CHE374 Cheatsheet

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1 Tips

Intuition:

2 Introduction and time value of money (PS1)

2.1 Interest

Definition: Money that is earned by investors (creditors/lenders) for allowing others (borrowers) to use their money.

2.2 Interest rate

Definition: The rate at which interest is earned (determined by risks):

$$i = \frac{I}{P} \tag{1}$$

- P: Principle amount (amount of money borrowed today)
- I: Total interest amount

2.3 Simple interest

Definition:

$$F_N = P + NPi = P(1 + Ni) \tag{2}$$

- F_N : Future amount in (time unit) N
  - N: Number of periods (e.g. years)
- **Key:** Applies only to the original principal.

Beginning of Period	Amount Lent	Interest Amount	Amount Owed at Period End
1	P	Pi	P+Pi
2	P+Pi	Pi	P+2Pi
3	P+2Pi	Pi	P+3Pi
…	…	…	…
N	P+(N-1)Pi	Pi	P+NPi = P(1+Ni)

Figure 1: Simple interest table.

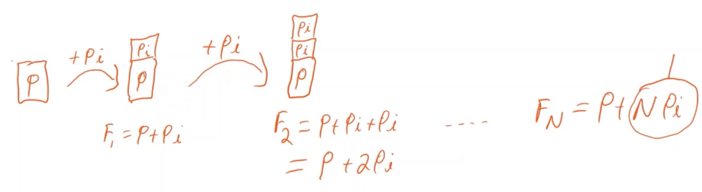


Figure 2: Derivation of simple interest.

2.4 Compound interest

Definition:

$$F_N = P(1 + i)^N \tag{3}$$

- **Key:** Applies to the principal and to all interest already accrued, so that you can earn interest on both.

Warning: Assume compound interest unless stated otherwise.

Beginning of Period	Amount Lent	Interest Amount	Amount Owed at Period End
1	P	Pi	P(1+i)
2	P(1+i)	P(1+i)i	$P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2i$	$P(1+i)^3$
…	…	…	…
N	$P(1+i)^{N-1}$	$P(1+i)^{N-1}i$	$P(1+i)^N$

Figure 3: Compound interest table.

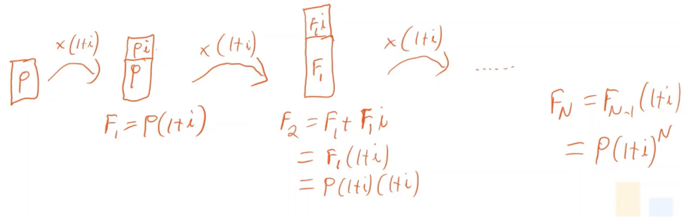


Figure 4: Derivation of compound interest.

2.5 Subperiod interest rate

Motivation: What if you can compound multiple times per year?

Definition: Fraction of the nominal interest rate:

$$i_s = \frac{r}{m} \tag{4}$$

- r: Nominal interest rate (usually for 1 year), which doesn't take compounding into account and is stated annually.
- m: Number of times compounded (subperiods) per year

2.6 Effective interest rate

Motivation: How would you compare investments with different compounding periods?

Definition: The equivalent interest rate if compounded only once over the stated time period (usually 1 year).

$$i_e = (1 + i_s)^m - 1 \tag{5}$$

- **Key:** Provides a measure of the annual interest cost, regardless of the compounding frequency. Whether interest is compounded monthly, quarterly, or continuously, the total amount of interest per year will be i_e.
  - r will be adjusted to ensure that the effective annual rate remains consistent.

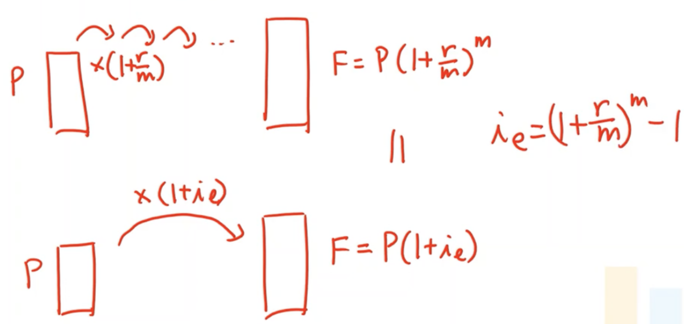


Figure 5: (Top) Subperiod interest rate. (Bot.) Equivalent interest rate if compounded only once per year. This is used to solve for the actual effective interest rate.

2.7 Continuous compound interest

Definition: The finite limit of i_e as the compounding period over one year becomes infinitesimally small:

$$i_e = \lim_{m\to\infty} \left(1 + \frac{r}{m}\right)^m - 1 = e^r - 1 \tag{6}$$

- **Key:** i_e increases as the compounding period decreases, but it reaches the finite limit eventually.
- **Careful:** Know when to use the continuous compounding and "regular" compounding formulas.

The general version over t years:

$$i_e = e^{rt} - 1 \tag{7}$$

2.8 Compound interest with subperiods

Definition:

$$F = P(1+i_s)^m = P(1+i_e) = Pe^{r_{cc}t} \tag{8}$$

- F: Future amount
- **Note:** For the same nominal interest rate, the more frequently you compound, the more you earn at the end of the year.
  - **Intuition:** You are collecting some of the interest along the way and reinvesting that back.

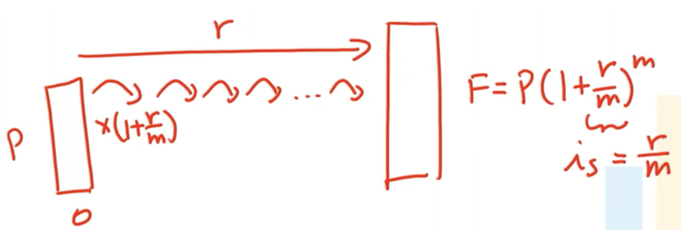


Figure 6: Compound interest in which there is interest that can occur within the nominal interest rate over m times.

2.9 L1 Takeaways

Intuition:
- Rates are always happening, analogous to $1m/s, km/h, mi/h$
- The only difference between the rates is just language. We are converting into different rates based on compounding and the period (but they are all the same).

Example: What is the final value of \$100 at an interest rate of 10% per year, compounded semi-annually?
1. **Break down the interest rate**: Since the interest is compounded semi-annually, the 10% annual interest rate is divided into two periods, each with a 5% interest rate per period.
2. **Calculate the future value after the first 6 months:**
$$\text{Value after 6 months} = 100 \times (1+0.05) = 100 \times 1.05 = 105.$$
3. **Calculate the future value after 1 year:** The interest is compounded again after another 6 months. The new value is obtained by applying another 5% interest on \$105:
$$\text{Value after 1 year} = 105 \times (1+0.05) = 105 \times 1.05 = 110.25.$$
4. **Final result:** The final amount is:
$$\boxed{110.25}$$

What is the effective annual interest rate, given a nominal rate of 10% per year, compounded semi-annually?
1. **Determine the effective annual interest rate**: The effective annual rate, r_{eff}, is the rate that gives the same final value if compounded annually. We solve for r_{eff} using:
$$100 \times (1 + r_{\text{eff}}) = 100(1+5\%)^2 = 110.25.$$
Solving for r_{eff}:
$$1 + r_{\text{eff}} = 1.1025 \quad \Rightarrow \quad r_{\text{eff}} = 0.1025 = 10.25\%.$$
2. **Final result**: Therefore, the effective annual interest rate is:
$$\boxed{10.25\%}$$

What is the continuously compounded interest rate, given a final value of \$110.25 after 1 year on an initial investment of \$100?

