

## The Empirical Implications of the Interest-Rate Lower Bound<sup>†</sup>

By CHRISTOPHER GUST, EDWARD HERBST, DAVID LÓPEZ-SALIDO,  
AND MATTHEW E. SMITH\*

*Using Bayesian methods, we estimate a nonlinear DSGE model in which the interest-rate lower bound is occasionally binding. We quantify the size and nature of disturbances that pushed the US economy to the lower bound in late 2008 as well as the contribution of the lower bound constraint to the resulting economic slump. We find that the interest-rate lower bound was a significant constraint on monetary policy that exacerbated the recession and inhibited the recovery, as our mean estimates imply that the zero lower bound (ZLB) accounted for about 30 percent of the sharp contraction in US GDP that occurred in 2009 and an even larger fraction of the slow recovery that followed. (JEL C11, C32, E12, E23, E32, E43, E52, G01)*

As a result of the financial crisis of 2007–2009, the US economy experienced a sharp contraction in activity followed by a markedly slow recovery. At the same time, the federal funds rate fell and remained at its effective lower bound for a protracted period of time. Accordingly, the zero lower bound (ZLB) constraint became a practical concern for monetary policymakers, limiting their ability to offset contractionary disturbances to the economy. In light of these developments, an

\*Gust: Division of Monetary Affairs, Federal Reserve Board, 20th Street and Constitution Avenue NW, Washington, DC 20551 (email: christopher.gust@frb.gov); Herbst: Division of Monetary Affairs, Federal Reserve Board, 20th Street and Constitution Avenue NW, Washington, DC 20551 (email: edward.p.herbst@frb.gov); López-Salido: Division of Monetary Affairs, Federal Reserve Board, 20th Street and Constitution Avenue NW, Washington, DC 20551 (email: david.lopez-salido@frb.gov); Smith: Hutchin Hill Capital, LP, 142 West 57th Street, 15th Floor, New York, NY 10019 (email: matt.e.smith@gmail.com). We thank Chris Sims and three anonymous referees for helpful comments, and Kathryn Holston for excellent research assistance. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. This paper represents the personal work product and views of Matthew E. Smith and have neither been reviewed nor endorsed by Hutchin Hill Capital, LP, its affiliates, any funds advised by them, or any of their officers, directors, employees, agents, partners, principals, owners, or shareholders (collectively, “Hutchin Hill Parties”). No representation or warranty is made by Matthew E. Smith or any Hutchin Hill Party about the completeness or accuracy of any information provided herein, and such parties expressly disclaim all liability for any of the information contained in these materials. These materials have been prepared for educational use only and may not be relied upon for any other purpose (including without limitation any commercial purpose). These materials do not constitute an offer, or the solicitation of an offer, to buy or sell any security or financial instrument or investment advisory services. No Hutchin Hill Party shall be liable or have any obligation with respect to these slides or any reliance thereon. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

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important question remains: what role did this constraint play in exacerbating the economic slump that occurred in the aftermath of the financial crisis?

In this paper, we address this question by formally estimating and evaluating a dynamic stochastic general equilibrium (DSGE) model. In doing so, the paper offers a methodological contribution by estimating a nonlinear DSGE model in which a lower bound is occasionally binding. We apply this methodology to the interest-rate lower bound, but our approach is more general and could be used in many different contexts, including models with financial constraints. When applied to the interest-rate lower bound, the technique allows us to identify the nature and size of disturbances that pushed the interest rate to the lower bound in the United States in late 2008 and quantify the role of the constraint in exacerbating the resulting economic slump. Moreover, our methodology provides a new perspective on the relative contributions of the endogenous and exogenous sources of business-cycle fluctuations.

We estimate a nonlinear version of a medium-scale model widely used in macroeconomics and monetary economics. In particular, we use a model similar to those of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), as these models have been successful in providing an empirically plausible account of key macroeconomic variables including output, consumption, investment, inflation, and the nominal interest rate. The model includes several real and nominal frictions including habit persistence in consumption, costly adjustment of investment, and costs to adjusting nominal wages and prices. Because of these features, the model has many state variables, and we demonstrate that it is possible albeit challenging to estimate a model of this size nonlinearly using Bayesian techniques.

Fluctuations in the model are driven by five shocks including those to total factor productivity, government spending, and monetary policy. We also include two other shocks that can be interpreted in a reduced-form manner as financial shocks. The first is the marginal efficiency of investment (MEI) shock discussed in Justiniano, Primiceri, and Tambalotti (2011), which affects the transformation through which goods can be turned into productive capital. As shown in Justiniano, Primiceri, and Tambalotti (2011), this shock can be viewed as a disturbance to the financial sector's ability to channel savings into investment. The second is a disturbance to the household's intertemporal Euler equation governing the purchase of a risk-free bond as in Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2015). Following Smets and Wouters (2007), we refer to this shock as a *risk premium* shock since it affects the spread between the risk-free rate and the return on risky assets.

