# ROB311 Quiz 3

Hanhee Lee

April 3, 2025

## Contents

# Turn-Taking Multi-Agent Decision Algorithms

## 1 Monte-Carlo Tree Search (MCTS)

**Algorithm:**

1. Selection: Traverse using an alternate policy until a node has unexplored children.

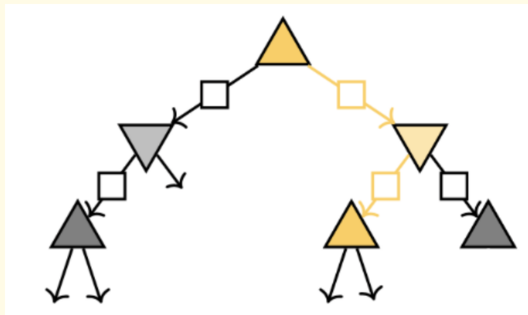

Figure 1

   - Our Agent (Upper Triangle): Uses UCB to choose the next node to explore
   - Other Agent (Down Triangle): Can't control their actions, so this agent picks w/ their own heuristic.
   - Square Boxes: Estimated values (i.e. $n$ and $\hat{q}$)
   - Ends when there is at least one action that hasn't been explored yet. In this case, two actions ahven't been explored.
   - Can skip expansion and simulation if the most recently expanded node is a terminal state.
2. Expansion: Expand an unexplored child; initialize $n(a)$ and $\hat{q}(s, a)$.

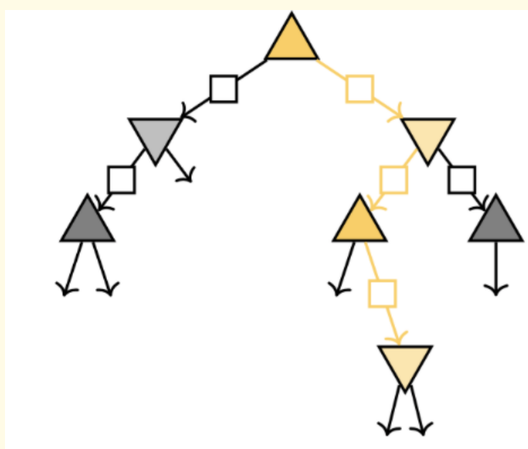

Figure 2

   - $\hat{q}(s, a)$ is initialized to 0 and $n(a)$ is initialized to 1 b/c we've visited this node once.
   - Randomly pick an unexplored action unless there is only one action left.
   - Can skip similuation if the most recently expanded node is a terminal state.
3. Simulation: Traverse using the random policy until a terminal node is reached.

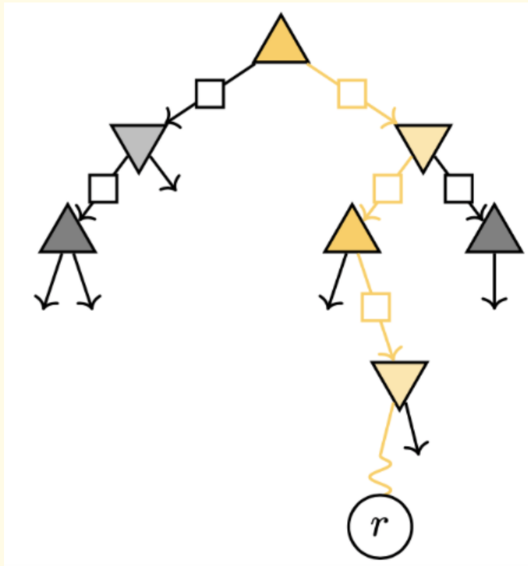


Figure 3

- Using random policy to simulate the game until a terminal state is reached (i.e. reward is obtained)

