ECE353 Lectures

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Contents

1 Prologue

Summary:
- This course will focus on planning
- Variables:
  - State: $\mathbf{x}(t)$
  - Action(s): $\mathbf{u}(t)$
  - Measurement: $\mathbf{y}_k^{(i)}$
  - Context: $\mathbf{z}_k^{(i)}$
  - Old Context: $\mathbf{z}_{k-1}^{(i)}$
  - Plan: $\mathbf{p}_k^{(i)}$
  - (i): Ith agent
- Conversion to DT is necessary because robots are digitalized system and then converted back to CT for execution.

1.1 Setup of Planning Problems

Summary: In a planning problem, it is assumed that:
- the environment is representable using a discrete set of states, $\mathcal{S}$
- for each state, $s \in \mathcal{S}$, each agent, i, has a discrete set of actions, $\mathcal{A}_i(s)$, with $\mathcal{A}(s) := \times_i \mathcal{A}_i(s)$ (joint action set)
- a **move** is any tuple, (s, a), where $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
- a **transition** is any 3-tuple, (s, a, s'), where $s, s' \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
  - the transition resulting from a move may be deterministic/stochastic
- $\text{rwd}_i(s, a, s')$ is agent i's reward for the transition, (s, a, s')
- a **path** is any sequence of transitions of the form

$$p = \left\langle (s^{(0)}, a^{(1)}, s^{(1)}), (s^{(1)}, a^{(2)}, s^{(2)}), \ldots \right\rangle$$

- each agent wants to realize a path that maximizes its own reward

Warning: $\mathcal{A}(s)$ is the joint action set of all agents at state s.

1.2 Components of a Robotic System

Summary:

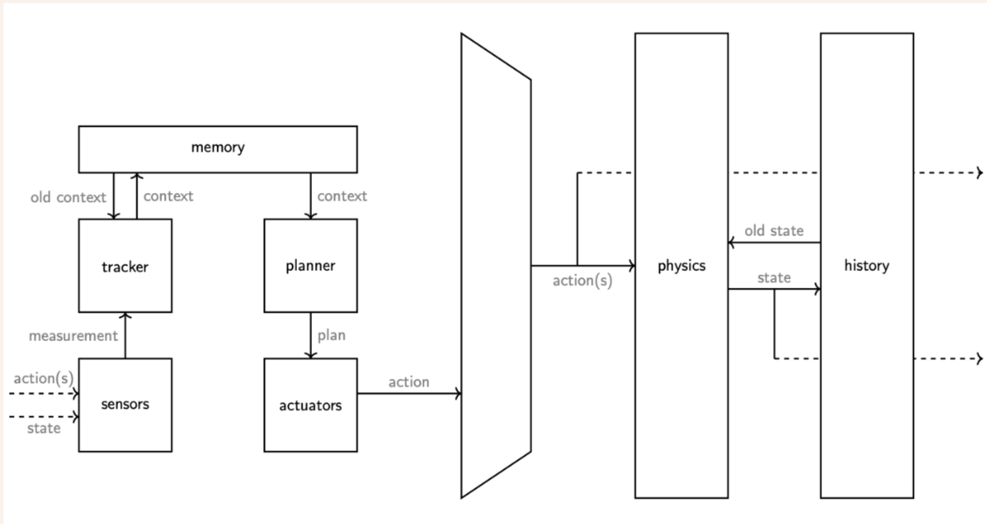


Figure 1: Components of a Robotic System (Words)

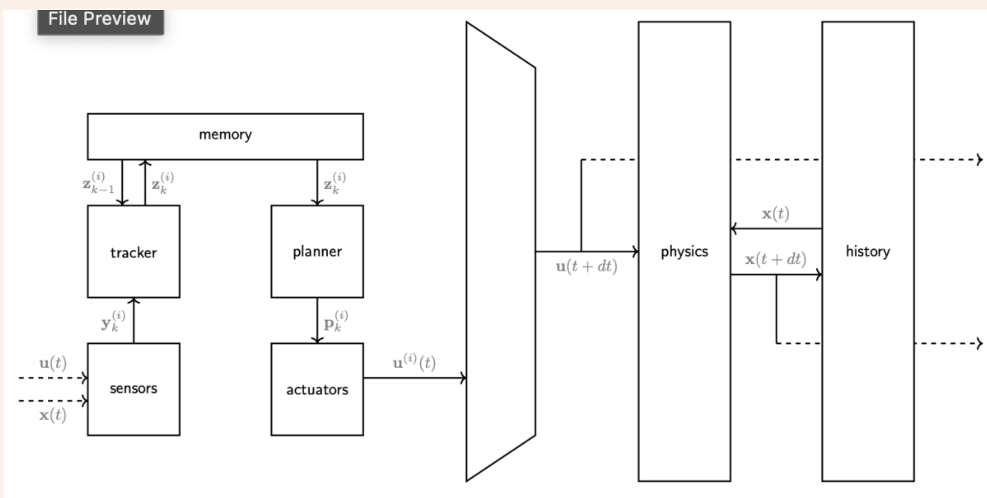


Figure 2: Components of a Robotic System (Math)

1.2.1 Overview (Robots, the Environment)

Definition:

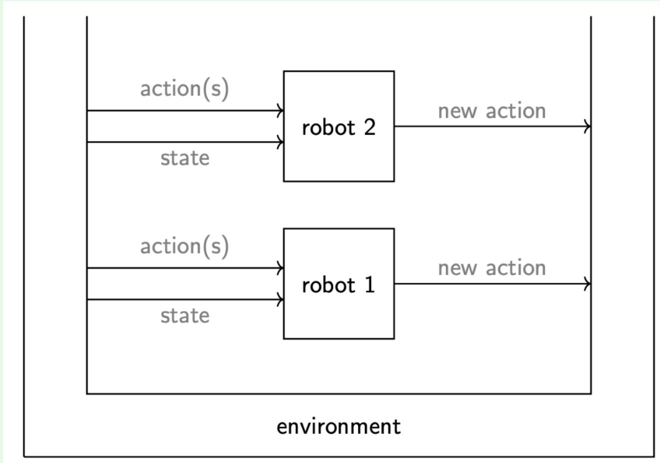


Figure 3: Overview (Robots, the Environment)

