ROB311 Quiz 1

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1 Constraint Satisfaction Problems

1.1 Setup of CSP

Definition: A **constraint satisfaction problem (CSP)** consists of:
- a set of **variables**, $\mathcal{V}$, where the domain of $V \in \mathcal{V}$ is $\text{dom}(V)$
- a set of **constraints**, $\mathcal{C}$, where the scope of $C \in \mathcal{C}$ is $\text{scp}(C) \subseteq \mathcal{V}$

1.2 Assignment

Definition: An **assignment** is a set of pairs, $\{(V, v)\}_{V \in \tilde{\mathcal{V}}}$, where $v \in \text{dom}(V)$, and $\tilde{\mathcal{V}} \subseteq \mathcal{V}$. It is **complete** if $\tilde{\mathcal{V}} = \mathcal{V}$, and **partial** otherwise.

1.3 Formulating a CSP as a Search Problem

Motivation: We don't formulate a CSP as a search problem because the path tree of all possible ways to build a complete assignment is too large. The number of paths in the three is

$$\mathcal{O}\left(|\mathcal{V}|! \times b^d\right)$$

- $b = \max_{V \in \mathcal{V}} |\text{dom}(V)|$
- $d = |\mathcal{V}|$

1.4 Consistent

1.4.1 Complete Assignment

Definition: A complete assignment, A, is **consistent** if it satisfies every constraint $\mathcal{C}$ with $\text{scp}(\mathcal{C}) \subseteq \tilde{\mathcal{V}}$.

Warning: A solution to a CSP is any complete and consistent assignment.

1.4.2 Partial Assignment

Definition: A (possibly partial) assignment, $\{(V, v)\}_{V \in \tilde{\mathcal{V}}}$, is **consistent** if it satisfies every constraint, $C \in \mathcal{C}$ such that $\text{scp}(C) \subseteq \tilde{\mathcal{V}}$.

1.4.3 k-Consistent

Definition: A CSP is ***k*-consistent** if for any consistent assignment of $k - 1$ variables, $\{(V, v)\}_{V \in \tilde{\mathcal{V}}}$, and any k^{th} variable, V', there is a value, $v' \in \text{dom}(V')$, so the assignment, $\{(V, v)\}_{V \in \tilde{\mathcal{V}}} \cup \{(V', v')\}$ is consistent.

Notes:
- Edge/Arc Consistent: $k = 2$

1.5 Constraint Satisfaction Algorithm

Algorithm:

```
A ← {}                                   ▷ initialize an empty assignment
for V ∈ 𝒱 do 𝒟(V) ← COPY(dom(V))
SATISFY(𝒱, 𝒞, 𝒟, A)
```

1.5.1 Satisfy

Algorithm:

```
procedure SATISFY(𝒱, 𝒞, 𝒟, A):
    if COMPLETE(A, 𝒱) then
        return A                                   ▷ a solution was found
    V ← REMOVE(𝒱, A)
    for v ∈ 𝒟(V) do                                ▷ try each value in V's current domain
        𝒟′ ← COPY(𝒟)                               ▷ cache the current domains for backtracking
        A ← A ∪ {(V, v)}
        𝒟(V) ← {v}
        𝒟, success ← ENFORCE(𝒱, 𝒞, 𝒟, V, k)
        if success then                            ▷ enforce k consistency
            A ← SATISFY(𝒱, 𝒞, 𝒟, A)                ▷ recursively continue if possible
            if A ≠ NULL then
                return A
        𝒟 ← 𝒟′                                     ▷ backtrack if not possible
        A ← A \ {(V, v)}
    return NULL                                    ▷ No solution found in this branch
```