In the model, the lower bound constraint is potentially an important factor influencing the stance of monetary policy and hence economic outcomes. By constraining the current interest rate, the lower bound limits the degree of monetary stimulus. However, monetary policy can still be effective in influencing expectations about future output and inflation, thereby affecting the current levels of prices and spending. Similarly, the current decisions of households and firms are influenced by the expectation that the constraint may be binding in future states.

Our solution method takes into account the effect that uncertainty about the ZLB has on the economic decisions of households and firms, as we do not impose certainty-equivalence or perfect foresight to solve the model. With this solution, we estimate the nonlinear model using Bayesian methods via a Markov chain

Monte Carlo (MCMC) algorithm. Since we cannot evaluate the likelihood using the Kalman filter, we use the particle filter. Although others have estimated nonlinear models with these techniques, this paper is the first to our knowledge to do so using a nonlinear model with an occasionally binding constraint.

Our empirical approach offers several advantages. First, as noted above, it explicitly takes into account the effect of future uncertainty about whether the ZLB will bind or not on the economic decisions of agents, and this uncertainty is accounted for in the estimation of the model. In contrast, others including Ireland (2011) estimate a linearized DSGE model without the constraint, which neglects to consider how the lower bound systematically changes monetary policy and thus the behavior of economic agents—a key aspect at the heart of our analysis. Moreover, we find evidence that the risk of a binding ZLB constraint affected economic behavior during the 2003–2004 episode in which the federal funds rate was near-but-never-at the constraint.

A second advantage of our approach is that it uses all available data, including the period over which the ZLB binds, to estimate the model. In contrast, an alternative approach is to estimate a linear model over a sample period in which the ZLB does not bind and use those estimates to simulate a nonlinear version of the model that imposes the constraint.<sup>1</sup> This approach precludes jointly estimating the parameters of the model and the shocks that occurred over the zero lower bound period and then using them to quantify the economic effects of the constraint, a central feature of our analysis.

We also find that there are important differences between the parameter estimates coming from the linearized version of the model and those coming from the nonlinear version, suggesting that inference based on the estimates of the linearized model is flawed. In particular, we find that the linear version of the model attributes an overly large role to the marginal efficiency of investment shock in explaining economic fluctuations. This result arises because the nonlinear version of the model has a stronger internal propagation mechanism than its linear counterpart. This stronger propagation reflects, in part, the nonlinearity associated with the ZLB but more importantly that the linear dynamics understate the sensitivity of investment to economic disturbances in models with costs to adjusting investment as in Christiano, Eichenbaum, and Evans (2005). Thus, our results highlight the importance of estimating the nonlinear model in order to properly quantify the relative contributions of endogenous and exogenous sources of business-cycle fluctuations.

We estimate the model using US quarterly data on output, consumption, and investment as well as inflation and the nominal interest rate from 1983 through 2014. For this sample period, there is a single episode spanning from 2009 until the end of the sample in which the nominal interest rate is effectively at the lower bound. We examine the model's estimated shocks during this episode and characterize which shocks were important in causing the Great Recession and in driving the nominal rate to the lower bound.

For the episode in which the interest rate was at the lower bound, the two financial disturbances—the risk premium and MEI shocks—were key driving forces

<sup>1</sup> See, for example, Christiano, Eichenbaum, and Trabandt (2015) and Aruoba, Cuba-Borda, and Schorfheide (2016).

pushing the interest rate down and keeping it at the lower bound during and after the Great Recession. However, the risk premium shock is the relatively more important of the two financial shocks, especially in explaining the large contraction in output, consumption, and investment that occurred. The productivity shock played a role in contributing to the slow economic recovery and in explaining why inflation did not fall more sharply during the Great Recession. Monetary policy shocks contributed little to the dynamics of inflation, output, and the nominal interest rate over the course of this episode. However, this does not necessarily imply that monetary policy was unimportant, as our results highlight that the systematic portion of the monetary policy rule was critical to understanding the dynamics of the economy during and in the aftermath of the financial crisis.

Using the estimates of the shocks, the central question we address is how much the lower bound constrained the ability of monetary policy to stabilize the economy. We answer this question by comparing our estimated model in which the lower bound is imposed to the hypothetical case in which monetary policy can act in an unconstrained manner. Our mean estimate from this counterfactual experiment implies that about 30 percent of the sharp contraction in output during the Great Recession was due to the interest-rate lower bound. The zero lower bound played an even larger role in accounting for the slow recovery following this contraction, as US GDP did not recover back to its pre-recessionary level until the end of 2012. In the absence of the ZLB constraint, our mean estimates imply that output would have recovered to its pre-recessionary level about a year earlier and that the ZLB explains a little more than one-half of the lower output over the 2008–2012 period. Moreover, the ZLB constraint was an important factor in contributing to inflation running persistently below the Federal Reserve's target over this period. An important caveat, associated with these estimates is that there is considerable uncertainty surrounding them.