Notes:
- Environment → previous actions + current state → robot → new action → environment

1.2.2 Robot (Sensors, Actuators, the Brain)

Definition:

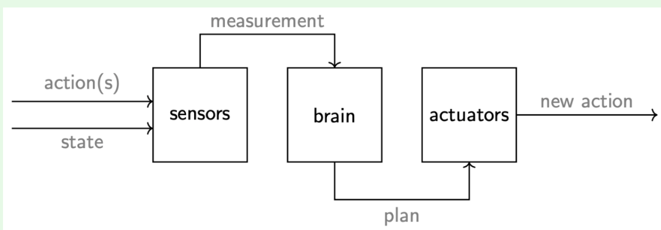


Figure 4: Robot (Sensors, Actuators, the Brain)

Notes:
- Measurements can be noisy and inaccurate if not a perfect sensor.
- Measurements go into the brain which can create a plan.

1.2.3 Brain (Tracker, Planner, Memory)

Definition:

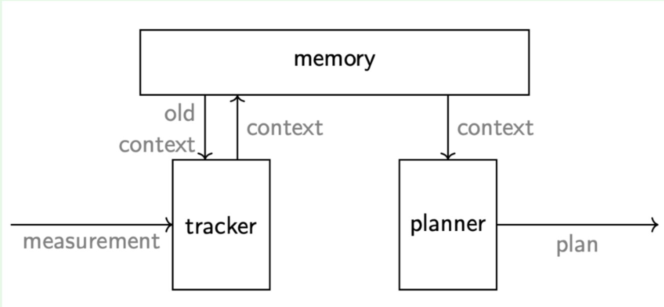


Figure 5: Brain (Tracker, Planner, Memory)

Notes:

- The tracker takes in the measurements and old context and updates the context.
- The planner takes in the context and creates a plan.
- The memory stores the context.

1.2.4 Environment (Physics, State)

Definition:

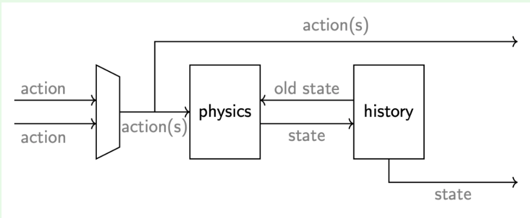


Figure 6: Environment (Physics, State)

1.3 Equations of a Robotic System

1.3.1 Sensing

Definition: Take a measurement:

$$\mathbf{y}^{(i)}(t) = \mathrm{sns}^{(i)}\big(\mathbf{x}(t), \mathbf{u}(t), t\big)$$

Convert the measurement into a discrete-time signal using a sampling period of $T^{(i)}$:

$$\mathbf{y}_k^{(i)} = \mathrm{dt}\big(\mathbf{y}^{(i)}(t), t, T^{(i)}\big)$$

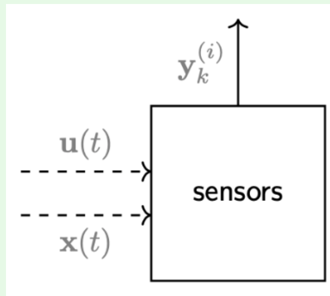


Figure 7: Sensing

1.3.2 Tracking

Definition: Track (update) the context:

$$\mathbf{z}_k^{(i)} = \mathrm{trk}^{(i)}\big(\mathbf{z}_{k-1}^{(i)}, \mathbf{y}_k^{(i)}, k\big)$$

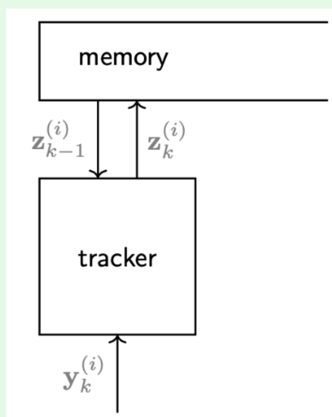


Figure 8: Tracking

1.3.3 Planning

Definition: Make a plan:

$$\mathbf{p}_k^{(i)} = \text{pln}^{(i)}\big(\mathbf{z}_k^{(i)}, k\big)$$

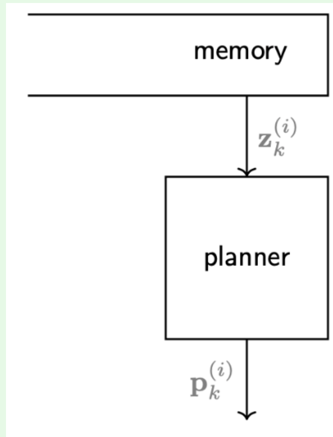


Figure 9: Planning

1.3.4 Acting

Definition: Convert the plan into a continuous-time signal using a sampling period of $T^{(i)}$:

$$\mathbf{p}(t) = \text{ct}\big(\mathbf{p}_k^{(i)}, t, T^{(i)}\big)$$

Execute the plan:

$$\mathbf{u}^{(i)}(t) = \text{act}^{(i)}\big(\mathbf{p}^{(i)}(t), t\big)$$

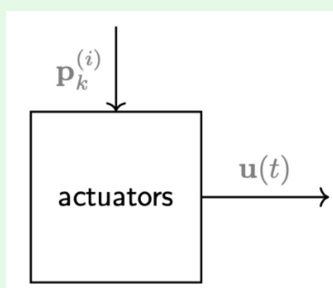


Figure 10: Acting

1.3.5 Simulating

Definition: Simulate the environment's response:

$$\dot{\mathbf{x}}(t) = \text{phy}\big(\mathbf{x}(t), \mathbf{u}(t), t\big)$$

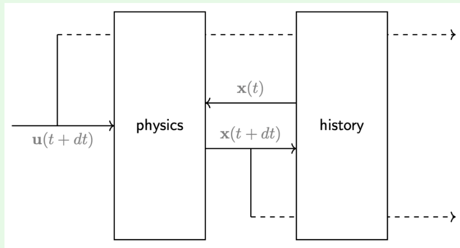


Figure 11: Simulating

2 Uninformed Search Algorithms

Summary:
- Not responsible for proofs, but know when to use each algorithm.