4. Back-propogation: Get the reward and reverse; update $n(a)$ and $\hat{q}(s,a)$.

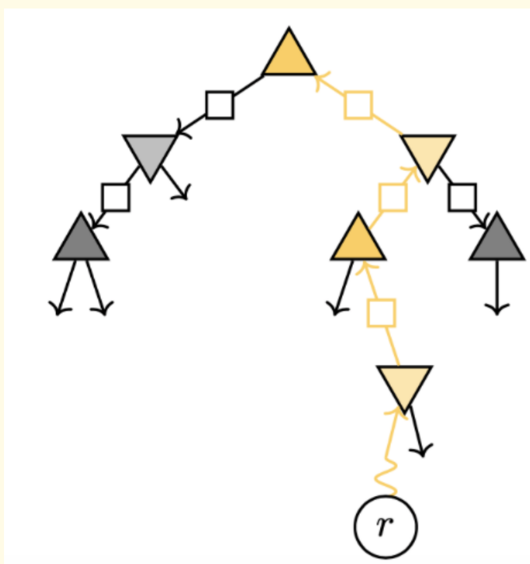


Figure 4

- Go up the path in yellow and update the values of $n(a)$ and $\hat{q}(s,a)$ for OUR agent only (i.e. the upper triangle)

**Warning:**
- Works for more than 2 agents.
- Don't need to know anyone else's reward function.
- Has to be turn taking but can be not alternating (i.e. immediate switch between agents)
- Can augment simultaneous actions
- Communication
- Works fo rnon-zero sum games.

### 1.1 1 Player vs. 2 Player Turn-Taking Game Tree

**Notes**:
- 1 Player:
  – 
- 2 Player:
  – 

## 1.2 Examples

### 1.2.1 1 Player Turn-Taking Game Tree

#### 1.2.2 2 Player Turn-Taking Game Tree

**Example**:

1. **Given:** Consider a simplified two-player turn-based game tree. You are currently at the root node $S_0$, which has three possible actions $a_1, a_2, a_3$. The current statistics of its children are as follows:

| **Action** | $N(s_0, a)$ | $\bar{X}(s_0, a)$ |
|---|---|---|
| $a_1$ | 10 | 0.6 |
| $a_2$ | 5 | 0.8 |
| $a_3$ | 0 | – |

   - $N(s_0, a)$: Number of times action $a$ has been selected at state $s_0$
   - $\bar{X}(s_0, a)$: Average reward obtained from action $a$ at state $s_0$
   - UCB $= \bar{X}(s_0, a) + \sqrt{\frac{\ln(t)}{N(s_0, a)}}$
     - $t$: Total number of actions taken at $s_0$
2. **Problems:**
   - If we were to use the UCB algorithm, which nodes get selected during the selection phase? Which node gets expanded during the expansion phase?
   - Suppose from the expanded node, simulation is performed until termination. A reward of +1 is obtained. Update the statistics at $s_0$ accordingly.
   - Then, repeat the question, assuming a reward of $-1$ is attained after the simulation phase.
3. **Solution:**
   (a) **Selection 1:** $s_0$ since we traverse until a node has unexplored children (i.e. $s_3$ is unexplored)
   (b) **Expansion 1:** $s_3$ is automatically expanded since it is the only unexplored child of $s_0$ w/ $N(s_0, a_3) = 1$ and $\bar{X}(s_0, a_3) = 0$
   (c) **Simulation 1:** Get a reward of +1
   (d) **Back Propogation 1:** For this edge from $s_0$ to $s_3$, we update the statistics as follows:
   - $N(s_0, a_3) = 1$
   - $\bar{X}(s_0, a_3) = \frac{1}{1} = 1$

   (e) **Selection 2:** $s_0$ and choose the action with the highest UCB value for $s_1$, $s_2$, and $s_3$:
   - $UCB(s_0, a_1) = 0.6 + \sqrt{\frac{\ln(16)}{10}} = 1.13$
   - $UCB(s_0, a_2) = 0.8 + \sqrt{\frac{\ln(16)}{5}} = 1.54$
   - $UCB(s_0, a_3) = 1 + \sqrt{\frac{\ln(16)}{1}} = 2.67$. Therefore, choose $s_3$ as part of the selection phase and assume it has unexplored children.

