FIT 5215 Deep Learning

Quiz for:
Stochastic Gradient Descent and Optimization

Tutor Team

Department of Data Science and AI
Faculty of Information Technology, Monash University
Email: trunglm@monash.edu/tuananh.bui@monash.edu

Question 1

- Consider the optimization problem to train a feed-forward NN

$$\min_\theta J(\theta) = \Omega(\theta) + \frac{1}{N}\sum_{i=1}^{N} CE\left(y_i, f(x_i;\theta)\right)$$

where $\theta = \left[\left(W^k, b^k\right)\right]_{k=1}^{L}$ and $\Omega(\theta) = \lambda \sum_k \sum_{i,j} \left(W_{i,j}^{\mathbf{k}}\right)^2 = \lambda \sum_k \left\|\boldsymbol{W}^{\mathbf{k}}\right\|_F^2$. Choose the correct answers. (MC)

- A. Minimizing $\Omega(\theta)$ encourages more weights $W_{i,j}^{\mathbf{k}}$ to approach to 0.
- B. Minimizing $\Omega(\theta)$ encourages complex models.
- C. Minimizing $\Omega(\theta)$ encourages simple models.
- D. Minimizing $\Omega(\theta)$ combats underfitting.
- E. Minimizing $\Omega(\theta)$ combats overfitting.

Question 2

- Consider the optimization problem to train a feed-forward NN

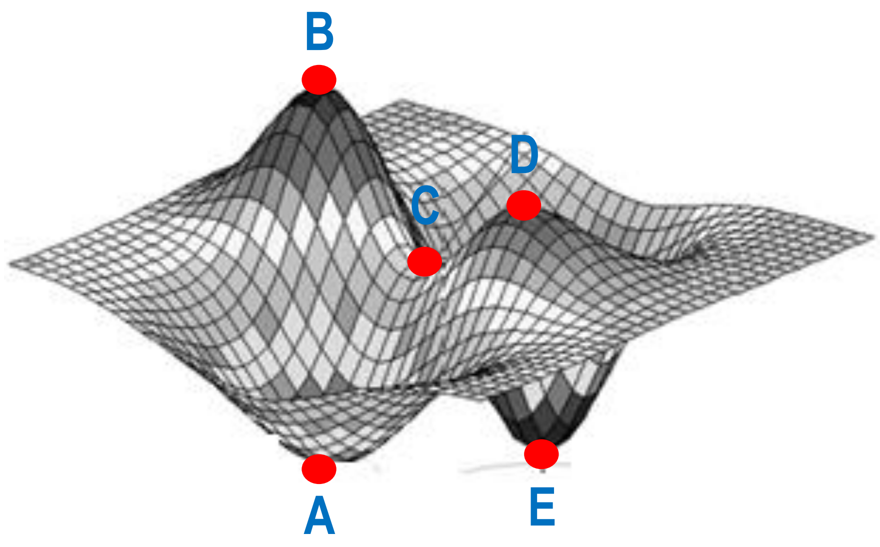
$$\min_\theta J(\theta) = \Omega(\theta) + \frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$$

where $\theta = \left[\left(W^k, b^k\right)\right]_{k=1}^{L}$ and $\boldsymbol{f}(\boldsymbol{x_i};\boldsymbol{\theta})$ returns the prediction probabilities for $\boldsymbol{x}_i$. Choose the correct answers. (MC)

- A. $\frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$ is known as a regularization term.
- B. $\frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$ is known as an empirical loss.
- C. Minimizing $\frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$ makes the model more fit to the training set.
- D. Minimizing $\frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$ makes the model more fit to the testing set.
- E. Minimizing $\frac{1}{N}\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} CE\big(y_i, f(x_i;\theta)\big)$ can lead to overfitting.

Question 3

- Given a loss surface as showing, which statements are correct? (MC)



- A. **A**, **B**, **C**, **D**, and **E** are critical points.
- B. **A**, **E**, and **C** are local minima, while **B**, **D** are local maxima.
- C. **B** and **D** are local minima, **A** and **C** are local maxima, and **C** is a saddle point.
- D. **C** is a saddle point, **B** and **D** are local maxima, while **A** and **E** are local minima.
- E. **B** is global maxima, **D** is local maxima, and **C** is a saddle point.

Question 4

- Let $f(w) = w^2 - 3w + 1$. Assume that we use gradient descent with the learning rate $\eta = 0.1$ to solve $\min_w f(w)$. At the iteration t, we are at $w_t = 2$. What is the value of w_{t+1} at the next iteration? (SC)

- A. $w_{t+1} = 1.$
- B. $w_{t+1} = 1.8$
- C. $w_{t+1} = 1.9.$
- D. $w_{t+1} = 2.1.$

f' = 2w - 3

f'(w_t) = 4 - 3 = 1

w_t+1 = w_t - eta (f'(w_t)

= 2 - 0.1(1) = 1.9

Question 5

- Given the function $f(w) = \frac{1}{1000}\sum_{i=1}^{1000}(w - x_i)^2$ where $x_i = i, \forall i = 1, \dots, 1000$. We need to solve $\min_w f(w)$ using stochastic gradient descent with the learning rate $\eta = 0.1$. Assume we sample a batch $b_1 = 1, b_2 = 3, b_3 = 5, b_4 = 7$ of indices and at the iteration t, we have $w_t = 10$. What is the value of w_{t+1} at the next iteration? (SC)

what is the update of weights using SGD (batches)

- A. $w_{t+1} = 8.9$.
- B. $w_{t+1} = 8.7$.
- C. $w_{t+1} = 8.5$.
- D. $w_{t+1} = 8.8$.

