The Digital Music Synthesizer

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Abstract

The digital music synthesizer (DMS) is a system that generates and manipulates signals to create musical sound using digital signal processing (DSP) techniques (wikipedia).

1 Introduction

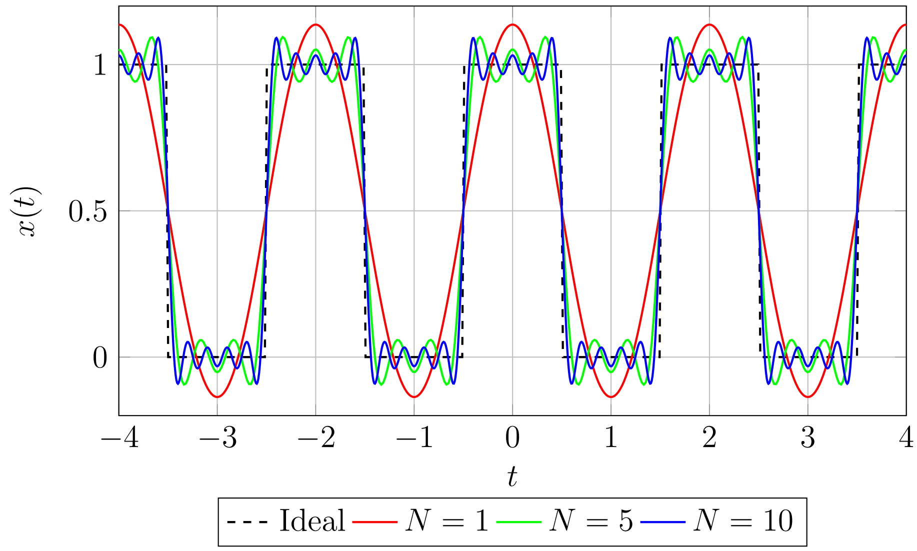
2 Mathematical Background

2.1 Common Waveforms for Digital Music Representation

The fundamental building blocks of digital music synthesis are the basic waveforms. These waveforms are used to create more complex sounds by combining them in various ways. The most common waveforms used in digital music synthesis are the sine wave, square wave, sawtooth wave, and triangle wave. For $x \in \mathbb{R}^{\mathbb{R}}$ (from mathworld and PS6)

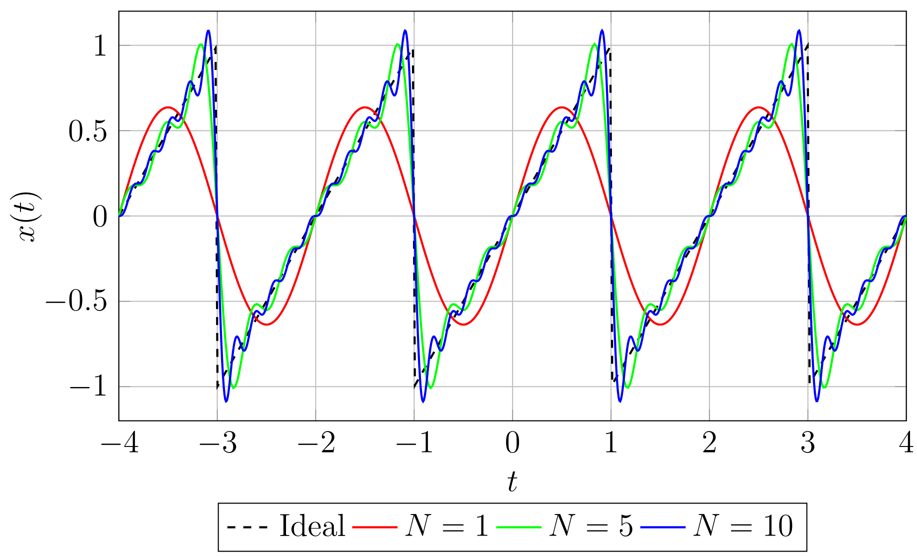
1. **Sine:**
$$x(t) = A\sin(2\pi ft + \phi) \tag{1}$$

2. **Square Wave:**
$$x(t) = \frac{1}{T} + \sum_{k\geq 1} \frac{2}{T} \operatorname{sinc}\left(\frac{k}{T}\right) \cos\left(2\pi \frac{k}{T} t\right) \tag{2}$$



3. **Sawtooth Wave:**

$$x(t) \sum_{k \geq 1} \frac{(-1)^{k-1} T}{\pi k} \sin\left(2\pi \frac{k}{T} t\right) \tag{3}$$



4. **Triangle Wave:**

$$x(t) = \sum_{k=1,3,5\ldots}^{\infty} \frac{8A}{\pi^2 k^2} (-1)^{(k-1)/2} \cos\left(2\pi \frac{k}{T} t\right) \tag{4}$$

- A: Amplitude of the waveform.
- f[Hz]: Frequency of the waveform.
- t: Time.
- ϕ[rad]: Phase offset.

- n: Harmonic number.
- T: Period of the waveform.

5. **Additive Synthesis:** Synthesizing sound by adding together N sinusoidal components by time-varying amplitude and frequency envelopes to get $y \in \mathbb{R}^{\mathbb{R}}$, (from Spectral audio signal processing III)

$$y(t) = \sum_{i=1}^{N} A_i(t) \sin(\theta_i(t)) \tag{5}$$

- $A_i(t) \in \mathbb{R}$: Amplitude of ith partial over time t
- $\theta_i(t) = \int_0^t \omega_i(t)dt + \theta_i(0) \in \mathbb{R}$
  - $\omega_i(t) = \frac{d\theta_i(t)}{dt} \in \mathbb{R}$: Radian frequency of ith partial vs. time.
  - $\phi_i(0) \in \mathbb{R}$: Phase offset of ith partial at time 0.

2.2 Fourier Analysis

From lecture

1. **Synthesis Equation (Inverse Fourier Transform):**

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\, df \tag{6}$$

2. **Analysis Equation (Fourier Transform):**

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt \tag{7}$$

2.3 Sampling

2.4 LTI Systems

From lecture

1. **Convolution Integral in Time Domain:**

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau \tag{8}$$

2. **Convolution Integral in Frequency Domain:**

$$Y(f) = X(f) \cdot H(f) \tag{9}$$

3 Digital Synthesis Pipeline

3.1 Waveform Generation

1. **Subtractive Synthesis:**

2. **Amplitude Modulation (AM) Synthesis:** From the theory and techniques of electronic music (Miller Puckette)

$$y(t) = A_c \left(1 + \frac{A_m}{A_c} \cos \omega_m t\right) \cos \omega_c t \tag{10}$$

3. **Frequency Modulation (FM) Synthesis:**

$$x(t) = A_c \sin\left[\omega_c t + \phi_c + A_m \sin(\omega_m t + \phi_m)\right] \tag{11}$$

- (A_c, ω_c, ϕ_c) : Specify the carrier sinusoid.
- (A_m, ω_m, ϕ_m) : Specify the modulator sinusoid.

3.2 Envelope Shaping

1. **Filtering:**

- **Low Pass Filter:** $H(j2\pi f) = \begin{cases} 1 & \text{if } |f| \leq f_c \\ 0 & \text{if } |f| > f_c \end{cases}$
- **High Pass Filter:** $H(j2\pi f) = \begin{cases} 1 & \text{if } |f| \geq f_c \\ 0 & \text{if } |f| > f_c \end{cases}$

3.3 Effects Processing

3.4 Digital-to-Analog Conversion

3.5 Example

4 Conclusion

Acknowledgement

References

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