Stéphane Crépey, Financial Modeling: A Backward Stochastic Differential Equations Perspective - Notes

Lee Wei Xuan

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Exercise 1.3.11

Follows from Exercise 1.3.12

Exercise 1.3.12

$\mathbb{1}_{\{\min(\nu,\theta)=n\}} = \mathbb{1}_{\{\nu=n\}} \cdot \mathbb{1}_{\{\theta\geq n\}} + \mathbb{1}_{\{\theta=n\}} \cdot \mathbb{1}_{\{\nu>n\}}$.

$\mathbb{1}_{\{\max(\nu,\theta)=n\}} = \mathbb{1}_{\{\nu=n\}} \cdot \mathbb{1}_{\{\theta\leq n\}} + \mathbb{1}_{\{\theta=n\}} \cdot \mathbb{1}_{\{\nu<n\}}$.

Exercise 1.3.18

No. Let $X_n \equiv 1$ for all n, then $S_n = n$. Fix $\epsilon_0 = 1/2$. Given $\delta > 0$, there exists $n_0 \in \mathbb{N}$ such that $n_0\delta \geq 1$. Pick $A \in \Omega$ (Maybe not always possible?) such that $\delta/2 \leq \mathbb{P}(A) < \delta$ and we have $\mathbb{E}(|S_n|\mathbb{1}_A) = n\mathbb{P}(A) \geq n\delta/2 \geq 1/2 = \epsilon_0$.