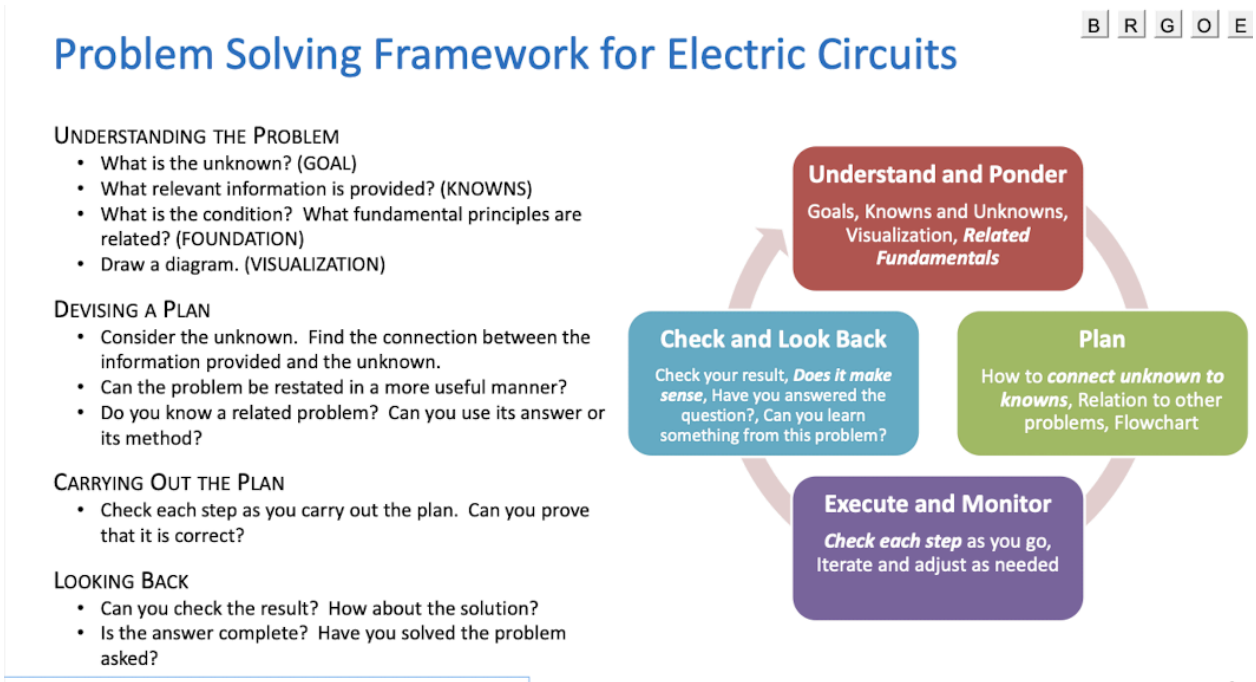
ECE159: Cheat Sheet

Processes

0.1 Problem Solving Framework for Electric Circuits:

Write all dependents in terms of the equation that relates it to the circuit.



0.2 Resistive Circuit Process:

1. Systematically reduce the resistive network so that the resistance seen by the source is R_{eq}
2. Determine the source current for a voltage source or the source voltage for a current source.
3. Expand the network, retracing the simplification steps, and apply Ohm's law, KVL, KCL, voltage and/or current division to determine all the currents and voltages in the circuit.

0.3 Nodal Analysis Solution Process:

1. Select one reference node (**be creative**)

2. Assign node voltages to all non-reference nodes ($N - 1\, nodes$).
3. Write a **constraint equation** for each independent and dependent voltage source.
4. Solve the easy nodes first, the ones with a voltage source connected to the reference node.
5. Apply KCL to each non-reference node not connected to a voltage source.
6. Apply KCL to each supernode.
7. Solve the system of $N - 1$ equations for all node voltages at the non-reference nodes.
8. Answer what the question asks with the information.

Tips:

- The node voltages control the direction of the current arrow (i.e. depending on the voltage polarity we choose will decide how the current flows into the passive component [left or right])
- We can change the direction of the current from KCL to KCL equation for the same current.

0.4 Mesh Analysis Solution Method:

1. Determine the number of meshes in the circuit.
2. Assign a mesh current to each (N meshes), and use **a consistent direction.**
3. If there are any current sources which are shared amongst two meshes, create a supermesh by combining those two meshes.
4. Write a constraint equation for each independent or dependent current source and for each supermesh ($N_l\, equations$).
5. Apply KVL to each mesh and/or supermesh ($N - N_l\, equations$).

6. Solve the set of N equations to find the unknown mesh currents.
7. Answer what the question asks with the information.

