ANTI-RESET WINDUP FOR PID CONTROLLERS

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Abstract. This paper describes and compares a number of ways of avoiding integrator windup for proportional-integral-derivative (PID) controllers. It covers both a number of ad hoc schemes and a general procedure to avoid windup, which admits a unification of the ideas. Design rules for anti-windup in general purpose PID controllers are derived and tested in simulations and experiments.

Keywords: PID control, process control, saturation, implementation

1. Introduction

Most control systems are designed for operation in the linear range. For large set point changes and disturbances the control signal will however be saturated, and then the system operates in open loop since the feedback path is broken. If the controller is unstable the breakup of the loop may give severe consequences. A PID controller is a typical example of a controller that may cause instability or poor transient output during saturation.

This paper presents design rules for anti-windup compensators in general purpose PID controllers. The rules are based on the controller's performance for a special disturbance. They are important for controllers that may be utilized on a variety of processes.

2. Anti-windup methods for PID controllers

A few methods for avoiding integrator windup will be briefly described. They are described in more detail in Åström and Rundqwist (1989).

Conditional integration methods

In these methods the integrator is updated only during certain conditions. The essential differences between the methods are the exact conditions for suspending and resuming integration, and how the integral part is treated when it is suspended. Examples of conditional integration in PID controllers are found in e.g. Fertik and Ross (1967), Gallun et al (1985) and Shinskey (1988).

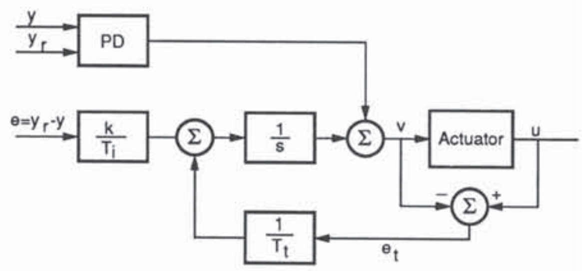

Figure 1. PID controller with anti-windup based on tracking.

Tracking or Backcalculation

The idea of backcalculation was proposed by Fertik and Ross (1967) for a velocity limited incremental PI algorithm. The idea is that the stored value is recomputed so that the controller output is exactly at the saturation limit. In Phelan (1977) the "Intelligent Integrator" is adjusted in the described way. During saturation the controller thus *tracks* given inputs and outputs. It was found advantageous not to reset the integrator in one sampling period but dynamically with a time constant T_t. Figure 1 shows a block diagram of a PID controller with tracking. Controllers with tracking are discussed e.g. in Åström (1987) and in Glattfelder and Schaufelberger (1983).

A general method

In a PID controller with filtering there are two states but only the integrator state is adjusted by the tracking mechanism. It may however be favorable to adjust both states during saturation. The idea can be generalized to n-th order controllers, see Åström and Wittenmark (1984). For a controller on state space form

$$\begin{aligned} \frac{dx}{dt} &= Fx + G_r y_r - G_y y \\ v &= Hx + D_r y_r - D_y y \end{aligned} \tag{1}$$

anti-windup is obtained by feedback from the difference between desired control signal v and the saturated control signal $u = \mathrm{sat}(v)$. The following controller is then obtained.

$$\begin{aligned} \frac{dx}{dt} &= Fx + G_r y_r - G_y y + M(u - v) \\ &= (F - MH)x + (G_r - MD_r)y_r \\ &\quad - (G_y - MD_y)y + Mu \\ v &= Hx + D_r y_r - D_y y \\ u &= \mathrm{sat}(v) \end{aligned} \tag{2}$$

where $F - MH$ has stable eigenvalues. This controller realization corresponds to an observer, and is therefor denoted the *observer approach.*

The conditioning technique, see Hanus et al (1987), is a special case of the observer approach, where $M = G_r D_r^{-1}$ in (2). The controller states x are then not directly affected by the reference signal y_r during saturation. The method requires that D_r^{-1} exists. The eigenvalues of $F - G_r D_r^{-1} H$ are equal to the transmission zeros of the controller.

3. Process and Controller

The properties of different methods will be investigated for a special case. Design rules will also be developed.

