

# ROB311 Quiz 2

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# 1 Bode Plots

## 1.1 Bode Plots

**Process:**

### 1.1.1 Constant Gain

### 1.1.2 Pole or Zero at $\omega = 0$

### 1.1.3 Non-Zero Pole or Zero

### 1.1.4 Complex Conjugate Poles

## 1.2 Robustness Margins

**Motivation:** Approximate the GM and PM from the Bode plot:

- $L(s)$  is a strictly proper rational fn.
- $L(s)$  has no poles in  $\mathbb{C}^+$  (no open loop variable poles)

### 1.2.1 Gain Margin

**Definition:**

$$|L(j\omega_{gc})| = 1 \iff |L(j\omega_{gc})|_{dB} = 0$$

### 1.2.2 Phase Margin

**Definition:**

$$|L(j\omega_{gc})| = 1 \implies |L(j\omega_{gc})|_{dB} = 0$$

## 2 Robustness Margins

### 3 Root Locus, Bode, and Nyquist

## 4 Control Design in the Frequency Domain

### 4.1 Goal

Motivation:

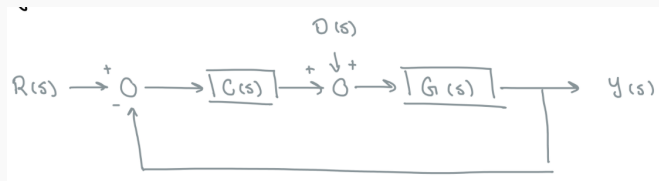


Figure 1

Design  $C(s)$  so that the feedback loop:

- BIBO Stable: Verify using Nyquist criterion
  - $\text{roots}(1 + C(s)P(s)) \subseteq \mathbb{C}^-$
  - $C(s)G(s)$  has no pole-zero cancellations in  $\overline{\mathbb{C}^+}$
- Satisfies certain performance specifications: Tune using Bode plots

### 4.2 Proportional Derivative (PD) Controller

Motivation: Increase PM at higher frequencies.

**Definition:**

$$C(s) = K(T_D s + 1) \quad (1)$$

- $K, T_D > 0$

Since  $U(s) = C(s)E(s)$ ,

$$u(t) = \underbrace{KT_D e(t)}_D + \underbrace{K e(t)}_P \quad (2)$$

#### 4.2.1 Bode Plot

Notes:

$$|C(j\omega)|_{\text{dB}} = 20 \log |K| + 20 \log |j\omega T_D + 1|$$

$$\angle C(j\omega) = \angle K + \angle(j\omega T_D + 1)$$

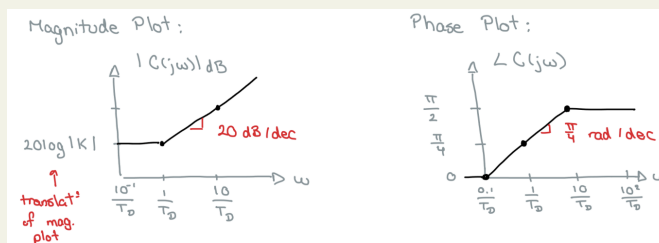


Figure 2

### 4.3 Proportional Integral (PI) Controller

**Motivation:** Increase the "system type" for better tracking (IMP) w/o affecting high frequencies.

**Definition:**

$$C(s) = K \left( 1 + \frac{1}{T_I s} \right) = K \frac{T_I s + 1}{T_I s} \quad (3)$$

- $K, T_I > 0$

Since  $U(s) = C(s)E(s)$ ,

$$u(t) = \underbrace{K e(t)}_P + \underbrace{\frac{K}{T_I} \int_0^t e(\tau) d\tau}_I \quad (4)$$

#### 4.3.1 Bode Plot

**Notes:**

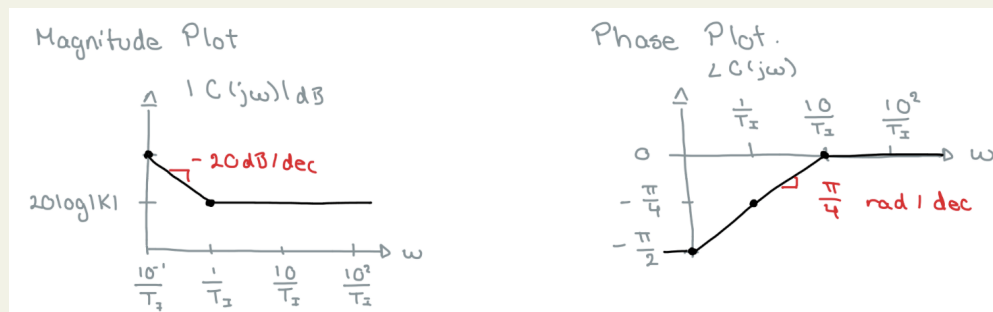


Figure 3

#### 4.3.2 Design Procedure

**Process:**

1. Choose  $K$  to meet asymptotic tracking or bandwidth (loosely increase  $\omega_{gc}$ ) requirements (often set  $K = 1$ )
2. Find the crossover frequency  $\omega_{gc}$  of  $KG(j\omega)$ . Suppose we are happy w/ the PM and  $\omega_{gc}$ .
3. Set  $\frac{1}{T_I} \ll \omega_{gc}$ . Typically want  $\frac{1}{T_I}$  b/w  $0.01\omega_{gc}$  and  $0.1\omega_{gc}$