1. **Apply the continuously compounding formula**: For continuous compounding, the formula to find the future value is:
$$F = Pe^{r_{cc} \cdot t},$$
where:
   - F is the final amount (\$110.25),
   - P is the initial principal (\$100),
   - r_{cc} is the continuously compounded interest rate, and
   - t is the time in years (1 year in this case).

   Substituting the known values into the formula:
$$100e^{r_{cc} \cdot 1} = 110.25.$$
2. **Solve for r_{cc}**: Dividing both sides by 100, we get:
$$e^{r_{cc}} = 1.1025.$$
Taking the natural logarithm of both sides to isolate r_{cc}:
$$r_{cc} = \ln(1.1025) \approx 0.09758 = 9.758\%.$$
3. **Final result**: Therefore, the continuously compounded interest rate is:
$$\boxed{9.758\%/\text{year}}$$
   - t: Important for continuous compounding.

2.10 How to solve for equivalent interest rates?

Process:
1. Identify the variables and values using the $r_{x/y}$ notation.
2. Set the effective interest rates equal to each other using variables using the general formula or CC version.
   - If using the continuous compound version and are looking for something other than $\%/year$, then you convert it by doing a simple unit change in the exponent
3. Solve for the variable of interest.

Intuition: $r_{x/y}$
- x: How much is interest compounded (e.g. monthly compounding, daily compounding, quarterly compounding, etc)
- y: How long of a period (e.g. per year, monthly, per month, etc)

Warning:
- Assume for any rate to be per year unless stated otherwise.
- Assume interest is compounded once per period (unless specified)

2.11 General formula

Definition:
$$r_{y/y} = \left(1 + \frac{r_{n/m}}{n_m}\right)^{n_y} - 1 \tag{9}$$
- n_m: # of compounding periods n in time m.
- n_y: # of compounding periods n per year

Intuition:
- Dividing by n_m gives you the interest rate applied at each compounding period within a certain time frame m.
- Raising to the power of n_y accumulates that periodic interest over a year, resulting in the effective interest rate. This will always be due to the first part (i.e. if $r_{x/y}$ then it is based on x to determine how many times we compound it over that year)
- If $\left(1+r_{x/x}\right)^{n_x}$, then n_x is the number of compounding periods in one year. You don't need to divide since it is already the interest rate per compounding period. Therefore, the only thing you need to do is find the accumulative interest over a year by raising it to the number of compounding periods per year.

3 Cash-flow diagrams and equivalence (PS2)

3.1 Categories of Cash-flows

Terminology:
- **First (Capital) Cost:** Expense to build/buy and install.
- **Revenues (Sales):** Receipts from sale of products or services.
- **Operation and Maintenance (O&M) Costs:** Expenses that are incurred on a regular basis.
- **Overhaul:** Major (capital) expenditure that occurs part way through the life of an asset.
- **Salvage Value:** Net receipt at project termination for sale/disposal of equipment.
- **Scrap value:** Value in materials of which item is made.
- **Disposal costs:** Costs to dispose of waste.
- **Project life-cycle costs:** Costs that occur at the start, during or end of a project.

3.2 Cash-Flow diagrams

Definition: A simple graph that summarizes the **timing** and **magnitude** of cash-flows.
- **X-axis:** Discrete time periods
- **Y-axis (implicit):** Size and direction of cash-flow.
- **Individual cash-flows (arrows):**
  - DOWN arrow is cash OUTFLOW (disbursements)
  - UP arrow is CASH inflow (receipts)

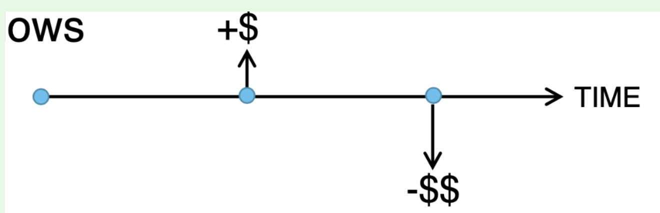


Figure 7: Cash flow diagrams.

3.2.1 The end of one period is the beginning of the next

Definition: N periods from now is end of period N and start of period $N+1$.

3.2.2 Conventions and Assumptions

Definition:
- **First costs:** Typically accounted for at *time 0*.
- **Cash-flows:**
  - Assumed to occur at the *end of the period*.
  - Occuring during the period are *summed* and accounted for at the *end of the period*.

- **Interest:** Compounded once per period (unless specified)

3.3 Types of cash flows

Definition:

- **Single payment(s) (receipts):** One-time cash flow at some time or period

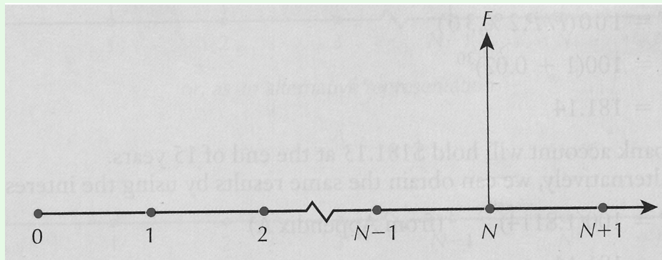


Figure 8: Single payment(s) / receipts

- **Perpetuity:** Cash flow of magnitude A that occurs at regular intervals until perpetuity (forever)

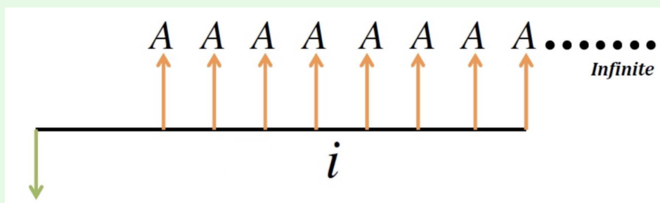


Figure 9: Perpetuity

- **Annuity (uniform payments or receipts):** Cash flow of magnitude A that occurs in regular intervals for N periods

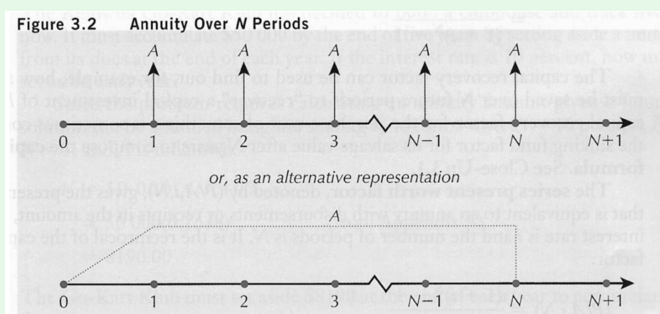


Figure 10: Annuity

- **Arithmetic gradient:** Cash flow of magnitude A in the first period that grows incrementally each period with magnitude G up to N periods, where A or G can be positive or negative.
  - Period 1: Mag $= A$
  - Period 2: Mag $= A + G$
  - Period 3: Mag $= A + 2G$
  - Period N: Mag $= A + (N-1)G$

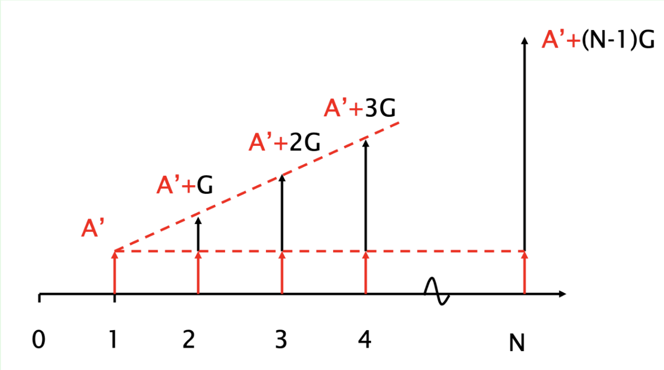


Figure 11: Arithmetic gradient with positive A.