Our paper is most closely related to Chung et al. (2012), who use counterfactual simulations in which policy is unconstrained to evaluate the cost of the ZLB in the context of a large-scale macroeconomic model. By comparing actual outcomes to the counterfactual outcomes of unconstrained simple policy rules, they argue that the ZLB was a significant constraint on monetary policy, contributing to lower inflation and a higher unemployment rate. We also perform a similar counterfactual exercise; however, our methodology differs significantly from theirs, including the fact that we estimate the policy rule used in our counterfactual analysis.

Our paper is also related to Christiano, Eichenbaum, and Trabandt (2015) and Campbell et al. (2017), who study the effectiveness of the Federal Reserve's forward guidance policies during the ZLB episode. Although this question is important in assessing the cost of the ZLB, these papers do not disentangle the effects of the ZLB from other frictions that contributed to the Great Recession as we do. Similarly, Wu and Xia (2016) also focus on evaluating the benefits of unconventional policy rather than the cost of the ZLB constraint. In doing so, they use a term structure model to construct a measure of the shadow or desired rate that policymakers would choose if they could ignore the ZLB constraint. We also construct such a measure, though our estimates are derived from an estimated structural model.

The rest of the paper proceeds as follows. The following section presents the macroeconomic model that we estimate, while Section II discusses how the model is

solved and estimated. The next three sections present the model's results. Section III discusses the model's parameter estimates and time series properties of the model. Section IV examines the estimated shocks that took the economy to the lower bound and that account for the Great Recession, and Section V quantifies the effects of the zero lower bound constraint on the economy. Section VI presents some conclusions.

## I. The Model

The model is similar to Christiano, Eichenbaum, and Evans (2005) or Smets and Wouters (2007). The economy consists of a continuum of households, a continuum of firms producing differentiated intermediate goods, a perfectly competitive final goods firm, and a government in charge of fiscal and monetary policy. We now lay out the objectives and constraints of the different agents as well as the sources of uncertainty.

### A. Firms

There is a continuum of monopolistically competitive firms producing differentiated intermediate goods. These goods are used as inputs by a perfectly competitive firm producing a single final good. The final good is produced by a representative, perfectly competitive firm with a constant-returns technology,  $Y_t = \left( \int_0^1 Y_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ , where  $Y_t(j)$  denotes intermediate good  $j \in [0, 1]$ , and  $\varepsilon_p > 1$  is a constant elasticity of demand. Profit maximization implies that the final goods producer's demand for good  $j$  is given by  $Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t$ , where  $P_t(j)$  denotes the price for intermediate good  $j$  and  $P_t$  denotes the aggregate price level.

The production function for good  $j$  is

$$(1) \quad Y_t(j) = K_t(j)^\alpha (Z_t N_t(j))^{1-\alpha},$$

where  $Z_t$  denotes the aggregate level of technology, and  $K_t(j)$  and  $N_t(j)$  denote the capital and labor services hired by firm  $j$ , respectively. Technology is assumed to evolve according to

$$(2) \quad Z_t = Z_{t-1} G_Z \exp(\epsilon_{Z,t}),$$

where  $\epsilon_{Z,t} \sim iid N(0, \sigma_Z^2)$  denotes a random disturbance to technological growth that causes it to depart from a deterministic growth rate,  $G_Z$ .

Intermediate goods firm  $j$  sells its output in a monopolistically competitive market and sets its nominal price  $P_t(j)$  subject to a quadratic cost to adjusting its price as in Rotemberg (1982). We depart from Christiano, Eichenbaum, and Evans (2005) and use Rotemberg contracts instead of Calvo contracts to model nominal price

and wage stickiness, as this choice helps reduce the dimensionality of the model's nonlinear state space.<sup>2</sup> A firm's cost to adjusting its price in period  $t$  is given by

$$\frac{\varphi_p}{2} \left( \frac{P_t(j)}{\tilde{\pi}_{t-1} P_{t-1}(j)} - 1 \right)^2 Y_t,$$

where the parameter  $\varphi_p \geq 0$  governs the size of the adjustment cost and  $Y_t$  denotes aggregate output. A producer's price change is indexed to  $\tilde{\pi}_{t-1} = \bar{\pi}^a \pi_{t-1}^{1-a}$  where  $\bar{\pi}$  denotes the central bank's inflation target and  $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ . The parameter  $a$  satisfies  $1 \geq a \geq 0$  and determines the extent to which price indexation is tied to the central bank's inflation target or to the lagged inflation rate. The cost of price adjustment makes the problem of the intermediate goods producer dynamic; that is, it chooses  $P_t(j)$  to maximize its expected present discounted value of profits,

$$E_0 \sum_{t=1}^{\infty} \beta^t \Lambda_t \left[ \left( \frac{P_t(j)}{P_t} - mc_t \right) Y_t(j) - \frac{\varphi_p}{2} \left( \frac{P_t(j)}{\tilde{\pi}_{t-1} P_{t-1}(j)} - 1 \right)^2 Y_t \right],$$

taking  $mc_t$ ,  $Y_t$ ,  $\tilde{\pi}_{t-1}$ , and  $\Lambda_t$  as given. In equilibrium, the variable  $\Lambda_t$  is equal to the marginal value of an additional unit of real profits to a household in period  $t$ , reflecting the ownership of the firm by a household in the economy. The variable  $mc_t$  denotes real marginal costs, which are the same for all of the intermediate goods producers and are given by  $mc_t = \frac{1}{\Phi} (r_t^k)^\alpha \left( \frac{W_t}{P_t} \right)^{1-\alpha}$ , where  $\frac{W_t}{P_t}$  and  $r_t^k$  denote the aggregate real wage and rental rate of capital services, respectively, and  $\Phi \equiv \alpha^\alpha (1 - \alpha)^{1-\alpha}$ .