The process

The process is two identical cylindrical cascaded tanks used for basic experiments with automatic control, see Åström and Östberg (1986). The control signal is pump speed, which determines the influent flow rate to the upper tank, and the process output is the level of the lower tank. A linearized state space model for the double tank is

$$\begin{aligned} \frac{dx}{dt} &= \begin{pmatrix} -\alpha & 0 \\ \alpha & -\alpha \end{pmatrix} x + \begin{pmatrix} \beta \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{aligned} \tag{3}$$

where x_1 and x_2 are upper and lower tank levels respectively. The control signal u is restricted to the interval $[0, 1]$. Parameters $\alpha = 0.015$ (s^{-1}) and $\beta = 0.05$ (s^{-1}), thus the time constant for a tank is approximately 70 seconds.

The controller

The PID controller has filtered derivative of the measurement signal $y(t)$ and a proportional part that only acts on a fraction b of the reference signal y_r, see Åström and Hägglund (1988). The controller thus have PI action in the feedforward path and PID action in the feedback path. Parameter b positions one closed loop zero which has great influence on the overshoot after a step change in the reference signal. A state space model for the controller is

$$\begin{aligned} \frac{dx}{dt} &= \begin{pmatrix} 0 & 0 \\ 0 & -\frac{N}{T_d} \end{pmatrix} x + \begin{pmatrix} \frac{K}{T_i} \\ 0 \end{pmatrix} y_r - \begin{pmatrix} \frac{K}{T_i} \\ \frac{N}{T_d} \end{pmatrix} y \\ u &= \begin{pmatrix} 1 & -KN \end{pmatrix} x + \begin{pmatrix} Kb \end{pmatrix} y_r \\ &\quad - \begin{pmatrix} K(1+N) \end{pmatrix} y \end{aligned} \tag{4}$$

where x_1 is the integral part and x_2 is a low pass filtered $-y$. The derivative part is $-KN(x_2 + y)$. (4) has the same structure as (1) and then many anti-windup methods can be described with (2).

Controller parameters are $K = 5$, $T_i = 40$ s, $T_d = 15$ s, $N = 5$ and $b = 0.3$, which gives an overshoot of 10 % and a natural frequency ≈ 0.05 rad/s for the three dominating closed loop poles.

4. Analysis

The three principal methods described in Section 2 will be analyzed with respect to stability and their performance for a special disturbance.

Tracking for (4) corresponds to

$$M = \begin{pmatrix} \frac{1}{T_t} & 0 \end{pmatrix}^T \tag{5}$$

in (2). The eigenvalues of $F - MH$ are then $-1/T_t$ and $-N/T_d$.

The observer approach for (4) needs selection of both the eigenvalues of $F - MH$. A simple choice is to make them equal, in this case $-\omega_0$, and then

$$M = \begin{pmatrix} \frac{\omega_0^2 T_d}{N} & \frac{T_d}{KN^2}\left(\omega_0 - \frac{N}{T_d}\right)^2 \end{pmatrix}^T \tag{6}$$

The conditioning technique for (4) gives a fixed anti-windup which is a special case of tracking with $T_t = bT_i$. This is due to the feedforward PI structure of (4). If the reference signal is differentiated conditioning does not correspond to tracking.

In conditional integration the integral part is kept constant until the control error changes sign or the controller desaturates. Thus $T_i = \infty$ in (4) during saturation. The feedback path is then a PD controller, i.e. a lead compensator.

Stability

A system with a saturating actuator can always be reduced to a standard configuration with a linear system having a nonlinear feedback. For time invariant monotonic nonlinearities sufficient conditions for stability can be obtained from the off-axis circle criterion (Narendra and Taylor, 1973).

If the nonlinear feedback element is a saturation the linear system $G^*(s)$ is given by

$$G^* = \frac{G_c G_p - W}{1 + W} \tag{7}$$

where G_p is the process transfer function, G_c is the feedback path of the controller and W describes the anti-windup. The two latter transfer functions, which follow from (2), are

$$W = H(sI - F)^{-1} M \tag{8}$$

$$G_c = H(sI - F)^{-1} G_y + D_y \tag{9}$$

THEOREM 1
If the linear system $G^*(s)$ has all poles in the open left half plane and has nonlinear feedback from a saturation the closed loop is absolutely stable provided that a straight line through the origin can be given a nonzero slope such that $G^*(i\omega) + 1$ is strictly to the right of the line. □

Proof: See Narendra and Taylor (1973), p. 169.