- **Geometric gradient:** Cash flow of magnitude A in the first period that grows at a rate G for each period, where A or G can be positive or negative.
  - Period 1: Mag $= A$
  - Period 2: Mag $= A(1+G)$
  - Period 3: Mag $= A(1+G)^2$
  - Period 4: Mag $= A(1+G)^{N-1}$

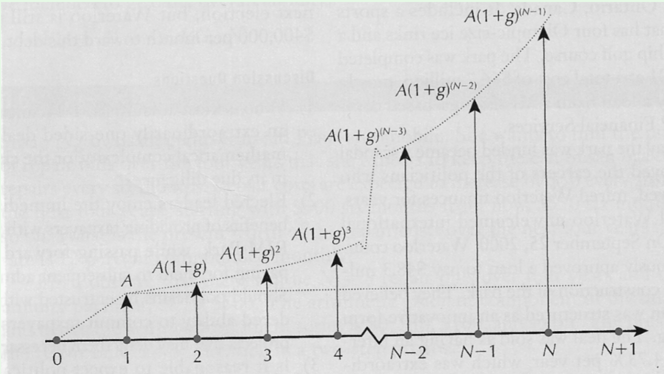


Figure 12: Geometric gradient series for receipts with positive growth

3.4 Equivalence

Definition:
- **Mathematical Equivalence:** A consequence of the mathematical relationship between time and money.
- **Market Equivalence:** A consequence of the ability to exchange one cash-flow for another at zero cost.
- **Decisional Equivalence -** Due to indifference on the part of the decision maker among available choices.

3.4.1 Mathematical equivalence

Definition: Equivalence in terms of future:
- Two cash-flows, P_t at time t and F_{t+N} at time $t+N$, are mathematically equivalent w.r.t. interest rate i, if:

$$F_{t+N} = P_t(1+i)^N \tag{1}$$

- If F_{t+N+M} (where M is a second number of periods) is equivalent to P_t, then:

$$F_{t+N+M} = P_t(1+i)^{N+M} \tag{2}$$

$$F_{t+N+M} = F_{t+N}(1+i)^M \tag{3}$$

Equivalence in terms of present

$$P_t = \frac{F_{t+N}}{(1+i)^N} \tag{10}$$

$$P_t = \frac{F_{t+N+M}}{(1+i)^{N+M}} \tag{11}$$

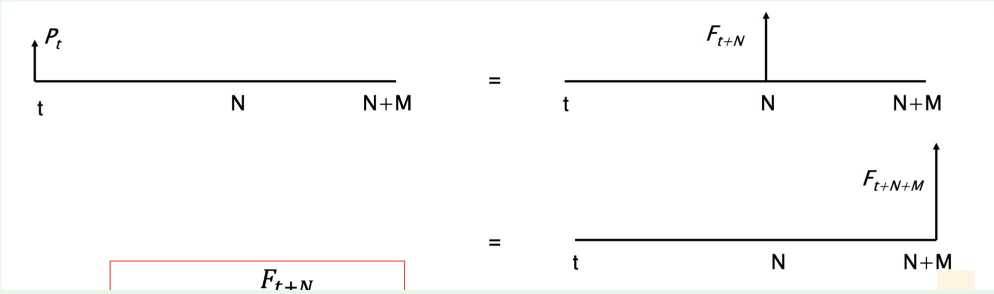


Figure 13: Mathematical equivalence.

3.4.2 Market equivalence

Definition: One can exchange cash-flows between present and future amounts:
- **Borrowing**: Exchanging a future cash-flow for a present one.
- **Lending/Investing**: Giving up a current cash-flow for a future one.

3.4.3 Decisional equivalence

Definition:
- For a decision maker, two cash-flows, P_t at time t and F_{t+N} at time $t+N$, are equivalent if they are indifferent between the two.
- Here, the interest rate is not a prior information.
- **Implied** interest rate relating to P_t and F_{t+N} can be calculated from the decision that the cash-flows are equivalent.

3.5 Calculate the present value of (regular) cash-flows

3.5.1 A factor approach

Definition: We assume equivalence:
1. Interest is compounded once per period.
2. Cash-flow occurs at the end of the period.
3. Time 0 is period 0 or the start of period 1.
4. All periods are the same length.

3.5.2 Equivalence Factors

Definition:

$$(X/Y, i, N) \text{ reads: What is } X \text{ given } Y, i, N$$

3.5.3 Factor notation

Definition:

- (X/Y, i%, N)
  - X and Y are chosen from the cash-flow symbols P, F, A, G, and Geom.
- If you have Y multiplied by a factor, you get the equivalent value of X.
  - e.g. $P = F(P/F, i, N)$.
- Convert from a present value P to a future cash-flow F in year N, then:

$$F = P(F/P, i, N)$$

- For the geometric gradient:

$$P = G(P/G, i, g, N) \quad \text{or} \quad P = G(P/\text{Geom}, i, g, N)$$

3.5.4 Economic equivalence factors

Definition: Equivalence factors are used to convert between different types of cash flows.

- $(F/P, i, N)$ **Compound amount factor**
- $(P/F, i, N)$ **Present worth factor**
- $(A/F, i, N)$ **Sinking fund factor**
- $(F/A, i, N)$ **Series compound amount factor**
- $(A/P, i, N)$ **Capital recovery factor**
- $(P/A, i, N)$ **Series present worth factor**
- $(P/G, i, N)$ **Arithmetic gradient to present worth**
- $(P/\text{Geom}, i, g, N)$ **Geometric gradient to present worth**

3.5.5 Relationship among factors (Invertibility):

Definition:

$$(X/Y,\, i,\, N) = \frac{1}{(Y/X,\, i,\, N)} \tag{12}$$

3.5.6 Compound amount factor:

Definition:

$$(F/P,\, i,\, N) = (1+i)^N \tag{13}$$

3.5.7 Present worth factor:

Definition:

$$(P/F,\, i,\, N) = \frac{1}{(1+i)^N} \tag{14}$$

3.5.8 Present value of a perpetuity (no factor):

Definition:

$$P = \frac{A}{i}, \quad A = Pi \tag{15}$$

- **Note:** Present value of this perpetuity is finite.

3.5.9 Series present worth factor:

Definition:

$$(P/A,\, i,\, N) = \left[\frac{1}{i} - \frac{1}{i(1+i)^N}\right] = \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \tag{16}$$

3.5.10 Present value of an arithmetic gradient:

Definition:

$$P = A\,(P/A,\, i,\, N) + G\,(P/G,\, i,\, N) \tag{17}$$

- Initial annuity A that is constant starting at $t = 1$
- Growth value G grows arithmetically starting at $t = 2$

$$(P/G,\, i,\, N) = \frac{1}{i^2}\left(1 - \frac{1+iN}{(1+i)^N}\right)$$

- Assumes no cash-flow at time 0.
- 4 possibilities besides $G = 0$
  1. $A > 0$ and $G > 0$ - means positive and increasing.
  2. $A > 0$ and $G < 0$ - means positive but decreasing.
  3. $A < 0$ and $G > 0$ - means negative but becoming less so.
  4. $A < 0$ and $G < 0$ - means negative and becoming more so.

3.5.11 Present value of a geometric series:

Definition:

$$(P/Geom,\, i,\, g,\, N) = \frac{1}{1+g}\left(P/A,\, i^0,\, N\right) \tag{18}$$

- $i^o = \dfrac{1+i}{1+g} - 1$

OR:

$$(P/Geom,\, i,\, g,\, N) = \frac{1 - \left(\frac{1+g}{1+i}\right)^N}{i-g} \tag{19}$$

- Growth rate: g

3.6 How to perform cash-flow analysis

Process:

1. Draw a cash-flow diagrams (optional)
2. Identify variables with corresponding values.
3. Find the interest rates that are appropriate for the factors.