1.5.2 Enforce: Enforcing k-Consistency

Algorithm: Pre-pruning: Enforce without assigning any variables.

```
procedure ENFORCE(𝒱, 𝒞, 𝒟, V, k):
    Q ← {C ∈ 𝒞 s.t. V ∈ scp(C)}                    ▷ initialize affected constraints
    while Q ≠ ∅ do
        C ← REMOVE(Q)
        for V′ ∈ scp(C) do       ▷ enforce consistency on each variable w.r.t. each affected constraint
            success ← ENFORCEVAR(k, V′, 𝒱, 𝒞, 𝒟)
            if not success then
                return False                       ▷ consistency could not be enforced
            Q ← Q ∪ {C′ ∈ 𝒞 | V′ ∈ scp(C′)}
    return True                                    ▷ consistency was enforced
```

1.5.3 EnforceVar: Enforcing k-Consistency

Algorithm:

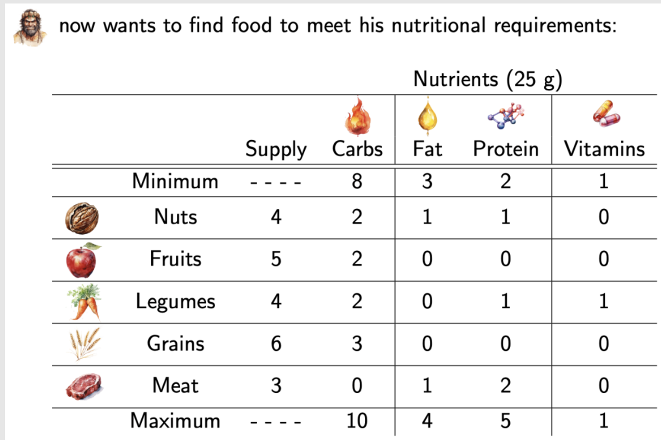
```
procedure ENFORCEVAR(𝒱, 𝒞, 𝒟, V, k):
    for v ∈ 𝒟(V) do
        for C ∈ 𝒞 do
            if V ∈ scp(C) and |scp(C)| ≤ k then
                flag ← False
                for A ∈ 𝒳 × 𝒟(V′) do
                    if A ∪ {(V, v)} ∈ 𝒞 then
                        flag ← True
                        break
                if not flag then
                    𝒟(V) ← 𝒟(V) \ {v}
                if 𝒟(V) = ∅ then
                    return False                   ▷ no valid domain values remain for V
    return True
```

1.6 Canonical Problems

Process: Setup of CSP:

1. Determine variables to track, domain of each variable, and constraints.

Example:


now wants to find food to meet his nutritional requirements:

	Supply	Carbs	Fat	Protein	Vitamins
		Nutrients (25 g)			
Minimum	- - - -	8	3	2	1
Nuts	4	2	1	1	0
Fruits	5	2	0	0	0
Legumes	4	2	0	1	1
Grains	6	3	0	0	0
Meat	3	0	1	2	0
Maximum	- - - -	10	4	5	1