Tracking: Theorem 1 is satisfied for $0 < T_t < \infty$. $G^*(s) + 1$ is strictly positive real (SPR) for $0 < T_t < 30$ s $\approx \frac{3}{4}T_i$. The Nyquist curves for some values of T_t are shown in Figure 2.

The observer approach: Theorem 1 is satisfied for 0.012 rad/s $< \omega_0 <$ 2.9 rad/s. For $\omega_0 <$ 0.012 rad/s $G^*(s)$ is conditionally stable. $G^*(s) + 1$ is SPR for 0.067 rad/s $< \omega_0 <$ 0.93 rad/s.

Conditional integration: Here

$$G^*(s) = K \frac{(N+1)s + N/T_d}{s + N/T_d} G_p \tag{10}$$

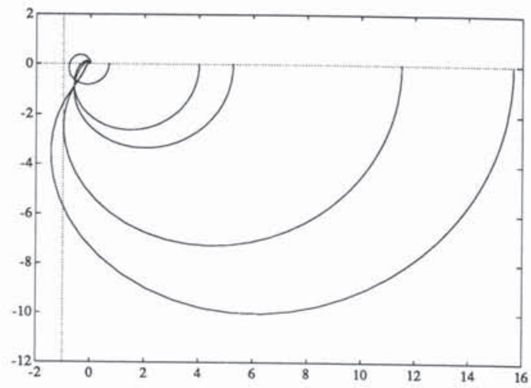

Figure 2. $G^*(i\omega)$ for tracking for (from right to left) $T_t = T_i$, $3/4T_i$, T_d, $0.3T_i$, $T_i/10$.

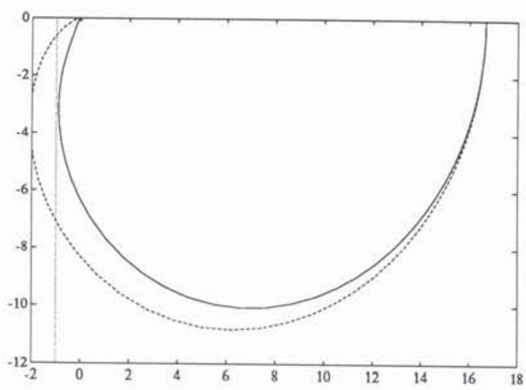

Figure 3. $G^*(i\omega)$ for conditional integration (solid line) and for $G^* = KG_p$ (dashed line).

and Theorem 1 is satisfied since $G^*(s) + 1$ is SPR, see Figure 3. This is due to the positive phase of the PD feedback path. If the derivative action is also inhibited during saturation then $G^*(s) = KG_p(s)$ still satisfies Theorem 1, see Figure 3, but $G^*(s) + 1$ is not SPR.

Approximate disturbance response

Another approach to determine tracking time constant T_t or observer bandwidth ω_0 is to compute the controller's response to a specific disturbance and then select T_t or ω_0 such that certain conditions are satisfied. A very critical experiment with the two cascaded tanks is *pouring a cup of water* in the lower tank. The result is a step change in the measurement $y(t)$. The controller's derivative part then roughly produces an impulse, which is the dominating component in the control signal. The impulse saturates the controller and then the anti-windup mechanism passes the impulse to the integrator.

Below the desired control output $v(t)$ is determined based on an approximate sawtooth shaped process output $y(t)$ such that

$$y(t) = \begin{cases} y_r + \Delta y + y't & \text{if } 0 < t < T_y \\ y_r & \text{otherwise} \end{cases} \quad (11)$$

where $\Delta y > 0$, $y' < 0$ and $\Delta y + y'T_y = 0$. For $t < 0$ stationarity is assumed. This approximation of $y(t)$ is only valid as long as the control signal is saturated at the lower limit. Detailed derivation of the results below are found in Rundqwist (1989).

