# CHE260: (Thermodynamics) Cheat Sheet

- Have the correct amount of significant digits (ie. lowest sig figs from question statement) on answers and round off answers at the end.
- Think intuitively about the problem to see if the numbers make sense.
- Figure out what type of question it is and the necessary equations.
- 1. *Find:* Read the problem and state what you are asked to find in your own words. This step may seem obvious, but it is surprising how often people misunderstand the question or miss essential information.
- 2. *Known:* List all the information and properties provided, as well as information about properties that remain constant (e.g. the volume in a isochoric process or the temperature in an isothermal process).
- 3. *Diagrams:* Draw a schematic diagram of the system. Mark the system boundary and decide whether the system is a control mass or volume. Show energy and mass transfers between the system and surroundings by arrows. Draw a process diagram if necessary.
- 4. Assumptions: Decide how you are going to model the system. List all assumptions that you make.
- 5. Governing Equations: Which conservation law are you going to apply? Depending on what you are trying to find, you may use principles of conservation of mass or energy. Write down the governing equations.
- 6. *Properties:* List all property values that are not given in the problem statement. This includes information extracted from tables or other sources.
- 7. *Solution:* Substitute known values of variables in the governing equations and solve them to find unknowns.
- 8. Answer: State the answer and confirm that it is what was asked for.
- 9. Discussion: Are your results reasonable? Can you draw any conclusions from them?
  - Isothermal doesn't mean adiabatic birch
  - When you only have one temperature and you have to make a guess for the average temperature, then just put it in your calculator to see if its reasonable given T2 or something.

#### **General Info**

$$N = \frac{kg \cdot m}{s^2} \mid Pa = \frac{N}{m^2} = \frac{J}{m^3} \mid J = \frac{kg \cdot m^2}{s^2}$$

## **Sphere**

$$\frac{4}{3}\pi r^3$$

In an isolated system, no work or enthalpy can be done. In an insulated system, no heat transfer can be done, but work can be done.

## **L2: Concepts and Definitions**

## 2.0 General Terminology of Systems

| Term                 | Definition   |
|----------------------|--|
| Thermodynamic System | A <b>thermodynamic system</b> (ie. system) can be any piece of matter or region of space.  |
|                      | Surroundings System boundary  System  System  Figure 2.5 Definition of system, surroundings and system boundary.   |
| Surroundings         | The <b>surroundings</b> are everything outside the system.   |
| Boundary             | <ul> <li>The system boundary is the surface that separates the system from the surroundings.</li> <li>Boundary is a surface without thickness. (eg. a container cannot be the boundary).</li> <li>Everything we include in the boundary has to be accounted for in the calculations.</li> <li>The boundary can be stationary or move as well.</li> </ul> |

## 2.1 Three Types of Systems

| Type of System | Definition |
|----------------|------------|
|----------------|------------|

## Closed System (Control Mass)

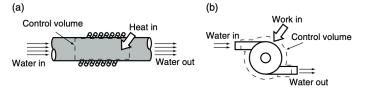
- Mass can't cross the system boundary (mass is fixed), while energy can cross the system boundary.
- The volume can be compressed or expanded.



**Figure 2.7** Closed systems: (a) a rock, (b) a sealed bottle and (c) gas contained in a cylinder fitted with a piston.

## Open System (Control Volume)

- Both energy and mass can cross the system boundary.
- The boundaries of the control volume may change shape and the system may also move in space.



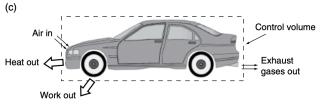


Figure 2.8 Open systems: (a) heated section of a water pipe, (b) a water pump and (c) a car.

#### **Isolated System**

- Both mass and energy can't cross the system boundary.
- The properties of an isolated system at **equilibrium** are constant and uniform.

Rigid, perfectly insulated and impervious walls

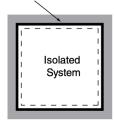


Figure 2.9 Isolated system, surrounded by rigid, perfectly insulated and impervious walls.

#### 2.2 Definition of Property

A property of a system is any attribute that can be measured without knowing the history of the system.

- **Eg. of Thermodynamic Properties:** mass, volume, temperature, energy.
- Infinitesimally small quantities use d for properties (eg. dx, dt), while  $\delta$  (eg.  $\delta W$ ) for non-properties.
  - This is to remind ourselves that we cannot integrate them in the same way we integrate properties.

#### 2.3 Definition of Path Function and State Function

Path functions depend on the path taken to get from the initial to the final state.

• **Eg.** heat and work.

State functions only depend on the initial and final states.

## 2.4 Intensive Property

**Intensive** properties are those that can be specified at a point within the system and are independent of system mass.

- Eg. Temperature and pressure.
- Lower case letters to denote intensive properties.

## 2.5 Extensive Property

**Extensive** properties depend on the size of the system.

- **Eg.** Mass, volume, and energy.
- Upper case letters to denote extensive properties

## 2.6 How to Convert Extensive Properties into Intensive Properties?

If we take any extensive property and divide it by the mass of the system we obtain a new intensive property (ie. specific).

#### 2.7 Steady State

A system whose properties do not change with time, even though it is exchanging energy or mass with its surroundings, is said to be in a **steady state**.

#### 2.8 Definition of Equilibrium vs. Non-equilibrium

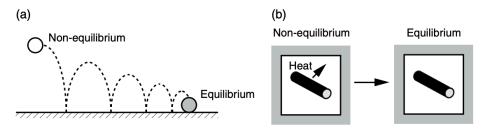
#### Equilibrium

All physical systems that are left isolated eventually reach a state of equilibrium where their properties do not change with time.

 External intervention is necessary to change the properties once it has reached equilibrium

#### Non-Equilibrium

Non-equilibrium state, where the properties of an isolated system change spontaneously



**Figure 2.13** (a) A ball bounces until it comes to rest and reaches equilibrium. (b) A heated metal rod in an insulated box loses heat until it reaches equilibrium when the temperature everywhere in the box is uniform.

## 2.9 Difference Between Steady State and Equilibrium

## **Steady State:**

A system that is interacting with its surroundings, but whose properties are not changing with time, is at steady state.

## Equilibrium:

An isolated system with constant properties is at equilibrium.

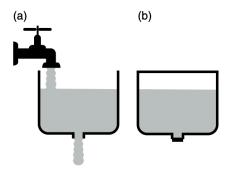
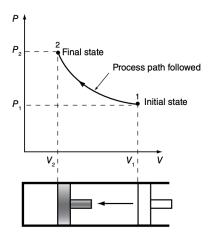


Figure 2.14 (a) A system at steady state, in which water flows into the sink at the same rate that it drains out. (b) An isolated system at equilibrium.

#### 2.11 Definition of Process

The change of a system from one state to another is known as a **process**.



**Figure 2.18** A fixed mass of gas in a cylinder is compressed by a piston from initial state 1 to final state 2. The process path shows the state of the system during the compression.

• The process path is shown by the line connecting these two points, which defines intermediate values of the two properties from initial to final state.

## 2.12 Different Types of Thermodynamic Processes

| Type of Process | Definition                                 |
|-----------------|--|
| Isothermal      | The temperature of the system is constant. |
| Isobaric        | The pressure of the system is constant.    |
| Isochoric       | The volume of the system is constant.      |

## 2.13 Quasi-Equilibrium Process

**Quasi-equilibrium process** is an infinitely slow one, in which the system approaches equilibrium at each stage.

- Allows us to define the properties of the system at every stage in the process.
- Whenever we represent a process by a curve on thermodynamic axes, we are making the assumption that the process is quasi-equilibrium.

