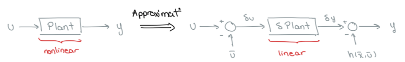
Modelling CS u: control input, y: plant output
State variable CS is in state variable form if
$\dot{x}_1 = f_1(t, x_1, \dots, x_n, u), \dots, \dot{x}_n = f_n(t, x_1, \dots, x_n, u)$
$y = h(t, x_1, \dots, x_n, u)$ is a collection of n 1st order ODEs.
Time-Invariant (TI) CS is TI if $f_i(\cdot)$ does not depend on t.
State space (SS) TI CS is in SS form if $\dot{x} = f(x, u), y = h(x, u)$ where $x(t) \in \mathbb{R}^n$ is called the state.
Single-input-single-output (SISO) CS is SISO if $u(t), y(t) \in \mathbb{R}$.
LTI CS in SS form is LTI if $\dot{x} = Ax + Bu,\ y = Cx + Du$
$A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times m},\ C \in \mathbb{R}^{p\times n},\ D \in \mathbb{R}^{p\times m}$
where $x(t) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^m,\ y(t) \in \mathbb{R}^p$.
Input-Output (IO) LTI CS is in IO form if
$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdot + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + \cdot + b_1 \frac{du}{dt} + b_0 u$$
where $m \leq n$ (causality)

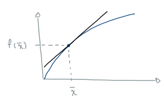
IO to SS Model 1. Define x s.t. highest order derivative in $\dot{x}$
2. Write $\dot{x} = Ax + Bu = f(x, u)$ by isolating for components of $\dot{x}$
3. Write $y = Cx + Du = h(x, u)$ by setting measurement output y to component of x
Equilibria y_d (steady state) b/c if $y(0) = y_d$ at $t = 0$, then $y(t) = y_d\ \forall t \geq 0$.

Equilibrium pair Consider the system $\dot{x} = f(x, u)$. The pair $(\bar{x}, \bar{u})$ is an equilibrium pair if $f(\bar{x}, \bar{u}) = 0$.
Equilibrium point $\bar{x}$ is an equilibrium point w/ control $u = \bar{u}$.
*If $u = \bar{u}$ and $x(0) = \bar{x}$ then $x(t) = \bar{x}\ \forall t \geq 0$ (i.e. a system that starts at equilibrium remains at equilibrium).
Find Equilibrium Pair/Point 1. Set $f(x, u) = 0$
2. Solve $f(x, u) = 0$ to find $(x, u) = (\bar{x}, \bar{u})$.
3. If specific $u = \bar{u}$, then find $x = \bar{x}$ by solving $f(x, \bar{u}) = 0$.

Linearization Nonlinear System Consider system $\dot{x} = f(x, u)$ w/ equ. pair $(\bar{x}, \bar{u})$, then error coordinates around equ. pair $\delta x = x - \bar{x}$, $\delta u = u - \bar{u}$, $\delta y = y - h(\bar{x}, \bar{u})$ w/
$$\delta\dot{x} = A\delta x + B\delta u,\ A = \frac{\partial f(\bar{x}, \bar{u})}{\partial \underline{x}} \in \mathbb{R}^{n_1 \times n_1},\ B = \frac{\partial f(\bar{x}, \bar{u})}{\partial u} \in \mathbb{R}^{n_1},$$
$$\delta y = C\delta x + D\delta u,\ C = \frac{\partial h}{\partial \underline{x}}(\bar{x}, \bar{u}) \in \mathbb{R}^{1\times n_1},\ D = \frac{\partial h(\bar{x}, \bar{u})}{\partial u} \in \mathbb{R}$$
*Only valid at equ. pairs.



Linear Approx. Given a diff. fcn. $f : \mathbb{R} \to \mathbb{R}$, its linear approx. at $\bar{x}$ is $f_{\text{lin}} = f(\bar{x}) + f'(\bar{x})(x - \bar{x})$.
*Remainder Thm: $f(x) = f_{\text{lin}} + r(x)$ where $\lim_{x\to\bar{x}} \frac{r(x)}{x-\bar{x}} = 0$.



*Note: Can provide a good approx. near $\bar{x}$ but not globally.
*Gen. $f : \mathbb{R}^{n_1} \to \mathbb{R}^{n_2}$, $f(x) = f(\bar{x}) + \frac{\partial f}{\partial x}(\bar{x})(x - \bar{x}) + R(x)$
*Jacobian: $\frac{\partial f}{\partial x}(\bar{x}) = \left[\frac{\partial f}{\partial x_1}(\bar{x}) \quad \dots \quad \frac{\partial f}{\partial x_{n_1}}(\bar{x})\right] \in \mathbb{R}^{n_2 \times n_1}$
Linearization Steps 1. Find equ. pair $(\bar{x}, \bar{u})$
2. Derive A, B, C, D and then evaluate at $(\bar{x}, \bar{u})$
3. Write $\delta\dot{x} = A\delta x + B\delta u$ and $\delta y = C\delta x + D\delta u$