Tracking From (2) with matrices given by (4) and (5) and $y(t)$ given by (11) the desired control output $v(t)$ has the form (if $N/T_d \neq 1/T_t$)

$$v(t) = v_0 + v_1 t + v_2 e^{-t/T_t} + v_3 e^{-tN/T_d} \quad (12)$$

where all v_i:s are functions of K, T_i, etc. but not of t. Now, approximate $v(t)$ by assuming

$$\frac{N}{T_d} \gg \frac{1}{T_t} \quad \text{and} \quad t > 3\frac{T_d}{N} \quad (13)$$

giving

$$\tilde{v}(t) = \tilde{v}_0 + \tilde{v}_1 t + \tilde{v}_2 e^{-t/T_t} \quad (14)$$

In $\tilde{v}(t)$ the fast mode is eliminated but not the resetting of the integrator at $t = 0$. We also assume $T_y \gg T_t$ and then

$$\tilde{v}(0) = u_0 + k\left(\frac{T_d}{T_t} - 1\right)\Delta y - kT_d y' \quad (15)$$

$$\tilde{v}(T_y) = u_{min} - ky'T_t^2\left(\frac{1}{T_t} - \frac{1}{T_i}\right) \quad (16)$$

where u_0 is the stationary control signal before the disturbance and u_{min} is the lower control limit. Two reasonable demands are that 1) $\tilde{v}(0) \leq u_0$ and 2) $\tilde{v}(T_y) \geq u_{min}$. A necessary but not sufficient condition for 1) is

$$T_t > T_d \quad (17)$$

A necessary and sufficient condition for 2) is

$$T_t \leq T_i \quad (18)$$

RESULT 1
For anti-windup by tracking the tracking time constant is to be chosen such that

$$T_d < T_t \leq T_i \quad (19)$$

□

For the controller in this paper we get 15 s $< T_t \leq$ 40 s.

The observer approach Here the desired control signal $v(t)$ has the form

$$v(t) = v_0 + v_1 t + v_2 e^{-\omega_0 t} + v_3 t e^{-\omega_0 t} \quad (20)$$

where $v(0)$ is independent of ω_0. Assume $T_y \gg \omega_0^{-1}$. Then

$$v(T_y) = u_{min} - \frac{ky'}{\omega_0^2}\left(\frac{N}{T_d} + \frac{1}{T_i} - \frac{2N}{\omega_0 T_i T_d}\right) \quad (21)$$

$v(T_y) \geq u_{min}$ if

$$\omega_0 \geq \frac{2}{T_d/N + T_i} \approx \frac{2}{T_i} \quad (22)$$

$\frac{dv}{dt}(0)$ however depends on ω_0 and a sufficient condition for $\frac{dv}{dt}(0) > 0$ is

$$\omega_0 \geq \frac{1}{2(N+1)}\left(\frac{1}{T_i} + \frac{N}{T_d}\right) \approx \frac{1}{2T_d} \quad (23)$$

The observer approach with $\omega_0 = N/T_d$ gives the same controller realization as tracking with $T_t = T_d/N$. This is a too short time constant, see (17), and thus N/T_d is above an acceptable upper limit for ω_0.

RESULT 2
For anti-windup by the observer approach it is required that

$$\max\left(\frac{1}{2T_d}, \frac{2}{T_i}\right) \leq \omega_0 < \frac{N}{T_d} \quad (24)$$

□

For the controller in this paper $\frac{1}{2T_d} = 0.033$ rad/s, $\frac{2}{T_i} = 0.050$ rad/s and $\frac{N}{T_d} = 0.333$ rad/s.

5. Evaluation

In this section simulations with a linear process model and experiments on a nonlinear process are used for evaluation of previous results.