## B. Households

We assume that there is a continuum of monopolistically competitive households indexed by  $i \in [0, 1]$  supplying a differentiated labor service,  $N_t(i)$ . This allows us to model nominal wage rigidities in an analogous manner to the nominal price rigidities described earlier. Household  $i$  sells labor services to a representative employment agency producing a single labor input,  $N_t$ , which is in turn supplied to the intermediate goods firms in a perfectly competitive labor market. The Dixit-Stiglitz aggregator for labor services is given by

$$N_t = \left( \int_0^1 N_t(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

with  $\varepsilon_w > 1$ . Profit maximization by the employment agency taking as given the aggregate wage,  $W_t$ , and each household's wage,  $W_t(i)$ , yields the following set of demand schedules:  $n_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t$ ,  $\forall i$ .

<sup>2</sup>With Rotemberg adjustment costs, the model has 12 state variables. If instead we used Calvo contracts, there would be two additional state variables reflecting the dispersion of wages across households and prices across firms.



A household's preferences are given by

$$(3) \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t(i) - \gamma C_{t-1}(i)) - \psi_L \frac{N_t(i)^{1+\sigma_L}}{1+\sigma_L} - \varphi_t^w(i) \right\},$$

where  $C_t(i)$  denotes the consumption of household  $i$ , and  $\varphi_t^w(i)$  is the loss in utility associated with adjusting the household's wage,  $W_t(i)$ . The parameter  $\gamma$  governs the importance of habits in consumption. Household  $i$  also experiences disutility from working, with the parameter  $\psi_L$  affecting the level of this disutility and  $\sigma_L$  affecting the Frisch elasticity of labor supply. Similar to the cost of adjusting prices, the wage adjustment cost is quadratic:

$$\varphi_t^w(i) = \frac{\varphi_w}{2} \left[ \frac{W_t(i)}{\tilde{\pi}_t^w W_{t-1}(i)} - 1 \right]^2,$$

where  $\varphi_w \geq 0$  governs the size of this cost. Wage contracts are indexed to productivity and inflation, since  $\tilde{\pi}_t^w = G_Z \bar{\pi}^{a_w} (\exp(\epsilon_{Z,t}) \pi_{t-1})^{1-a_w}$  with the parameter  $a_w \in [0, 1]$ .

A household's budget constraint in period  $t$  is

$$(4) \quad \begin{aligned} C_t(i) + I_t(i) + \frac{B_{t+1}(i)}{R_t \eta_t P_t} + a(u_t(i)) \bar{K}_t(i) \\ = \frac{W_t(i)}{P_t} n_t(i) + r_t^k K_t(i) + \frac{B_t(i)}{P_t} + \frac{D_t}{P_t} - T_t. \end{aligned}$$

A household receives labor income from the employment agency and income from purchases of a risk-free nominal bond,  $B_t(i)$ . Each household receives the same amount of dividends,  $D_t$ , from the economy's intermediate goods producers and pays the same amount of lump-sum taxes,  $T_t$ . Household  $i$  also owns capital,  $\bar{K}_t(i)$ , which it combines with its desired level of capital utilization,  $u_t(i)$ , to transform into capital services,  $K_t(i) = u_t(i) \bar{K}_t(i)$ . To utilize physical capital and transform it into capital services, a household incurs the cost  $a(u_t(i)) \bar{K}_t(i)$ , where the function  $a(u_t(i))$  is given by

$$(5) \quad a(u_t(i)) = \frac{r^k}{\sigma_a} \{ \exp(\sigma_a(u_t(i) - 1)) - 1 \}.$$

A household purchases the final good and uses it for consumption,  $C_t(i)$ , or investment,  $I_t(i)$ . Nominal bonds are purchased at the price  $\frac{1}{R_t \eta_t P_t}$ , where  $R_t$  denotes the nominal interest rate. Following Smets and Wouters (2007) and Christiano, Eichenbaum, and Trabandt (2015),  $\eta_t$  is an exogenous disturbance to a household's return on the risk-free bond. Fisher (2015) discusses how an increase in  $\eta_t$ , to first order, can be interpreted as a rise in the demand for risk-free bonds. This risk premium shock affects the spread between the nominal rate and the return on risky assets, and is assumed to evolve according to

$$(6) \quad \ln(\eta_t) = \rho_\eta \ln(\eta_{t-1}) + \epsilon_{\eta,t},$$

where  $\epsilon_{\eta,t} \sim iid N(0, \sigma_\eta^2)$ .