#### 2.14 Definition of Thermal Reservoir

A **thermal reservoir** is defined as a system whose temperature remains constant in spite of heat transfer to or from it.

• Thermal reservoirs from which we withdraw heat are *heat sources* and those to which we transfer heat are *heat sinks*.

## L3: Thermodynamic System Properties

## 3.0 Relationship Between Mass (m) and Mass in (N)

$$m[kg] = N[kmol] \cdot M[kg/kmol]$$

•  $N = 6.022 \times 10^{23}$  molecules

#### 3.1 Pressure

 ALL CALCULATIONS OF GAS PROPERTIES ARE DONE USING ABSOLUTE PRESSURE VALUES

#### **Gas Pressure:**

$$P = \frac{F}{A} [Pa]$$

#### **Pressure Units:**

$$10^5 Pa = 1 \ bar \& 1 \ atm = 1.01325 \ bar$$

#### **Manometers:**

Manometers measure the difference between absolute and atmospheric pressure is

$$P_{abs} - P_{atm} = \rho g h$$

- P<sub>abs</sub> is absolute pressure
- ullet  $P_{atm}$  is atmosphere pressure
- $\rho$  is density.

#### **Gauge Pressure:**

$$P_{gauge} = P_{abs} - P_{atm}$$

## 3.2 Definition of Thermal Equilibrium

Heat will flow from higher temperature object to lower temperature object in close contact. When the two objects reach the same temperature, there will be no more heat transfer and they are then said to be in **thermal equilibrium**.

## 3.3 Ideal Gas Equation

$$PV = NR_uT = mRT$$

- $R_u = 8.314 \frac{kJ}{kmol \cdot K}$  (Universal Gas Constant)
- $R = \frac{R_u}{M}$  (Particular Gas Constant)

## 3.5 Internal Energy

The **internal energy** (U) is the total microscopic energy of all the molecules in a substance. It is an extensive property, given for a monatomic ideal gas (ie. noble gases) by

$$U = \frac{3}{2} N R_{u} T$$

• The internal energy of a unit mass of an ideal gas is a function only of its temperature

## 3.6 The $\Delta$ in Not Specific/Specific Internal Energy of an Ideal Gas

For any ideal gas, if we change the temperature of an ideal gas from  $T_1$  to  $T_2$  its energy changes by

$$U_2 - U_1 = \frac{3}{2}NR_u(T_2 - T_1) = \frac{3}{2}mR(T_2 - T_1) = \frac{3}{2}mc(T_2 - T_1)$$

- c is the specific heat.
- For the specific internal energy, divide by the mass.

## 3.7 Properties of Liquids and Solids

Liquids and solids can be assumed to be incompressible with constant specific volume so that

$$v = constant$$

## L4: Energy and the First Law of Thermodynamics

## **✓**4.1 Energy

Energy is an **extensive property** of all thermodynamic systems. A system possesses energy if it is capable of lifting a weight.

## **✓**4.2 Microscopic & Macroscopic Forms of Energy:

Potential energy and kinetic energy are macroscopic forms of energy.

Changes in position and velocity respectively.

While internal energy includes all microscopic forms of energy storage.

 Changes in temperature, pressure, chemical composition, electrical or magnetic state.

## **✓**4.3 Energy:

• For a stationary system, KE and PE are constant and only U changes.

## Gravitational Potential Energy (PE)

$$PE = mgz[J]$$

The height z of the system is measured above an **arbitrary datum plane** to define PE = 0.

|   | <ul> <li>PE raised above this level is positive. Below is negative.</li> </ul>   |
|---|--|
| Change in Gravitational<br>Potential Energy | $d(PE) = d(mgz)$ $\Delta PE = mg\Delta z \ [J] \ if \ m \ and \ g \ are \ constant.$ • $\Delta PE$ does not depend on the choice of the level.     |
| Kinetic Energy (KE)                         | $KE = \frac{1}{2}mv^2 [J]$   |
| Change in Kinetic Energy                    | $\Delta KE = \frac{1}{2}m(v_2^2 - v_1^2)[J]$   |
| Total Energy                                | The <b>total energy</b> (E) is an extensive property that is $E = PE + KE + U[J]$  |
| PE per Unit Mass                            | $pe = \frac{PE}{m} = gz\left[\frac{J}{kg}\right]$  |
| KE per Unit Mass                            | $ke = \frac{KE}{m} = \frac{1}{2}v^2 \left[\frac{J}{kg}\right]$   |
| Specific Internal Energy                    | $u = \frac{U}{m} \left[ \frac{J}{kg} \right]$  |
| Specific Energy                             | The <b>specific energy</b> (e) is an intensive property that is $e = \frac{E}{m} = pe + ke + u = gz + \frac{1}{2}v^2 + u\left[\frac{J}{kg}\right]$ |

## **✓**4.4 Energy Transfer

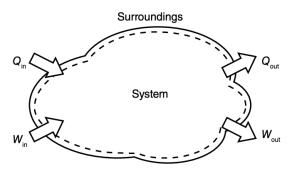
Energy can be transferred to or from a system in two ways: as **work** (W) or **heat** (Q).

## **✓**4.5 Difference Between Work and Heat:

**Heat** transfer is an exchange of energy that occurs due to a temperature difference between the system and its surroundings. All other forms of energy transfer are classified as **work**.

## **✓**4.6 How to Make an Energy Balance?

- 1. Define the system and mark its boundary with a dashed line.
- 2. Show all energy transfers across the system boundary, either as heart or work, with arrows.
  - a. Energy transfers to a system from the surroundings, either as heat or work, are **positive**.
  - b. Energy transfers from a system to the surroundings, either as heat or work are **negative**.
  - c. If you are not sure whether energy is added to or removed from the system in a particular problem, make a guess. If you guess incorrectly, you will get a negative answer, showing that the actual direction is the opposite of what you assumed.



## **✓**4.7 Heat and Work Equations:

| Term     | Definition   |
|----------|--|
| Net Heat | The <b>net heat</b> added to a system in which heat transfer occurs through several streams                                  |
|          | $Q_{net} = \sum_i Q_i [J]$ • $Q_{net} > 0$ means more heat is added to the system than it loses to surroundings. Vice versa. |

| Net Work                                     | $W_{net} = \sum_{i} W_{i}[J]$ • $W_{net} > 0$ means more work is supplied to the system that it does on the surroundings. Vice versa.   |
|--|---|
| Power  | <b>Power</b> is the rate of doing work ( $\dot{W}$ ) $\dot{W} = \frac{\delta W}{dt} [Watts = \frac{J}{s}]$  |
| Mechanical Work Power                        | Mechanical work done by a constant force F through an infinitesimal distance dx is $\delta W = F dx$ The power expended in applying the force is $\dot{W} = \frac{\delta W}{dt} = F \frac{dx}{dt} = F v \left[ Watts \right]$ • Where v is the velocity of the point of application of the force. |
| Rate of Heat Transfer                        | $Q = \frac{\delta Q}{dt} [W]$   |
| Heat Transfer Per Unit<br>Mass of the System | $q = \frac{Q}{m} \left[ \frac{J}{kg} \right]$ • Heat is not a property, so q is not an intensive property.  |
| Work Done Per Unit<br>Mass of the System     | $w = \frac{W}{m} \left[ \frac{J}{kg} \right]$ • Work is not a property, so w is not an intensive property.  |

## 4.8 Definition of Heat Transfer

**Heat transfer** (Q) is a mode of energy transport that occurs when a temperature difference exists between the system and surroundings.