2.1 Setup

Definition: In a search problem, it is assumed that:
- There is only one agent (us).
- For each state, $s \in S$, we have a discrete set of actions, $\mathcal{A}(s)$.
- The transition resulting from a move, (s, a), is deterministic; the resulting state is $tr(s, a)$.
- $cst(s, a, tr(s, a))$ is our cost for the transition, $(s, a, tr(s, a))$.
- We want to realize a path that minimizes our cost.

A search problem may have no solutions, in which case, we define the solution as `NULL`.

2.2 Search Graphs

Definition: In a search graph (a graph representing a search problem):
- S is defined by the vertices.
- $\mathcal{G}$ is a subset of the vertices.
- $s^{(0)}$ is some vertex.
- $tr(\cdot, \cdot)$ and $\mathcal{T}$ are defined by the edges.
- $cst(\cdot, \cdot, \cdot)$ is defined by the edge weights.

2.3 Path Trees

Definition: A search algorithm explores a tree of possible paths.
- In such a tree, each node represents the path from the root to itself.
  - The node may also include other info (such as the path's origiin, cost, etc).

2.4 Search Algorithms

Definition: All search algorithms follow the template below:

```
𝒪 ← {(⟨⟩, 0)}                                          ▷ initialize a set of open nodes
SEARCH(𝒪)
```

- $\langle\rangle$ is the empty path, and 0 is the cost of the empty path.

```
procedure SEARCH(𝒪)
    if 𝒪 = ∅ then
        return NULL                     ▷ the search algorithm failed to find a path to a goal
    n ← REMOVE(𝒪)                                                      ▷ "explore" a node n
    if DST(n) ∈ 𝒢 then
        return n                              ▷ the search algorithm found a path to a goal
    for n′ ∈ CHL(n) do
        𝒪 ← 𝒪 ∪ {n′}                                  ▷ "expand" n and "export" its children
    SEARCH(𝒪)
```

- Explore: Remove a node from the open set.
- Exapnd: Generate the children of the node.
- Export: Add the children to the open set.

Warning: The key difference is in the order that REMOVE($\cdot$) removes nodes.

2.4.1 Characteristics of a Search Algorithm

Definition: We want to choose `REMOVE(·)` so that the algorithm exhibits the following characteristics:

Characteristic	Description
Halting	Terminates after finitely many nodes explored
Sound	Returned (possibly NULL) solution is correct
Complete	Halting and sound when a non-NULL solution exists
Optimal	Returns an optimal solution when multiple exist
Time Efficient	Minimizes the nodes **explored**/expanded/exported
Space Efficient	Minimizes the nodes simultaneously open

- Will be using explored for time efficiency.

The characteristics of the algorithm also depend on several properties of the path tree over which it searches. These properties include:

- Branching factor: b ($b < \infty$), the maximum number of children a node can have.
- Depth: d, the length of the longest path.
- Length of the shortest solution: l^*
- Cost of the cheapest solution: c^*
- Cost of the cheapest edge: ϵ

We want to choose `REMOVE(·)` so that the algorithm exhibits the aforementioned characteristics for as many path trees as possible.

2.4.2 Breadth First Search (BFS)

Definition: Explores the least-recently expanded open node first.

Property	Description
Halting	$d < \infty$ non-NULL
Sound	always
Complete	always
Optimal	constant cst
Time	b^{l^*}
Space	b^{l^*+1}

2.4.3 Depth First Search (DFS)

Definition: Explores the most-recently expanded open node first.

Property	Description
Halting	$d < \infty$
Sound	always
Complete	$d < \infty$
Optimal	never
Time	b^d
Space	bd

2.4.4 Iterative Deepening DFS (IDDFS)

Definition: Same as DFS but with iterative deepening.

Property	Description
Halting	always
Sound	always
Complete	always
Optimal	constant cst
Time	b^{l^*}
Space	bl^*

2.4.5 Cheapest-First Search (CFS)

Definition: Explores the cheapest open node first.

Property	Description
Halting	$d < \infty$ non-NULL
Sound	yes
Complete	$\epsilon > 0$
Optimal	$\epsilon > 0$
Time	$b^{c^*/\epsilon}$
Space	$b^{c^*/\epsilon+1}$

2.5 Modifications to Search Algorithms

2.5.1 Depth-Limiting

Definition: Depth limit of $d_{\max}$ to any search algorithm by modifying SEARCH(·) as follows:

```
procedure SEARCHDL(𝒪, d_max):
    if 𝒪 = ∅ then
        return NULL                        ▷ the search algorithm failed to find a path to a goal
    n ← REMOVE(𝒪)                                                        ▷ "explore" a node, n
    if dst(n) ∈ 𝒢 then
```

```
        return n                                  ▷ the search algorithm found a path to a goal
    for n′ ∈ chl(n) do                                ▷ "expand" n and "export" its children
        if len(n′) ≤ d_max then                              ▷ unless the child is too long
            O ← O ∪ {n′}
    SEARCHDL(O, d_max)
```

2.5.2 Iterative Deepening

Definition: Iteratively increase the depth-limit, $d_{\max}$, to any search algorithm w/ depth-limiting, by placing SEARCHDL(·) in a wrapper, SEARCHID(·):

```
procedure SEARCHID():
    n ← NULL
    d_max ← 0
    ▷ while a solution has not been found, reset the open set, run the search algorithm, then increase the depth-limit
    while n = NULL do
        O ← {(⟨⟩, 0)}
        n ← SEARCHDL(O, d_max)
        d_max ← d_max + 1
    return n
```

Warning: Increasing $d_{\max}$ can be done in different ways.