Intuition:

- If you are looking for X given Y, but you have Z given X and Y given Z, then multiply as $(X/Y) = (Z/X)(Y/Z)$
- Use invertibility when you can.
- To discount a value back to the present value, multiply by $(P/F, i, N)$ where N is the amount of time you want to discount back.

3.6.1 Examples

Example: Claudia wants to deposit an amount P now such that she can withdraw an equal amount of $\$2,000$ each year for the first 5 years and then $\$3,000$ for the following 3 years. Calculate P if the interest earned is 8% per year.

1. **Phase 1:** Calculate the present value of the first set of withdrawals ($2,000$ for 5 years). The present value of an annuity is given by the formula:
$$P_1 = 2000 \times (P/A, 8\%, 5)$$
Where $(P/A, 8\%, 5)$ is the annuity factor for 5 periods at 8
2. **Phase 2:** Calculate the present value of the future withdrawals ($3,000$ for 3 years), but first, we need to compute the future value at the end of year 5. The future value of these withdrawals at the end of year 5 is:
$$F_5 = 3000 \times (P/A, 8\%, 3)$$
3. Now, discount F_5 back to the present (time 0) using the present value of a single sum formula:
$$P_2 = F_5 \times (P/F, 8\%, 5)$$
Where $(P/F, 8\%, 5)$ is the present value factor for 5 periods at 8
4. **Total Present Value:** The total present value P is the sum of the present values from phase 1 and phase 2:
$$P = P_1 + P_2$$

We calculate F_5 (the future value at the end of year 5) and then discount it to find P_2 for the following reasons:

- **Why Calculate F_5?**
The \$3,000 withdrawals start in year 6. To account for these future payments, we first determine their value at the end of year 5, denoted as F_5, since these withdrawals begin after year 5.
- **Why Discount F_5 to Find P_2?**
Once F_5 is known, we discount it to the present (year 0) to find P_2, which represents the portion of the deposit needed today to fund the future withdrawals.
- **Summary:**
  1. Calculate F_5: The future value of the \$3,000 withdrawals at the end of year 5.
  2. Discount F_5: Convert it to P_2 using present value factors.
- **Why Not Use a Single Formula for \$3,000 Withdrawals?**
Since the \$3,000 withdrawals start after year 5, their value at year 0 differs from the value of the earlier \$2,000 withdrawals. Directly using a single annuity formula for the \$3,000 withdrawals would neglect the fact that these payments occur later.

4 Cash-flow analysis: Bonds (PS3)

4.1 Mortgage Terms

Terminology:

1. **Principle:** The amount of money you borrow to pay for a real property.
2. **Down Payment:** The fraction of the cost of the real property that you pay upfront yourself. (Usually 20%)
3. **Loan-to-Value Ratio (LTV):** Ratio of mortgage loan to value of the property.
4. **Mortgage Rate:** The interest rate charged on the mortgage. Compounding period usually matches frequency of payments.
5. **Amortization Period:** Time horizon for mortgage payment.
6. **Term:** Duration of time where the mortgage rate is fixed. When term ends, re-evaluate how much you still owe, then use new interest rate to calculate monthly payment based on time left in amortization period.

4.2 Net amount owed at end of term:

Definition:

$$\begin{aligned} \text{Net} &= P\left(F/P,\, \frac{i}{N},\, t \times N\right) - A\left(F/A,\, \frac{i}{N},\, t \times N\right) \\ &= P(1+i)^{t\times N} - A\left[\frac{(1+i)^{t\times N}-1}{i}\right] \end{aligned} \tag{20}$$

- P: Mortgage principle
- A: Regular mortgage payment (usually per month)
- i: Mortgage rate per annum based
- N: Number of payment periods per year
- t: Number of years in term

4.3 Net monthly payment:

Definition:

$$A = P\left(A/P,\, \frac{i}{N},\, t \times N\right) = A\left[\frac{i(1+i)^{t\times N}}{(1+i)^{t\times N}-1}\right] \tag{21}$$

- P: Mortgage principal (or what is left)
- A: Regular mortgage payment (usually per month)
- i: Mortgage rate per annum
- N: Number of payment periods per year
- t: Number of years in amortization (or what is left)

4.4 Bond Terms

Terminology:

- **Bond:** A type of loan where the creditor pays a stated amount at specified intervals for a defined period (*Coupon Payments*), plus a final amount at a specified date (*Face Value*).
- **Coupon Rate:** The rate used to calculate coupon payments.
- **Coupon Payments:** Regular payments made over the course of a bond's lifetime. Amount is determined by coupon rate and frequency of payment (per the same time unit as the coupon rate).

$$\text{Coupon Amount} = (\text{Coupon Rate}) \times \frac{\text{Face Value}}{\text{Payment Frequency}} \tag{22}$$

- **Yield:** Hypothetical interest rate of a bond given a purchase price. Solved using interpolation.
- **Bond Price:**

$$\begin{aligned} P &= A\left(P/A,\, \frac{i}{m},\, N\right) + F\left(P/F,\, \frac{i}{m},\, N\right) \\ &= A\left[\frac{(1+\frac{i}{m})^N - 1}{\frac{i}{m}(1+\frac{i}{m})^N}\right] + F\left[\frac{1}{(1+\frac{i}{m})^N}\right] \end{aligned} \tag{23}$$

- i: Yield
- m: Frequency of coupon payments per time unit (e.g. year)
- N: Number of periods to maturity ($m \times$ time unit)
- A: Value of coupon payment

5 Risk, reward, and arbitrage (PS4)

5.1 Terms

Terminology:

- **Valuation:** Analytical process of determining future cash flows.
- **Financial Risk:** Uncertainty in a future payoff.
- **Variance of Returns:** The variance in the rate of return from a vector of return rates of a given stock, company, portfolio, etc.

$$\sigma_i^2 = \mathrm{Var}(\vec{R}_i), \quad \vec{R}_i = \begin{bmatrix} r_{t_1} \\ r_{t_2} \\ \vdots \\ r_{t_m} \end{bmatrix}$$

- **Volatility (σ_i):** The standard deviation of the variance of return; is a form of risk (and therefore uncertainty).
- **Market Portfolio (MP):** Portfolio representing the whole market, often estimated through a stock index.
- **Systematic Risk:** Risk associated with the market as a whole, e.g. effect of economy on sales and stock value.
- **Idiosyncratic Risk:** Risk independent of the economy and specific to a company.
- **Arbitrage:** Taking advantage of a price difference between two or more markets to achieve a risk-free gain.
- **Forward Contract (Forwards):** An obligation to buy or sell a certain asset:
  - At a specified price
  - At a specified time
- **Futures Contracts:** Similar to forwards except settled daily (not just at maturity), so they can be bought and sold, and are also traded on exchanges.