- Have mesh currents go in the same direction to make the signs familiar and easy to do without headwork.
- For current mesh analysis, the resistor is always positive for voltage because of PSC
- **REMEMBER PASSIVE ELEMENTS HAS CURRENT GO INTO POSITIVE SO BOTH WAYS.**

0.5 Procedure for Finding the Thevenin or Norton equivalent circuit:

1. Remove the load and replace with either an **open-circuit** to find the **open-circuit voltage** (Thevenin equivalent), or replace with a **short-circuit** to find the **short-circuit current** (Norton equivalent)
2. Determine the **Thevenin equivalent resistance** of the network. There are **three different ways.**
   a. **A circuit with only independent sources.**
      i. Short out all voltage sources and open all current sources. Then find the equivalent resistance of the resulting resistive network.
   b. **A circuit containing only dependent sources.**
      i. Apply an independent voltage (1V) or current (1A) source to the output terminals (**with the load removed**). Determine the current through the voltage source, or the voltage across the current source to find R_{Th}.
      ii. **Note:** The equivalent circuit will be just a Thevenin resistance.
      iii. **Always be careful** in not **separating** the dependent source from its controlling variable.
   c. **A circuit containing both independent and dependent sources.**

i. Determine both the open-circuit voltage and the short-circuit current to find $R_{Th} = \frac{V_{OC}}{I_{SC}}$.

3. The load can now be reconnected across the terminals of the **equivalent circuit**, which **replicates the original circuit.**

0.6 Solving a General First-Order Circuit:

1. Draw circuit for $t < 0\ s$, and determine the initial value of the inductor's current or the capacitor's voltage.
2. Draw the circuit for $t = 0 +\ s$, and determine the initial value of the desired current or voltage, $i(0 +)$ or $v(0 +)$.
   a. To do this we need to first replace the inductor with a current source (with a value of $i_L(0 +)$), or replace the capacitor with a voltage source (with a value of $v_c(0 +)$). Then we can solve the circuit at this instant of time ($t = 0 +\ s$) for the initial values that we are looking for.
3. Draw the circuit for $t > 0\ s$, and determine the Thevenin resistance "seen" by the inductor of the capacitor.
   a. From this we can calculate the time constant, $\tau = \frac{L}{R_{th}}$ for RL circuits and $\tau = R_{th}C$ for RC circuits.
4. Draw the circuit for $t \to \infty\ s$, and determine the final current or voltage value, $i(\infty)$ or $v(\infty)$.
   a. To do this we replace the inductor with a short circuit or the capacitor with an open circuit.

0.7 AC Steady-State Circuit Analysis:

i. Converting the circuit from the time-domain to frequency-domain by:
   a) Converting all source values from time-domain sinusoidal values to phasor values.
   b) Converting all passive elements into impedances using the frequency ω of the source: $\mathbf{Z}_R = R\ \Omega$, $\mathbf{Z}_C = \frac{-j}{\omega C}\ \Omega$, and $\mathbf{Z}_L = j\omega L$
ii. Analyzing the converted circuit in the frequency domain using *all of the techniques* we have developed for DC circuit analysis
iii. Converting the resulting phasor voltages and currents into time-domain sinusoids using original source frequency ω:

$$\mathbf{V} = V_o\angle\theta = V_o e^{j\theta} \quad \rightarrow \quad v(t) = \mathrm{Re}\{\mathbf{V}e^{j\omega t}\} = \mathrm{Re}\{V_o e^{j\theta} e^{j\omega t}\} = V_o \cos(\omega t + \theta)$$

How to Check your Answers:

- Find the power of each element and then use conservation of energy to make sure that it's 0.
- Apply both mesh and nodal analysis
- When doing **complex calculations:**
  - Store values
  - Use brackets carefully

Equations:

Week 1:

1.1 Voltage:

$$v_{ab} = \frac{Change\ in\ Potential\ Energy}{Charge} = \frac{dw}{dq}$$

$$[V] = [\frac{J}{C}]$$

1.2 Current:

$$i = \frac{Charge}{Time} = \frac{dq}{dt}$$

$$[A] = [\frac{C}{s}]$$

1.3 Power:

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$[W] = [\frac{J}{s}]$$

1.4 Energy:

$$w = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt$$

$$[W] = [\frac{J}{s}]$$

- Energy absorbed (positive)
- Energy delivered (negative)

1.5 The Law of Conservation of Energy

$$\sum_{i=1}^{n} p_i = 0$$

1.6 Passive Sign Convention:

If a positive current enters the positive side of the voltage for that element, then energy is absorbed by that element. Otherwise, energy is delivered.

