ROB311 Quiz 2

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1.1 Bode Plots

Process:

1.1.1 Constant Gain

1.1.2 Pole or Zero at $\omega = 0$

1.1.3 Non-Zero Pole or Zero

1.1.4 Complex Conjugate Poles

1.2 Robustness Margins

Motivation: Approximate the GM and PM from the Bode plot:

- $L(s)$ is a strictly proper rational fn.
- $L(s)$ has no poles in $\mathbb{C}^+$ (no open loop variable poles)

1.2.1 Gain Margin

Definition:

$$|L(j\omega_{gc}) = 1| \iff |L(j\omega_{gc})|_{dB} = 0$$

1.2.2 Phase Margin

Definition:

$$|L(j\omega_{gc})| = 1 \implies |L(j\omega_{gc})|_{dB} = 0$$

2 Robustness Margins

3 Root Locus, Bode, and Nyquist

4 Control Design in the Frequency Domain

4.1 Goal

Motivation:

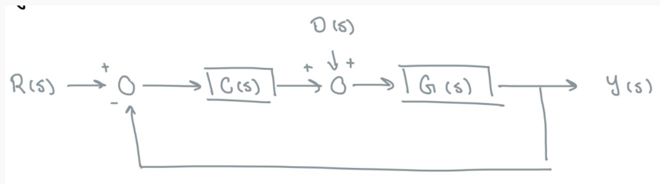

Figure 1

Design $C(s)$ so that the feedback loop:

- BIBO Stable: Verify using Nyquist criterion
  - roots$(1 + C(s)P(s)) \subseteq \mathbb{C}^-$
  - $C(s)G(s)$ has no pole-zero cancellations in $\overline{\mathbb{C}^+}$
- Satisfies certain performance specifications: Tune using Bode plots

4.2 Proportional Derivative (PD) Controller

Motivation: Increase PM at higher frequencies.

Definition:

$$C(s) = K(T_D s + 1) \tag{1}$$

- $K, T_D > 0$

Since $U(s) = C(s)E(s)$,

$$u(t) = \underbrace{KT_D e(t)}_{D} + \underbrace{Ke(t)}_{P} \tag{2}$$

4.2.1 Bode Plot

Notes:

$$|C(j\omega)|_{\text{dB}} = 20 \log |K| + 20 \log |j\omega T_D + 1|$$

$$\angle C(j\omega) = \angle K + \angle(j\omega T_D + 1)$$

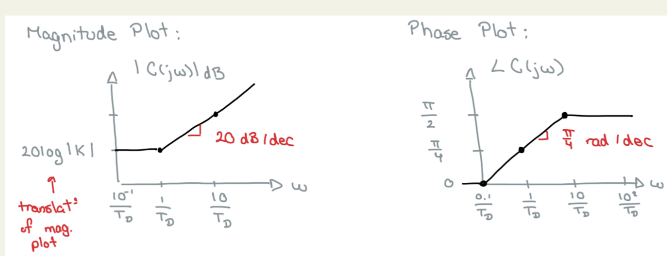

Figure 2

4.3 Proportional Integral (PI) Controller

Motivation: Increase the "system type" for better tracking (IMP) w/o affecting high frequencies.

Definition:

$$C(s) = K\left(1 + \frac{1}{T_I s}\right) = K\frac{T_I s + 1}{T_I s} \tag{3}$$

- $K, T_I > 0$

Since $U(s) = C(s)E(s)$,

$$u(t) = \underbrace{Ke(t)}_{P} + \underbrace{\frac{K}{T_I}\int_0^t e(\tau)d\tau}_{I} \tag{4}$$

4.3.1 Bode Plot

Notes:

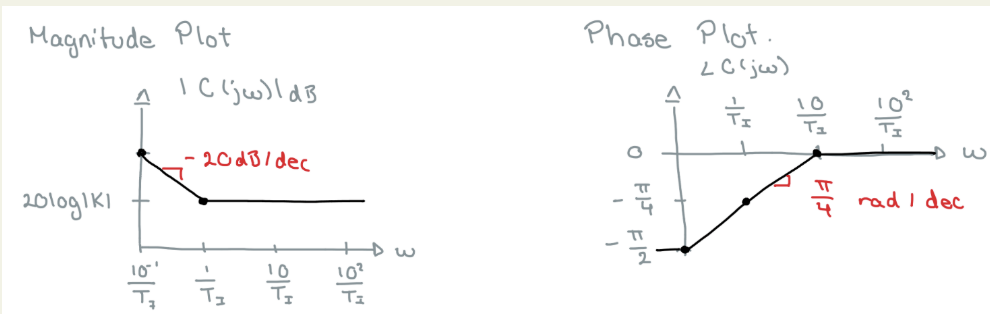

Figure 3

4.3.2 Design Procedure

Process:

1. Choose K to meet asymptotic tracking or bandwidth (loosely increase w_{gc}) requirements (often set $K = 1$)
2. Find the crossover frequency ω_{gc} of $KG(j\omega)$. Suppose we are happy w/ the PM and ω_{gc}.
3. Set $\frac{1}{T_I} \ll \omega_{gc}$. Typically want $\frac{1}{T_I}$ b/w $0.01\omega_{gc}$ and $0.1\omega_{gc}$

4.4 Proportional Integral Derivative (PID) Controller

Definition:

$$C(s) = K(T_D s + 1)\left(1 + \frac{1}{T_I s}\right) = K_p + \frac{K_I}{s} + K_D s \tag{5}$$

- $K, T_I, T_D > 0$

4.4.1 Design Procedure

Process:

1. Design K, T_D (i.e. the PD controller) to increase the PM.
2. Design T_I (i.e. the PI controller) to increase system type (satisfy IMP) w/o affecting high frequencies.

4.5 Examples

Example:

1. **Given:** $G(s) = \dfrac{1}{j\omega(j\omega+1)}$, $C(s) = K(T_D s + 1)$
2. **Problem:** Sketch Bode plots of $C(s)G(s)$ for PD controllers:
   - $K = 1$, $T_D = 10 \to 20\log|K| = 0$
   - $K = 10$, $T_D = 10 \to 20\log|K| = 20$
   - Corner frequency: $\omega_c = \dfrac{1}{T_D} = 10^{-1}$
3. **Solution:**

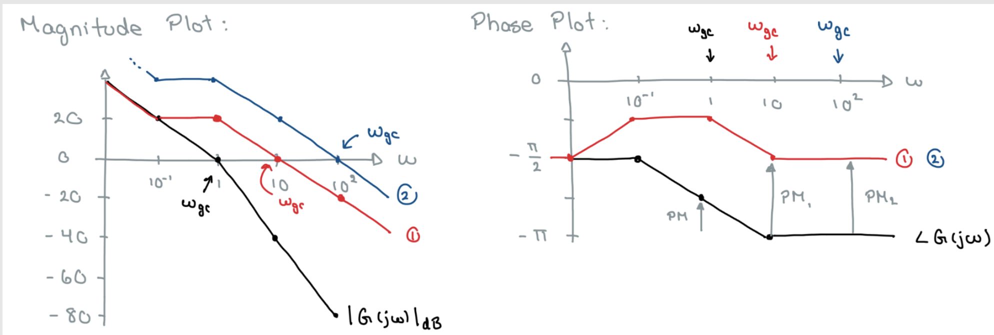

Figure 4

5 Lead Controller

Motivation:
- A lead controller approximates a PD controller as $\alpha \to 0$.
- Used to add phase at a particular frequency $\omega_{\max}$ in order to increase the PM (decreases the % OS).

Definition:

$$C(s) = K\frac{T_s + 1}{\alpha T_s + 1} \tag{6}$$

- $K, T > 0$
- $0 < \alpha < 1$

5.1 Bode Plot

Notes:

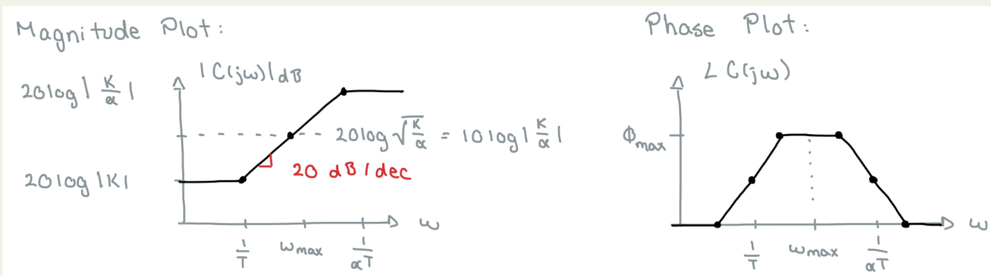

Figure 5

- $w_{\max} = \dfrac{1}{\sqrt{\alpha}T}$
- $\phi_{\max} = \alpha \sin\left(\dfrac{1-\alpha}{1+\alpha}\right)$