## **4.5 Work**

## 4.5.0 Definition of Work

**Work** is energy transfer across the boundary of a closed system in the absence of any temperature difference.

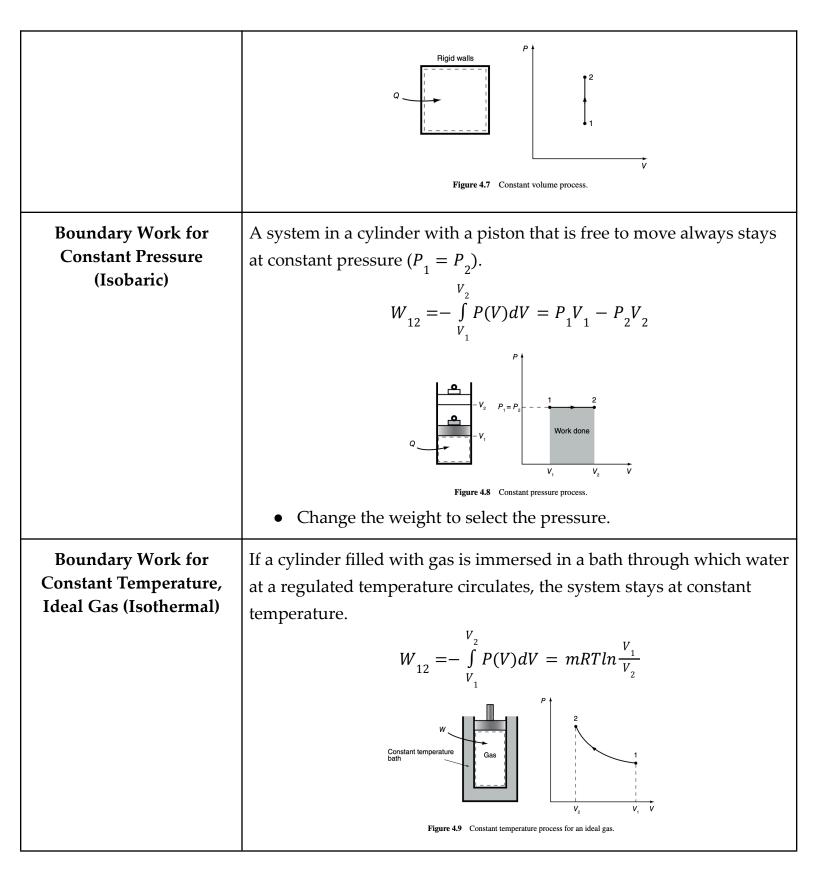
## **✓**4.5.1 Definition of Boundary Work

When a force acts on the boundaries of a system and deforms them, so that the system is either compressed or expanded, **boundary work** is done since the point of application of the force moves through a finite distance.

## **✓**4.5.2 Different Kinds of Boundary Work

• Note: It is possible to have 2 processes that begin and finish with the same pressure and volume, but require different amounts of work.

| Term  | Definition  |
|---|---|
| Boundary Work                                       | The work done in compressing or expanding a gas from state 1 to state 2, in which the volume changes from $V_1$ to $V_2$ is $W_{12} = -\int\limits_{V_1}^{V_2} P(V) dV$ • If the system expands, then $W_{12} < 0$ and does work on the surroundings. • If the system compresses, then $W_{12} > 0$ and surroundings do work on the system. |
| Boundary Work for<br>Constant Volume<br>(Isochoric) | A constant volume process is one in which heat is added to a system confined within rigid walls $(dV = 0)$ $W_{12} = -\int_{V_1}^{V_2} P(V)dV = 0$  |

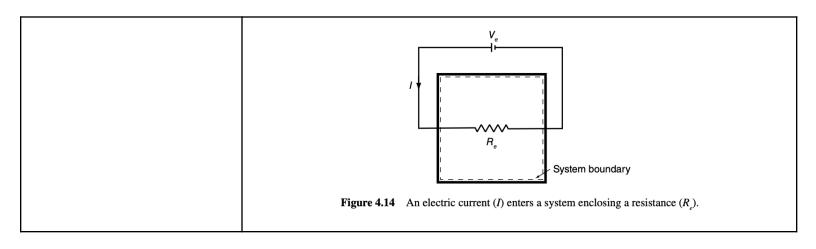


| <b>extropic process</b> can be modelled in which $PV^n = C$ where n and constants.         |
|--|
| $W_{12} = \frac{P_2 V_2 - P_1 V_1}{n - 1} $ for $n \neq 1$                                 |
| Isothermal: $W_{12} = P_1 V_1 ln \frac{V_1}{V_2} = P_2 V_2 ln \frac{V_1}{V_2}$ for $n = 1$ |
| Figure E4.7 Polytropic expansion of gas in a cylinder.                                     |
|  |

## 4.5.3 Different Kinds of Work

| Term                                | Definition  |
|-------------------------------------|---|
| Flow Work                           | <b>Flow work</b> is required to completely drive the fluid element into the control volume by using a force that must act through a distance: $W = FI - PAI - PV[I]$  |
|                                     | $W_{flow} = FL = PAL = PV [J]$ (a) Fluid element $FV = FV$ Control volume $F = FV = FV$ Fluid element $FV = FV$ Fluid element $FV = FV$ Fluid element $FV = FV$ Fluid element   |
|                                     | Figure 4.10 Fluid element (a) before and (b) after entering the control volume.   |
|                                     | <ul> <li>When fluid enters a control volume the surroundings do work on it and this flow work increases the total energy of the system.</li> <li>Conversely, when fluid leaves a control volume the system has to do work on the fluid to force it out. The fluid transports the</li> </ul> |
| Flow Work Per Unit Mass<br>of Fluid | flow work with it and the system energy decreases. $w_{flow} = \frac{w_{flow}}{m} = Pv\left[\frac{J}{kg}\right]$  |

| Torque                         | When a constant force (F) acting at a radius (r) about a shaft makes it rotate, the torque produced is $\tau = Fr  [Nm]$ |
|--------------------------------|--|
|                                | Figure 4.12 Torque produced in a shaft by a force $(F)$ acting at radius $(r)$ .   |
| Shaft Work                     | If the shaft revolves $N_r$ times, the force acts through a distance $2\pi rN_r$ .                                       |
|                                | The work done by the shaft is  |
|                                | $W_{shaft} = 2\pi N_r \tau$  |
| Rotational Speed               | $\omega = \frac{dN_r}{dt} [s^{-1}]$  |
| Power Transmitted by the Shaft | $\dot{W}_{shaft} = 2\pi\omega\tau [Watts]$   |
| Spring Work                    | The work required to displace the end of the spring from $x_1$ to $x_2$ is   |
|                                | $W_{spring} = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} Kx dx = \frac{1}{2} K(x_2^2 - x_1^2)$                             |
| Electrical Work                | The rate at which energy enters is   |
|                                | $\dot{W}_{elec} = V_e I = I^2 R_e = \frac{V_e^2}{R_e}$   |
|                                | I is electric current entering a control mass.   |
|                                | R <sub>e</sub> is electrical resistance     V is the voltage applied.  |
|                                | $ullet$ $V_e$ is the voltage applied.  |



## **✓ 4.6 The First Law for a Control Mass**

The energy of a system can be measured, relative to a reference state whose energy we arbitrarily assume to be zero. Heat and work are the only two ways of transferring energy across the boundaries of a closed system.

## **V**Definition of Work

The **work** (W) done on a system in an adiabatic process equals its change in energy ( $\Delta E$ ).

## **V**Definition of Heat Transfer

The **heat** (Q) transferred to a closed system is defined as being equal to its change in energy, less any work done during the process.