5.2 Capital Asset Pricing Model (CAPM)

Definition:

$$E[R_c] = r_f + \beta_c \left(E[R_{mp}] - r_f\right) \tag{24}$$

- $E[R_c]$: Expected rate of return for a company
- r_f: Risk-free rate
- β_c: Measure of risk for the company, systematic risk, related to market risk
- $E[R_{mp}]$: Expected rate of return of the market portfolio, represents the whole market

Relationships for β:

$$\beta_i = \frac{\sigma_{i,MP}}{\sigma_{MP}^2} = \rho_{i,MP}\frac{\sigma_i}{\sigma_{MP}} \tag{25}$$

- $\sigma_{i,MP}$: Covariance between i^{th} company and market portfolio (MP)
- σ_{MP}^2: Variance of MP
- $\rho_{i,MP}$: Correlation of returns between i^{th} company and MP
- σ_i: Volatility of i^{th} company
- σ_{MP}: Volatility of MP

5.3 Replication (FIX)

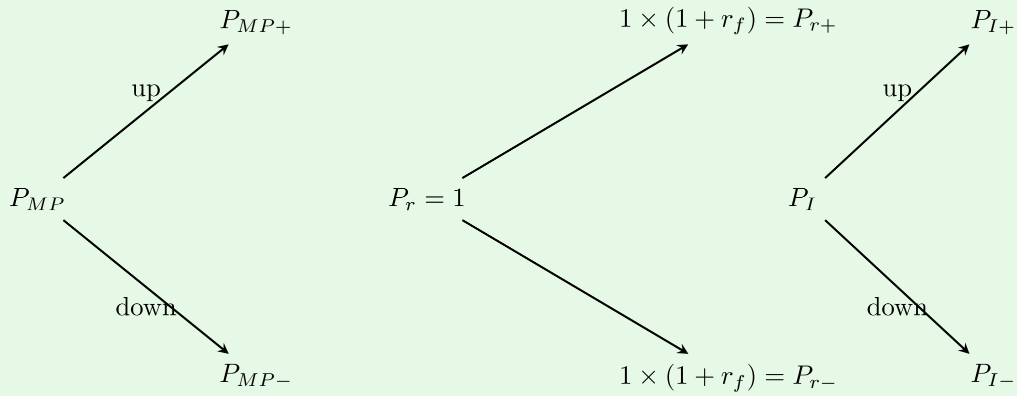
Definition:

- Given present market price P_{MP} will take on certain values if it goes up or down: P_{MP+}; P_{MP-}
  - Probability of market going up is X, down is $(X-1)$ (interchangeable)
- Given project/investment/etc. price P_I will take on certain values depending on if market goes up or down: P_{I+}; P_{I-}

Want to find present value of project:

- Risk-free rate is r_f

The following networks are Market, R_f, and Project:



- Replication wants to model a portfolio with the same risk-value as the project:
  - a: # of MP shares
  - b: # of risk-free shares/bonds/etc.

$$\begin{aligned} &\text{Up:} \quad aP_{MP+} + bP_{r+} = P_{I+} \\ &\text{Down:} \quad aP_{MP-} + bP_{r-} = P_{I-} \end{aligned} \quad \text{Solve for } a, b$$

$$P_I = a \times P_{MP} + b \times P_r \quad \text{with} \quad P_r = 1$$

- Can also find β:

$$E[R_{MP}] = \frac{P_{MP+}X + P_{MP-}(X-1)}{P_{MP}}$$

$$E[R_I] = \frac{P_{I+}X + P_{I-}(X-1)}{P_I}$$

$$\therefore \quad \beta = \frac{E[R_I] - r_f}{E[R_{MP}] - r_f}$$

5.4 Forward Rate

Definition: Given rates for investments between $t = 0$ and $t = t_1$, or $t = t_2$:

$$r_{0,t_1}, \quad r_{0,t_2}$$

The interest forward rate t_1 years from $t = 0$ for a duration of $(t_2 - t_1)$ years is:

$$r_{t_1,t_2} = \frac{r_{0,t_2} \cdot t_2 - r_{0,t_1} \cdot t_1}{t_2 - t_1} \tag{26}$$

6 Comparison methods - PW, AW, FW (PS5)

6.1 Terminology

Terminology:

- **Independent:** Expected costs and benefits of each project do not depend on whether or not the other one is chosen.
- **Mutually Exclusive:** When one project is chosen, all the others are excluded.
- **Related but not Mutually Exclusive:**
  - Not mutually exclusive: You can select more than one (budget permitting).
  - Related: Selecting one may affect the selection of another option.
- **MARR:** Minimum acceptable rate of return/hurdle rate

  - The "do nothing" option, i.e. the rate of return if you were to not invest in a project.
  - Type of discount rate.
- **Present Worth (PW):** Present value of benefits minus costs, discounted at MARR.
  - The amount by which a project is beating the best alternative expressed in today's value.
  - $PW > 0$: Acceptable
  - $PW < 0$: Unacceptable
- **Annual Worth (AW):** The equivalent annuity of PW, with MARR as discount rate.
  - $AW > 0$: Acceptable
  - $AW < 0$: Unacceptable

6.2 Evaluating mutually exclusive projects

Process:

1. Define the time horizon.
2. Develop cash flows for each alternative.
3. Calculate the PW using MARR.
4. Compare the PWs and pick the best.
   - Higher PW is better.

6.3 Comparing different lives

Example:

6.3.1 Repeated lives - PW

- Assume project repeats itself, and use least common multiple as time horizon.
  - Repeats with same cash flow.
- Compare PW at end of time horizon

6.3.2 Repeated lives - AW

- Compare equivalent annuity of PW for individual lives.
  - AW is equivalent annuity, so magnitude of annuity stays the same when the project is repeated.

6.3.3 Study period

- Specify a time period for comparison.
- For projects that last longer than study period, assume can terminate them early and adjust salvage value if necessary.
  - Calculate PW for new period of affected projects.
- Uses fixed time horizon.

7 Comparison methods - IRR (PS6)

7.1 Comparison methods terminology

Terminology:

- **Internal Rate of Return (IRR):** The discount rate at which the present worth of a project is equal to 0.
  - More profitable projects have higher IRR.
- **Simple Investment:** When all negative cash flows occur before all positive cash flows.
  - Can have multiple IRR when cash flows are not simple.
- **Payback Period:** Time it takes for the sum of revenues/savings to equal the initial investment.
- **Discounted Payback Period:** Time it takes for the sum of present worth (PW) of revenues/savings to equal the initial investment.
- **Incremental IRR:** Evaluates the difference (increment) between two mutually exclusive alternatives.

- **De Facto MARR:** The IRR of the project that, when summing the financial commitments (FC) of all projects by highest IRR, is the last to be taken on before exceeding the allotted budget.

7.2 IRR calculation

Process:

Analytical method

1. Write PW equation for all cash flows using explicit formulas for cash flow factors, leaving discount rate as i.
2. Enter the equation into Desmos, iteratively solve for $i > 0$ that makes PW equal to 0.
3. If $IRR > MARR$, project is worthwhile.
4. If $IRR < MARR$, project is not worthwhile.

Excel method

1. Enter cash flows into Excel starting at year 0.
2. Use IRR function: $= IRR(cashflows, i\% \quad guess)$
3. If $IRR > MARR$, project is worthwhile.
4. If $IRR < MARR$, project is not worthwhile.

7.3 Payback period calculation

7.3.1 Non-Discounted

Definition:

1. Find the year when the sum of revenues/savings equals the initial investment.
2. If between periods, interpolate:

$$\frac{N - y_1}{y_2 - y_1} = \frac{FC - c_1}{c_2 - c_1} \Rightarrow N = (y_2 - y_1)\frac{FC - c_1}{c_2 - c_1} + Y_1 \tag{27}$$

- N: Payback period (a decimal value when interpolating)
- FC: First cost/initial investment (positive)
- y_1: Lower bound of the period of interpolation ($y_1 \leq y_2$)
- y_2: Upper bound of the period of interpolation
- c_1: Cumulative sum of revenues/savings at period $y_1 (c_1 \leq c_2)$
- c_2: Cumulative sum of revenues/savings at period y_2

7.3.2 Discounted

Definition:

1. Discount revenue at year y_N by $(P/F, i, N)$.
2. Add discounted revenue to cumulative discounted revenue of the previous year; the sum is the cumulative discounted revenue in year y_N.
3. Visually identify the discounted payback period or interpolate as seen in the Non-Discounted section.

7.4 Incremental IRR

Process:

1. Order alternatives in increasing order of FC (First cost).
2. Start with the "do-nothing" alternative.
3. Using ΔFC and ΔA between the current choice and the option being evaluated as the FC and A, if the resultant $IRR > MARR$, switch to that alternative as the reference.
4. Repeat step 3 for the rest of the options.
5. Final choice is the most profitable project.

