# ROB311 Quiz 2

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## Contents

# 1 Bode Plots

## 1.1 Bode Plots

**Process:**

### 1.1.1 Constant Gain

### 1.1.2 Pole or Zero at $\omega = 0$

### 1.1.3 Non-Zero Pole or Zero

### 1.1.4 Complex Conjugate Poles

## 1.2 Robustness Margins

**Motivation:** Approximate the GM and PM from the Bode plot:

- $L(s)$ is a strictly proper rational fn.
- $L(s)$ has no poles in $\mathbb{C}^+$ (no open loop variable poles)

### 1.2.1 Gain Margin

**Definition:**

$$|L(j\omega_{gc}) = 1| \iff |L(j\omega_{gc})|_{dB} = 0$$

### 1.2.2 Phase Margin

**Definition:**

$$|L(j\omega_{gc})| = 1 \implies |L(j\omega_{gc})|_{dB} = 0$$

## 2 Robustness Margins

# 3 Root Locus, Bode, and Nyquist

# 4 Control Design in the Frequency Domain

## 4.1 Goal

**Motivation**:

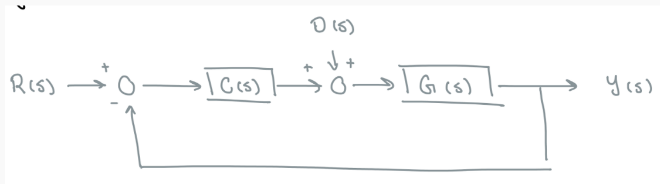


Figure 1

Design $C(s)$ so that the feedback loop:

- BIBO Stable: Verify using Nyquist criterion
  - roots$(1 + C(s)P(s)) \subseteq \mathbb{C}^-$
  - $C(s)G(s)$ has no pole-zero cancellations in $\overline{\mathbb{C}^+}$
- Satisfies certain performance specifications: Tune using Bode plots

## 4.2 Proportional Derivative (PD) Controller

**Motivation**: Increase PM at higher frequencies.

**Definition**:

$$C(s) = K(T_D s + 1) \tag{1}$$

- $K, T_D > 0$

Since $U(s) = C(s)E(s)$,

$$u(t) = \underbrace{KT_D e(t)}_{D} + \underbrace{Ke(t)}_{P} \tag{2}$$

### 4.2.1 Bode Plot

**Notes**:

$$|C(j\omega)|_{\text{dB}} = 20\log|K| + 20\log|j\omega T_D + 1|$$

$$\angle C(j\omega) = \angle K + \angle(j\omega T_D + 1)$$

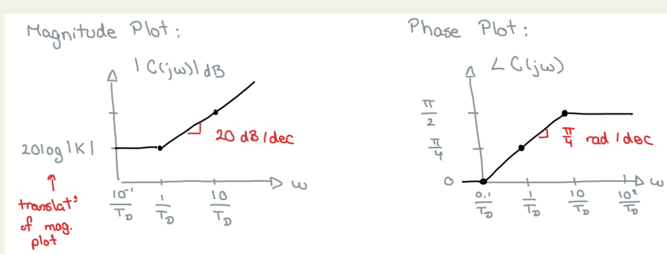


Figure 2

## 4.3 Proportional Integral (PI) Controller

**Motivation:** Increase the "system type" for better tracking (IMP) w/o affecting high frequencies.

**Definition:**

$$C(s) = K\left(1 + \frac{1}{T_I s}\right) = K\frac{T_I s + 1}{T_I s} \tag{3}$$

- $K, T_I > 0$

Since $U(s) = C(s)E(s)$,

$$u(t) = \underbrace{Ke(t)}_{P} + \underbrace{\frac{K}{T_I}\int_0^t e(\tau)d\tau}_{I} \tag{4}$$

### 4.3.1 Bode Plot

**Notes:**

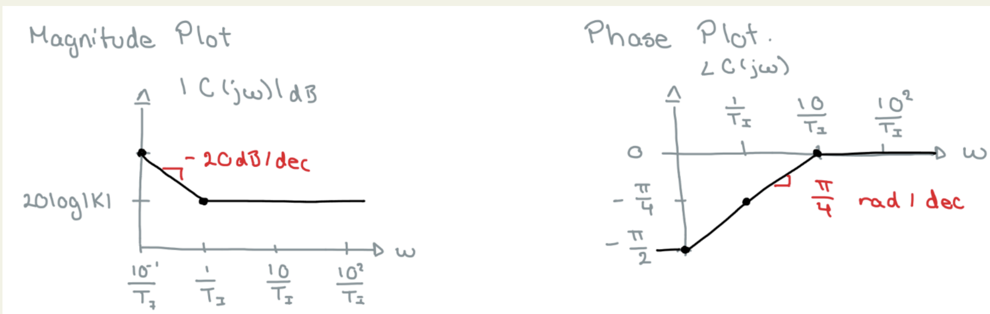


Figure 3

### 4.3.2 Design Procedure

**Process:**

1. Choose $K$ to meet asymptotic tracking or bandwidth (loosely increase $w_{gc}$) requirements (often set $K = 1$)
2. Find the crossover frequency $\omega_{gc}$ of $KG(j\omega)$. Suppose we are happy w/ the PM and $\omega_{gc}$.
3. Set $\dfrac{1}{T_I} \ll \omega_{gc}$. Typically want $\dfrac{1}{T_I}$ b/w $0.01\omega_{gc}$ and $0.1\omega_{gc}$

## 4.4 Proportional Integral Derivative (PID) Controller

**Definition:**

$$C(s) = K(T_D s + 1)\left(1 + \frac{1}{T_I s}\right) = K_p + \frac{K_I}{s} + K_D s \tag{5}$$

- $K, T_I, T_D > 0$

### 4.4.1 Design Procedure

**Process:**

1. Design $K$, $T_D$ (i.e. the PD controller) to increase the PM.
2. Design $T_I$ (i.e. the PI controller) to increase system type (satisfy IMP) w/o affecting high frequencies.