   (f) **Expansion 2:** Not enough info but assume we expand an unexplored child.
   (g) **Simulation 2:** Get a reward of $-1$
   (h) **Back Propogation 2:** For this edge from $s_0$ to $s_3$, we update the statistics as follows:
   - $N(s_0, a_3) = 2$
   - $\bar{X}(s_0, a_3) = \frac{1 + (-1)}{2} = 0$

**Example**:

1. **Given:** Consider (partial) 2-player turn-taking game-tree in which 21 iterations of MCTS have already been performed:

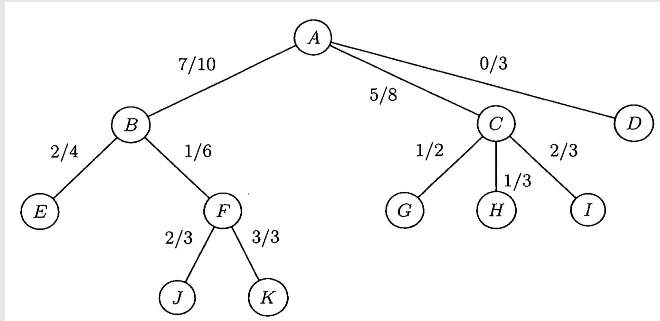


Figure 5

   - Total reward: Numerator
   - Total number of times action $a$ has been selected at state $s$: Denominator
2. **Problem:** If we use UCB to rank order state-action pairs, which of the following states will be chosen during the 22nd selection phase.
3. **Solution:**
   - $\text{UCB}(AB) = 7/10 + \sqrt{\frac{\ln(21)}{10}} = 1.25$
     - $\text{UCB}(BE) = 2/4 + \sqrt{\frac{\ln(10)}{4}} = 1.26$
     - $\text{UCB}(BF) = 1/6 + \sqrt{\frac{\ln(10)}{6}} = 0.79$
   - $\text{UCB}(AC) = 5/8 + \sqrt{\frac{\ln(21)}{8}} = 1.24$
   - $\text{UCB}(AD) = 0/3 + \sqrt{\frac{\ln(21)}{3}} = 1.01$
   - Therefore, choose A,B,E by selecting the nodes with the highest UCB values.

**Example**:

1. **Given:** Consider (partial) 2-player turn-taking game-tree in which 9 iterations of MCTS have already been performed:

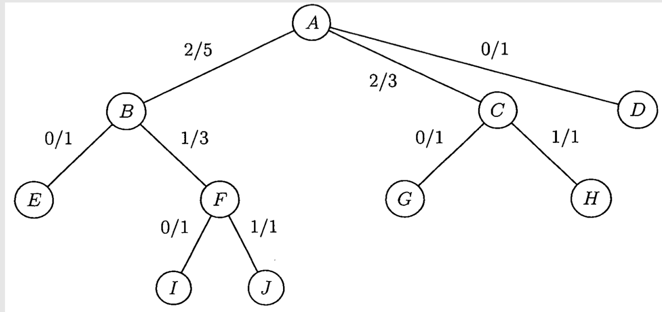


Figure 6

   - **Fix:** $CG$ has 0/2 not 0/1 and $CH$ has 0/1 not 1/1
2. **Problem:** Suppose path chosen during the 10th selection phase had the state sequence $\langle A, C, H \rangle$ (i.e. $H$ is the state expanded during the 10th expansion phase)
   - The simulation phase lasts for 12 transitions, after which a terminal state is reached.
   - The reward to the last turn-taker was +4.
   - Find $q(A, \langle A, B \rangle)$, $q(A, \langle A, C \rangle)$, $q(C, \langle C, H \rangle)$
3. **Solution:**
   - Assuming P1 starts at $A$, then P2 goes at $C$, then P1 goes at $H$, that means after 12 transitions (**even number**), P1 is the last turn-taker, therefore, P1 gets the reward of +4.
   - **Backpropogation:**
     - $N(C, \langle C, H \rangle) = 1,\ X(C, \langle C, H \rangle) = 4$ so 4/1
     - $N(A, \langle A, C \rangle) = 4,\ X(A, \langle A, C \rangle) = 2 + 4 = 6$ so 6/4
     - $q(A, \langle A, B \rangle) = 2/5 = 0.4$
     - $q(A, \langle A, C \rangle) = 6/4 = 1.5$
     - $q(C, \langle C, H \rangle) = 4/1 = 4$