8 Depreciation, Financial accounting (PS7)

8.1 Depreciation terms and variables

Terminology:

- **Depreciation:**
  1. Diminish in value over time.
  2. Reduce the recorded value of an asset over a predetermined period.
     - Not a cash flow!
- **Cost Basis:** The value against which depreciation is measured. Usually based on First Cost.
- **Market Value:** Actual value of an asset if sold in a free market. Usually cannot be observed until the item is actually sold.
- **Book Value:** The value calculated for accounting purposes according to an agreed-upon model.
- BV_t**:** Book value at time t (end of year)

$$BV_t = BV_{t-1} - D_t = BV_0 - \sum_{k=1}^{t} D_k \tag{28}$$

- **B:** Basis, AKA first cost, original purchase price.

$$B = BV_0$$

- **S:** Salvage value, AKA selling cost, market value (not always interchangeable).
- D_t**:** Depreciation in year t.
- **N:** Depreciable life of the asset. Not necessarily equal to the useful life.
- **d:** Proportion of asset value lost to depreciation.
  - See Declining Balance Method.
- **Loss on Disposal:** One-time additional depreciation value. Accounts for lower salvage value than predicted by depreciation.
- **Recaptured Depreciation:** One-time negative depreciation value. Accounts for higher salvage value than predicted by depreciation.
  - If higher than cost basis, the difference between market value and salvage value is called capital gain, and the difference between predicted value and cost basis is recaptured depreciation.

8.2 Reasons for depreciation

Definition:

Asset Deterioration

- **Use-Related Physical Loss:** As something is used, the more it/its parts wear out. AKA "wear and tear."
- **Time-Related Physical Loss:** Even if not used, things will deteriorate over time due to natural or other factors.

Asset Obsolescence

- **Functionally-Related Loss:** Loss that occurs without physical changes.
  - E.g. Car styles may change, computers become more powerful.

8.3 Straight line method

Definition:

$$D_t = \frac{B - S}{N} \tag{29}$$

$$BV_t = B - tD_t = B - t\left(\frac{B - S}{N}\right) \tag{30}$$

8.4 Declining balance method

Definition:

$$D_t = (BV_{t-1}) \cdot d \tag{31}$$

$$BV_t = BV_{t-1} - (BV_{t-1}) \cdot d = B(1-d)^t \tag{32}$$

Rate Selection

$$S = B(1-d)^N \tag{33}$$

$$d = 1 - \sqrt[N]{\frac{S}{B}} \tag{34}$$

Double Declining Balance: Double what the straight-line method would have been.

$$d = \frac{2}{N} \tag{35}$$

8.5 Sum of years' digits (SOYD)

Definition:

- Faster than straight line during early years, slower than straight line during later years.
- Arbitrarily divides depreciation into chunks.

$$SOYD = \sum_{k=1}^{N} k \tag{36}$$

$$D_t = \frac{N-t+1}{SOYD} \cdot (B-S) \tag{37}$$

$$BV_t = BV_{t-1} - D_t \tag{38}$$

8.6 Unit of production method

Definition: Assumes depreciation is a function of equipment use rather than time

$$D_t = \frac{\text{production in year } t}{\text{lifetime production}} \cdot (B-S) \tag{39}$$

$$BV_t = BV_{t-1} - D_t \tag{40}$$

8.7 CCA depreciation

Definition:

Capital Cost Allowance (CCA): Amount depreciated in a given year

$$CCA_N = \text{CCA Rate} \times \left(\frac{1}{2}\text{This year's addition} + UCC_{N-1}\right) \tag{41}$$

- UCC_{N-1}: UCC from last year

Undepreciated Capital Cost (UCC): Book value in a given year

$$UCC_N = UCC_{N-1} + \text{This year's addition} - CCA \tag{42}$$

8.8 Re-evaluated service life

Definition: If service life differs from that assumed, re-evaluate service life and depreciate at faster or slower rate from then on.

- Do not change previous book values

9 Financial accounting (PS8)

9.1 Liquidity ratios

Definition:
Current Ratio: Measures the company's ability to meet short-term debt obligations, paying current liabilities with current assets.

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}} \tag{43}$$

- Higher the ratio the more current assets available to pay off current debt.
- Numbers below 1 could be a sign of concern.

Acid Test Ratio: Shows company's ability to pay off debts if all of them were due immediately.

$$\text{Acid-Test Ratio} = \frac{\text{Cash} + \text{Short-term Investments} + \text{Net current receivables}}{\text{Current Liabilities}} \tag{44}$$

9.2 Efficiency ratios

Definition:
Inventory Turnover: Measure of the number of times the average level of inventory is sold during the year.

$$\text{Inventory Turnover} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory over Period}} \tag{45}$$

- A high number indicates an ability to quickly sell inventory.

Days' Inventory: Measures speed at which inventory is sold.

$$\text{Days' Inventory} = \frac{\text{Average Inventory}}{\left(\frac{\text{Cost of Goods Sold}}{365}\right)} \tag{46}$$

- 365 is 1 year period.
- Lower value indicates more efficient operation.

Accounts Receivable Turnover: Measures how quickly a company collects money from its customers; its ability to collect cash from credit customers.

$$\text{Accounts Receivable Turnover} = \frac{\text{Net Credit Sales}}{\text{Average Net Accounts Receivable}} \tag{47}$$

Alternatively, can use total sales:

$$\text{Accounts Receivable Turnover} = \frac{\text{Total Sales}}{\text{Average Net Accounts Receivables}} \tag{48}$$

Days' Receivables: Number of days that an invoice is outstanding before payment is collected.
- Inverse of receivables turnover multiplied by number of days in period being analyzed.

$$\text{Days' Receivables} = \frac{\text{Average Receivables}}{\left(\frac{\text{Sales}}{365}\right)} \tag{49}$$

- 365 is 1 year period.

9.3 Leverage ratios

Definition:
Debt Ratio: Proportion of assets financed with debt.

$$\text{Debt Ratio} = \frac{\text{Total Liabilities}}{\text{Total Assets}} \tag{50}$$

Debt to Equity Ratio:

$$\text{Debt Equity Ratio} = \frac{\text{Total Liabilities}}{\text{Total Equity}} \tag{51}$$

Equity Ratio:

$$\text{Equity Ratio} = \frac{\text{Equity}}{\text{Total Assets}} \tag{52}$$

Times Interest Earned: Measures the number of times that operating income can cover interest expenses.
- Operating income is after operating expense.

$$\text{Times Interest Earned} = \frac{\text{Operating Income}}{\text{Interest Expense}} \tag{53}$$

Alternatively, use earnings before tax and income (EBIT) instead of operating income:

$$\text{Times Interest Earned} = \frac{\text{EBIT}}{\text{Interest Expense}} \tag{54}$$

9.4 Profitability ratios

Definition:
Profit Margin: Percentage of each sales dollar earned as net income.

$$\text{Profit Margin} = \frac{\text{Net Income}}{\text{Net Sales}} \tag{55}$$

Return on Assets (ROA): Measures how well a company is making money based on all the finance resources committed to the firm.

$$\text{ROA (First Form)} = \frac{\text{Net Income}}{\text{Average Assets}} \tag{56}$$

$$\text{ROA (Second Form)} = \frac{\text{Net Income} + \text{Interest} \cdot (1 - \text{Tax Rate})}{\text{Average Assets}} \tag{57}$$

- Asset = liabilities + equity
- Tax Rate = $\frac{\text{Income Tax}}{\text{Income before Tax}}$

Return on Shareholders' Equity (ROE): Measures how much the company has earned on funds invested by shareholders.

$$\text{ROE} = \frac{\text{Net Income}}{\text{Average Equity}} \tag{58}$$

Earnings Per Share (EPS): Measures the profitability of a company on a per share basis.

$$\text{EPS} = \frac{\text{Net Income}}{\text{Total Shares Outstanding}} \tag{59}$$

9.5 Performance ratios

Definition:
Price to Earnings (P/E): Relates a company's share price to its EPS.

$$P/E = \frac{\text{Share Price}}{\text{EPS}} \tag{60}$$

- High P/E could mean overvaluation or expectations of high growth rates.
- Not used for companies with no or negative earnings.
- Would expect higher P/E for company with more debt compared to equivalent company with less debt.