1.7 Ohm's Law

$$v = iR$$

$$[V] = [A][\Omega]$$

1.8 Conductance:

$$G = \frac{1}{R} = \frac{i}{v}$$

1.9 Power Dissipated in Resistor

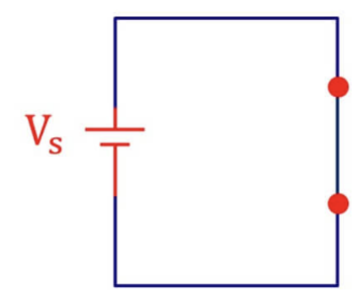
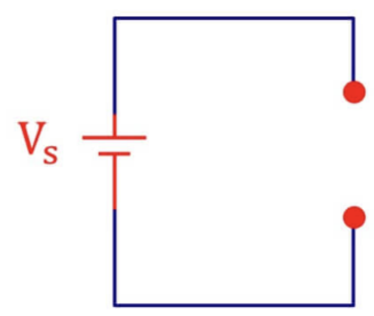
$$p = vi = i^2R = \frac{v^2}{R}$$

1.10 Relation Between Branches, Nodes, and Loops:

$$b = l + n - 1$$

- b is a **branch,** which is a specific element of the circuit.
- n is a **node,** which is a point of a circuit where two or more branches meet.
- l is a **loop,** which is any closed path within a circuit.

1.11 Short Circuit vs. Open Circuit:

Type of Circuit	Image	Info
Short Circuit		$v = 0$
Open Circuit		$i = 0$

1.12 Equivalent Resistances and Conductances:

- Parallel resistors: Have the same voltage (share two nodes).
- Series resistors: Have the same current (share one node).

Connection	**Equivalent Resistances**	**Equivalent Conductances**

Series	$R_{eq} = R_1 + R_2 + ... + R_n$	$G_{eq} = (\frac{1}{G_1} + \frac{1}{G_2} + ... + \frac{1}{G_n})^{-1}$
Parallel	$R_{eq} = (\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n})^{-1}$	$G_{eq} = G_1 + G_2 + ... + G_n$

2.1 Kirchhoff's Current Law:

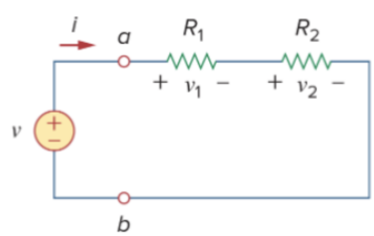
$$\sum_{Node\,?} i_{in} = \sum_{Node\,?} i_{out} \text{ or } \sum_{j=1}^{N} i_j = 0 \; (N = number\ of\ wires\ connected\ to\ node)$$

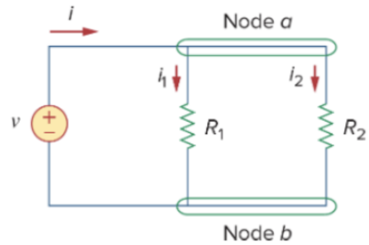
2.2 Kirchhoff's Voltage Law:

$$\sum_{Loop\,?} v_{rises} = \sum_{Loop\,?} v_{drops} \text{ or } \sum_{j=1}^{N} v_j = 0 \; (N = number\ of\ elements\ in\ that\ loop)$$

2.3 Voltage Division and Current Division Principle:

- Watch for signs while using these principles.

Division	Image	Equations
Voltage		$v_1 = \frac{R_1}{R_1+R_2} v_0$ $v_2 = \frac{R_2}{R_1+R_2} v_0$ **For N series resistors:** $v_n = \frac{R_n}{R_1+R_2+...+R_n} v$

Current		$i_1 = \frac{R_2}{R_1+R_2} i_0$ $i_2 = \frac{R_1}{R_1+R_2} i_0$ **For N parallel resistors:** $i_n = \frac{G_n}{G_1+G_2+...+G_N} i_0$

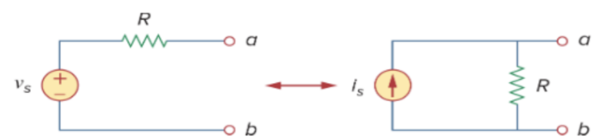
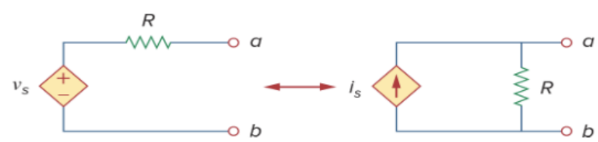
3.1 Nodal and Mesh Analysis Method By Inspection:

Type of Linear Circuits	Image	Where:
Only independent **current** sources	$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$	G_{kk} - Sum of conductances that are attached to node k. $G_{kj} = G_{jk}$ − Negative of the sum of the conductances directly connecting nodes k and j. v_k − The unknown voltage at node k. i_k − Sum of all independent current sources directly connected to node k, with currents entering the node being treated as positive.
Only independent **voltage** sources.	$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$	R_{kk} − Sum of the resistances in mesh k. $R_{kj} = R_{jk}$ − Negative of the sum of the resistance in common with meshes k and j. i_k − Unknown mesh current for mesh k in the clockwise

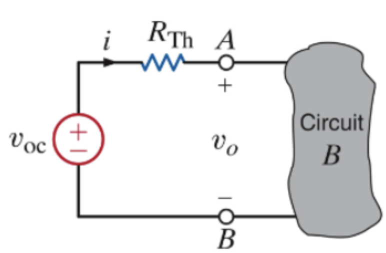
		direction. v_k – Sum taken clockwise of all independent voltage sources in mesh k, with a voltage rise treated as positive.