Figure 1: Information

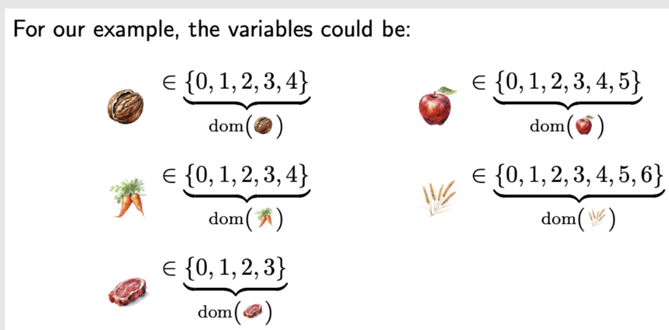


Figure 2: Variables

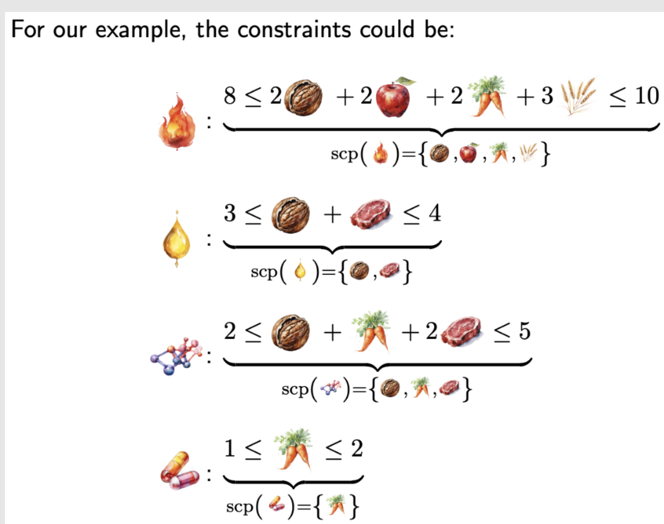


Figure 3: Constraints

Process: How to build a hyper-graph?

1. Circle the variables that appear in constraint $C_i\ \forall i$.

Example:

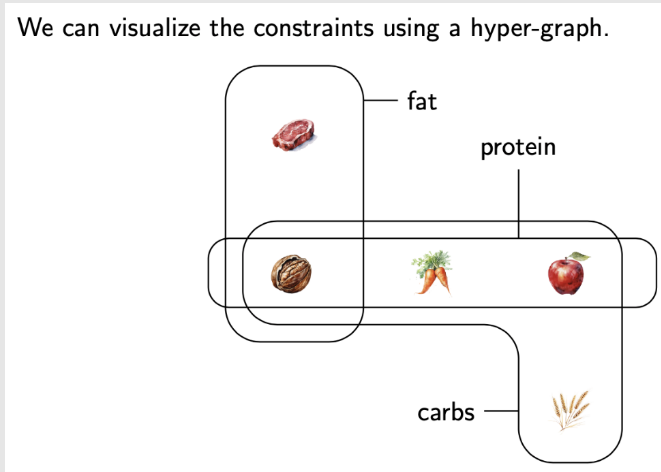


Figure 4

Process: How to Enforce k-Consistency?

1. Given $\mathcal{V}$ w/ $\text{dom}(V) = \{v_1, \ldots, v_{|\text{dom}(V)|}\}\ \forall V \in \mathcal{V}$ and $\mathcal{C}$ w/ $\text{scp}(C) = \{V_1, \ldots, V_{|\text{scp}(C)|}\}\ \forall C \in \mathcal{C}$.
2. Remove all constraints that have $k+1$ or more variables and add the rest to a queue.
3. **Pre-pruning:** For each remaining $C \in \mathcal{C}$, do the following:
   (a) For each $V \in \text{scp}(C)$, do the following:
      i. For each $v \in \text{dom}(V)$, do the following:
      - Fix V to v.
      - For the other $V \in \text{scp}(C)$, check if the constraint is satisfied by trying all combinations (need only one).
      - **Key:** If the constraint is not satisfied, then remove the value from $\text{dom}(V)$. Add any affected constraints back to the queue.
4. Repeat until the queue is empty.

Warning: Can think of checking as picking $k-1$ variables, then choosing any value for the k^{th} variable that satisfies all constraints. While enforcing is fixing a variable to a value, then checking if there is a combination for the other variables that satisfies all constraints.

Warning: Enforcing k-consistency is enforcing $k-1, \ldots, 1$-consistency.

Process: How to determine a solution to a CSP?

1. After pre-pruning the domains.
2. Assign variables in alphabetical order and values in numerical order.
3. Prune the pre-pruned domains.
4. If you can assign all variables, then you have a solution. If you have domain wipeout, backtrack.
5. Repeat the process until you find all solutions.