A household's purchases of investment augment the physical stock of capital according to

$$(7) \quad \bar{K}_{t+1}(i) = (1 - \delta) \bar{K}_t(i) + \mu_t(1 - S_t(i)) I_t(i),$$

where  $\delta$  is the depreciation rate of capital and  $\mu_t$  is an exogenous disturbance to the marginal efficiency of transforming final goods into tomorrow's physical capital. This shock is assumed to evolve according to

$$(8) \quad \ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \epsilon_{\mu,t},$$

where  $\epsilon_{\mu,t} \sim iid N(0, \sigma_\mu^2)$ . As discussed in Justiniano, Primiceri, and Tambalotti (2011), this shock can be interpreted as a reduced-form way of capturing financial frictions that affect the transformation of aggregate savings into investment. In accumulating capital, households must also incur an adjustment cost,  $S_t(i)$ , to transform investment into capital available for production next period. Following Christiano, Eichenbaum, and Evans (2005), these adjustment costs are given by

$$(9) \quad S_t(i) = \frac{\varphi_I}{2} \left( \frac{I_t(i)}{G_Z I_{t-1}(i)} - 1 \right)^2.$$

### C. Monetary and Fiscal Policies

The central bank must set the nominal rate  $R_t$  in accordance with its lower bound constraint,

$$(10) \quad R_t = \max[1, R_t^N],$$

where  $R_t^N$  denotes the notional or desired rate that the central bank would like to set in the absence of the constraint. The notional rate is set according to

$$(11) \quad R_t^N = (R_{t-1}^N)^{\rho_R} \left[ R \left( \frac{\pi_t}{\bar{\pi}} \right)^{\gamma_\pi} x_{gt}^{\gamma_x} \left( \frac{Y_t}{G_Z Y_{t-1}} \right)^{\gamma_g} \right]^{(1-\rho_R)} \epsilon_{R,t},$$

with  $0 < \rho_R < 1$ ,  $\gamma_\pi \geq 0$ ,  $\gamma_g \geq 0$ , and  $\gamma_x \geq 0$ . In the above,  $R = \beta^{-1} G_Z \bar{\pi}$  denotes the interest rate in the nonstochastic steady state and  $x_{gt}$  denotes the output gap. The variable  $\epsilon_{R,t}$  is an exogenous disturbance to the policy rule satisfying  $\epsilon_{R,t} \sim iid N(0, \sigma_R^2)$ .

The output gap depends on the deviations of utilization and labor from their nonstochastic steady-state values,  $x_{gt} = u_t^\alpha \left( \frac{N_t}{\bar{N}} \right)^{(1-\alpha)}$ , where  $u_t$  is the aggregate level of capital utilization, whose nonstochastic steady-state value is equal to 1, and



$N$  denotes the nonstochastic steady-state level of labor. This concept of the output gap departs from Smets and Wouters (2007), who use the natural level of output (i.e., the level of output in the absence of price and wage rigidities). Use of such a concept would considerably increase the size of the nonlinear state space needed to solve the model so our use of  $x_{gt}$  in part reflects that it is computationally convenient. In addition, in practice a central bank may rely on estimates of an output gap concept more closely related to  $x_{gt}$  than the natural level of output, and we find that when we use this concept of the output gap, it tracks reasonably well the measure constructed by the Congressional Budget Office (for a comparison, see the online Appendix).

Government spending is determined exogenously as a time-varying fraction of output:  $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ , where  $g_t$  evolves according to

$$(12) \quad \ln(g_t) = (1 - \rho_g) \ln g + \rho_g \ln(g_{t-1}) + \epsilon_{g,t},$$

where  $\epsilon_{g,t} \sim iid N(0, \sigma_g^2)$  and  $g$  denotes the steady-state share of government spending. Finally, the government budget constraint is assumed to be satisfied on a period-by-period basis so that  $G_t = T_t$ .

#### D. Market Clearing

We focus on a symmetric equilibrium in which all intermediate goods producing firms and all households make the same decisions, and we denote aggregate variables by dropping a variable's dependence on  $i$  or  $j$ . In a symmetric equilibrium, the aggregate production function is given by

$$(13) \quad Y_t = K_t^\alpha [Z_t N_t]^{1-\alpha},$$

since  $K_t = K_t(j)$ ,  $N_t = N_t(j) \forall j$ . The market clearing condition for the final good is

$$(14) \quad C_t + I_t + G_t + \frac{\varphi_p}{2} \left[ \frac{\pi_t}{\tilde{\pi}_{t-1}} - 1 \right]^2 Y_t + a(u_t) \bar{K}_t = Y_t,$$

where we have used that  $u_t = u_t(i)$ ,  $C_t = C_t(i)$ , and  $I_t = I_t(i)$  for all  $i$  in a symmetric equilibrium.