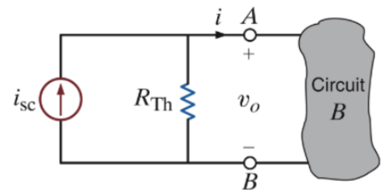
4.1 Source Transformation:

- Voltage sources connected in series in a single branch can be combined into a single source.
- Current sources connected in parallel with each other can be combined into a single source.

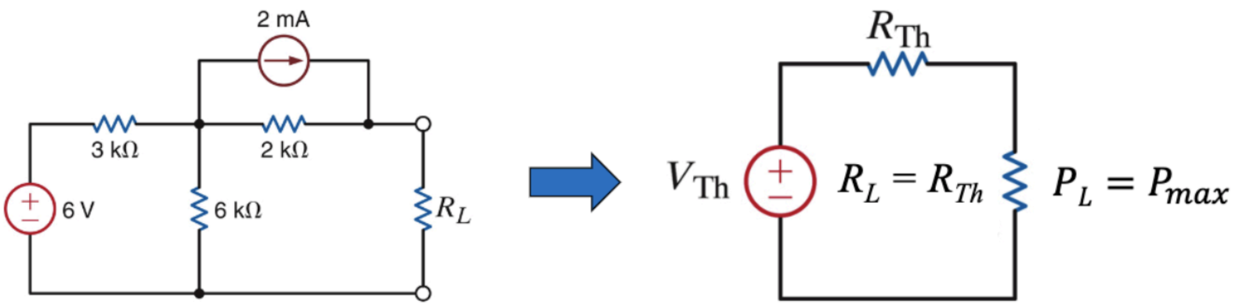
Type:	Image:
Independent	
Dependent	

4.2 Thevenin's and Norton's Equivalents:

Type	Image	Info
Thevenin		$R_{eq} = R_{Th} = R_N = \frac{v_{OC}}{i_{SC}}$

Norton		

5.1 Maximum Power Transfer:



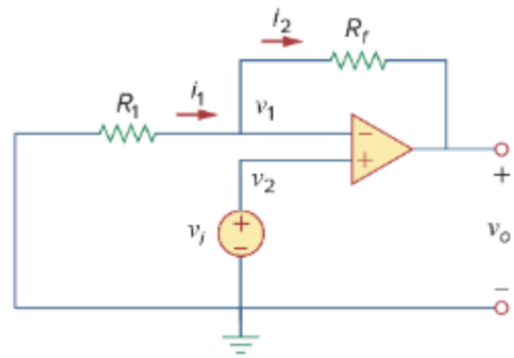
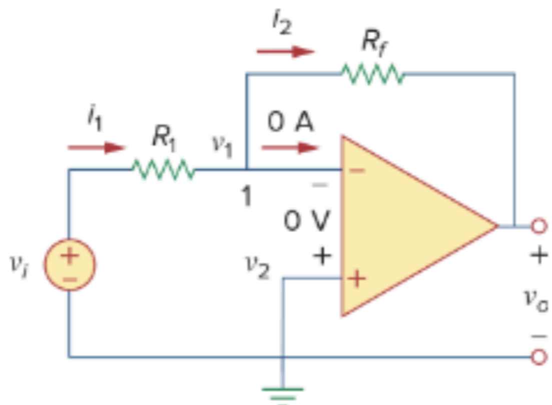
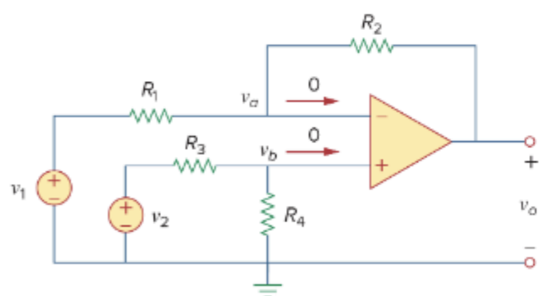
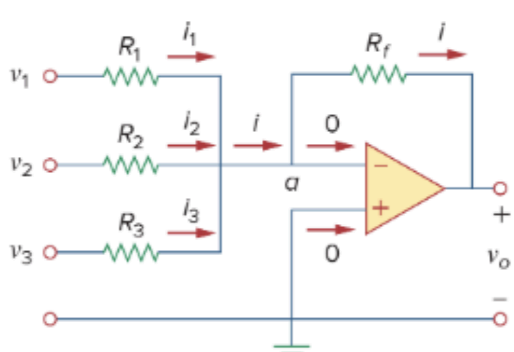
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{I_N^2 R_{Th}}{4}$$