## First Law for a Control Mass (First Law of Thermodynamics)

The change in energy of a closed system equals the net energy transferred to it in the form of work and heat

$$Q + W = \Delta E = \Delta PE + \Delta KE + \Delta U$$

- $\bullet \quad W = W_{_{PV}} + W^{^{o}}$
- Heat transfer (Q) and work (W) from surroundings to the system are positive following our sign convention.

## First Law for Infinitesimal Amounts

The first law for infinitesimal amounts of heat transfer ( $\delta Q$ ) and work ( $\delta W$ ) which produce an infinitesimal change in system energy (dE):

$$\delta Q + \delta W = dE$$

## **Rate Equation**

If the infinitesimal energy change (dE) takes place over an infinitesimal interval of time (dt), then

Q+ 
$$\dot{W} = \frac{dE}{dt}$$

- Q and W are the rates of energy transfer to the system in the form of heat and work.
- The amount of energy stored in the system is a property, and it changes at a rate of  $\frac{dE}{dt}$

## 4.7 Enthalpy

## 4.7.0 Enthalpy

$$H = U + PV[J]$$

• Since U and PV are both extensive properties, H is also an extensive property.

## 4.7.1 Specific Enthalpy

Specific enthalpy (h) is an intensive property defined as

$$h = u + Pv\left[\frac{J}{kg}\right]$$

## ✓ What Does Enthalpy Measure?

Enthalpy (H) measures the capacity of a fluid to do work.

## 4.7.2 Heat Supplied to an Isobaric Systems (Constant Pressure)

The heat supplied to a constant pressure system equals its change in enthalpy

$$Q_{12} = H_2 - H_1[J]$$

## **✓**4.7.3 Heat Required Per Unit Mass of the System

$$q_{12} = h_2 - h_1 \left[ \frac{J}{kg} \right]$$

## **✓**4.8 Specific Heats

## 4.8.0 Definition of Specific Heat

The **specific heat** (c) is the amount of energy required to raise the temperature of a unit mass of a substance by one degree.

## **✓**4.9 Specific Heats of Ideal Gases

| Type                                 | Definition  • NOTE WERE DERIVING THESE FROM THE DEFINITIONS THEMSELVES.   |
|--------------------------------------|---|
| c <sub>v</sub> for Ideal Gases       | The internal energy (U) of an ideal gas is a function only of its temperature (T). If $u=u(T)$ , and not $u=u(T,P)$ we do not require a partial differential and can write $c_v(T)=\frac{du}{dT}$ • For an ideal gas, $c_v=\frac{3}{2}R$ • True for all processes, not just constant volume |
| Specific Enthalpy of an<br>Ideal Gas | For an ideal gas, $Pv = RT$ , then $h = u + RT$ • For an ideal gas, $h = \frac{5}{2}RT$   |
| c <sub>p</sub> for Ideal Gases       | For an ideal gas, $Pv = RT$ , so the enthalpy of an ideal gas is  |

h = u + RT. Since specific internal energy (u) and specific enthalpy (h) is a function only of temperature and gas constant (R) is a constant, therefore,

$$c_p(T) = \frac{dh}{dT}$$

- For an ideal gas,  $c_p = \frac{5}{2}R$
- True for all processes, not just constant pressure.

## 4.9.3 Change in Specific Internal Energy for Ideal Gases

$$\Delta u = u_2 - u_1 = \int_{T_1}^{T_2} c_v(T) dT$$

- Appendix 4 for some common gases.
- **✓** 4.9.4 Change in Specific Enthalpy for Ideal Gases

$$\Delta h = h_2 - h_1 = \int_{T_1}^{T_2} c_p(T) dT$$

- Check appendix one for values of specific heat of gases.
- Appendix 4 for some common gases.

## **✓** 4.9.5 How are the Two Specific Heats of an Ideal Gas Related?

For an ideal gas,

$$c_p = c_v + R$$

For molar properties

$$\overline{c}_{v} - \overline{c}_{v} = R_{u}$$

Derivation in textbook.

#### 4.9.4 Specific Heat Ratio

$$\gamma = \frac{c_p}{c_v} = \frac{c_p}{c_v}$$

• For **monatomic ideal gases**, the specific heat ratio is  $\frac{5}{3}$ 

## $\checkmark$ 4.10 Which should you use, $c_p$ or $c_v$ ?

- We do not require that the volume or pressure remain constant during a
  process to use c<sub>v</sub> and c<sub>p</sub> values to calculate changes in u and h.
  - Since it only depends on the initial and final states, not the path followed.
- It is always true for an ideal gas undergoing any thermodynamic process, even if it is not under constant volume or constant pressure conditions that the change in specific internal energy and specific enthalpy is 4.9.3 and 4.9.4 equations.

## **✓**4.12 Specific Heats of Liquids and Solids

We cannot do compression or expansion work on an incompressible substance, since its volume does not change. The internal energy of an incompressible substance is therefore a function of temperature alone, u = u(T)

## **4.12.0** Definition of Incompressible Substances

**Incompressible substances,** whose specific volume remains constant.

## ✓ 4.12.1 Specific Heat of Liquids and Solids

For an incompressible substance such as a solid or liquid,  $c_p$  and  $c_v$  are always equal

$$c_{p}(T) = c_{p}(T) = c(T)$$

- Check appendix 2 and 3 for typical values of c for common solids and liquids.
- Derivation is in the textbook.

**✓**4.12.2 Specific Change in Internal Energy for Solids and Liquids

$$\Delta u = u_2 - u_1 = c \int_{T_1}^{T_2} dT = c(T_2 - T_1)$$

## **✓**4.12.2 Specific Change in Enthalpy for Solids and Liquids

If the specific heat c is constant, independent of temperature, then

$$\Delta h = h_2 - h_1 = c(T_2 - T_1) + v(P_2 - P_1)$$

# L7: Steady Mass Flow Through a Control Volume, The First Law for Steady Flow Through a Control Volume, & Steady Flow Devices

## **▼7.0 Steady Mass Flow Through a Control Volume**

Turbines, pumps, compressors and heat exchangers are devices that exchange energy with the surroundings in the form of work or heat while fluids flow through them. They all operate under steady flow conditions in which fluid enters at the same rate as it leaves, so that no mass accumulates inside them.

7.0.0 The Rate at Which Fluid Enters the Control Volume:

$$\dot{\mathbf{m}} = \frac{\delta m}{dt} = \rho A \mathbf{V} = \frac{A}{v} \mathbf{V} \left[ \frac{kg}{s} \right]$$

• Since specific volume  $v = \frac{1}{\rho}$  because mass is being divided.

7.0.1 Mass Balance:

$$\sum_{i} \dot{\mathbf{m}}_{i} = \sum_{e} \dot{\mathbf{m}}_{e}$$

## 7.1 The First Law for Steady Mass Flow Through a Control Volume

• Look in the textbook to see where the formula comes from.