Tracking

In Figure 4 some tracking time constants are compared in simulations with start-up and the *cup of water* disturbance. I terms of the IAE (integral-absolute-error) criterion the choice $T_t = \sqrt{T_i T_d}$ gives the best result for the disturbance. $T_t = T_i$ gives an acceptable result with slightly larger undershoot. When $T_t \leq T_d$ (as when using the conditioning technique for this particular controller) the controller desaturates too quickly and gives a prolonged period of too high level

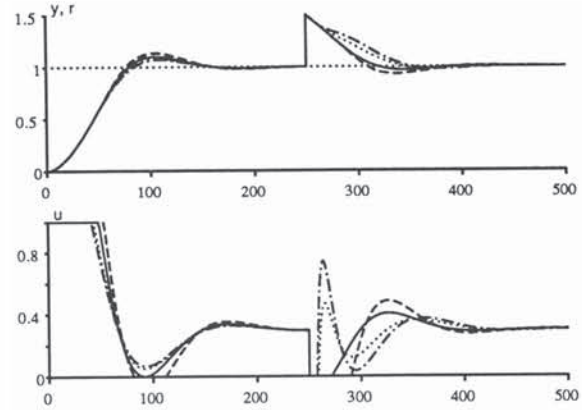

Figure 4. Simulation of start-up and disturbance with anti-windup by tracking. Tracking time constants are $T_t = \sqrt{T_i T_d}$ (solid), $T_t = T_i$ (dashed), $T_t = T_d$ (dotted) and $T_t = bT_i$, i.e. the conditioning technique (dash-dotted). Reference signal is also dotted.

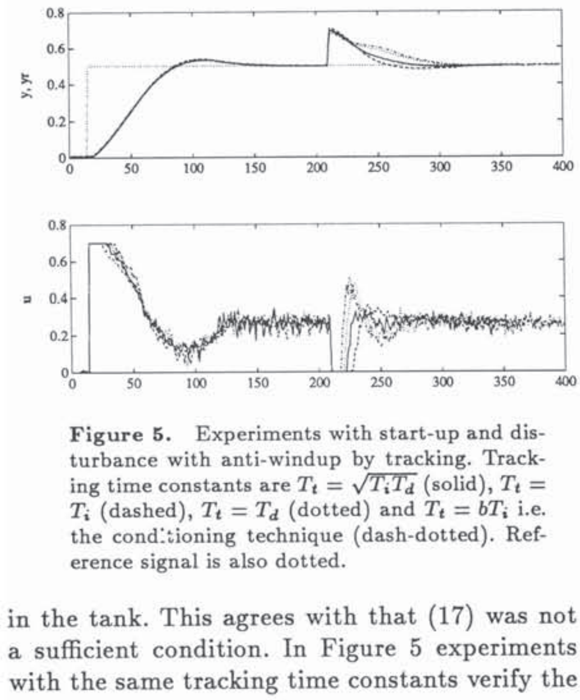

Figure 5. Experiments with start-up and disturbance with anti-windup by tracking. Tracking time constants are $T_t = \sqrt{T_i T_d}$ (solid), $T_t = T_i$ (dashed), $T_t = T_d$ (dotted) and $T_t = bT_i$ i.e. the conditioning technique (dash-dotted). Reference signal is also dotted.

in the tank. This agrees with that (17) was not a sufficient condition. In Figure 5 experiments with the same tracking time constants verify the results.

The drawback of having too small a constant T_t is shown in Figure 6. The fast resetting of the controller gives two results: 1) too quick desaturation of the controller, and 2) saturation at the *upper limit*, causing a severe performance deterioration. There is not much difference in the start-up. The IAE criterion is smallest for $T_t \approx 0.2T_i$. Thus $T_t = \sqrt{T_i T_d}$ (which often becomes $\frac{1}{2}T_i$) seems to be a reasonable choice.

The observer approach

In Figure 7 some values of ω_0 are compared for the observer approach. For the disturbance the IAE criterion is smallest for $\omega_0 = 0.064$ rad/s. The two lower limits in (24) give larger

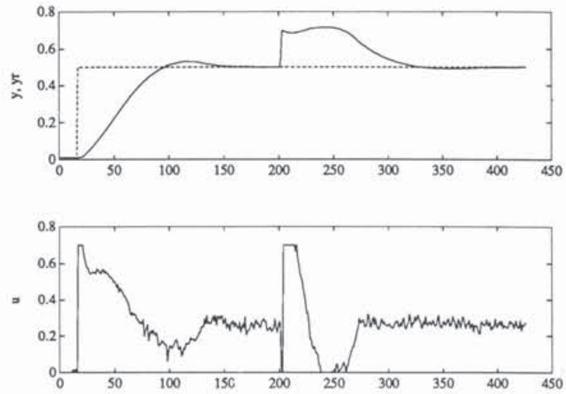

Figure 6. Experiment with start-up and disturbance with anti-windup by tracking, where tracking time constant $T_t \ll T_i$. Reference signal is dashed.