## II. Solution and Econometric Inference

#### A. Model Solution

The model is solved for a minimum state variable solution using a projection method similar to Christiano and Fisher (2000) and Judd et al. (2014), and the details of the method are provided in an online technical Appendix.<sup>3</sup> Because we

<sup>3</sup>The minimum state variable solution rules out equilibria with additional state variables such as a sunspot or those with nonstationary dynamics. See McCallum (1999) for a discussion. However, Benhabib, Schmitt-Grohé,

are estimating the model, it might be tempting to use a computationally efficient solution algorithm that respects the nonlinearity in the Taylor rule but log-linearizes the remaining equilibrium conditions. This approach has been used by Bodenstein, Guerrieri, and Gust (2013) and Guerrieri and Iacoviello (2015), among others. Unfortunately, we find that this approach performs poorly when applied to our model, as the linearized dynamics poorly approximate the nonlinear dynamics even for moderately sized shocks. Instead, as discussed in the online technical Appendix, we develop a computationally efficient algorithm that is easy to parallelize and ideal for solving a nonlinear, medium-scale DSGE model with an occasionally binding constraint. Using this solution algorithm, we show that there are differences between the nonlinear and linear versions of the model and that they have important implications for the estimates of the model's parameters and the propagation of shocks. Another attractive feature of our solution method is that it takes into account the uncertainty about the likelihood that the economy will be at the lower bound, as it does not rely on perfect foresight or certainty equivalence.

### B. Data and Estimation

The solution algorithm characterizes a transition equation for the model variables,  $s_t$ , as a function,  $\Phi$ , of its past realization,  $s_{t-1}$ , and current innovations to the shocks,  $\epsilon_t$ , which we write as

$$(15) \quad s_t = \Phi(s_{t-1}, \epsilon_t; \theta), \quad \epsilon_t \sim N(0, I),$$

where

$$s_t = [\bar{k}_{t+1}, y_t, c_t, i_t, \pi_t, R_t^N, w_t, \eta_t, \mu_t, g_t, y_{t-1}, c_{t-1}, i_{t-1}],$$

$$\epsilon_t = [\epsilon_{\eta,t}, \epsilon_{\mu,t}, \epsilon_{Z,t}, \epsilon_{g,t}, \epsilon_{R,t}], \text{ and}$$

$$\theta = [\beta, \bar{\Pi}, g_z, \alpha, \rho_R, \gamma_{\Pi}, \gamma_g, \gamma_x, \gamma, \sigma_L, \sigma_a, \varphi_I, \varphi_p, \varphi_w, a, a_w, \rho_g, \rho_\mu, \sigma_\eta, \sigma_\mu, \sigma_Z, \sigma_g, \sigma_R].$$

Within the model variables  $s_t$ ,  $\bar{k}_{t+1}$ ,  $y_t$ ,  $c_t$ ,  $w_t$ , and  $i_t$  denote the level of the capital stock, output, consumption, real wage, and investment scaled by the level of technology,  $Z_t$ .

Our goal is to determine plausible values for the parameters,  $\theta$ , for tracking and explaining the US macroeconomic experience since the early 1980s. Specifically, we base the empirical analysis on quarterly US data from 1983:I to 2014:I, using five time series to estimate the model. As measures of real activity, we use per-capita GDP growth, per-capita consumption growth, and per-capita investment growth.

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and Uribe (2001) emphasize that there is a second steady state that is deflationary due to the imposition of the ZLB on the Taylor rule. Our solution algorithm in principle does not rule out dynamics that fluctuate around this deflationary steady state. But, in practice, we use the linearized solution as the initial guess for the solution algorithm and find that the solution algorithm never converged to this unintended deflationary equilibrium: for every estimated parameter draw that we examined from the posterior distribution, the ergodic mean of inflation was greater than zero. Moreover, such deflationary outcomes do not characterize the US data over our sample period. As discussed in Christiano and Eichenbaum (2012), this deflationary equilibrium is also not E-learnable.

Inflation is measured as the change in the log of the GDP deflator, while our measure of the short-term interest rate is given by the quarterly average of the daily three-month US Treasury bill (quarterly) rate, following Ireland (2011). Additional details about the data are provided in the online Appendix.

We assume that the observables contain measurement error, so the relationship to our model variables is given by

$$(16) \quad \begin{bmatrix} \text{Output Growth} \\ \text{Consumption Growth} \\ \text{Investment Growth} \\ \text{Inflation} \\ \text{Interest Rate} \end{bmatrix} = \begin{bmatrix} \ln \left( \frac{y_t G_Z}{y_{t-1}} \right) + \epsilon_{Z,t} \\ \ln \left( \frac{c_t G_Z}{c_{t-1}} \right) + \epsilon_{Z,t} \\ \ln \left( \frac{i_t G_Z}{i_{t-1}} \right) + \epsilon_{Z,t} \\ \ln(\pi_t) \\ \ln(R_t) \end{bmatrix} + \text{measurement error},$$

which we write compactly as

$$(17) \quad y_t = \Psi(s_t, \epsilon_t; \theta) + u_t, \quad u_t \sim N(0, \Sigma_u).$$