Process: Checking k-Consistency

1. Enforce k-consistency.
2. If you have to pre-prune, then not k-consistent.

Example: Pre-Pruning Domains

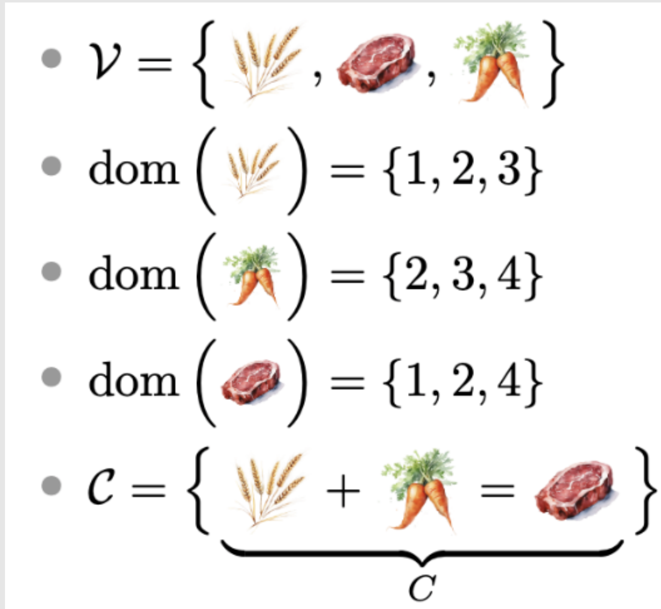


Figure 5

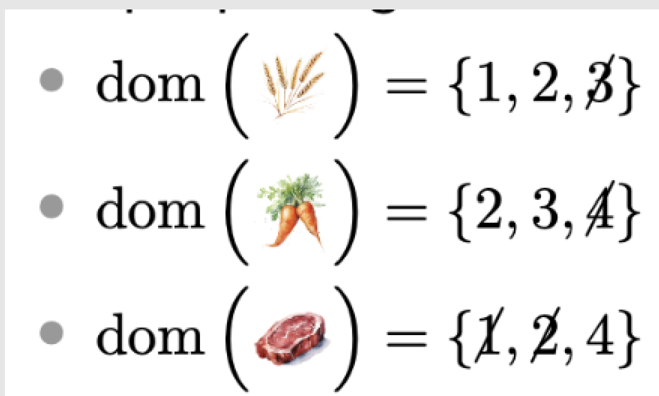


Figure 6: Pre-pruning. Since only one constraint, it is also pruning.

Example:

1. **Given:** Consider a CSP in which $\mathcal{V} = \{A, B, C, D, E\}$, where:

$$\begin{aligned} \text{dom}(A) &= \{0, 1, 2, 3, 4\} \\ \text{dom}(B) &= \{0, 1, 2, 3, 4\} \\ \text{dom}(C) &= \{0, 1, 2, 3\} \\ \text{dom}(D) &= \{0, 1, 2, 3, 4, 5\} \\ \text{dom}(E) &= \{0, 1, 2, 3, 4, 5, 6\} \end{aligned}$$

   and $\mathcal{C} = \{C_1, C_2, C_3, C_4\}$, where:

$$\begin{aligned} &C_1 : 8 \leq 2a + 2b + 2d + 3e \leq 10 \\ &C_2 : 3 \leq a + c \leq 4 \\ &C_3 : 2 \leq a + b + 2c \leq 5 \\ &C_4 : 1 \leq b \leq 2 \end{aligned}$$

2. **Problem:** Solve the following CSP using $k = 4$ consistency. Pre-prune the domains using $k = 4$ consistency. Assign variables in alphabetical order and values in numerical order.

Example: 4-Consistency Pre-Pruning

Queue	**Fixed Value**	**Satisfactory Combination?**
		$C_4 : 1 \leq b \leq 2$
$\{C_2, C_3, C_1\}$	$b = 0,\ b = 1,\ b = 2,\ b = 3,\ b = 4$	No, Yes, Yes, No, No

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, 3\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$
- $\{C_2, C_3, C_1\}$

		$C_2 : 3 \leq a + c \leq 4$
$\{C_3, C_1\}$	$a = 0,\ a = 1,\ a = 2,\ a = 3,\ a = 4$	Yes, Yes, Yes, Yes, Yes
-	$c = 0,\ c = 1,\ c = 2,\ c = 3$	Yes, Yes, Yes, Yes