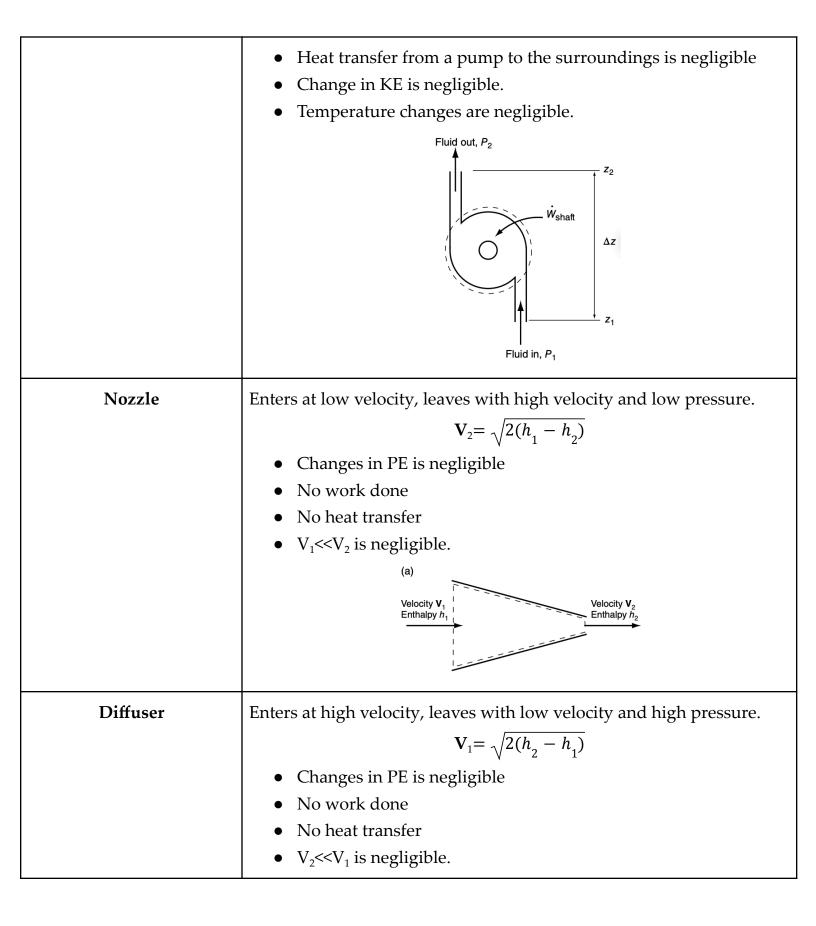
| Type | Definition |
|------|------------|
|------|------------|

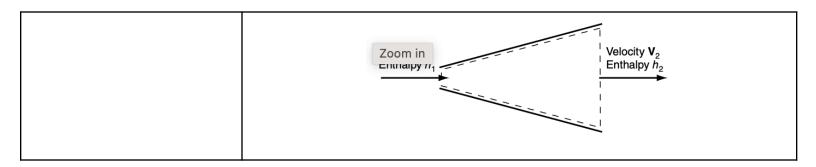
| General Energy Balance                   | $\sum_{out} \dot{m}(h + \frac{1}{2}\hat{V}^2 + gz) - \sum_{in} \dot{m}(h + \frac{1}{2}\hat{V}^2 + gz) = \dot{Q} + \dot{W}_{o}$   |
|--|--|
| Energy Balance                           | $\dot{\mathbf{E}}_{1}=\dot{\mathbf{E}}_{2}$  |
|  | The rate of energy input equals the rate of energy output.   |
|  | È includes energy due to the fluid entering and leaving the control volume, and also any heat or work transfers:                 |
|  | $\dot{Q} + \dot{W} + \dot{m}(h_1 + \frac{v_1^2}{2} + gz_1) = \dot{m}(h_2 + \frac{v_2^2}{2} + gz_2) [W]$                          |
|  | $\dot{Q} + \dot{W} = \dot{m}[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)][W]$   |
| Energy Balance Per Unit<br>Mass of Fluid | $q + w = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$   |
|  | • $q = \frac{\dot{Q}}{\dot{m}} \left[ \frac{J}{kg} \right]$ is heat transfer per unit mass of fluid flowing                      |
|  | through the control volume  • $w = \frac{\dot{W}}{\dot{m}} \left[ \frac{J}{kg} \right]$ is the work done per unit mass of fluid. |

## 7.2 Steady Flow Devices

| Type     | Definition   |
|----------|--|
| Turbines | Enters with high pressure, leaves with low pressure.         |
|          | $\dot{W}_{shaft} = \dot{m}(h_2 - h_1)$                       |
|          | • $\dot{W}_{shaft}$ < 0 so does work on the surroundings     |
|          | Turbines are insulated, so Q is negligible                   |
|          | • Changes in KE and PE are negligible where v1=v2 and z1=z2. |

|            | (a) Fluid in, $h_1$ Wishaft  Turbine  Fluid out, $h_2$  |
|------------|---|
| Compressor | Enters with low pressure, leaves with high pressure. w $\dot{W}_{shaft} = \dot{m}(h_2 - h_1)$ • $\dot{W}_{shaft} > 0$ sp does work on the system • Turbines are insulated, so $Q$ is negligible • Changes in KE and PE are negligible.  • These can be negligible because v1=v2 and z1=z2 • There can be a balancing of forces. |
| Pumps      | Raise pressure.   |





## L8: Converting Heat to Work, Entropy, 2nd Law & Rev and Irrev Processes

## 8.0 Definition of Macroscopic Entropy

| Entropy Definition     |   |
|------------------------|---|
| Entropy                | Deminion  |
|                        | $Entropy\ change\ = rac{	ext{Heat\ transferred}}{	ext{Temperature}}$ |
|                        | • Use the temperature of the system boundary.                         |
|                        | For efficient energy conversion, we want to minimise entropy          |
|                        | increase.   |
| Entropy                | $dS = \frac{\delta Q}{T} \left[ \frac{J}{K} \right]$                  |
|                        | T [K] is the absolute temperature.                                    |
| Change in Entropy      | $\Delta S = \int_{1}^{2} dS = \int_{1}^{2} \frac{\delta Q}{T}$        |
|                        | If thermal reservoir  |
|                        | $\Delta S = \frac{1}{T} \int_{1}^{2} \delta Q = \frac{Q_{12}}{T}$     |
|                        | Temperature remains constant.   |
|                        | • $Q_{12}$ is the total amount of heat added to or removed from the   |
|                        | system.   |
| Rate of Entropy Change | $\dot{S} = \frac{\dot{Q}}{T} \left[ \frac{W}{K} \right]$              |

#### 8.1 Entropy Generated Between Two Thermal Reservoirs

$$S_{gen} = \Delta S_B - \Delta S_A$$

- Remember that these entropy changes are absolute values
- We cannot have  $S_{gen}$ <0 (destroyed) because then  $\Delta T$  < 0 (flowing from low to high temp.)

#### 8.2 Summary of Entropy

- Entropy is an extensive property.
- Entropy is created when heat is transferred through a finite temperature difference.
- Energy is conserved but entropy can be generated, but not destroyed.
- Heat increases the entropy, while work does not change the entropy.

## 8.3 Second Law of Thermodynamics

The entropy of an isolated system will increase until the system reaches a state of equilibrium. The entropy of an isolated system in equilibrium remains constant.

$$dS_{isolated} > 0$$
 for an isolated system not in equilibrium  $dS_{isolated} = 0$  for an isolated system in equilibrium  $dS_{isolated} < 0$  not possible for an isolated system

#### 8.4 Reversible Process vs. Irreversible Process

A **reversible process** is a process that produces no entropy.

$$S_{qen} = 0$$
 for a reversible process

An **irreversible process** is a process that produces entropy.

$$S_{qen} > 0$$
 for an irreversible process

• All real processes are irreversible.

#### 8.5 Isentropic Processes

Processes that are both **reversible** (no entropy generation  $S_{gen} = 0$ ) and **adiabatic** (no entropy transfer) have  $\Delta S = 0$ .

#### 8.6 Internal Reversibility vs. External Reversibility

**Internally reversibility** is when no entropy is generated within the system.

$$dS = \left(\frac{\delta Q}{T}\right)_{int.\,rev.}$$

**External irreversibility** is when entropy is created outside the system.

$$dS = \left(\frac{\delta Q}{T}\right)_{irr} + dS_{gen}$$

#### 8.7 State Postulate

All isolated systems reach a state of equilibrium. The equilibrium state of a pure, simple compressible substance is completely described by its mass (m), volume (V) and internal energy (U).