2.5.3 Cost-Limiting

Definition: Cost limit of $c_{\max}$ to any search algorithm by modifying SEARCH(·) as follows:

```
procedure SEARCHCL(O, c_max):
    if O = ∅ then
        return NULL                               ▷ the search algorithm failed to find a path to a goal
    n ← REMOVE(O)                                                        ▷ "explore" a node, n
    if dst(n) ∈ G then
        return n                                  ▷ the search algorithm found a path to a goal
    for n′ ∈ chl(n) do                                ▷ "expand" n and "export" its children
        if cst(n′) ≤ c_max then                         ▷ unless the child is too expensive
            O ← O ∪ {n′}
    SEARCHCL(O, c_max)
```

2.5.4 Iterative-Inflating

Definition: Iteratively increase the cost limit, $c_{\max}$, to any search algorithm with cost-limiting, by placing SEARCHCL(·) in a wrapper, SEARCHII(·):

```
procedure SEARCHII():
    n ← NULL
    c_max ← 0
    ▷ while a solution has not been found, reset the open set, run the search algorithm, then increase the cost-limit
    while n = NULL do
        O ← {(⟨⟩, 0)}
        n ← SEARCHCL(O, c_max)
        c_max ← c_max + ϵ
    return n
```

Warning: Increasing $c_{\max}$ can be done in different ways.

2.5.5 Intra-Path Cycle Checking

Definition: Do not expand a path if it is cyclic. Modify SEARCH($\cdot$) as follows:

```
procedure SEARCH(𝒪):
    if 𝒪 = ∅ then
        return NULL
    n ← REMOVE(𝒪)
    if dst(n) ∈ 𝒢 then
        return n
    for n′ ∈ chl(n) do                    ▷ "expand" n and "export" its children
        if not CYCLIC(n′) then                   ▷ unless the child is cyclic
            𝒪 ← 𝒪 ∪ {n′}
    SEARCH(𝒪)
```

- Optimately of an algorithm is preserved provided $\epsilon > 0$.

2.5.6 Inter-Path Cycle Checking

Definition: We modify SEARCH($\cdot$) as follows:

```
procedure SEARCH(𝒪, 𝒞):
    if 𝒪 = ∅ then
        return NULL
    n ← REMOVE(𝒪)
    𝒞 ← 𝒞 ∪ {n}                                   ▷ add n to the closed set
    if dst(n) ∈ 𝒢 then
        return n
    for n′ ∈ chl(n) do                    ▷ "expand" n and "export" its children
        if n′ ∉ 𝒞 then              ▷ unless the child's destination is closed
            𝒪 ← 𝒪 ∪ {n′}
    SEARCH(𝒪, 𝒞)
```

and then call the algorithm as follows:

```
𝒪 ← {(⟨⟩, 0)}
𝒞 ← ∅                                  ▷ initialize a set of closed vertices
SEARCH(𝒪, 𝒞)
```

2.6 Informed Search Algorithms

2.6.1 Estimated Cost

Definition: $\text{ecst}(\cdot)$, to estimate the total cost to a goal given a path, p, based on the following:
- Cost of path p: $\text{cst}(p)$
- Estimate of the extra cost needed to get to a goal from $\text{dst}(p)$: $\text{hur} : S \to \mathbb{R}_+$
  - $\text{hur}(s)$ estimates the cost to get to $\mathcal{G}$ from s and $\text{hur}(p)$ means $\text{hur}(\text{dst}(p))$.

Example: Some common choices for $\text{ecst}(\cdot)$ include:
1. $\text{ecst}(p) = \text{hur}(p)$; called nearest-first search (NFS)
2. $\text{ecst}(p) = \text{cst}(p) + \text{hur}(p)$; called A* (A-star)

2.7 Characteristics of an Informed Search Algorithm

Definition:
1. Heuristic: $\text{hur}(\cdot)$
2. Cost estimation: $\text{ecst}(\cdot)$

2.7.1 Heuristics

2.7.2 Heuristic-First Search (HFS)

2.7.3 A-Star Search (A*)

2.7.4 Iterative Inflating A-Star Search (IIA*)

2.7.5 Designing Heuristics via Problem Relaxation

2.7.6 Combining Heuristics

2.8 Anytime Search Algorithms

2.9 Formulating a Search Problem

2.10 Canonical Examples

Process: How to setup a search problem?

1. Givne a search graph, we need to define the following:
   - $\mathcal{S}$: set of vertices
   - $\mathcal{G}$: goal states (subset of $\mathcal{S}$)
   - $s^{(0)}$: initial state
   - $\mathcal{T}$: set of edges (defined by $\text{tr}(\cdot,\cdot)$)
     - $\text{tr}(\cdot,\cdot)$: transition function
   - $\text{cst}(\cdot,\cdot,\cdot)$: cost function (defined by edge weights)

Example:

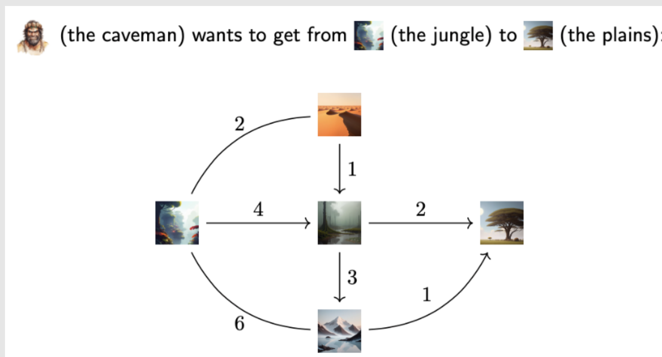


Figure 12

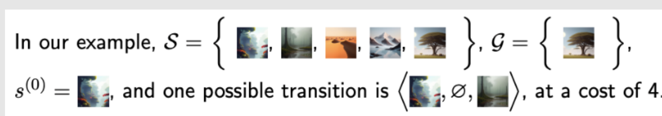


Figure 13

Example:

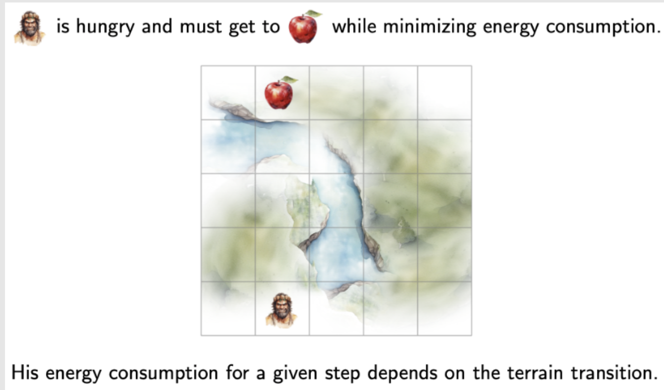


Figure 14

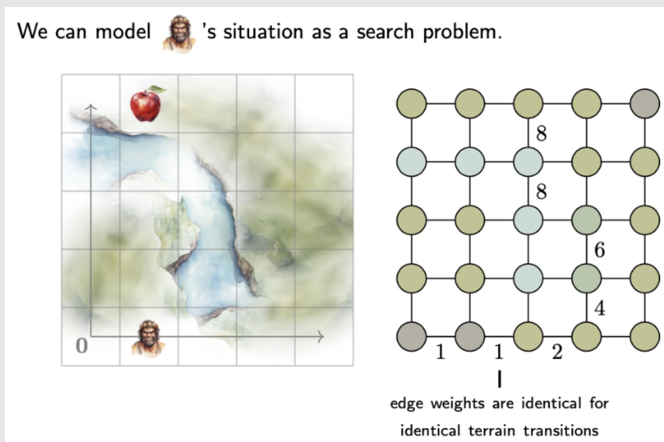


Figure 15

- $\mathcal{S} = \{0, \dots, 4\}^2$
- $\mathcal{G} = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$
- $s^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Process: How to setup a path tree?

1. Start at $s^{(0)}$
2. Choose a path until you reach a goal state.
3. Repeat until you have found all paths (probably infinite).

Example:

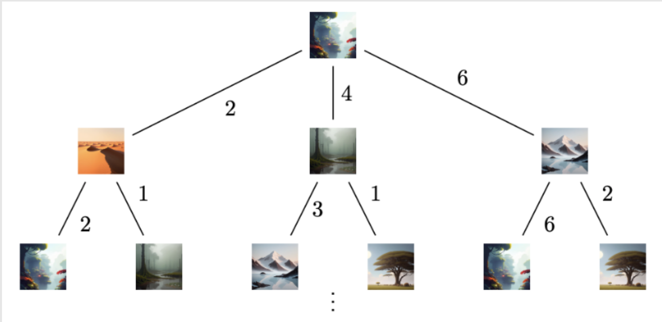


Figure 16

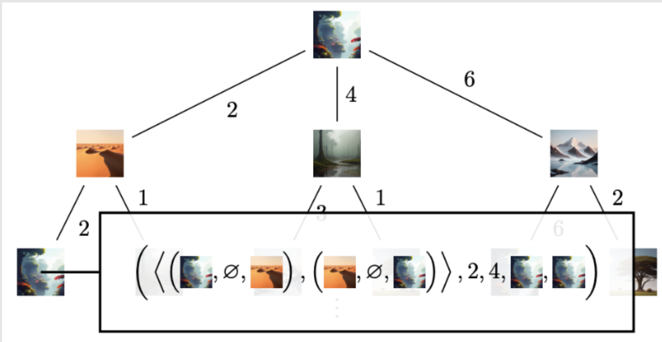


Figure 17

Process: When to use each algorithm?

1. Find properties needed for the problem and match them to the characteristics of the algorithm.
2. Choose the algorithm that best matches the properties.
    - **BFS:**
    - **DFS:**
    - **IDDFS:**
    - **CFS:**
    - **A*:**

Example:

Process: How to Trace Through a Search Algorithm

1.

Example: BFS

Example: DFS

Example: IDDFS

Example: CFS

Example: HFS

Example: A*

Example: IIA*

Example: WA*

Process: How to Figure Out Soln. w/o Performing Search Algorithm?

1.

Example:

Process: How to Prove Consistent/Admissible Given a Search Graph?

Admissible:

1. Given $\text{hur}(s)$ and search graph with $\text{cst}(s, a, \text{tr}(s, a))$ or $\text{rwd}(s, a, \text{tr}(s, a))$. If consistent, then it is admissible.
2. Check $\forall s \in \mathcal{G}$, $\text{hur}(s) = 0$. If not, then it is not admissible.
3. For each $s \in \mathcal{S}$, calculate $\text{hur}^*(s)$ (i.e. actual cost of optimal soln.) using the search graph.
   (a) Start at s and choose path that gives the lowest cost or highest reward to $s \in \mathcal{G}$.
4. Check if $\text{hur}(s) \leq \text{hur}^*(s)\ \forall s \in \mathcal{S}$. If not, then it is not admissible.
5. Repeat $\forall s \in \mathcal{S}$.
6. If all are true, then it is admissible.

Consistent:

1. Given $\text{hur}(s)$ and search graph with $\text{cst}(s, a, \text{tr}(s, a))$ or $\text{rwd}(s, a, \text{tr}(s, a))$.
2. Check $\forall s \in \mathcal{G}$, $\text{hur}(s) = 0$. If not, then it is not consistent.
3. For each $s \in \mathcal{S}$, calculate $\text{hur}(s) - \text{hur}(\text{tr}(s, a))$.
   (a) check if it is $\leq \text{cst}(s, a, \text{tr}(s, a))$ or $\geq \text{rwd}(s, a, \text{tr}(s, a))$. If not, then it is not consistent.
   (b) Repeat $\forall a \in \mathcal{A}(s)$
4. Repeat $\forall s \in \mathcal{S}$.
5. If all are true, then it is consistent.

Warning: Be careful of bidirectional edges bc for consistency you need compute the cost of the heuristic edge in both directions.

Example:

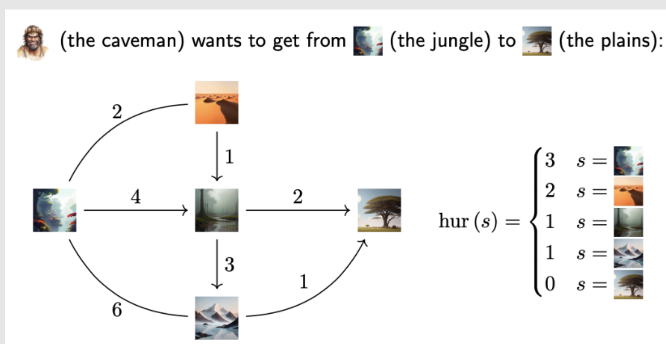


Figure 18: Jungle ($s^{(0)}$), Desert, Swamp, Mountain, Plains (Goal)

Admissible:

1. $s =$**Plains:** $\text{hur}(\text{Plains}) = 0$
2. $s =$**Jungle:** $\text{hur}(\text{Jungle}) = 3 \leq \text{hur}^*(\text{Jungle}) = 2 + 1 + 2 = 5$
3. $s =$**Desert:** $\text{hur}(\text{Desert}) = 2 \leq \text{hur}^*(\text{Desert}) = 1 + 2$
4. $s =$**Swamp:** $\text{hur}(\text{Swamp}) = 1 \leq \text{hur}^*(\text{Swamp}) = 2$
5. $s =$**Mountain:** $\text{hur}(\text{Mountain}) = 1 \leq \text{hur}^*(\text{Mountain}) = 1$
6. Therefore, it is admissible.