We estimate  $\theta$  using Bayesian methods. Herbst and Schorfheide (2015) and the references therein provide background on Bayesian estimation of (linear and nonlinear) DSGE models; here we give a brief overview and leave the details to the online technical Appendix. Stacking the five observed time series as  $Y_{1:T}$ , Bayesian inference is characterized by a posterior distribution over the parameters,  $p(\theta | Y_{1:T})$ , that is proportional to the product of the likelihood for  $Y_{1:T}$  and prior distribution for  $\theta$ ,<sup>4</sup>

$$p(\theta | Y_{1:T}) \propto \underbrace{p(Y_{1:T} | \theta)}_{\text{likelihood}} \times \underbrace{p(\theta)}_{\text{prior}}.$$

Constructing the posterior distribution is conceptually straightforward but computationally very challenging. The transition equation defined by equation (15) along with the observation equation (17) define a nonlinear state space model. Since the observables are only a subset of the model variables, we are required to integrate out the unobserved states to compute the likelihood. Unlike a linear (Gaussian) state space model, we cannot evaluate these integrals analytically via the Kalman filter. Instead, we use a (Sequential) Monte Carlo technique known as a particle filter. This simulation-based approach provides us with an estimate of the likelihood,  $\hat{p}(Y_{1:T} | \theta)$ , for a given parameter draw,  $\theta$ , by constructing discrete approximations (*particles*) to the distributions of the underlying states.<sup>5</sup>

<sup>4</sup>In a slight abuse of notation,  $Y_t$  denotes aggregate output at date  $t$  and  $Y_{1:T}$  denotes the five observed time series from 1983:I to 2014:I.

<sup>5</sup>There is a long literature on particle filtering—see Creal (2012) for a survey. We use a very basic version of the particle filter, the so-called bootstrap particle filter described in Gordon, Salmond, and Smith (1993) with some minor modifications. The online technical Appendix contains the details on the implementation of the particle filter.

We also resort to simulation-based methods to elicit draws from the posterior. Markov-chain Monte Carlo (MCMC) techniques construct a Markov chain  $\{\theta^i\}_{i=1}^N$  whose invariant distribution coincides with the posterior distribution of interest,  $p(\theta | Y_{1:T})$ . We use the so-called random walk Metropolis-Hastings (RWMH) algorithm first used in DSGE model estimation by Schorfheide (2000) and Otrok (2001). A complication here is that, like Fernández-Villaverde and Rubio-Ramírez (2007), we are using MCMC with a particle-filter-based estimate of the likelihood. We appeal to Andrieu, Doucet, and Holenstein (2010), who show formally that, despite the use of the particle filter, the Markov chain will still have the desired posterior distribution as its invariant distribution. For more details on the MCMC algorithm, including hyperparameters and run time, see the online technical Appendix.

*Measurement Errors.*—As mentioned above, we include measurement error in the observation equation (16). One reason for its presence is feasibility: without measurement error—or with only very small measurement errors—our particle filter degenerates and the accuracy of the likelihood estimate becomes very poor.<sup>6</sup> The measurement error is defined as

$$\Sigma_u = m_e \times \text{diag}(\mathbf{V}[Y_{1:T}]),$$

where we set  $m_e = 0.25$  for the baseline parameter estimates of the model and  $m_e = 0.1$  for a lower measurement error case. These values imply that, for each observable, the variance of its measurement error is 25 percent ( $m_e = 0.25$ ) or 10 percent ( $m_e = 0.1$ ) of its total variance over the estimation period, respectively. As discussed in the online technical Appendix, either of these values ensures that the particle filter estimates are stable, and we present results for two values of  $m_e$  to highlight how measurement error affects our estimates.

The inclusion of measurement error represents a departure from the majority—though not all—of the literature estimating linear DSGE models. In particular, it means that our estimates are more uncertain than they otherwise might be. The benefit of this approach, though, is that we are able to conduct inference using the full nonlinear structure of the model, including the zero lower bound. Given that the dynamics of the nonlinear model differ considerably from the linearized dynamics, we think that it is a trade-off worth making.

*Parallelization.*—While most of the technical aspects are relegated to the online technical Appendix, it is worth mentioning some of the key innovations that we make in order to estimate a nonlinear, medium-scale DSGE model. The principal obstacle in our empirical exercise is time. It can take a long time to solve the model and evaluate the particle filter. Finally, the MCMC estimation strategy requires solving the model and evaluating the particle filter many times in an iterative fashion. For both the solution of the model and evaluation of the particle filter, we rely on extensive parallelization to reduce computational time. The parallelization of the projection method used to construct the model solution is straightforward. The parallelization

<sup>6</sup>Another important reason, as discussed later, is that measurement error can absorb some of the model misspecification.