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, 3\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$
- $\{C_3, C_1\}$

		$C_3 : 2 \leq a + b + 2c \leq 5$
$\{C_1\}$	$a = 0,\ a = 1,\ a = 2,\ a = 3,\ a = 4$	Yes, Yes, Yes, Yes, Yes
-	$b = 1,\ b = 2$	Yes, Yes
-	$c = 0,\ c = 1,\ c = 2,\ c = 3$	Yes, Yes, Yes, No

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$
- $\{C_1, C_2\}$

Example: 4-Consistency Continued:

Queue	Fixed Value	Satisfactory Combination?

$$C_1 : 8 \leq 2a + 2b + 2d + 3e \leq 10$$

$\{C_2\}$	$a = 0$, $a = 1$, $a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes, Yes, Yes
-	$b = 1$, $b = 2$	Yes, Yes
-	$d = 0$, $d = 1$, $d = 2$, $d = 3$, $d = 4$, $d = 5$	Yes, Yes, Yes, Yes, Yes, No
-	$e = 0$, $e = 1$, $e = 2$, $e = 3$, $e = 4$, $e = 5$, $e = 6$	Yes, Yes, Yes, No, No, No, No

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_2\}$

$$C_2 : 3 \leq a + c \leq 4$$

$\{\}$	$a = 0$, $a = 1$, $a = 2$, $a = 3$, $a = 4$	No, Yes, Yes, Yes, Yes
-	$c = 0$, $c = 1$, $c = 2$	Yes, Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_3, C_1\}$

$$C_3 : 2 \leq a + b + 2c \leq 5$$

$\{C_1\}$	$a = 1$, $a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes, Yes
-	$b = 1$, $b = 2$	Yes, Yes
-	$c = 0$, $c = 1$, $c = 2$	Yes, Yes, No

- $\text{dom}(A) = \{\cancel{0}, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_1, C_2\}$

Example: 4-Consistency Continued:

Queue	Fixed Value	Satisfactory Combination?

$$C_1 : 8 \leq 2a + 2b + 2d + 3e \leq 10$$

$\{C_2\}$	$a = 1$, $a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes, Yes
-	$b = 1$, $b = 2$	Yes, Yes
-	$d = 0$, $d = 1$, $d = 2$, $d = 3$, $d = 4$	Yes, Yes, Yes, Yes, No
-	$e = 0$, $e = 1$, $e = 2$	Yes, Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, \cancel{4}, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_2\}$

$$C_2 : 3 \leq a + c \leq 4$$

$\{\}$	$a = 1$, $a = 2$, $a = 3$, $a = 4$	No, Yes, Yes, Yes
-	$c = 0$, $c = 1$	Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, \cancel{1}, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, \cancel{4}, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_3, C_1\}$

$$C_3 : 2 \leq a + b + 2c \leq 5$$

$\{C_1\}$	$a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes
-	$b = 1$, $b = 2$	Yes, Yes
-	$c = 0$, $c = 1$	Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, \cancel{1}, 2, 3, 4\}$, $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$, $\text{dom}(D) = \{0, 1, 2, 3, \cancel{4}, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{C_1\}$

$$C_1 : 8 \leq 2a + 2b + 2d + 3e \leq 10$$

$\{\}$	$a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes
-	$b = 1$, $b = 2$	Yes, Yes
-	$d = 0$, $d = 1$, $d = 2$, $d = 3$	Yes, Yes, Yes, No
-	$e = 0$, $e = 1$, $e = 2$	Yes, Yes, No

- $\text{dom}(A) = \{\cancel{0}, \cancel{1}, 2, 3, 4\}$, $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$, $\text{dom}(D) = \{0, 1, 2, \cancel{3}, \cancel{4}, \cancel{5}\}$
- $\text{dom}(E) = \{0, 1, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}\}$
- $\{\}$

