HW 8 M362M, Fall 2024

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Problem 1

```
n_sim <- 100000
first_wet_simulator <- function() {
  # output: (number of trips before gets wet, -1 / 1 / 2 )
  # -1 : Never got wet
  # 1  : Got wet home >> office
  # 2  : Got wet office >> home
  umbrellas <- c(2, 2)
  # Observe for 10000 days (avoid infinity if never wet)
  for (day in 1:10000) {
    morning_rain <- (sample(c(0, 1), 1, replace = TRUE, prob = c(0.95, 0.05)))
    if (morning_rain == 1) {
      if (umbrellas[1] > 0) {
        umbrellas[1] <- umbrellas[1] - 1
        umbrellas[2] <- umbrellas[2] + 1
      } else {
        return(c(2 * (day - 1), 1))
      }
    }
    evening_rain <- (sample(c(0, 1), 1, replace = TRUE, prob = c(0.8, 0.2)))
    if (evening_rain == 1) {
      if (umbrellas[2] > 0) {
        umbrellas[1] <- umbrellas[1] + 1
        umbrellas[2] <- umbrellas[2] - 1
      } else {
        return(c(2 * day - 1, 2))
      }
    }
  }
  return(c(2 * day, -1))
}
results <- replicate(n_sim, first_wet_simulator())
# Expected number of trips before gets wet
(mean(results[1, ]))
## [1] 38.19132
# Probability first time gets wet is office >> home
(mean(results[2, ] == 2))
## [1] 0.98916
```

Problem 2

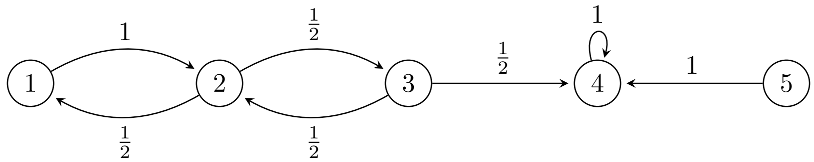

Figure 1: Markov Chain-Counterexample for (1), (2), (3), (4)

Consider the Markov chain shown above.

- (1) is **FALSE**, let $C = \{1, 2, 3\}$, it is not closed because $p_{34} > 0$ but $4 \notin C$.
- (2) is **FALSE**, let $C = \{4\}$ then $C^c = \{1, 2, 3, 5\}$ is not closed as $p_{34} > 0$.
- (3) is **FALSE**, $\{5\}$ is a class but transient because $\sum_{n=1}^{\infty} p_{55}^{(n)} = 0 \neq +\infty$.
- (4) is **FALSE** because $p_{13} = 0$.

Problem 3

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Let P be the matrix shown above.

- (1) is **FALSE** because $P^6 = I$ but $P \neq I$.
- (2) is **FALSE**, we have two classes $\{1, 2\}$ and $\{3, 4, 5\}$.
- (3) is **TRUE**. Let S be the finite state space of the Markov chain. For $k \in \mathbb{N}, P^{nk} = (P^n)^k = I$. Therefore $p_{ii}^{(nk)} = 1$ for all $i \in S$. Then

$$\sum_{k=1}^{\infty} p_{ii}^{(k)} \geq \sum_{k=1}^{\infty} p_{ii}^{(nk)} = +\infty$$

- (4) is **FALSE**, period of state 1 is 2 but the period of state 3 is 3.

End of Homework