. complex conjugate poles.



- *dominant poles, additional poles.
 2. Real part of zeros are -ve & very diff. from the real part of the two dom. poles.

- Internal Stablity: $\dot{x}=Ax$ is

 1. Stable if $\forall x(0) \in \mathbb{R}^n$, the soln. x(t) is bdd; that is, $\exists M>0$ s.t. $\|x(t)\| \leq M \ \forall t \geq 0$.

 2. Asymp. Stable if it's stable & $\forall x(0) \in \mathbb{R}^n$, the soln. x(t) converges to the origin; that is, $\lim_{t\to\infty} x(t) = 0$.

 3. Unstable if it's not stable; that is, $\exists x(0) \in \mathbb{R}^n$ s.t. x(t) is not bdd.
- Asymptotic Stablity Thm. x = Ax is A.S. iff $\operatorname{eig}(A) \subseteq \mathbb{C}^- \equiv \{s \in \mathbb{C} \mid \operatorname{Re}(s) < 0\}$, i.e. open left half plane (OLHP). Instability Thm. If \exists an eigenvalue λ of $A \le M$ w/ $\operatorname{Re}(\lambda) > 0$, then

- Instability Thm. If \exists an eigenvalue x = Ax is unstable. x = Ax is unstable. Fact: Zeros of $s^2 + a_1s + a_0$ are in \mathbb{C}^- iff $a_1, a_0 > 0$. Internal Stability 1. Linearize around (\bar{x}, \bar{u}) w/ $\bar{u} = 0$. 2. Find A and determine $\operatorname{eig}(A) = \lambda$ s.t. $\operatorname{det}(sI A) = 0$. 3. Check if $\operatorname{eig}(A) \subseteq \mathbb{C}^-$ for asymptotic stability 4. Check if $\operatorname{Re}(\operatorname{eig}(A)) > 0$ for instability. BIBO Stability: An LTI system w/ 0 i.c. is Bounded Input Bounded Output (BIBO) stable if for any bdd input u(t), the output u(t) is also bdd.
- Bounded Output (BIBO) stable if for any bdd input u(t), the output y(t) is also bdd.

 BIBO Unstable: An LTI system w/0 i.c. is BIBO unstable if it's not BIBO stable; that is, \exists a bdd u(t) s.t. y(t) is not bdd.

 BIBO Stable Thm. A system y(s) = G(s)U(s) is BIBO stable iff poles $(G(s)) \subseteq \mathbb{C}^-$.

 Lemma: If p is a pole of G(s), then p is an eig(A). I.e. poles $(G(s)) := \{p \in \mathbb{C} \mid p$ is a pole of $G(s) \} \subseteq \text{eig}(A)$.

 *Pole-0 Cancellation: eig(A) need not be a pole of G(s).

- *Pole-0 Cancellation: $\operatorname{eig}(A)$ need not be a pole of G(s). Thm. If $\operatorname{eig}(A) \subseteq \mathbb{C}^-$, then $\forall B, C, D$ the TF G(s) is BIBO stable. That is, internal asymptotic stability \Rightarrow BIBO stability. BIBO Stability 1. Find G(s) from SS form and determine poles. 2. Check if $\operatorname{poles}(G(s)) \subseteq \mathbb{C}^-$. 1. Check if $\operatorname{eig}(A) \subseteq \mathbb{C}^-$ since internal asymptotic stability \Rightarrow BIBO stability. Routh-Hurwitz: Consider $a(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_0$. * $s^n \mid 1 \quad a_{n-2} \quad a_{n-4} \quad a_{n-6} \quad \cdots \quad 0$ * $s^n \mid 1 \quad a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad a_{n-7} \quad \cdots \quad 0$ * $s^n \mid 1 \quad a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad a_{n-7} \quad \cdots \quad 0$

- $*s^{n-2} \mid b_1 \quad b_2 \quad b_3 \quad \cdots \\ *s^{n-3} \mid c_1 \quad c_2 \quad \cdots$

 \mathbb{C}^- iff the 1st col of Routh array has no sign changes. The # of

- sign changes is equal to the # of roots of $a(s) \in C^+ := \{s \in \mathbb{C} : \text{Re}(s) > 0\}$.

 *If 1st element of a row is 0, Rooth array cannot be completed.

 FVT v1: Suppose $Y(s) = \mathcal{L}\{y(t)\}$ is a proper rational fcn. If $y(\infty) := \lim_{t \to \infty} y(t)$ exists and is finite, then $y(\infty) = \lim_{s \to 0} sY(s)$ FVT v2: Suppose $Y(s) = \mathcal{L}\{y(t)\}$ is a proper rational fcn. Moreover, suppose either:
- 1. $poles(Y(s)) \subseteq \mathbb{C}^-$
- 2. Y(s) has only one pole at s=0 and all other poles are in \mathbb{C}^- . Then $y(\infty):=\lim_{m\to 0} sY(s)$ exists and is finite and satisfies $y(\infty)=\lim_{m\to 0} sY(s)$. FVT 1. Does $y(\infty)$ exist? Check if pole at s=0, then compute
- Rooth Array to see if poles are in \mathbb{C}^- . 2. Compute $\lim_{s\to 0} sY(s)$ if it exists.