## 8.8 Fundamental Equation

The fundamental equation which contains all the information needed to calculate all properties of a system is

$$S = S(U, V, m)$$

## 8.9 Specific Entropy

$$s = \frac{S}{m}$$

#### 8.10

The state of a simple compressible system is fixed if we know any two independent, intensive properties

$$s = s(u, v) | u = u(s, v) | v = v(u, s)$$

## L9: Equilibrium in a Gas, A Simple Example, & Molecular Definition of Entropy

#### 9.0 Microstates and Macrostates

**Microstate:** List the possible arrangements of each particle.

n particles can be arranged in 2<sup>n</sup> ways

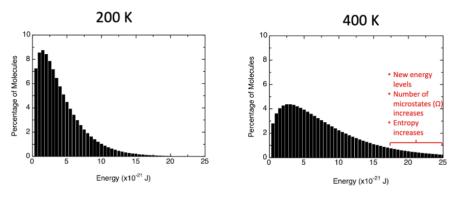
Total number of microstates, 
$$\Omega_{total} = 2^n$$

**Macrostate:** List of properties that can be measured macroscopically which remain unchanged (Pressure, Temperature, Mass).

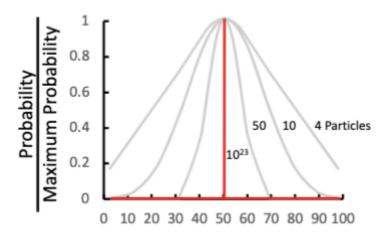
#### 9.1 Maxwell-Boltzmann Equilibrium Distribution

If gas temperature is raised, new energy levels appear, the number of microstates and entropy increases.

Assume all microstates are equally probable



9.2 Probability Distribution with System With Large Number of Particles All microstates correspond to the equilibrium state.



% of Particles on Left

- Equilibrium macrostate is the most probable one with the largest number of microstates for fewer particles.
  - For a large number of particles, all microstates correspond to the equilibrium state  $\Omega_{eq} \approx \Omega_{tot} = \Omega$  (refer to this).

## 9.3 Molecular/Statistical/Microscopic Definition of Entropy

$$S = kln(\Omega)$$

•  $k = 1.38 \times 10^{-23} \left[ \frac{J}{K} \right]$  is Boltzmann constant.

#### 9.4 The Number of Microstates in a Monatomic Gas

$$\Omega(U, V, n) = f(n)V^n U^{3n/2}$$

- f is a function of n only, where n is the number of atoms.
- U is total internal energy
- V is volume
- $\Omega$  is the number of microstates.

## 9.5 Microscopic Entropy Change of a Monatomic Ideal Gas

$$\Delta S = S_2 - S_1 = kln(\Omega_2) - kln(\Omega_1) = kln(\frac{\Omega_2}{\Omega_1})$$

Using 
$$\Omega(U, V, n) = f(n)V^n U^{3n/2}$$

$$\Delta S = nk \left[ ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}ln\left(\frac{U_2}{U_1}\right) \right]$$
Using  $U = \frac{3NR_uT}{2}$  and  $nk = NR_u$ 

$$\Delta S = NR_u \left[ ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}ln\left(\frac{T_2}{T_1}\right) \right] = mR \left[ ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}ln\left(\frac{T_2}{T_1}\right) \right]$$

#### 9.6 Statistical 2nd Law of Thermodynamics

Entropy increases as an isolated system goes from a less probable state to a more probable state.

Since we know  $\Omega_f >> \Omega_i$  because there's more arrangement of particles when in equilibrium, then

$$\Delta S = k ln(\frac{\Omega_f}{\Omega_i}) > 0$$

• The probability of going to a high to low probable state is very low.

## L10: 5.9 Third Law of Thermodynamics, Production of Entropy, Heat and Work: A Microscopic View, & Order and Uncertainty

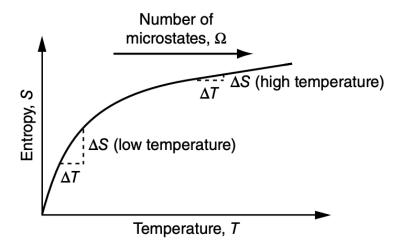
## 10.0 Third Law of Thermodynamics

The entropy of a pure substance in thermodynamic equilibrium is zero at a temperature of absolute zero.

## 10.1 Production of Entropy

Once entropy has been created in an isolated system it cannot disappear. The only way to reduce the entropy of a system is to transfer it to the surroundings.

## 10.3 Variation of Entropy with Temperature



• At low temperature a given change  $\Delta T$  largely increases in entropy, than at high temperatures.

# L11: 6.1 The Postulates of Classical Thermodynamics, 6.2 Thermal Equilibrium and Temperature, 6.3 Mechanical Equilibrium and Pressure, 6.4 Gibbs Equation,

- Fundamental Concepts: Mass, volume, energy, and entropy
- **Postulates to Describe Thermodynamics:** State Postulate, 1st, 2nd, and 3rd Law

## 11.0 Definition of Temperature

$$T = \left(\frac{\partial U}{\partial S}\right)_{m,V}$$

- Intensive property since it's the ratio of extensive properties.
- This satisfies our expectations for temperature
  - Equal temperatures => equilibrium
  - Heat transfer from higher to lower temperature.

· Measure of how a given around of E will change the entropy of a closed system at constant V.

· At high temps, adding energy produces a small change in entropy.

· At low temps, some amount of energy produces a big change in entropy.

S

If I fin & temps.)

A

Monontonic idual gas  $N \sim N^{\frac{3}{2}N} \sim T^{3/2}$ 

## 11.1 Mechanical Equilibrium and Pressure

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{m,V} = \left(\frac{\partial S}{\partial v}\right)_{V}$$

## 11.2 Gibbs Equation

$$ds = \frac{1}{T}du + \frac{P}{T}dv$$

#### 11.3 Reversible Heat Transfer

#### **Reversible Process**

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int.\,rev.}$$

If constant temperature

$$\Delta S = \frac{1}{T} \int_{1}^{2} (\delta Q)_{rev} = \frac{Q_{12}}{T}$$

**Irreversible Process** 

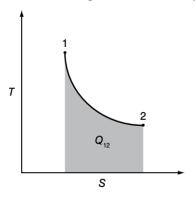
$$\Delta S = \int_{1}^{2} \left(\frac{\delta Q}{T}\right)_{irr} + S_{gen}$$

## 11.4 T-S Diagram

Since we know from a reversible process that

$$Q_{12} = \int_{1}^{2} T dS \mid q_{12} = \frac{Q_{12}}{m} = \int_{1}^{2} T ds$$

ullet  $Q_{12}$  is the total heat transfer during the internally reversible process.



## L12: 6.5 Entropy Changes in Solids and Liquids, 6.6 Entropy Changes in Ideal Gases & 6.7 Isentropic Processes

## 12.0 Entropy Changes in Solids and Liquids

$$\Delta s = cln(\frac{T_2}{T_1}) \ since \ c_p \approx c_v = c$$

- Isentropic process,  $\Delta s = 0$  since  $T_2 = T_1$
- Interpolate
- For incompressible material.

## 12.1 Entropy Changes in Ideal Gases

## 1. Temperature and Specific Volume

$$\Delta s = s_2 - s_1 = c_v ln(\frac{T_2}{T_1}) + Rln(\frac{v_2}{v_1})$$

- Assume c<sub>v</sub> is constant.
- This accounts for the other definition of the entropy change for a monatomic ideal gas in statistics.