- C. $\theta_{t+1} = \theta_t - \frac{\eta}{b}\sum_{k=1}^{b}\nabla_\theta l(x_{i_k}, y_{i_k}; \theta_t)$.

- $\tilde{f}(w) = \frac{1}{4}\left[(w - x_{b_1})^2 + (w - x_{b_2})^2 + (w - x_{b_3})^2 + (w - x_{b_4})^2\right]$
  $= \frac{1}{4}[(w-1)^2 + (w-3)^2 + (w-5)^2 + (w-7)^2]$
- $\tilde{f}'(w) = \frac{1}{4}(8w - 32) = 2w - 8$
- $w_{t+1} = w_t - \eta\tilde{f}'(w_t) = 10 - 0.1 \times (2 \times 10 - 8) = 8.8$

Question 6

- Consider the optimization problem: $\min_{\theta} L(D;\theta) := \frac{1}{N}\sum_{i=1}^{N} l(x_i, y_i; \theta)$ with θ is the model parameter and $D = \{(x_1, y_1), \dots, (x_N, y_N)\}$ is a training set. Let us sample a batch of indices $i_1, \dots, i_b$ uniformly from $\{1, \dots, N\}$. Which statements are correct about the update rule of stochastic gradient descent? (SC)

  - A. $\theta_{t+1} = \theta_t - \frac{\eta}{N}\sum_{i=1}^{N} \nabla_\theta l(x_i, y_i; \theta_t)$.
  - B. $\theta_{t+1} = \theta_t + \frac{\eta}{N}\sum_{i=1}^{N} \nabla_\theta l(x_i, y_i; \theta_t)$.
  - C. $\theta_{t+1} = \theta_t - \frac{\eta}{b}\sum_{k=1}^{b} \nabla_\theta l\left(x_{i_k}, y_{i_k}; \theta_t\right)$.
  - D. $\theta_{t+1} = \theta_t + \frac{\eta}{b}\sum_{k=1}^{b} \nabla_\theta l\left(x_{i_k}, y_{i_k}; \theta_t\right)$.

Question 7

Given 4 implementations as below. What are f1/f2/f3/f4? (SC).

- ❑ A. sigmoid/tanh/softmax/relu
- ❑ B. softmax/tanh/relu/sigmoid
- ❑ C. sigmoid/tanh/relu/softmax
- ❑ D. softmax/tanh/sigmoid/relu

```python
def f1(x):
    return 1. / (1 + np.exp(-x))


def f2(x):
    return (np.exp(2*x)-1.) / (np.exp(2*x)+1.)

def f3(x):
    return x*(x>0)

def f4(x):
    return np.exp(x) / np.sum(np.exp(x))
```

Question 8

Which is $\frac{\partial h}{\partial \bar{h}}$ *if* $h = \sigma(\bar{h})$, $\bar{h}$ *is a vector and* σ *is an activation function?* (SC)

- ❑ A. $\sigma'(\bar{h})$
- ❑ B. $\mathrm{d}iag(\bar{h})$
- ❑ C. $\mathrm{d}iag(\sigma(\bar{h}))$
- ❑ D. $\mathrm{d}iag(\sigma'(\bar{h}))$

Question 9

Which is $\frac{\partial h}{\partial \bar{h}}$ *if* $h = \sigma(\bar{h})$, $\bar{h}$ = [-1, 1, 2] *and* σ *is ReLU activation function?* (SC)

ANS: C

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 10

Which is $\frac{\partial l}{\partial \bar{h}}$ *if* $h = \sigma(\bar{h})$, $\bar{h}$ *=[-1,1,2],* σ *is ReLU activation function and* $\frac{\partial l}{\partial h}$ = [1,1,1]? (SC)

dl/dh x dh/dh_bar = [1,1,1] x diag([0,1,1]) = [0,1,1]

A. [-1, 1, 2]
B. [0, 1, 2]
C. [0, 1, 1]
D. [-1, 1, 1]

Question 11

Which is $\frac{\partial l}{\partial W}$ if $\bar{h} = Wx + b$, $\frac{\partial l}{\partial \bar{h}}$ = [0,1,1], $x = [1,2,3]^T$, $b = [0,1,2]^T$? (SC)

$A = 6$

$B = [1, 2, 3]$

$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

dl/dw
= dl/dhbar x dhbar/dw
= [0,1,1] * x = [0,1,1] x [1,2,3]^T

$$\frac{\partial l}{\partial W} = \frac{\partial l}{\partial \bar{h}} \frac{\partial \bar{h}}{\partial W} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \times [1,2,3]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

From Kevin Clark's note (https://web.stanford.edu/class/cs224n/readings/gradient-notes.pdf), section 3.5:

$$\frac{\partial J}{\partial \mathbf{z}} = \delta, \mathbf{z} = \mathbf{W}\mathbf{x} \Rightarrow \frac{\partial J}{\partial \mathbf{W}} = \delta^T \mathbf{x}^T$$

Replace J by l and $\mathbf{z}$ by $\bar{h}$, and apply the same flow of calculation in the above note, we will have:

$$\frac{\partial l}{\partial \bar{h}} = [0, 1, 1] = \delta, \bar{h} = \mathbf{W}\mathbf{x} + \mathbf{b} \Rightarrow \frac{\partial l}{\partial \mathbf{W}} = \delta^T \mathbf{x}^T = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \times [1\ 2\ 3]$$