Example: 4-Consistency Post-Pre-Pruning:

$$C_1 : 8 \leq 2a + 2b + 2d + 3e \leq 10$$
$$C_2 : 3 \leq a + c \leq 4$$
$$C_3 : 2 \leq a + b + 2c \leq 5$$
$$C_4 : 1 \leq b \leq 2$$

Solution	**Updated Necessary Domains After Assignment**
$A = 2$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1$	$\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1$	$\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1, D = 0$	$\text{dom}(E) = \{\cancel{0}, 1\}$
$A = 2, B = 1, C = 1, D = 0, E = 1$	**Solution Found**
$A = 2$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1$	$\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1$	$\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1, D = 1$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 2, B = 1, C = 1, D = 1, E = 0$	**Solution Found**
$A = 2$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1$	$\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1$	$\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 1, C = 1, D = 2$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 2, B = 1, C = 1, D = 2, E = 0$	**Solution Found**
$A = 2$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{\cancel{0}, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 2, B = 2$	$\text{dom}(C) = \{\cancel{0}, \cancel{1}\}$, $\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
-	**No Solution Found**
$A = 3$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 3, B = 1$	$\text{dom}(C) = \{0, \cancel{1}\}$, $\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0$	$\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0, D = 0$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0, D = 0, E = 0$	**Solution Found**
$A = 3$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 3, B = 1$	$\text{dom}(C) = \{0, \cancel{1}\}$, $\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0$	$\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0, D = 1$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 1, C = 0, D = 1, E = 0$	**Solution Found**
$A = 3$	$\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1\}$, $\text{dom}(D) = \{0, 1, 2\}$, $\text{dom}(E) = \{0, 1\}$
$A = 3, B = 2$	$\text{dom}(C) = \{0, \cancel{1}\}$, $\text{dom}(D) = \{0, \cancel{1}, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 2, C = 0$	$\text{dom}(D) = \{0, \cancel{1}, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 2, C = 0, D = 0$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 3, B = 2, C = 0, D = 0, E = 0$	**Solution Found**
$A = 4$	$\text{dom}(B) = \{1, \cancel{2}\}$, $\text{dom}(C) = \{0, \cancel{1}\}$, $\text{dom}(D) = \{0, 1, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 4, B = 1$	$\text{dom}(C) = \{0, \cancel{1}\}$, $\text{dom}(D) = \{0, \cancel{1}, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 4, B = 1, C = 0$	$\text{dom}(D) = \{0, \cancel{1}, \cancel{2}\}$, $\text{dom}(E) = \{0, \cancel{1}\}$
$A = 4, B = 1, C = 0, D = 0$	$\text{dom}(E) = \{0, \cancel{1}\}$
$A = 4, B = 1, C = 0, D = 0, E = 0$	**Solution Found**

Example: 3-Consistency

Fixed Value	**Satisfactory Combination?**
$C_4: 1 \leq b \leq 2$	
$b=0$, $b=1$, $b=2$, $b=3$, $b=4$	No, Yes, Yes, No, No

- $\text{dom}(A) = \{0,1,2,3,4\}$
- $\text{dom}(B) = \{\cancel{0},1,2,\cancel{3},\cancel{4}\}$
- $\text{dom}(C) = \{0,1,2,3\}$
- $\text{dom}(D) = \{0,1,2,3,4,5\}$
- $\text{dom}(E) = \{0,1,2,3,4,5,6\}$

Fixed Value	**Satisfactory Combination?**
$C_2: 3 \leq a + c \leq 4$	
$a=0$, $a=1$, $a=2$, $a=3$, $a=4$	Yes, Yes, Yes, Yes, Yes
$c=0$, $c=1$, $c=2$, $c=3$	Yes, Yes, Yes, Yes

- $\text{dom}(A) = \{0,1,2,3,4\}$
- $\text{dom}(B) = \{\cancel{0},1,2,\cancel{3},\cancel{4}\}$
- $\text{dom}(C) = \{0,1,2,3\}$
- $\text{dom}(D) = \{0,1,2,3,4,5\}$
- $\text{dom}(E) = \{0,1,2,3,4,5,6\}$