Dividend Yield: Shows how much a company pays out relative to its stock price.

$$\text{Dividend Yield} = \frac{\text{Dividend per Share}}{\text{Price per Share}} \tag{61}$$

- Mature and stable companies most likely to pay dividends.
- New and high-growth companies more likely to reinvest earnings instead of paying dividends.

Dividend Payout Ratio:

$$\text{Dividend Payout Ratio} = \frac{\text{Dividends}}{\text{Net Income}} = \frac{\text{Dividends per Share}}{\text{EPS}} \tag{62}$$

Market Capitalization: Total dollar market value of a company's outstanding shares of stock.

$$\text{Market Cap} = \text{Price per Share} \times \text{Shares Outstanding} \quad (63)$$

10 Taxation (PS9)

10.1 Taxable income (FIX)

Definition:

$$\text{Taxable Income} = \text{Revenue} - \text{Expenses} \quad (64)$$

10.2 Flat corporate tax

Definition:

$$\text{Corporate Tax Payable} = \text{Taxable Income} \times \text{Tax Rate} \quad (65)$$

- Paying tax results in a negative cash flow.
- Tax savings result in a positive cash flow.

10.3 Types of revenue

Terminology:
- Sales revenues
- Interest revenues (interest earned)
- Capital gains: $Salvage\,Value - Book\,Value$

10.4 Types of expenses

Terminology:
- Cost of goods sold (raw materials)
- General expenses/SG&A (salaries)
- Interest expenses (debt)
- Depreciation expenses
- Capital losses
- Not included
  - Dividends
  - Asset purchases
  - Others

10.5 Discounting after-tax cash flow requires lower rate of return

Definition: Modified Rates:

$$\text{(After tax) MARR} = \text{MARR (before tax)} \times (1 - \text{tax rate}) \quad (66)$$

$$\text{(After tax) IRR} = \text{IRR (before tax)} \times (1 - \text{tax rate}) \quad (67)$$

10.6 Capital cost allowance (CCA)

Definition: Amount depreciated in a given year.

$$\text{CCA}_N = \text{CCA Rate} \times \left(\frac{1}{2} \text{ This year's addition} + \text{UCC}_{N-1}\right) \tag{68}$$

- UCC_{N-1}: UCC from last year

10.7 Undepreciated capital cost (UCC)

Definition: Book value in given year.

$$\text{UCC}_N = \text{UCC}_{N-1} + \text{This year's addition} - \text{CCA} \tag{69}$$

10.8 CCA pooling

Definition: All assets in a class are pooled together, with depreciation expenses based on UCC of all assets in that class.

10.9 CCA half-year rule

Definition:
- Additions in the current year are depreciated at half the CCA rate.
- Carried-over UCC is depreciated at the normal rate.

10.10 Tax savings from depreciation

Definition:

$$\text{Tax savings} = \text{CCA} \times t \tag{70}$$

- t: Tax rate

10.11 CCA rules on disposition (selling asset)

Definition:
- If other items remain in pool: **Open Book**
  - Pool not closed upon sale of asset.
  - UCC reduced by sales proceeds (S).
- If no other items in pool: **Closed Book**
  - If $S <$ Book Value (BV): **Terminal loss**, claim BV $- S$ as expense, reduce taxable income by loss.
  - If $S >$ BV and $S <$ Cost (C): **Recapture**, report $S -$ BV as income, increase taxable income by $S -$ BV.
  - If $S >$ C: **Capital Gain**.
  - UCC must always be zero after the pool is closed.

10.12 Calculating PW with taxes: explicit method

Process:
1. **Find After-Tax MARR:**

$$\text{(After-tax) MARR} = \text{MARR (before-tax)} \times (1 - \text{tax rate}) \tag{71}$$

2. **Calculate After-Tax Revenue, Find Present Worth Over Lifespan:**

$$A = \text{Revenue} \times (1 - t) \tag{72}$$
$$\text{PW}(A) = A\,(P/A, i\%_{\text{MARR}}, N) \tag{73}$$

3. **Find FC**

4. **Find tax savings from depreciation, discount appropriately to find PW of all tax savings over N years:**

$$\text{CCA}_N = \text{CCA Rate} \times \left(\frac{1}{2}\text{ This year's addition} + \text{UCC}_{N-1}\right) \tag{74}$$
$$\text{UCC}_N = \text{UCC}_{N-1} + \text{This year's addition} - \text{CCA}_N \tag{75}$$
$$(P/A, i, N) = \frac{1}{i} - \frac{1}{i(1+i)^N} = \frac{(1+i)^N - 1}{i(1+i)^N} \tag{76}$$

5. **Depend on Whether Open or Closed Book:**
   - Open book:
     - Discount S to PW.
   - Closed book:
     - Claim/report gain/recapture/loss by finding $|S - \text{BV}|$ and calculating taxed or tax savings.

6. **Sum results of steps 1-5:**
   - Evaluate like MARR evaluation

10.13 Calculating PW with taxes: tax benefit factor

10.13.1 Tax benefit factor:

Definition: For every dollar spent, the present worth (PW) of future tax savings is τ dollars:

$$\tau = \frac{\text{PW(tax savings)}}{FC} \tag{77}$$

Depends on depreciation method:

1. **Declining Balance:**

$$\tau_{db} = \frac{td}{i+d} \quad \text{(After-tax MARR)} \tag{78}$$

   - t: Tax rate
   - d: Depreciation rate
   - i: After-tax MARR

2. **Declining Balance with Half-Year Rule:**

$$\tau_{\frac{1}{2}} = \frac{td}{i+d} \cdot \frac{1 + i/2}{1+i} \tag{79}$$

   - Applies to CCA asset purchases

10.13.2 Effective first cost:

Definition: Reduced first cost due to tax savings:

$$\text{PW(FC)} = -\text{FC} + \text{FC} \times \tau_{1/2} = -\text{FC}(1 - \tau_{1/2}) \tag{80}$$

10.13.3 Effective salvage value:

Definition: Reduction in salvage value due to loss of tax benefits associated with disposition:

$$\text{PW}(S) = (S - R \times \tau_{db})(P/F, i, N) \tag{81}$$

- S: Original salvage value
- R: Amount reduced in asset pool
  - $R = S$: Open book
  - $R = UCC$: Closed book

Process:

1. **Find Effective Fixed Cost (FC):**
$$\text{PW(FC)} = -\text{FC} \cdot (1 - \tau_{1/2}) \tag{82}$$
2. **Find After-Tax Revenues:**
$$A = \text{Revenue} \cdot (1 - t) \tag{83}$$
$$\text{PW}(A) = A \cdot (P/A, i\%_{\text{MARR}}, N) \tag{84}$$
3. **Find Effective Salvage Value:**
   - Open Book:
$$\text{PW}(S) = S \cdot (1 - \tau_{db}) \cdot (P/F, i\%, N) \tag{85}$$
   - Closed Book:
$$R = UCC_N, \quad T = \begin{cases} +(BV - S) \cdot t & \text{if losses} \\ -(S - BV) \cdot t & \text{if gains/recapture} \end{cases} \tag{86}$$
$$\text{PW}(S) = (S - R \times \tau_{db} + T) \cdot (P/F, i\%, N) \tag{87}$$
     - $T > 0$: Losses
     - $T < 0$: Gains/recapture
4. **Sum all values from steps 1–3:**
   - Evaluate like MARR evaluation.

