ECE368 Cheatsheet

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1 L1 (LG-IPPR 1.1, 1.2; Murphy 2.1 – 2.3)

Summary:

FAQ:
- How to study? Practice, practice.
- What textbooks? Use 2024 version of Murphy, Leon Garcia as main reference, Bishop, 4th textbook is intro.
- How is HW graded? Effort, and tutorials are used to explain soln.

1.1 Sample Space

Motivation: If you have 4 sheeps and a flea, the probability that starting from sheep 1, the flea will jump to sheep 4 in 10 steps is 0.2.
- Ambigious as there are 2 different interpretations for the sample space (i.e. space of probability is not clear):
  - Set of sheeps
  - Set of number of steps

1.2 Probability Definitions

Definition:
- **Random Experiment:** An outcome (realization) for each run.
- **Sample Space Ω:** Set of all possible outcomes.
- **Events:** (measurable) subsets of Ω.
- **Probability of Event A:** $P[A] \equiv P[\text{'outcome is in A'}]$.

Example: Roll Fair Die
- $\Omega = \{1, 2, 3, 4, 5, 6\}$.
- $P[\text{'even number'}] = \frac{1}{2}$.

1.3 Axioms of Probability

Definition:
1. $P[A] \geq 0$ for all $A \in \Omega$.
2. $P[\Omega] = 1$.
3. If $A \cap B = \emptyset$, then $P[A \cup B] = P[A] + P[B]$ for all $A, B \in \Omega$.

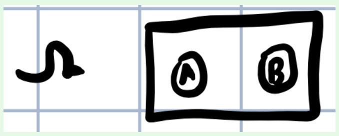

Figure 1: 3rd Axiom

1.4 Conditional Probability

Definition:

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \tag{1}$$

- $|$: Given event (data/obs.).

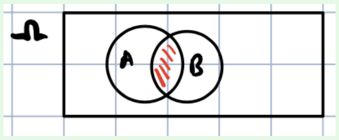

Figure 2: Conditional Probability

Notes:
- Changing sample space to B.
- Conditional probability satisfy the 3 axioms, can be viewed as probability measure on new sample space B.

1.4.1 Consequences of Conditional Probability

Definition:

$$P[A \cap B] = P[A|B]P[B] = P[B|A]P[A] \tag{2}$$

1.4.2 Independence

Definition: A and B are independent iff

$$P[A \cap B] = P[A]P[B] \iff P[A|B] = P[A] \iff P[B|A] = P[B] \tag{3}$$

1.4.3 Importance of Labelling

Example: **Toss 2 Fair Coins**

1. **Given:** Given that one of the coins is heads, what is the probability that the other coin is tails?
2. **Wrong Solution:** $\frac{1}{2}$ since $\{HH, HT, TH, TT\}$, so $P[T|H] = \frac{1}{2}$, which assumes that the coins are distinguishable (i.e. coin #1 is heads)
3. **Correct Solution:** $\frac{2}{3}$ since $\{HH, HT, TH\}$ as we didn't specify which coin was heads, so $P[T|H] = \frac{2}{3}$, which assumes that the coins are indistinguishable.

1.5 Total Probability

Definition: If $H_1, \ldots, H_n$ form a partition of Ω, then

$$P[A] = \sum_{i=1}^{n} P[A|H_i]P[H_i] \tag{4}$$

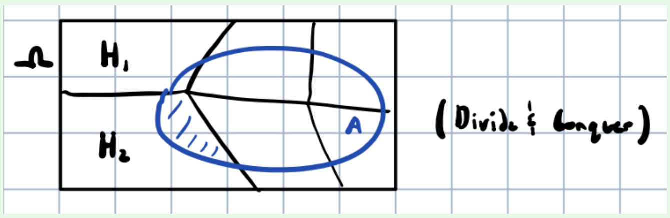

Figure 3: Total Probability

1.6 Bayes' Rule

Definition:

$$P[H_k|A] = \frac{P[H_k \cap A]}{P[A]} = \frac{P[A|H_k]P[H_k]}{\sum_{i=1}^{n} P[A|H_i]P[H_i]} \quad (5)$$

1.6.1 Posteriori Probability, Priori Probability (Prior), Likelihood

Definition:
- **Posteriori:** $P[H_k|A]$.
- **Priori:** $P[H_k]$.
- **Likelihood:** $P[A|H_k]$.

Example: Suppose a lie detector is 95% accurate, i.e. $P[\text{'out=truth'}|\text{'in=truth'}] = 0.95$ and $P[\text{'out=lie'}|\text{'in=lie'}] = 0.95$. It says that Mr. Ernst is lying. What is the probability Mr. Ernst is actually lying.
- **Observation:** $A =$ 'out=lie'.
- **Hypothesis:** $H_0 =$ 'in=lie' and $H_1 =$ 'in=truth'.
- **Solution:** $P[H_0|A] = \frac{P[A|H_0]P[H_0]}{P[A|H_0]P[H_0] + P[A|H_1]P[H_1]} = \frac{0.95 \times P[H_0]}{0.95 \times P[H_0] + 0.05 \times (1 - P[H_0])}$.
- $H_0 = 0.01$**:** i.e. 1% of the population are liars, then $P[H_0|A] = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.05 \times 0.99} = 0.16$.

