

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 7

Problem 1. Specify the classes of the following Markov chains and determine which are transient and which recurrent. List the absorbing states.

$$P_1 = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 1/2 & 0 & 1/2 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

Write down R code to simulate each of these chains. Consider the initial conditions $\pi_0 = \delta_i$ for each $i \in \mathcal{S}$. Does the long time behavior depend on the initial state?

Problem 2. Let $\{X_n\}_{n=0}^\infty$ be a Markov chain on $\mathcal{S} = \{1, 2, 3, 4, 5\}$ with transition matrix given by

$$P = \begin{bmatrix} 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- (1) Sketch the transition graph for this chain.
- (2) Classify the states (find classes).
- (3) List all closed sets of states.
- (4) A **return set** $R(i)$ for state i is

$$R(i) = \{n \in \mathbb{N} : p_{ii}^n > 0\}.$$

Find the return sets for all states ($i = 1, 2, 3, 4, 5$).

- (5) Compute p_{ij}^n , for $i = 2, j = 5$ and $n = 1, 2, 3, 4, 5$ by counting all possible paths joining 2 and 5 in n steps and adding their probabilities. (*Note: Do not compute P^n for $n = 1, 2, 3, 4, 5$ for this problem.*)
- (6) Write R code to check the previous answer.