11 Inflation (PS10)

11.1 Inflation terms

Terminology:

- **Inflation:** A rise in the average price of goods and services over time, reflecting a decline in the purchasing power of the dollar.
- **Deflation:** A decrease in the average price of goods and services over time.
- **Consumer Price Index (CPI):** A measure that examines the weighted average of prices of a basket of goods and services which are of primary consumer needs.
- **CPI Base Year:** 2002
- **CPI Base Year Index:** 100
  - Index for any other year indicates the number of dollars needed in that year to buy the basket of goods that cost \$100 in 2002.
- **Actual (current, nominal) dollars:** Expressed in the monetary units at the time the cash flow occurs.
- **Real (constant) dollars:** Expressed in the monetary units of constant purchasing power, and must always

be associated with a particular date.
- **Purchasing Power Ratio:** Ratio of actual investment value over price of good when base values are identical.
- **Actual Interest Rate** (i, i_A): Observed interest rate based on actual dollars.
- **Real Interest Rate** (i', i_R): Interest rate based on dollars of constant purchasing power.

$$1 + i_R = \frac{1 + i_A}{1 + f} \tag{88}$$

- f: Inflation rate

11.2 Calculating CPI & inflation

11.2.1 CPI index

Definition:

$$\text{Index} = \left(\frac{\text{Basket Value in Year } N}{\text{Basket Value in Base Year}} \right) \times 100 \tag{89}$$

11.2.2 Inflation rate from CPI index

Definition:

$$1 + f = \frac{\text{Index in Year } N_2 - \text{Index in Year } N_1}{\text{Index in Year } N_1};\ N_2 > N_1 \tag{90}$$

- f: Inflation from year N_1 to N_2.

$$1 + f_N = \frac{\text{CPI}_N - \text{CPI}_{N-1}}{\text{CPI}_{N-1}} \tag{91}$$

- f_N: Inflation in year N from CPI.

11.2.3 Average inflation from CPI

Definition:

$$1 + f_{N_1 \to N_2} = \frac{\text{CPI}_{N_2}}{\text{CPI}_{N_1}} \tag{92}$$

$$(1 + f_{\text{avg}})^{N_2 - N_1} = 1 + f_{N_1 \to N_2} \tag{93}$$

11.3 Real value

11.3.1 Real rate

Definition:

$$1 + r_{real} = \frac{1 + r_{actual}}{1 + f} \tag{94}$$

- f: Inflation rate
- r_{actual}: Actual rate of growth (e.g. of investment)

If continuously compounding:

$$e^{r_{real}} = e^{r_{actual} - f} \tag{95}$$

$$\therefore r_{real} = r_{actual} - f \tag{96}$$

11.3.2 Real value from purchasing power

Definition:

$$\text{Real Value}_N = \text{CPI}_o \times \text{PP}_N = \text{CPI}_o \times \left(\frac{1 + r_{actual}}{1 + f}\right)^N = \text{CPI}_o \times (1 + r_{real})^N \tag{97}$$

11.4 Economic valuation with inflation

11.4.1 Actual vs. real values

Definition:

Actual values
- Must adjust for inflation.
- Use actual MARR: i_A
- Most market interest rates given with actual rates

Real values
- Do not adjust for inflation.
- Use real MARR: i_R

11.4.2 Cash flow with inflation

Definition:
- If A is given in actual dollars:
$$\text{PW} = -\text{FC} + A(P/A, i_A, N) \tag{98}$$
- If A is given in real dollars:
$$\text{PW} = -\text{FC} + A(P/A, i_R, N) \tag{99}$$

11.4.3 Loan with inflation

Process:
1. Convert loan interest rate to effective annual rate:
$$1 + i_e = \left(1 + \frac{r}{m}\right)^m \tag{100}$$
2. Find real effective interest rate:
$$1 + i_R = \frac{1 + i_A}{1 + f} \tag{101}$$

11.4.4 Bond with inflation

Process:
1. Convert to actual effective rate i_A (yield):
$$1 + i_R = \frac{1 + i_A}{1 + f} \tag{102}$$
2. Convert to interest rate with compounding period matching coupon amounts:
$$1 + i_A = \left(1 + \frac{r}{m}\right)^m \tag{103}$$
3. Calculate bond price:
$$\text{Coupon amount} = \frac{\text{Coupon rate} \times \text{Face Value}}{\text{Payment Frequency}} \tag{104}$$

$$P = A\left(P/A, \frac{i}{m}, N\right) + F\left(P/F, \frac{i}{m}, N\right) \tag{105}$$

$$P = A\left(\frac{(1+\frac{i}{m})^N - 1}{\frac{i}{m}(1+\frac{i}{m})^N}\right) + F\left(\frac{1}{(1+\frac{i}{m})^N}\right) \tag{106}$$

- i: Yield (i_A).
- m: Frequency of coupon payments per time unit (e.g., year).
- N: Number of periods to maturity ($m \times$ time unit).
- A: Value of each coupon payment.

11.5 Inflation with tax benefits

Depreciation factors are in actual dollars, so actual interest rates must be used in tax benefit factors.

11.5.1 Tax benefit factors with inflation

Process:

1. Calculate the actual rate:
$$1 + i_A = (1 + i_R)(1 + f) \tag{107}$$
2. If S is given in today's dollars, convert to actual dollars. Alternatively, use the shortcut:
$$S_A = S(1 + f)^N \tag{108}$$
3. Calculate present worth (PW):
$$PW = -FC \times CTF + S_A \times CSF\,(P/A, i_A, N) \tag{109}$$
$$= -FC \times CTF + S\frac{(1+f)^N}{(1+i_A)^N} \times CSF \tag{110}$$
$$= -FC \times CTF + S\frac{1}{(1+i_R)^N} \times CSF \tag{111}$$
   - If S is given in real dollars, only need to discount at real MARR:
$$PW = -FC \times CTF + S_R(P/F, i_R, N) \times CSF \tag{112}$$
   - Always use actual values for tax benefit factors:
$$\tau_{db} = \frac{td}{i+d};\ CSF = 1 - \tau_{db} \tag{113}$$
   - t: Tax rate
   - d: Depreciation rate
   - i: After-tax MARR
$$\tau_{1/2} = \frac{td}{i+t_d} \cdot \frac{1+i/2}{1+i};\ CTF = 1 - \tau_{1/2} \tag{114}$$
4. If benefits are present (annuity or geometric sequence):

   Multiply further by $(P/\text{geom}, i_A, f, N)$ if using actual values.
   - Find i_R if **geometric**, discount at i_R increasing/decreasing at $+/-\ g\%$:
$$PW = G\,(P/\text{geom}, i_R, g_R, N) \tag{115}$$
     - Discount if value given for end of year and is actual:
$$PW = \frac{G}{1+f}\,(P/\text{geom}, i_R, g_R, N) \tag{116}$$
     - Taxes if revenue:
$$PW = G\,(P/\text{geom}, i_R, g_R, N)\,(1-t) \tag{117}$$

- If **annuity**, find PW using i_R:
$$PW = A\,(P/A, i_R, N) \tag{118}$$
  - Discount if value given for end of year and is actual:
$$PW = \frac{A}{1+f}\,(P/A, i_R, N) \tag{119}$$
  - Taxes if revenue:
$$PW = (P/A, i_R, N)\,(1-t) \tag{120}$$
- Sum to PW of FC and SV to get total PW.

12 Replacement (PS11)