## 2. Pressure and Specific Volume

$$\Delta s = s_2 - s_1 = c_v ln(\frac{P_2}{P_1}) + c_p ln(\frac{v_2}{v_1})$$

## 3. Temperature and Pressure

$$\Delta s = s_2 - s_1 = c_p ln(\frac{T_2}{T_1}) - Rln(\frac{P_2}{P_1})$$

- Assume c<sub>p</sub> is constant.
- Divide by mass for specific entropy.

## 12.2 Isentropic Processes

• Check reversibility and adiabatic first.

$$\gamma = \frac{c_p}{c_v}$$

Temperature and Specific Volume

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

**Temperature and Pressure** 

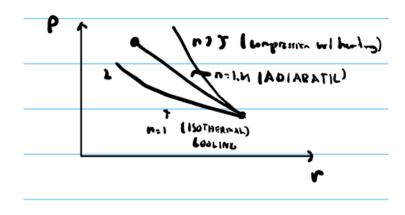
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Pressure and Specific Volume

$$\frac{\frac{P_2}{P_1}}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$$

$$P_1 v_1^{\gamma} = P_2 v_2^{\gamma} = const.$$

 The isentropic compression or expansion of an ideal gas is a polytropic process.



## L13: Entropy Balances for Control Masses, & Entropy Balances for Control Volumes

### Tips:

- Make sure you know if something is a thermal reservoir or not. This can screw you over.
- Also think about doing energy balances and also entropy balances.
- If no input entropy (heater), then  $S_{in}$  is 0, and no output entropy (insulated), then  $S_{out}$  is 0.
- Don't just assume that an object will go to the same temperature, it could an in-between.
- When you are doing entropy calculations for a thermal reservoir (ie. surroundings), ensure that you are getting the heat gain/loss from the perspective of the surroundings (BE CARE OF SIGN).
- You can do it either way, as long as you are consistent. Often it becomes confusing to use the sign for Q since it depends on what you are defining as your system. In this problem we have two systems, the house and the exterior. Q is negative for the house (it is being removed) and positive for the exterior (it is being added). We calculate  $\Delta S$  for each system can then add them up to get the net change in S, which gives us Sgen. That is a typo in the last line. It should read Sgen =  $\Delta Se + \Delta Si$
- Alternately we forget the sign convention and keep the value of Q positive all the time. We can then calculate the value of ΔS for each system, and then give each term a +ve or -ve sign depending on whether heat is added or removed. It is essential the same process. You can do it either way, but make sure you are consistent.

## 13.0 Entropy Balance for Control Mass

## **Rate of Entropy Transfer**

$$\dot{S} = \frac{\dot{Q}}{T}$$

• T is the temperature of the boundary.

#### Entropy Balance for a Control Mass with a Rate Basis

$$\frac{dS}{dt} = \dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen}$$

•  $\frac{dS}{dt} = 0$  for steady state.

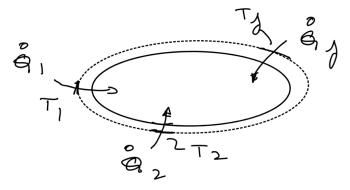
#### **Entropy Balance for a Control Mass**

$$\Delta S = S_{in} - S_{out} + S_{gen}$$

- $S_{in}$  is any any input entropy,  $S_{out}$  is any output entropy (unless insulated), and  $\Delta S$  is the system changing.
- If T constant (usually surroundings) have constant temp. then  $\frac{Q}{T}$
- SIGN CONVENTION ALREADY BUILT IN, TAKE ABSOLUTE VALUE

$$\frac{dS}{dt} = \sum_{j} \frac{\dot{Q}_{j}}{T_{j}} + \dot{S}_{gen}$$

- $T_j$  is the instantaneous temperature of the **boundary across which heat** transfer  $\dot{Q}_j$  occurs.
- In control mass, the only way for entropy to enter or leave the system is with heat transfer.



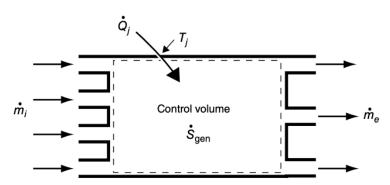
## 13.1 Entropy Balance for Control Volume

$$\dot{S}_{gen} = \sum_{e} \dot{m}_{e} S_{e} - \sum_{i} \dot{m}_{i} S_{i} - \int \frac{\delta \dot{Q}}{T}$$

• The last term will use the Q from the perspective of the system to surroundings.

$$\dot{S}_{gen} = \sum_{e} \dot{m}_{e} s_{e} - \sum_{i} \dot{m}_{i} s_{i} + \sum_{j} \frac{\dot{Q}_{j}}{T_{j}}$$

- SIGN CONVENTION ALREADY BUILT IN, TAKE ABSOLUTE VALUE
- $\sum_{out} \dot{ms} \sum_{in} \dot{ms}$  is fluid stream
- $\int \frac{\delta \dot{Q}}{T}$  is surroundings
- $\dot{Q} = 0$  (adiabatic)
- $\dot{S}_{gen} = 0$  (reversible)
- $\frac{dS}{dt} = 0$  (steady state)
- In control volume, entropy may enter and leave with **mass** and **heat** transfer



## L14: Isentropic, Steady-Flow Devices, Isentropic Efficiencies, & Bernoulli's Equation

#### 14.0 Efficiencies

- Usually have to solve the system twice.
  - With isentropic conditions.
  - With actual conditions.
- Or work backwards with efficiency.

| Efficiencies | Definition |
|--------------|------------|
|--------------|------------|

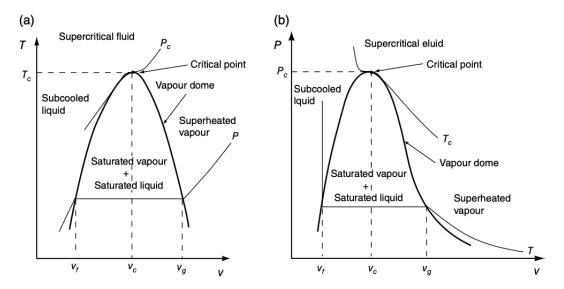
| General Isentropic | For systems that produce work $ w  \le  w _{isentropic} $ :   |
|--------------------|---|
| Efficiency         | · ·   |
| _                  | $\eta = \frac{\frac{\left \frac{\dot{w}}{\dot{m}}\right }{\left \frac{\dot{w}}{\dot{m}}\right }}{\left \frac{\dot{w}}{\dot{m}}\right }$ |
|                    | $\left  \frac{W}{} \right _{isentropic}$  |
|                    | Turbine: Using isentropic conditions, we will get the highest   |
|                    | possible amount of work.  |
|                    | For systems that are supplied with work $W_{isentropic} \leq W$ :   |
|                    | $\frac{\dot{W}}{\dot{m}}$   |
|                    | $\eta = \frac{\overline{\dot{m}}_{isentropic}}{\underline{\dot{W}}_{i}}$  |
|                    | <ul> <li>Pump &amp; Compressors: Using isentropic conditions, we will get</li> </ul>  |
|                    | the lowest possible amount of work.   |
|                    |   |
| Turbine Efficiency | $\eta_t = \frac{T_2 - T_1}{T_2 - T_1} < 1$  |
|                    | • Assuming $\Delta KE = \Delta PE = 0$ and an adiabatic process.  |
| Compressor or Pump | $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_2}$   |
| Efficiency         |   |
|                    | • Assuming $\Delta KE = \Delta PE = 0$ and an adiabatic process.  |
| Nozzle Efficiency  | $\frac{\overline{v}_2^2}{2}$ $\frac{-2}{v_2}$   |
|                    | $\eta_{nozzle} = \frac{\frac{\overline{v_2}}{2}}{\frac{\overline{v_2}}{v_{2s}}} = \frac{\overline{v_2}}{\overline{v_2}}$                |
|                    | 2 25  |
|                    | • Assuming $\Delta PE = W = 0$ and an adiabatic process.  |
|                    | Nozzle: Using isentropic conditions, we get the highest possible  |
|                    | exit velocity.  |
|                    | Diffuser: Using isentropic conditions, we get the highest exit  |
|                    | pressure.   |