Consistent:

1. $s =$**Plains:** $\text{hur}(\text{Plains}) = 0$
2. $s =$**Jungle:**
   (a) $\text{hur}(\text{Jungle}) - \text{hur}(\text{Desert}) = 3 - 2 = 1 \leq \text{cst}(\text{Jungle}, \cdot, \text{Desert}) = 2$
   (b) $\text{hur}(\text{Jungle}) - \text{hur}(\text{Swamp}) = 3 - 1 = 2 \leq \text{cst}(\text{Jungle}, \cdot, \text{Swamp}) = 4$
   (c) $\text{hur}(\text{Jungle}) - \text{hur}(\text{Mountain}) = 3 - 1 = 2 \leq \text{cst}(\text{Jungle}, \cdot, \text{Mountain}) = 6$
3. $s =$**Desert:**
   (a) $\text{hur}(\text{Desert}) - \text{hur}(\text{Jungle}) = 2 - 3 = -1 \leq \text{cst}(\text{Desert}, \cdot, \text{Jungle}) = 2$
   (b) $\text{hur}(\text{Desert}) - \text{hur}(\text{Swamp}) = 2 - 1 = 1 \leq \text{cst}(\text{Desert}, \cdot, \text{Swamp}) = 1$
4. $s =$**Swamp:**
   (a) $\text{hur}(\text{Swamp}) - \text{hur}(\text{Mountain}) = 1 - 1 = 0 \leq \text{cst}(\text{Swamp}, \cdot, \text{Mountain}) = 3$

   (b) $\text{hur}(\text{Swamp}) - \text{hur}(\text{Plains}) = 1 - 0 = 1 \leq \text{cst}(\text{Swamp}, \cdot, \text{Plains}) = 2$
5. $s =$**Mountain:**
   (a) $\text{hur}(\text{Mountain}) - \text{hur}(\text{Jungle}) = 1 - 3 = -2 \leq \text{cst}(\text{Mountain}, \cdot, \text{Desert}) = 6$
   (b) $\text{hur}(\text{Mountain}) - \text{hur}(\text{Plains}) = 1 - 0 = 1 \leq \text{cst}(\text{Mountain}, \cdot, \text{Plains}) = 1$
6. Therefore, it is consistent.

Process: How to Design Heuristic via Problem Relaxation?
1. Make an assumption to simplify the problem as a relaxed problem.
2. Find the cost of the optimal solution of the relaxed problem, $\text{cst}_{\text{rel}}(s)$.
3. HOW TO FIND THE COST OF THE OPTIMAL SOLUTION?

Example:

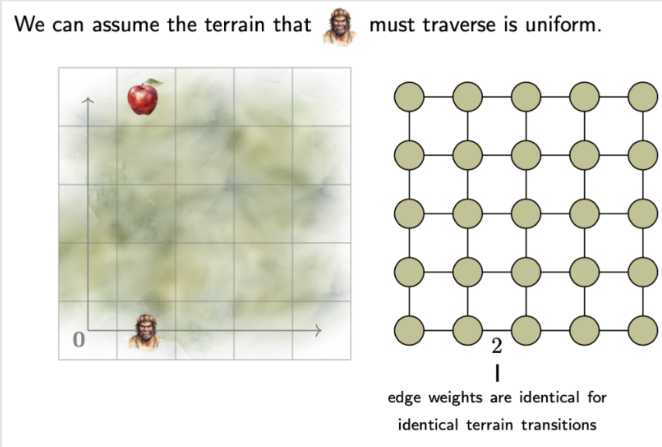


Figure 19

3 Informed Search Algorithms

Summary:

Example: Different ways to formulate the CSP problem.
- How can you formualte the CSP problem in a different way? Can I get a specific example?
  - The domain could be set to everything, then set the constraints later.
- What is the constraint graph showing? Grouping the variables
- How do you check consistency in a CSP?
- Why can you use any search algorithm when you formulate this as a search problem?
- What does a node contain? A node contans a path.
  - How does that match the example on slide 10. It does.
- Why is formulalting a CSP problem as a search problem a bad idea? B/c you have to search through all possible combinations, but if you find a constraint then you can prune the search space.
  - A lot easier to see if there is a solution or not. But in a search problem, you see if there's a solution and how to get to it.

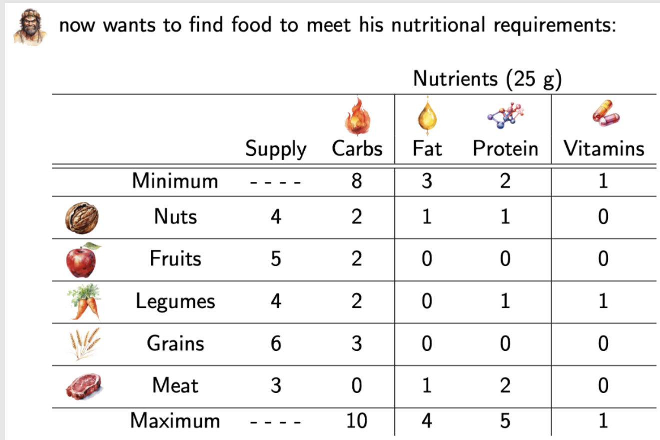
3.1 Admissible and Consistent

Summary: Want a way to learn heuristics.

Process: How to Setup a CSP?

1.