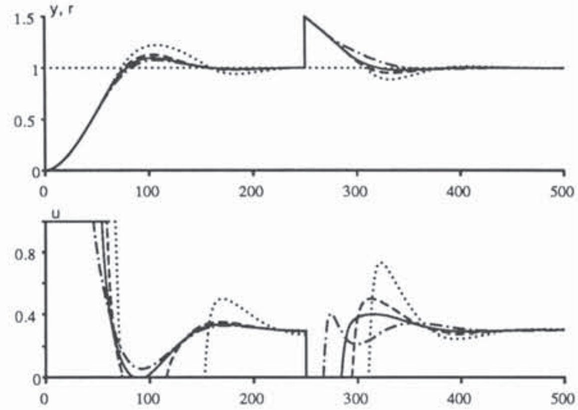

Figure 7. Simulation of start-up and disturbance with anti-windup by the observer approach. Observer poles are given by $\omega_0 = 0.064$ rad/s (solid), 0.050 rad/s (dashed), 0.033 rad/s (dotted) and 0.100 rad/s (dash-dotted). Reference signal is also dotted.

undershoot. For $\omega_0 = 0.050$ rad/s the result is acceptable, but not for $\omega_0 = 0.033$ rad/s. $\omega_0 = 0.10$ rad/s gives an acceptable response without undershoot, but for higher values of ω_0 the tank level is too high during a prolonged period. For start-up $\omega_0 = 0.14$ rad/s (not shown) gives the minimal IAE value. Thus ω_0-values in the interval 0.05–0.10 rad/s, here corresponding to $2/T_i$–$4/T_i$, seem to be reasonable.

Conditional integration

In Figure 8 conditional integration is tested. The method handles both start-up and the disturbance properly. Conditional integration and the best choices of tracking time constant T_t and observer poles ω_0 give almost identical result for the disturbance. For start-up conditional integration gives slightly less overshoot. Thus the three methods are essentially equal in their anti-windup capability.

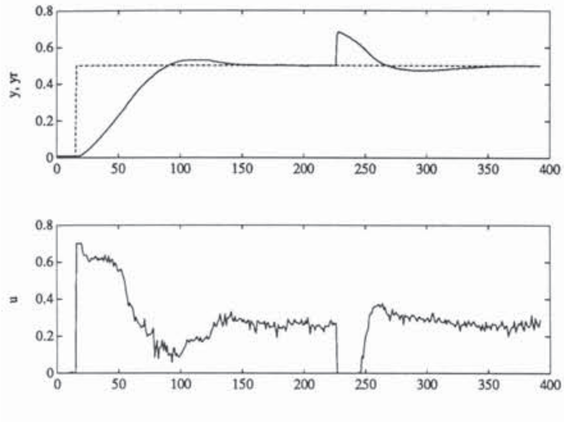

Figure 8. Experiment with start-up and disturbance with anti-windup by conditional integration. Reference signal is dashed.

6. Conclusions

All three anti-windup methods give stable closed loops for the chosen process and controller. For tracking the results from the disturbance analysis are shown to be reliable both in simulations and in experiments. The results also agree with well-known rules of thumb. Similar results were obtained for the observer approach. All three anti-windup methods have, with well chosen parameters, almost identical performance.

The *cup of water* disturbance is a very special and strongly exciting disturbance for a controller with large high frequency gain, e.g. a controller with derivative action. If such a controller may receive this type of disturbance it needs careful anti-windup. This explains the fairly restrictive results for the choice of tracking time constant and observer poles. These results are important for general purpose PID controllers, where the manufacturer cannot assume anything about the type of disturbances in an application. For special purpose PID controllers which will not receive this type of disturbances faster anti-windup is feasible.

It was also demonstrated that a set point oriented anti-windup method, e.g. the conditioning technique, may result in fairly poor performance for disturbances when the feedback and feedforward paths of the controller are too different.

7. Acknowledgements

The author wishes to thank Karl Johan Åström for fruitful discussions and suggestions.

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