5.2 Characteristics of Ideal Op-Amp Model:

1. Infinite input resistance ($R_{in} = \infty$)
2. Zero output resistance ($R_{out} = 0$)
3. Infinite voltage gain ($A_0 = \infty$)
4. Zero input current ($i_+ = i_- = 0$)
5. Zero input voltage ($v_d = v_{in} = v_+ - v_- = 0\ since\ v_+ = v_-$)

5.3 Op-Amp Circuits:

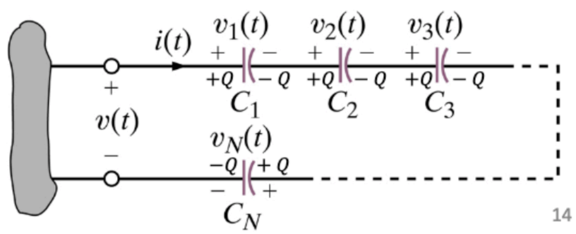
Type of Amplifiers	Circuit	Important Details

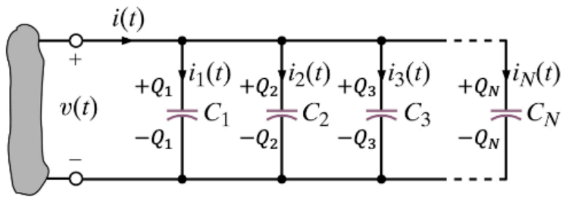
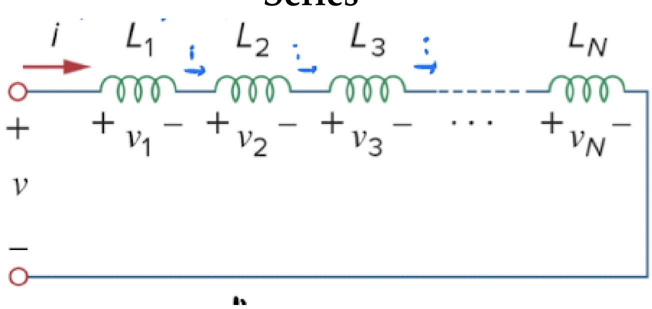
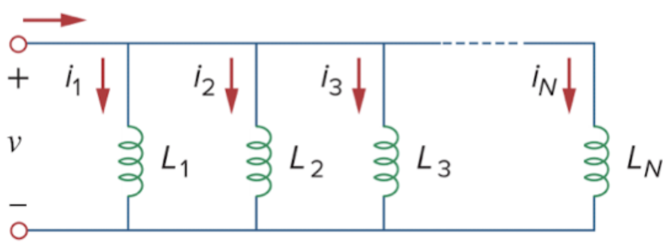
Non-inverting		Input source voltage source is connected to the “+” terminal Output voltage is given by $v_0 =+ \left(1 + \frac{R_f}{R_1}\right)v_i$
Inverting		Input source voltage is connected to the “-” terminal Output voltage is given by $v_0 =- \frac{R_f}{R_1}v_i$
Difference		Output voltage is related to the difference of voltage sources attached to “+” and “-” terminals. Output voltage is given by $v_0 = \frac{R_2}{R_1}(v_2 - v_1)$ if $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
Summing		Output voltage is related to the weighted sum of the voltage sources attached to the “-” terminal. Output voltage is given by $v_0 =- \left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$

6.1 Characteristics of Capacitors:

Characteristic:	**Equation:**
Energy stored in a capacitor	$w_C = \frac{1}{2}Cv^2$
Current	$i_C(t) = C\frac{dv_c(t)}{dt}$
Voltage	$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt$
Power	$p_C(t) = v_C(t)i_C(t) = Cv_C\frac{dv_C}{dt}$
Stored Energy	$w_C(t) = \int_{-\infty}^{t} p_L(\tau)d\tau = \frac{1}{2}Cv_C^2(t)$
After the initial charging up, a capacitor will look like an open-circuit to a DC source (i.e. $i_C = 0$) and will have a stored energy according to this equation.	