### 4.5 Examples

**Example**:

1. **Given:** $G(s) = \dfrac{1}{j\omega(j\omega+1)}$, $C(s) = K(T_D s + 1)$
2. **Problem:** Sketch Bode plots of $C(s)G(s)$ for PD controllers:
   - $K = 1$, $T_D = 10 \rightarrow 20\log|K| = 0$
   - $K = 10$, $T_D = 10 \rightarrow 20\log|K| = 20$
   - Corner frequency: $\omega_c = \dfrac{1}{T_D} = 10^{-1}$
3. **Solution:**

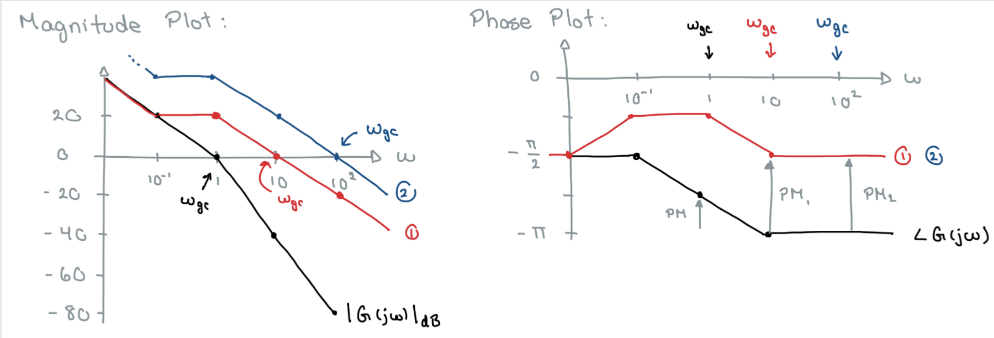


Figure 4

# 5 Lead Controller

**Motivation**:

- Approximates a PD controller as $\alpha \to 0$.
- Used to add phase at a particular frequency $\omega_{\max}$ in order to increase the PM (decreases the % OS).
- Increases $\omega_{gc}$

**Definition**:

$$C(s) = K\frac{T_s + 1}{\alpha T_s + 1} \tag{6}$$

- $K, T > 0$
- $0 < \alpha < 1$

## 5.1 Bode Plot

**Notes**:

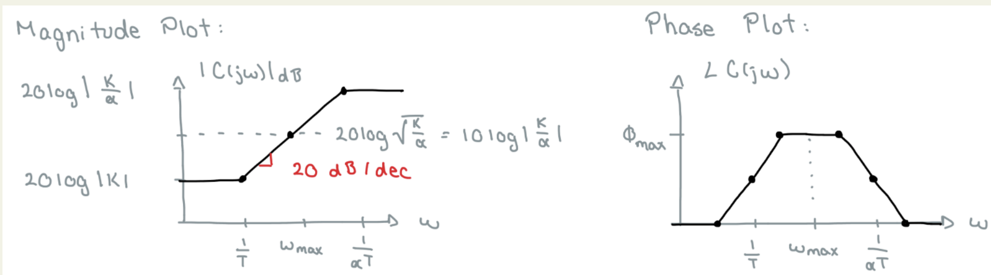


Figure 5

- $w_{\max} = \dfrac{1}{\sqrt{\alpha}T}$
- $\phi_{\max} = \alpha \sin\left(\dfrac{1-\alpha}{1+\alpha}\right)$

## 5.2 Design Procedure

**Process:**

1. Choose $K$ to satisfy asymptotic tracking (i.e. steady-state error) specifications or bandwidth (i.e. $\omega_{gc}$) requirements.
2. Draw Bode plot of $KG(j\omega)$ and find PM. Given a desired $\text{PM}_{\text{desired}}$, let $\Delta\text{PM} = \text{PM}_{\text{desired}} - \text{PM}$. Set $\phi_{\max} = \Delta\text{PM} + \text{Extra Margin}$
   - Rule of thumb: Extra Margin $= 30°$
     - Why? Since lead controller increases $\omega_{gc}$, this ends up decreasing the resulting PM. So to compensate, we add the extra margin.
3. $\alpha = \dfrac{1 - \sin(\phi_{\max})}{1 + \sin(\phi_{\max})}$
4. Solve for the frequency $\omega_{gc}^{\text{new}}$ s.t. $20\log|KG(j\omega_{gc}^{\text{new}})| = -20\log\left|\dfrac{1}{\sqrt{\alpha}}\right| = 10\log|\alpha|$
5. Impose $\omega_{\max} = \omega_{gc}^{\text{new}}$ and solve for $\dfrac{1}{T} = \sqrt{\alpha}\omega_{gc}^{\text{new}}$
6. Draw Bode plot of $C(j\omega)G(j\omega)$ and verify the PM is adequate.
7. Draw Nyquist plot to verify BIBO stability.