### 4.4 Proportional Integral Derivative (PID) Controller

**Definition:**

$$C(s) = K(T_D s + 1) \left( 1 + \frac{1}{T_I s} \right) = K_p + \frac{K_I}{s} + K_D s \quad (5)$$

- $K, T_I, T_D > 0$

#### 4.4.1 Design Procedure

**Process:**

1. Design  $K, T_D$  (i.e. the PD controller) to increase the PM.
2. Design  $T_I$  (i.e. the PI controller) to increase system type (satisfy IMP) w/o affecting high frequencies.

## 4.5 Examples

**Example:**

1. **Given:**  $G(s) = \frac{1}{j\omega(j\omega + 1)}$ ,  $C(s) = K(T_D s + 1)$
2. **Problem:** Sketch Bode plots of  $C(s)G(s)$  for PD controllers:
  - $K = 1$ ,  $T_D = 10 \rightarrow 20 \log |K| = 0$
  - $K = 10$ ,  $T_D = 10 \rightarrow 20 \log |K| = 20$
  - Corner frequency:  $\omega_c = \frac{1}{T_D} = 10^{-1}$
3. **Solution:**

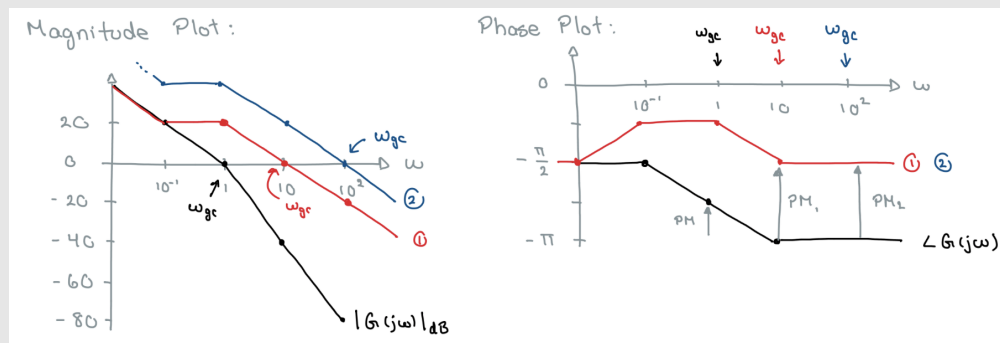


Figure 4

## 5 Lead Controller

### Motivation:

- Approximates a PD controller as  $\alpha \rightarrow 0$ .
- Used to add phase at a particular frequency  $\omega_{\max}$  in order to increase the PM (decreases the % OS).
- Increases  $\omega_{gc}$

### Definition:

$$C(s) = K \frac{T_s + 1}{\alpha T_s + 1} \quad (6)$$

- $K, T > 0$
- $0 < \alpha < 1$

### 5.1 Bode Plot

#### Notes:

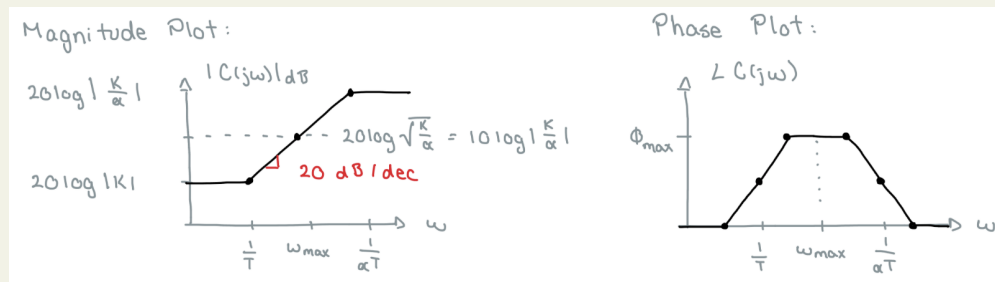


Figure 5

- $\omega_{\max} = \frac{1}{\sqrt{\alpha T}}$
- $\phi_{\max} = \alpha \sin \left( \frac{1 - \alpha}{1 + \alpha} \right)$

### 5.2 Design Procedure

#### Process:

1. Choose  $K$  to satisfy asymptotic tracking (i.e. steady-state error) specifications or bandwidth (i.e.  $\omega_{gc}$ ) requirements.