6.2 Capacitors and Inductors in Series and Parallel:

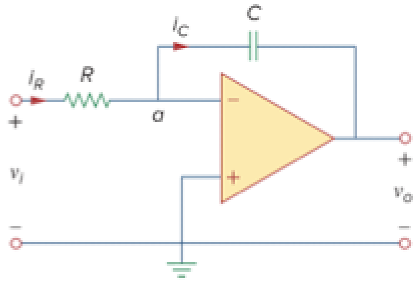
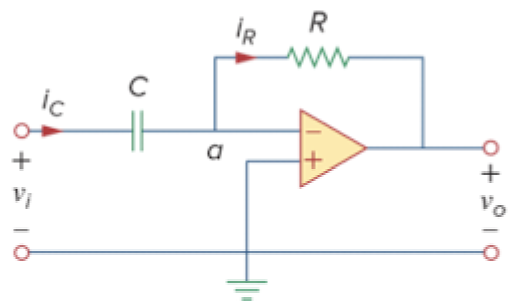
<table>
<tr><th>Type</th><th>Characteristics</th></tr>
<tr><td colspan="2">Capacitors</td></tr>
<tr><td>Series

</td><td>$C_{eq} = (\frac{1}{C_1} + \frac{1}{C_2} +...+ \frac{1}{C_N})^{-1}$</td></tr>
<tr><td>Parallel</td><td>$C_{eq} = C_1 + C_2 +...+ C_N$</td></tr>
</table>

<table>
<tr><td>
</td><td></td></tr>
<tr><td colspan="2">Inductors</td></tr>
<tr><td>Series

</td><td>$L_{tot} = L_1 + L_2 + ... + L_N$</td></tr>
<tr><td>Parallel

</td><td>$L_{tot} = (\frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_N})^{-1}$</td></tr>
</table>

6.3 Characteristics of Inductors:

Characteristic:	Equation:
Voltage	$v(t) = L\frac{di}{dt}$
Power	$p_L(t) = v_L(t)i_L(t) = Li_L\frac{di_L}{dt}$
Stored Energy	$w_L(t) = \int_{-\infty}^{t} p_L(\tau)d\tau = \frac{1}{2}Li_L^2(t)$
After the initial "charging up", an inductor will look like a short circuit to a DC source.	

6.4 Integrator and Differentiator:

Op Amp Integrator 	$v_0(t) = \frac{-1}{RC}\int_0^t v_i(t)dt$
Op Amp Differentiator 	$v_0(t) =- RC\frac{dv_i}{dt}$

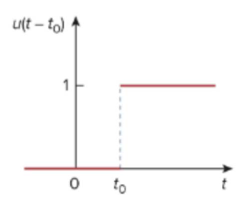
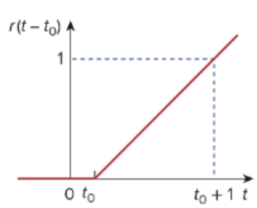
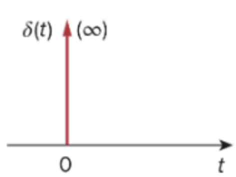
7.1 Behaviour of Inductors and Capacitors:

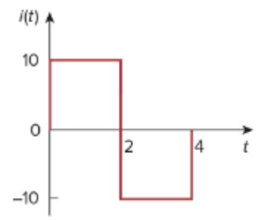
Inductor	$v_L(t) = L\frac{di_L(t)}{dt}$, $with\ i_L(0\ -) = i_L(0\ +)$	Current through an inductor cannot change instantaneously. When a switch has been closed for a long time, an inductor "looks like" a short circuit
Capacitor	$i_C(t) = L\frac{dv_C(t)}{dt}$, $with\ v_C(0\ -) = v_C(0\ +)$	Voltage through a capacitor cannot change instantaneously. When a switch has been closed for a long time, a capacitor "looks like" an open circuit.

7.2 First-Order Equations:

Type	Equations
First order circuit	$$v(t) = v(\infty) + [v(t_s^+) - v(\infty)]e^{-\frac{(t-t_s)}{\tau}}$$ $$i(t) = i(\infty) + [i(t_s^+) - i(\infty)]e^{-\frac{(t-t_s)}{\tau}}$$ $$\tau = R_{Th}C, \tau = \frac{L}{R_{Th}}$$
IF Method	$$\rho(t) = e^{\int P(t)dt}$$ $$v(t) = \frac{1}{\rho(t)}[\int \rho(t)f(t)dt + C]$$

7.3 Unit-Step, Ramp, and Delta Functions:

Function	Example	Info
Unit-step	 $u(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$	Can be used to represent the action of a switch in a transient circuit.
Ramp	 $r(t-t_0) = \begin{cases} 0, & t \le t_0 \\ t-t_0, & t \ge t_0 \end{cases}$	Another common input for electric circuits.
Delta	 $\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$	Represent an infinitesimally small burst of energy. It is the derivative of the

		unit-step function. It can be used to "sample" a function through integration: $\int_a^b f(t)\delta(t - t_0)dt = f(t_0)$
Gate	 $i(t) = 10[u(t) - 2u(t-2) + u(t-4)]$	The 3 functions above can be used to represent more complex inputs to electric circuits.