Fixed Value	**Satisfactory Combination?**
$C_3: 2 \leq a + b + 2c \leq 5$	
$a=0$, $a=1$, $a=2$, $a=3$, $a=4$	Yes, Yes, Yes, Yes, Yes
$b=1$, $b=2$	Yes, Yes
$c=0$, $c=1$, $c=2$, $c=3$	Yes, Yes, Yes, No

- $\text{dom}(A) = \{0,1,2,3,4\}$
- $\text{dom}(B) = \{\cancel{0},1,2,\cancel{3},\cancel{4}\}$
- $\text{dom}(C) = \{0,1,2,\cancel{3}\}$
- $\text{dom}(D) = \{0,1,2,3,4,5\}$
- $\text{dom}(E) = \{0,1,2,3,4,5,6\}$

Example:

Fixed Value	**Satisfactory Combination?**
	$C_2 : 3 \leq a + c \leq 4$
$a = 0$, $a = 1$, $a = 2$, $a = 3$, $a = 4$	No, Yes, Yes, Yes, Yes
$c = 0$, $c = 1$, $c = 2$	Yes, Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

	$C_3 : 2 \leq a + b + 2c \leq 5$
$a = 1$, $a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes, Yes
$b = 1$, $b = 2$	Yes, Yes
$c = 0$, $c = 1$, $c = 2$	Yes, Yes, No

- $\text{dom}(A) = \{\cancel{0}, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

	$C_2 : 3 \leq a + c \leq 4$
$a = 1$, $a = 2$, $a = 3$, $a = 4$	No, Yes, Yes, Yes
$c = 0$, $c = 1$	Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, \cancel{1}, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

	$C_3 : 2 \leq a + b + 2c \leq 5$
$a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes
$b = 1$, $b = 2$	Yes, Yes
$c = 0$, $c = 1$	Yes, Yes

- $\text{dom}(A) = \{\cancel{0}, \cancel{1}, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, \cancel{2}, \cancel{3}\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

4. **Conclusion:** $\text{dom}(A) = \{2, 3, 4\}$, $\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1\}$, $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$, $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

Example: 2-Consistency

Fixed Value	**Satisfactory Combination?**
$C_4 : 1 \leq b \leq 2$	
$b = 0$, $b = 1$, $b = 2$, $b = 3$, $b = 4$	No, Yes, Yes, No, No

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, 3\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

Fixed Value	**Satisfactory Combination?**
$C_2 : 3 \leq a + c \leq 4$	
$a = 0$, $a = 1$, $a = 2$, $a = 3$, $a = 4$	Yes, Yes, Yes, Yes, Yes
$c = 0$, $c = 1$, $c = 2$, $c = 3$	Yes, Yes, Yes, Yes

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, 3\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

4. **Conclusion:** $\text{dom}(A) = \{0, 1, 2, 3, 4\}$, $\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1, 2, 3\}$, $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$, $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

Example: 1-Consistency

Fixed Value	**Satisfactory Combination?**
$C_4 : 1 \leq b \leq 2$	
$b = 0$, $b = 1$, $b = 2$, $b = 3$, $b = 4$	No, Yes, Yes, No, No

- $\text{dom}(A) = \{0, 1, 2, 3, 4\}$
- $\text{dom}(B) = \{\cancel{0}, 1, 2, \cancel{3}, \cancel{4}\}$
- $\text{dom}(C) = \{0, 1, 2, 3\}$
- $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$
- $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$

4. **Conclusion:** $\text{dom}(A) = \{0, 1, 2, 3, 4\}$, $\text{dom}(B) = \{1, 2\}$, $\text{dom}(C) = \{0, 1, 2, 3\}$, $\text{dom}(D) = \{0, 1, 2, 3, 4, 5\}$, $\text{dom}(E) = \{0, 1, 2, 3, 4, 5, 6\}$