#### 14.1 Bernoulli's Equation

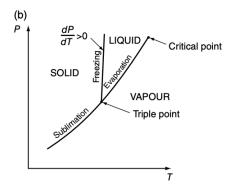
$$\frac{\frac{P_{1}}{\rho} + \frac{\overline{V}_{1}^{2}}{2} + gz_{1} = \frac{P_{2}}{\rho} + \frac{\overline{V}_{2}^{2}}{2} + gz_{2}}{\frac{P}{\rho} + \frac{\hat{V}_{2}^{2}}{2} + gz = constant}$$

## L15: Phase Change on P-v and T-v Diagrams, Quality, and Property Tables

#### 15.0 Phases



**Figure 7.9** Liquid–vapour phase change shown on (a) *T-v* and (b) *P-v* diagrams.



- Subcooled liquid => Incompressible liquid
- Superheated vapour => Ideal gas

| Phases                                  | Definition  |
|---|---|
| Subcooled Liquid (T <t<sub>sat)</t<sub> | Two properties are required (T & P) [One phase: Liquid]                   |
| Saturated Mixture (T=T <sub>sat</sub> ) | One property is required & quality (T or P) [Two phases: Liquid + Vapour] |
| Superheat Vapour (T>T <sub>sat</sub> )  | Two properties are required (T & P) [One phase: Gas]                      |
| Saturated Liquid (x=0)                  | One property is required (T or P) [Only liquid]                           |
| Saturated Vapour (x=1)                  | One property is required (T or P) [Only vapour]                           |

## L16: Solving Problems Using Property Tables

## Tips:

- If isentropic,  $\Delta s \Rightarrow s_1 = s_2$
- If you pass the critical point, look at A6.
- If properties lie between, use linear interpolation.
- This is a first-law control mass problem of the type that we have seen in Chapter 4, the only difference being that we now also consider the possibility of a phase change occurring within the fluid. To determine whether this happens, we have to fix the initial and final state of the water in the system.
- Saturated:  $\phi_f < \phi < \phi_g$
- Superheated:  $\phi_g < \phi$
- Subcooled:  $\phi_f > \phi$

## **Equations**

| Equations | <b>Definition:</b> f: saturated liquid   g: saturated vapour |
|-----------|--|
| Mass      | $m = m_f + m_g$  |

| Quality       | The mass fraction of a (vapour, liquid) mixture comprised of vapour   |
|---------------|---|
|               | $x = \frac{m_g}{m}$   |
|               | $1 - x = \frac{m_f}{m}$   |
|               | $x = \frac{\Phi - \Phi_f}{\Phi_g - \Phi_f}$ , where $\Phi = v$ , $h$ , $s$ , $u$ , $etc$ (any intensive property) |
| Properties    | $v = (1 - x)v_f + xv_g$   |
|               | $u = (1 - x)u_f + xu_g$   |
|               | $h = (1 - x)h_f + xh_g$   |
|               | $s = (1 - x)s_f + xs_g$   |
| Interpolation | $\frac{Y_{avg} - Y_1}{X_{avg} - X_1} = \frac{Y_2 - Y_1}{X_2 - X_1}$   |

## L17: Heat Engines, Perpetual Motion Machines, & Carnot Engine

## 17.0 Heat Engine:

Any device that works in a cycle and does work on the surroundings as long as heat is supplied to it.

## 17.1 Thermal Efficiency

$$\eta_{th} = \frac{W_{net}}{Q_{H}} = \frac{\dot{W}_{net}}{\dot{Q}_{H}}$$

## 17.2 Perpetual Motion Machines

Any engine that violates either 1st or 2nd law of Thermodynamics cannot exist.

#### 17.3 Note

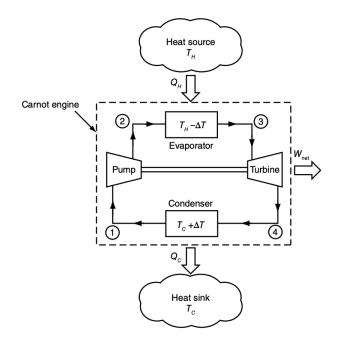
Over a cycle, all properties go to their initial value.

#### 17.4 Kelvins-Planck Statement

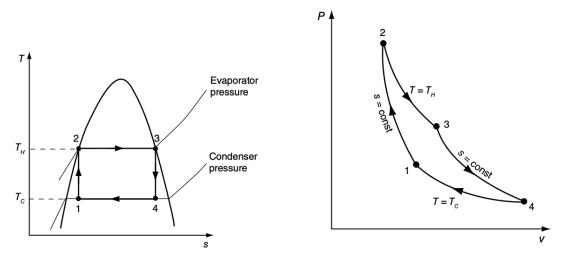
It is impossible for any device operating in a cycle to receive heat from a high temperature source and produce a net amount of work without recycling heat to a low temperature sink.

## 17.5 Carnot Engine (Control Volume)

- All processes are reversible and no entropy is generated.
- Heat and work transfers are reversible.



## 17.6 Diagrams



**Figure 8.6** A two-phase Carnot engine cycle shown on a *T-s* diagram. **Figure 8.8** A Carnot cycle shown on a *P-ν* diagram.

- **T-S Diagram:** Area under 2-3 is heat added, and area under 4-1 is heat rejected.
- P-v Diagram: Area under the curve shows the work done per unit mass during the cycle.

$$w_{net} = (T_H - T_C)(\Delta s_{23})$$

## 17.7 Carnot Efficiency

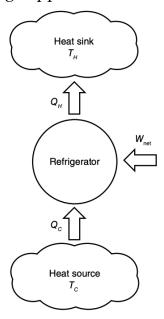
$$\eta_{Rev} = \eta_{Carnot} = 1 - \frac{T_C}{T_H}$$

## L18: Refrigerators and Heat Pumps, & Carnot Principles

• Always convert to Kelvin.

## 18.0 Refrigerators (Reverse Heat Engine)

Device that takes heat from a low temperature region and transfers it to a high temperature region while being supplied with work.



## 18.1 Coefficient of Performance for Refrigerator and Heat Pump

$$COP = \frac{\textit{Heat exchange with the conditioned space}}{\textit{Supplied work}}$$

Refrigerator:

$$COP_{R} = \frac{Q_{C}}{W_{net}}$$

**Heat Pump:** 

$$COP_{HP} = \frac{|Q_H|}{W_{net}} = \frac{1}{1 - \frac{T_c}{T_H}}$$

#### **Reverse Carnot:**

$$COP_{Rev} = COP_{Reverse\ Carnot} = \frac{1}{\frac{T_H}{T_C} - 1}$$

• Note:  $COP_{R/HP} \leq COP_{Rev}$ 

#### 18.2 Clausius Statement

Always need to supply a net amount of work to a refrigerator.

## **18.3 Carnot Principles**

- 1) The efficiency of a reversible heat engine is always greater than that of an irreversible engine operating between the same temperatures.
- 2) The efficiency of all reversible engines between the same two temperatures are the same.