Example:



now wants to find food to meet his nutritional requirements:

	Supply	Carbs	Fat	Protein	Vitamins
		Nutrients (25 g)			
Minimum	- - - -	8	3	2	1
Nuts	4	2	1	1	0
Fruits	5	2	0	0	0
Legumes	4	2	0	1	1
Grains	6	3	0	0	0
Meat	3	0	1	2	0
Maximum	- - - -	10	4	5	1

Figure 20

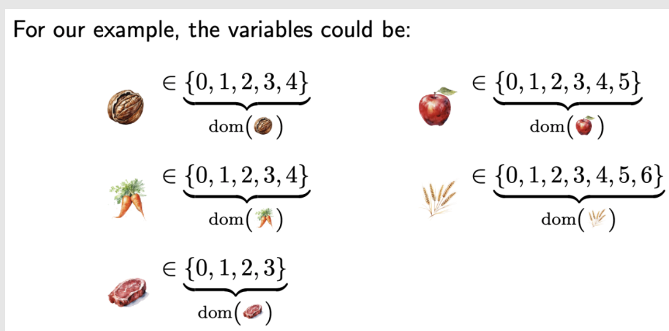


Figure 21

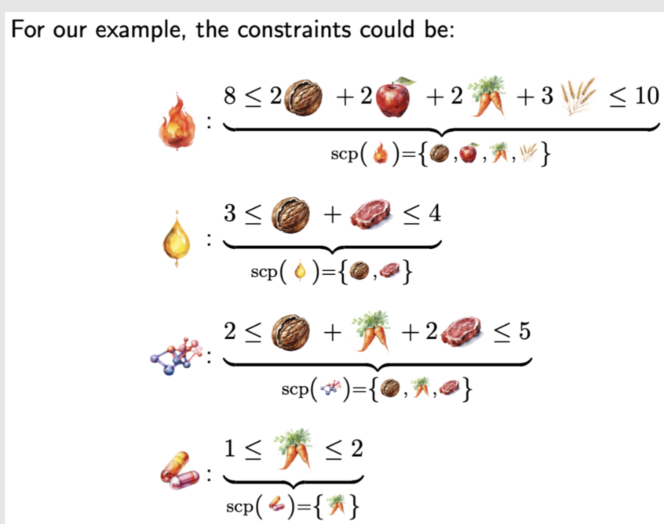


Figure 22

Process: How to build a hyper-graph?

1. Circle the variables that appear in constraint C_i.

Example:

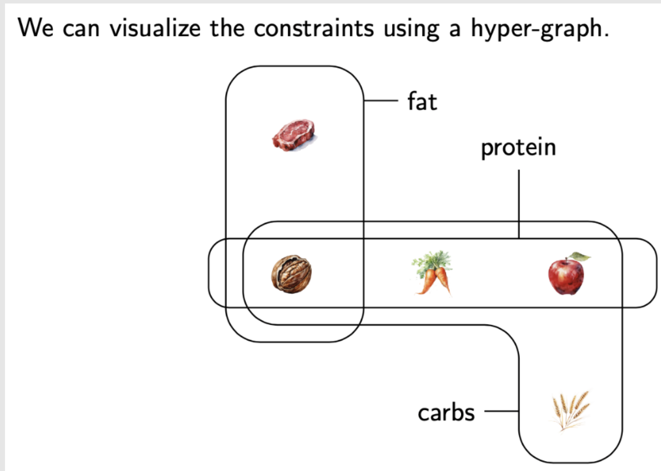


Figure 23

Process: How to build a path tree?

1.

Example:

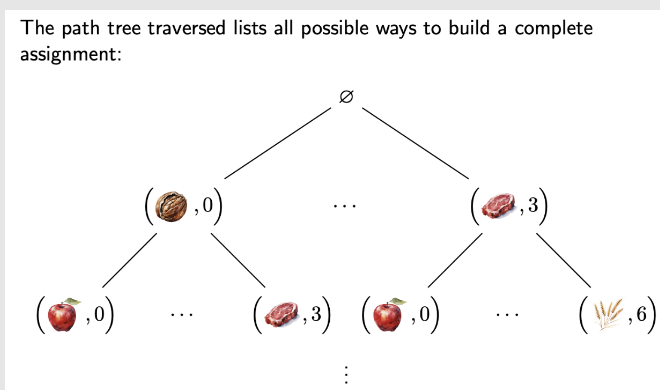


Figure 24

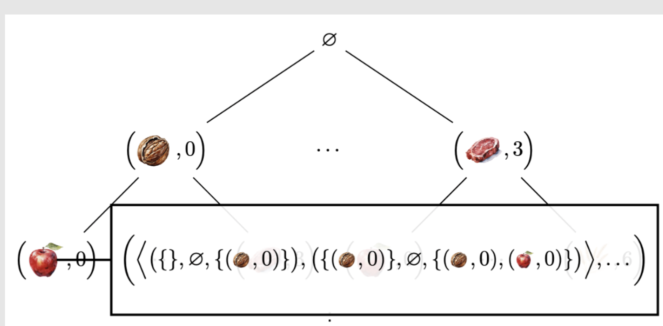


Figure 25

Process: How to determine a solution to a CSP?

1.

Example:

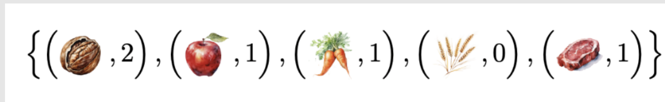


Figure 26

Process: How to check k-Consistency? FIS

1. Given a set of variables $\mathcal{V}$ w/ $\text{dom}(V) = \{v_1, \ldots, v_{\text{len}(V)}\}$ $\forall V \in \mathcal{V}$ and a set of contraints $\mathcal{C}$ w/ $\text{scp}(C) = \{V_1, \ldots, V_{\text{len}(C)}\}$ $\forall C \in \mathcal{C}$, check k-consistency.
2. For each $C \in \mathcal{C}$, do the following:
   (a) For $V \in \text{scp}(C)$, fix V to a value in $\text{dom}(V)$.
      i. For the other $V \in \text{scp}(C)$, check if the constraint is satisfied by trying all combinations.
   (b) If there is one combination that doesn't satisfy the constraint, then the CSP is not k-consistent.
   (c) Repeat $\forall V \in \text{scp}(C)$.
3. Repeat $\forall C \in \mathcal{C}$.
4. If all constraints are satisfied, then the CSP is k-consistent.