Warning: Need to know priori probability.

1.6.2 Interpretation of Bayes' Rule

Definition:

1.7 Random Variables

Motivation: **Coin Toss** Mapping of each outcome to a real number, i.e. $w \in \Omega$ is the outcome of a coin toss, and $X(w) = 1$ if heads and $X(w) = 0$ if tails.

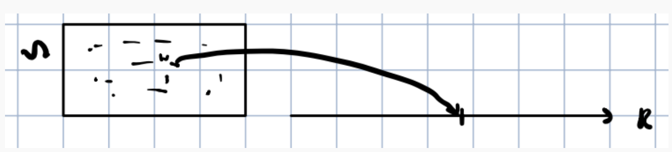

Figure 4: Random Variables

- Mapping is deterministic function. RV is not random or variable.

Definition: Mapping from Ω to $\mathbb{R}$.

1.7.1 Cumulative Distribution Function (CDF) of RV

Definition:

$$F_X(x) \equiv P[X \leq x] \quad (6)$$

1.7.2 Discrete RV PMF

Definition:

$$P_X(x_j) \equiv P[X = x_j] \quad j = 1, 2, 3, \dots \tag{7}$$

Example: Binonmial RV w/ (n, p)

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{8}$$

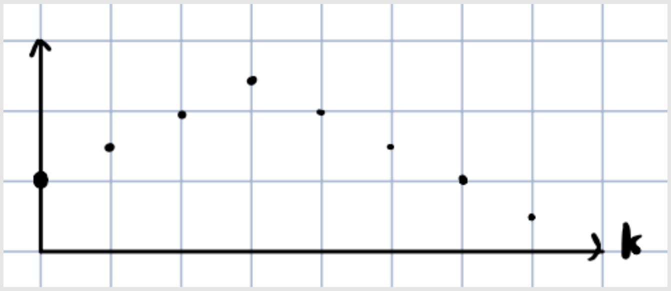

Figure 5: Binomial RV

1.7.3 Continuous RV PDF

Definition:

$$f_X(x) \equiv \frac{d}{dx} F_X(x) \tag{9}$$

$$P[x < X < x + dx] = f_X(x)dx \tag{10}$$

Example: Gaussian RV w/ (μ, σ^2)

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{11}$$

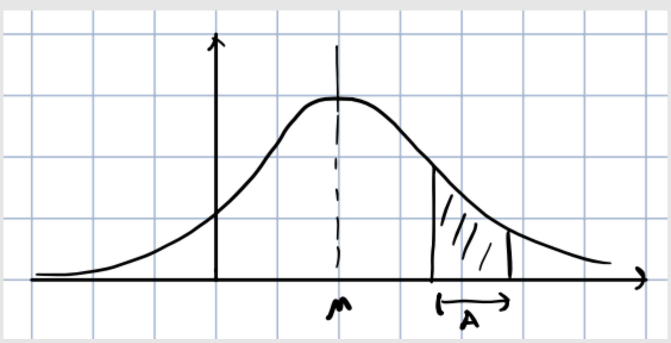

Figure 6: Gaussian RV

- $p[X \in A] = \int_A f_X(x)dx$.
- **Note:** Discrete RV has pdf w/ δ functions.

1.7.4 Conditional PMF/PDF

Definition:

$$P_X(x|A) \tag{12}$$

$$f_X(x|A) \tag{13}$$

Example: Continuous

$$f(x|X>a) = \begin{cases} \frac{f_X(x)}{P[X>a]} & \text{if } x > a \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Example: Geometric RV Geometric RV X w/ success probability p

$$P_X(k) = (1-p)^{k-1}p \tag{15}$$

- **Memoryless Property:** $P_X[k|X>m] = \frac{p(1-p)^{k-1}}{(1-p)^m} = p(1-p)^{k-m-1}$. So it only cares about the additional trials.

1.8 Expected Values

Definition:

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)dx \overset{\text{If int. values}}{=} \sum_k k f_X(k) \tag{16}$$

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) f_X(x)dx \overset{\text{If int. values}}{=} \sum_k h(k) f_X(k) \tag{17}$$

$$\text{Var}[X] = E[(X-E[X])^2] = E[X^2] - E[X]^2 \tag{18}$$

$$E[X|A] = \int_{-\infty}^{\infty} x f_X(x|A)dx \tag{19}$$

Example: Lottery Ticket

1. **Given:** Buying one lottery ticket per week
   - Each ticket has $10^{-7} = p$ chance of winning the jackpot.
   - X = '# of weeks to win jackpot'.
2. **Problem:** What is the expected number of weeks to win the jackpot?
3. **Solution:** $E[X] = \sum_{k=1}^{\infty} k(1-p)^{k-1}p = \ldots = \frac{1}{p} = 10^7$ weeks.
4. **Extension (Memoryless Property):** If I have already played for 999999 weeks, what is the expected number of weeks to win the jackpot? $E[X - 999999|X > 999999] = E[X] = 10^7$ weeks.

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