7.4 First-Order Transient Op Amp Circuits:

Analysis of First-Order Transient Circuits with Op Amps
1. Using Differential Equations: • Once the initial conditions for the capacitor voltage are known, then the circuit can be analyzed for $t > 0\ s$. • Redraw the circuit for $t > 0\ s$, and then apply KCL at the input node to relate the input to the output. Make use of the fact that $i_C = C\frac{dv_C}{dt}$. • Solve the resulting differential equation, for either the output voltage or the voltage across the capacitor.
2. Using Standard Form: • Knowing that the solution will be of the form: $x(t) = x(\infty) + [x(0+) - x(\infty)]e^{\frac{-t}{\tau}}$ • We can find the individual values by considering the circuit at the critical times.

8.1 Leading vs. Lagging

Leading	If sinusoid A leads sinusoid B, then sinusoid

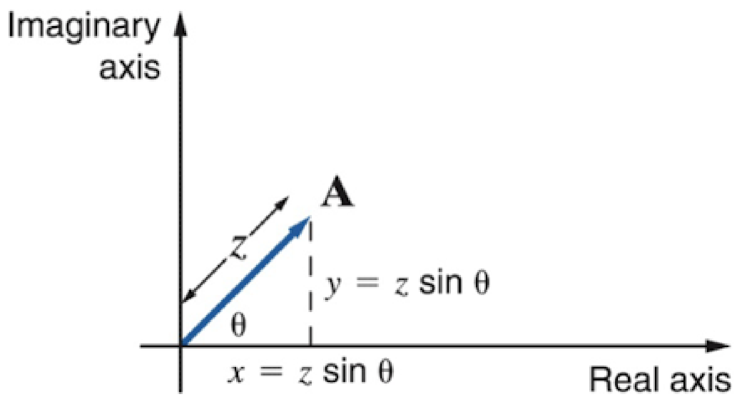
	A will reach a value before sinusoid B reaches that value.
Lagging	If sinusoid A lags sinusoid B, then sinusoid A will reach a value after sinusoid B reaches that value.

8.2 Sinusoidal Characteristics

Sinusoidal Signals: $v(t) = V_0 cos(\omega t + \varphi)$	
Amplitude (V_0)	The maximum value that the sinusoidal signal achieves
Period (T)	The time in seconds that it takes for the signal to return to the same value.
Frequency $(f = \frac{1}{T})$	This is the inverse of the period (Hz)
Angular Frequency $(\omega = 2\pi f)$	This is the frequency with units of radians/second (rad/s).
Phase (θ)	This is the "shift" in radians of the function from its reference with $\theta = 0$.

8.3 Complex Numbers:

EXPONENTIAL	POLAR	RECTANGULAR
$ze^{j\theta}$	$z\angle\theta$	$x+jy$
$\theta = \tan^{-1} y/x$	$\theta = \tan^{-1} y/x$	$x = z\cos\theta$
$z = \sqrt{x^2 + y^2}$	$z = \sqrt{x^2 + y^2}$	$y = z\sin\theta$



8.4 Phasors:

Phasors keeps track of the amplitude and phase of a sinusoidal signal in a circuit.	
Differentiating	Differentiating a time-domain signal is equivalent to **multiplying** a frequency-domain phasor by $j\omega$. $\frac{di}{dt} \rightarrow j\omega I$
Integrating	Integrating a time-domain signal is equivalent to **dividing** a frequency-domain phasor by $j\omega$. $\int idt \rightarrow \frac{I}{j\omega}$

9.1 Impedances:

$$Z = \frac{V}{I} = R(\omega) + jX(\omega)\Omega$$

- The real part of the impedance is the **resistance.**
- The imaginary part of the impedance is the **reactance.**

9.2 Admittances:

$$Y = \frac{I}{V} = G(\omega) + jB(\omega)\,S$$

- The real part of the impedance is the **conductance.**

- The imaginary part of the impedance is the **susceptance.**
- Note $G(\omega) \neq \frac{1}{R(\omega)}$

9.3 Time Domain to Phasor Domain of Passive Elements:

Resistor • Voltage and current in phase	$R\ \Omega$
Capacitor • Voltage lags current by 90°	$\frac{-j}{\omega C}\ \Omega$
Inductor • Voltage leads current by 90°	$j\omega L\ \Omega$

9.4 Parallel and Series Combinations for Z and Y:

Impedance	
Series	$\overline{Z} = \overline{Z}_1 + \overline{Z}_2 + \overline{Z}_3$
Parallel	$\overline{Z} = (\frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3})^{-1}$
Admittance	
Series	$\overline{Y} = (\frac{1}{\overline{Y}_1} + \frac{1}{\overline{Y}_2} + \frac{1}{\overline{Y}_3})^{-1}$
Parallel	$\overline{Y} = \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3$

9.5 KVL and KCL

These concepts hold in AC analysis.