Process: How to Enforce k-Consistency? FIX

1. Given a set of variables $\mathcal{V}$ w/ $\text{dom}(V) = \{v_1, \ldots, v_{\text{len}(V)}\}$ $\forall V \in \mathcal{V}$ and a set of contraints $\mathcal{C}$ w/ $\text{scp}(C) = \{V_1, \ldots, V_{\text{len}(C)}\}$ $\forall C \in \mathcal{C}$, enforce k-consistency.
2. For each $C \in \mathcal{C}$, do the following:
   (a) For $V \in \text{scp}(C)$, fix V to a value in $\text{dom}(V)$.
      i. For the other $V \in \text{scp}(C)$, check if the constraint is satisfied by trying all combinations. If the constraint is not satisfied, then remove the value from $\text{dom}(V)$.
   (b) Repeat $\forall V \in \text{scp}(C)$.
3. Check the resulting $\text{dom}(V) = \{v_1, \ldots, v_{\text{len}(V)}\}$ $\forall V \in \mathcal{V}$ w/ the other constraints.
4. Repeat $\forall C \in \mathcal{C}$.

Example:

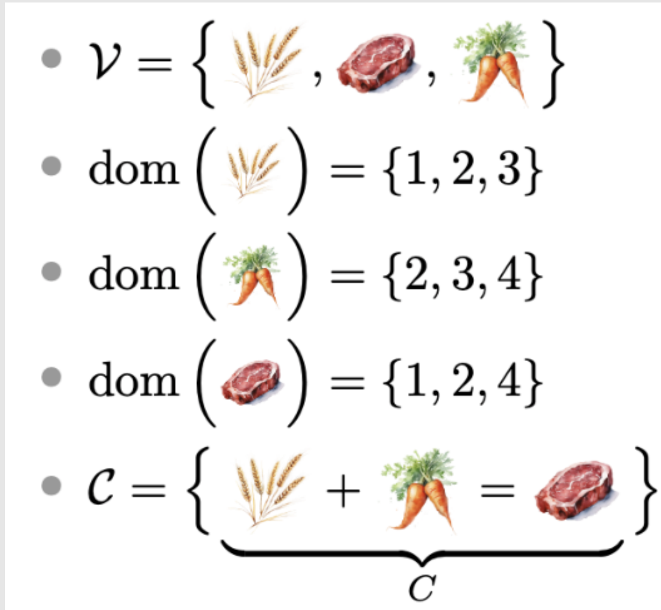


Figure 27

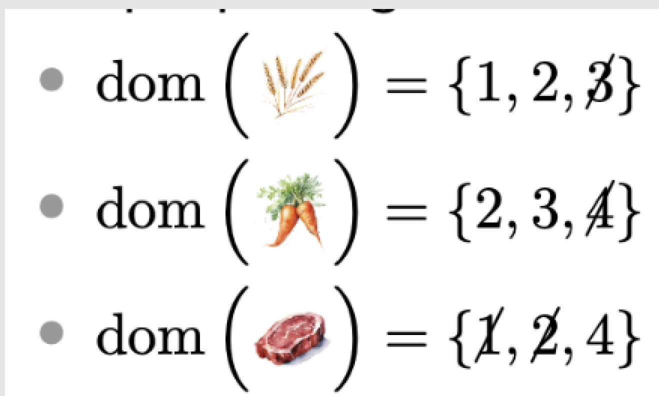


Figure 28

4 Probability Review

5 Learning Problems

Definition: In a learning problem, we assume that there is some (unknown) relationship,

$$f : \mathcal{X} \to \mathcal{Y}$$

s.t. $x \mapsto_f y$

Find $h : \mathcal{X} \to \mathcal{Y}$ (hypothesis) s.t. $h \approx f$, given some data about f:
- $\text{in}(\mathcal{D}) = \{x \text{ s.t. } (x, y) \in \mathcal{D}\}$
- $\text{out}(\mathcal{D}) = \{y \text{ s.t. } (x, y) \in \mathcal{D}\}$

5.1 Classification vs. Regression Problems

Definition:
- **Classification Problems:** $\mathcal{X} \subseteq \mathbb{R}^n$ and $\mathcal{Y} \subseteq \mathbb{N}$
- **Regression Problems:** $\mathcal{X} \subseteq \mathbb{R}^n$ and $\mathcal{Y} \subseteq \mathbb{R}$

5.2 Feature Spaces

Definition: It is often easier to learn relationships from high-level features (instead of the raw input).

5.3 Feasibility of Learning

Motivation: More than one function (hypothesis) may be consistent with the data.

Notes: So it may appear that finding the correct one should be impossible.

5.3.1 Probably Approximately Correct (PAC) Estimations

Example: Take N i.i.d. samples (i.e. take out a ball from an urn, record its color, and put it back in).
- $\nu \to \mu$ (empirical distribution $\to$ true distribution) as $N \to \infty$

5.3.2 Hoeffding's Inequality

Definition: Let μ denote the probability of an event, and ν denote its relative frequency in a sample size N. Then, for any $\epsilon > 0$,

$$P(|\nu - \mu| > \epsilon) \leq 2e^{-2\epsilon^2 N} \tag{1}$$

- ν: Relative frequency in the sample (known)
- μ: Probabillity of drawing a blue ball (unknown)
- $N \to \infty$: $\nu \to \mu$
- ϵ: How close we want ν to be to μ
- $\epsilon \to 0$: Probability will be 1
- $\epsilon \to \infty$: $\nu \to \mu$
- $\mu \overset{?}{\approx} \nu$: μ is probably approximately equal to nu.

Warning: We can approximate the true distribution with high probability by taking a large enough sample size, NOT guaranteeing that we can find the true distribution.

- Don't need to know where this theorem comes from.

Consider determining the class of a randomly chosen target point. If we ask a K-ary question about the points in $\mathcal{D}$

5.3.3 PAC Learning

5.4 Decision Trees

5.4.1 Structure of a Decision Tree

6 Probabilistic Inference Problems