Intro: **Random Experiment:** An outcome for each run.
Sample Space Ω: Set of all possible outcomes.
Event: Subsets of Ω.
Prob. of Event A: $P(A) = \frac{\text{Number of outcomes in A}}{\text{Number of outcomes in } \Omega}$
Axioms: $P(A) \geq 0 \; \forall A \in \Omega$, $P(\Omega) = 1$,
If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B) \; \forall A, B \in \Omega$
Cond. Prob. $P(A|B) = \frac{P(A \cap B)}{P(B)}$
$* \; P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$
Independence: $P(A|B) = P(A) \Leftrightarrow P(A \cap B) = P(A)P(B)$
Total Prob. Thm: If $H_1, H_2, \ldots, H_n$ form a partition of Ω, then $P(A) = \sum_{i=1}^{n} P(A|H_i)P(H_i)$.
Bayes' Rule: $P(H_k|A) = \frac{P(H_k \cap A)}{P(A)} = \frac{P(A|H_k)P(H_k)}{\sum_{i=1}^{n} P(A|H_i)P(H_i)}$
*Posteriori: $P(H_k|A)$, Likelihood: $P(A|H_k)$, Prior: $P(H_k)$
1 RV: **CDF:** $F_X(x) = P[X \leq x]$
PMF: $P_X(x_j) = P[X = x_j] \; j = 1, 2, \ldots$
PDF: $f_X(x) = \frac{d}{dx} F_X(x)$
*$P[a \leq X \leq b] = \int_a^b f_X(x)\, dx$ IS THIS CORRECT?
Cond. PMF: $P_X(x|A) = P[X = x|A] = \frac{P[X=x, A]}{P[A]}$ IS THIS CORRECT?
Cond. PDF: $f_X(x|A) = \frac{f_{X,A}(x,a)}{f_A(a)}$ IS THIS CORRECT?
Exp.: $E[h(X)] = \int_{-\infty}^{\infty} h(x) f_X(x) dx \mid \sum_{k=-\infty}^{\infty} h(k) f_X(k)$
Variance: $\text{Var}[X] = E[(X-\mu)^2] = E[X^2] - E[X]^2$
Cond. Exp.: $E[X|A] = \int_{-\infty}^{\infty} x f_X(x|A)\, dx$
2 RVs: **Joint PMF:** $P_{X,Y}(x,y) = P[X = x, Y = y]$
Joint PDF: $f_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y)$
*$P[(X,Y) \in A] = \int \int_{(x,y)\in A} f_{X,Y}(x,y)\, dx\, dy$
Expectation: $E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)\, dx\, dy$
Correlation: $E[XY]$
Covariance: $\text{Cov}[X,Y] = E[(X-\mu_X)(Y-\mu_Y)] = E[XY] - E[X]E[Y]$
Correlation Coeff.: $\rho_{X,Y} = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)\left(\frac{Y-\mu_Y}{\sigma_Y}\right)\right] = \frac{\text{Cov}[X,Y]}{\sigma_X \sigma_Y}$
Marginal PMF: $P_X(x) = \sum_{j=1}^{\infty} P_{X,Y}(x, y_j)$
Marginal PDF: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy$
Conditional PMF: $P_{X|Y}(x|Y) = P[X = x|Y = y] = \frac{P_{X,Y}(x,y)}{P_Y(y)}$
Conditional PDF: $f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$
Bayes' Rule $f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{f_{X|Y}(x|y) f_Y(y)}{\int_{-\infty}^{\infty} f_{X|Y}(x|y') f_Y(y')\, dy'}$
*$P_{Y|X}(y|x) = \frac{P_{X,Y}(x,y)}{P_X(x)} = \frac{P_{X|Y}(x|y) P_Y(y)}{\sum_{j=1}^{\infty} P_{X|Y}(x|y_j) P_Y(y_j)}$
Independent: $f_{X|Y}(x|y) = f_X(x) \; \forall y \Leftrightarrow f_{X,Y}(x,y) = f_X(x) f_Y(y)$
*If independent, then uncorrelated.
Uncorrelated: $\text{Cov}[X,Y] = 0 \Leftrightarrow \rho_{X,Y} = 0$
Orthogonal: $E[XY] = 0$
Conditional Expectation: $E[Y] = E[E[Y|X]]$ or $E[E[h(Y)|X]]$
*$E[E[Y|X]]$ w.r.t. X | $E[Y|X]$ w.r.t. Y.