ROB311 Quiz 2

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Contents

Probabilistic Inference Problems

1 Probability Review

1.1 Bayesian Network

Definition: Vertices represent random variables and edges represent dependencies between variables.

1.1.1 Junction

Definition: A **junction** consists of three vertices, X_1, X_2, and X_3, connected by two edges, e_1 and e_2:

- Both arrows pointing in one direction
- Both arrows pointing in opposite directions
- One arrow pointing in each direction

Warning: Want to look for causal relationships. Arrows, what's causing what, what's influencing what.

1.1.2 Causal Chain

Definition: A causal chain is a junction of the following form:

- X_1 and X_2 are dependent. X_2 is dependent on X_1. Vice versa. From a causal perspective, X_1 is influencing X_2. Subtle difference, just bc $X_1 \to X_2$.
- X_2 and X_3 are dependent.
- X_1 and X_3 are dependent.
  - Given X_2, X_1 and X_3 are independent. Why? X_2's door closes when you know X_2, so X_1 and X_3 are independent.

Warning: X_1 is influeincing X_2 and X_2 is influencing X_3.

1.1.3 Common Cause

Definition: A common cause is a junction of the form:

Notes:

- X_1 and X_3 are dependent.
  - Given X_2, X_1 and X_3 are independent. Why? X_2 whether you smoke or not, X_1 whether you have yellow teeth, X_3 whether you have lung cancer, if you don't know X_2, if they have yellow teeth, then they might smoke, then they might have lung cancer. If you know X_2, yellow teeth and lung cancer are independent b/c you already know if they smoke or not, and yellow teeth implies smoke,

1.1.4 Common Effect

Definition: A common effect is a junction of the form:

Notes:

- X_1 and X_3 are independent.
- Given X_2 or any of X_2's descendents, X_1 and X_3 are dependent.

Warning: Just b/c you don't know something about the middle variable, then it can be independent

Example: X_2 Grass being wet, X_1 raining, and X_3 sprinkler being on.

- If you know the grass is wet, you know that either the sprinkler is on or it's raining.
  - If it didn't have the sprinkler on, then it must have rained.
  - If it didn't rain, then the sprinkler must have been on.
  - So this means that X_1 and X_3 are dependent given X_2.
- If you don't know the grass is wet, then X_1 and X_3 are independent b/c you don't know if it rained or the sprinkler was on.

Example:

1. **Given:** Caveman is deciding whether to go hunt for meat. He must take into account several factors:
   - Weather
   - Possibility of over-exertion
   - Possibility encountering lion

   These factors can result in Cavemen's death. His decision will ultimately depend on the **chances** of his death.
2. **Binary Variables:**
   - $W = \{\text{Sun}, \text{Rainy}\}$: Weather
   - H: Whether the Cavemen goes hunting or not.
   - L: Whether the Cavemen encounters a lion or not.
   - T: Whether the Cavement is tired or not.
   - D: Whether the Cavemen dies or not
3. **Problem:** Cavemen must decide whether to go hunting or not.
   - He must consider the conditional probabilities (i.e. dependence) of each event.

Warning: Have to be discrete.

Example:
1. **Given:** Bayesian network.

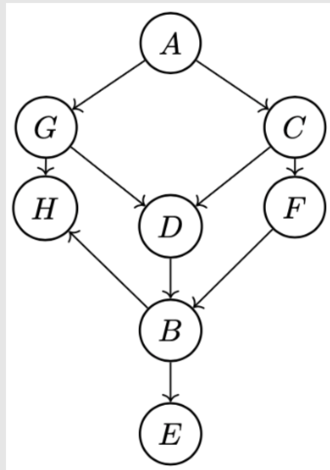


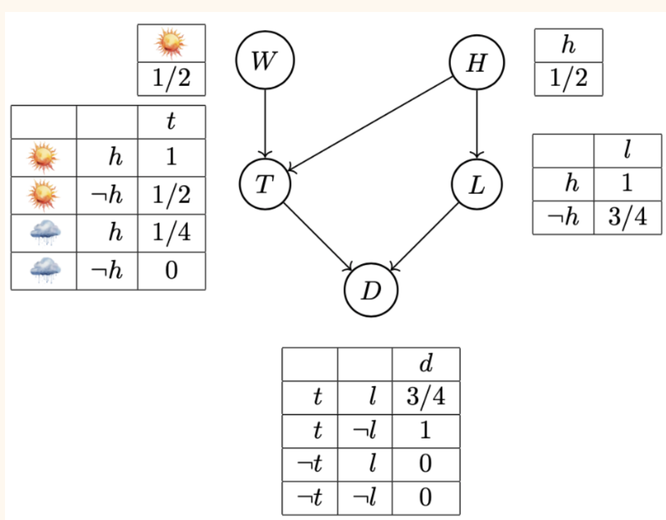
Figure 1

2. **Problem:** A and E are
   - independent if $\mathcal{K} =$
   - not necessarily independent for $\mathcal{K} =$

Process:
1.

Example: **Bayesian Inference**
1. **Given:**
2. **Problem:**

Process:
1. **Given:**



☀
1/2

		t
☀	h	1
☀	$\neg h$	1/2
🌧	h	1/4
🌧	$\neg h$	0

h
1/2

	l
h	1
$\neg h$	3/4

		d
t	l	3/4
t	$\neg l$	1
$\neg t$	l	0
$\neg t$	$\neg l$	0

Figure 2

Example: Inference via Sampling

1. **Given:**
2. **Problem:**