9.6 Sine to Cosine:

A time-domain signal that is *expressed in terms of $sin(\omega t)$*, needs to be first converted into a cosine signal before conversion into the frequency domain. To do this we can remember:

$$v(t) = V_o \sin(\omega t) = V_o \cos(\omega t - 90°) \quad \rightarrow \quad \mathbf{V} = V_o \angle -90°$$

10.1 Instantaneous Power through an element:

$$p(t) = v(t)i(t) = \frac{1}{2}V_0 I_0 cos(\theta_v - \theta_i) + \frac{1}{2}V_0 I_0 cos(2\omega t + \theta_v + \theta_i)$$

10.2 Average Power:

$$p_{avg} = \frac{1}{2}V_0 I_0 cos(\theta_v - \theta_i) = \frac{1}{2}V_0 I_0 cos(\theta_z) = \frac{1}{2}Re[VI^*]$$

$$p_{avg} = V_{rms} I_{rms} cos(\theta_z)$$

$$[W]$$

For a resistive load:

$$p_{avg} = \frac{1}{2}|I|R = \frac{1}{2}\frac{|V|^2}{R} = \frac{1}{2}|V||I|$$

10.3 Maximum Average Power:

$$Z_L = Z_{Th}^* = R_{Th} - jX_{Th}$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}$$

- For maximum power transfer to load R_L, $R_L = |Z_{Th}|$

10.4 RMS:

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)}$$

$\frac{V_{Pk}}{\sqrt{2}}$ or $\frac{I_{Pk}}{\sqrt{2}}$ for sinusoids

10.5 Apparent Power:

$$S = \frac{1}{2}V_{peak} I_{peak} = |V_{rms}||I_{rms}| = \sqrt{P^2 + Q^2}$$

$$[VA]$$

10.6 Relationship Between Average Power and Apparent Power:

$$P = Scos(\theta_z)$$

10.7 Power Factor

$$pf = \frac{P}{S} = cos(\theta_v - \theta_i) = cos(\theta_z)$$

- This is a dimensionless quantity
- The power factor is *leading* if the current leads the voltage, meaning the load is *capacitive*
- The power factor is *lagging* if the current lags the voltage, meaning the load is *inductive*

11.1 Active and Reactive Power:

$$P = V_{rms} I_{rms} cos(\theta_v - \theta_i)$$

$$Q = jV_{rms} I_{rms} sin(\theta_v - \theta_i)$$

11.1 Complex Power:

$$\overline{S} = (Real\ power) + j(Reactive\ power) = \frac{1}{2} V I^* = V_{rms} I^*_{rms}$$

$$\overline{S} = P + jQ = S\angle\theta_Z$$

11.2 Relationship Between Apparent Power and Complex Power:

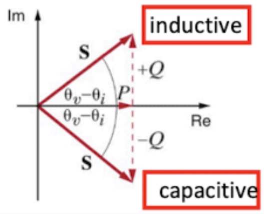
$$|\widehat{S}| = S = \frac{1}{2} V_{peak} I_{peak} = V_{rms} I_{rms}$$

11.3 Impedance's Complex Power:

$$\overline{S} = |I_{rms}|^2 Z = \frac{|V_{rms}|^2}{Z^*}$$

11.4 Power Triangle

3. The *power triangle* summarizes complex power and its relationships:



1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

11.5 Alternate Expressions:

$$Q = Ptan(cos^{-1}(pf))$$

11.5 Summary

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$
$$= |\mathbf{V}_{\text{rms}}|\ |\mathbf{I}_{\text{rms}}| \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\ |\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\ \cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\ \sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

11.6 Power Factor Correction:

To *increase the power factor* from $\text{pf}_1 = \cos\theta_1$ to $\text{pf}_2 = \cos\theta_2$ for a load that dissipates a real power given by P, we need a capacitance of:

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

$$C = \frac{Q_c}{\omega V_{rms}^2}$$

12.0

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Aiding

Opposing:

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Transient vs. Steady-State Component and Natural vs. Forced Response:

Transient vs. Steady-State	In general, a first-order circuit with a switch will have a response where all of the voltages and currents in the circuit have a **transient component** (usually after 5τ) and a **steady-state component.**
Natural Response	The **natural response** of a circuit relates to the energy stored in the capacitor and inductor at $t = 0\ s$. This is the

	response of the circuit that is caused by a “charged up” storage element releasing its energy to the rest of the circuit after the switch is moved.
Forced Response	The **forced response** of a circuit is caused by the independent sources in the circuit after the switch has moved.