of the particle filter, on the other hand, is more difficult, as the filter requires frequent communication amongst all particles. The online technical Appendix shows how to circumvent many of the problems associated with naïve parallelization. With these modifications to the solution algorithm and particle filter, we leverage a large distributed computing environment to sample from our posterior quickly.<sup>7</sup>

### III. Estimation Results

#### A. Parameter Estimates

Bayesian inference requires that we specify a prior distribution. Many of our marginal prior distributions are the same as in Justiniano, Primiceri, and Tambalotti (2011). The prior distributions for the Rotemberg adjustment cost parameters are specified to be consistent with prior beliefs that nominal wages are more rigid than prices. More details about the prior distributions are shown in the online Appendix. We also fix several parameters. In particular, the elasticity of the demand for labor inputs,  $\epsilon_w$ , and the elasticity of the demand for intermediate goods,  $\epsilon_p$ , are not estimated and are fixed at 6. The depreciation rate,  $\delta$ , is set to 0.025, and the nonstochastic steady-state share of government spending,  $g$ , is set equal to 20 percent. The preference parameter  $\psi_L$  is normalized to 1.

Estimating the nonlinear model is a challenging task, particularly for a model of this size, and this task is made more complicated because of the zero lower bound constraint, whose presence makes it difficult to solve the model for highly persistent risk premium shocks. To simplify the estimation of the model, we fix  $\rho_\eta$  at 0.85, a relatively high value that helps the model fit the data. In particular, this value is consistent with the microeconomic evidence presented in Gilchrist and Zakrajšek (2012), whose measure of the excess bond premium has an autocorrelation of 0.84 over our sample period. Moreover, a value of  $\rho_\eta = 0.85$  is not at odds with the aggregate data: as shown in the online Appendix, the likelihood function is increasing in  $\rho_\eta$ . However, as  $\rho_\eta$  gets close to 1, it becomes more difficult to compute a solution to the model for a wide range of parameter configurations, considerably complicating the estimation of the model.<sup>8</sup>

Table 1 presents the means and the intervals bracketed by the fifth and ninety-fifth percentiles of the marginal posterior distributions of the parameters for the baseline value of the measurement error ( $m_e = 0.25$ ). The mean estimates of the discount factor and the deterministic trend to the technology growth rate are just below 1 and 0.5 percent on a quarterly basis, respectively, implying a real rate of about 2.6 percent on an annualized basis in the nonstochastic steady state. The central bank's inflation target is estimated to have a mean of about 2.5 percent, which is higher than the current 2 percent target of the Federal Reserve, reflecting that the sample mean of inflation from 1983–2014 is close to 2.5 percent.

<sup>7</sup>We could, in principal, enjoy further gains by modifying the MCMC algorithm along the lines of Smith (2011), who develops a surrogate MCMC algorithm wherein the particle filter is not evaluated when the likelihood is predicted to be a low value. See Gust et al. (2012) for details.

<sup>8</sup>Whether it is possible to compute a solution for  $\rho_\eta$  close to 1 depends importantly on the other model parameters and in particular the adjustment costs for investment. Characterizing the parameter space for which we can find a solution to the model is an extremely difficult task given that the model is nonlinear and has many parameters.

TABLE 1—POSTERIOR DISTRIBUTION OF PARAMETERS

Parameter	Mean	[05, 95]	Parameter	Mean	[05, 95]
<i>Steady state</i>					
$100(\beta^{-1} - 1)$	0.14	[0.06, 0.23]	$100(\bar{\pi} - 1)$	0.61	[0.54, 0.68]
$100 \log(G_z)$	0.50	[0.46, 0.54]	$\alpha$	0.19	[0.16, 0.22]
<i>Policy rule</i>					
$\rho_R$	0.70	[0.59, 0.78]	$\gamma_\pi$	1.67	[1.21, 2.14]
$\gamma_g$	0.73	[0.39, 1.07]	$\gamma_x$	0.14	[0.07, 0.24]
<i>Endogenous propagation</i>					
$\gamma$	0.70	[0.63, 0.76]	$\sigma_L$	2.00	[1.01, 3.17]
$\sigma_a$	5.32	[3.78, 7.09]	$\varphi_I$	3.70	[2.24, 5.21]
$\varphi_p$	100.41	[65.10, 136.88]	$1 - a$	0.56	[0.36, 0.76]
$\varphi_w$	4,420.49	[1,693.15, 8,356.34]	$1 - a_w$	0.51	[0.29, 0.72]
<i>Exogenous processes</i>					
$\rho_G$	0.67	[0.29, 0.96]	$100 \sigma_G$	0.15	[0.11, 0.20]
$\rho_{\mu_I}$	0.80	[0.64, 0.92]	$100 \sigma_{\mu_I}$	2.39	[1.43, 3.70]
$100 \sigma_\eta$	0.44	[0.34, 0.54]	$100 \sigma_Z$	0.56	[0.38, 0.80]
$100 \sigma_R$	0.18	[0.14, 0.24]			

Note: Table reports the mean, fifth, and ninety-fifth percentiles of the posterior distribution estimated by pooling 4 MCMC chains with 50,000 draws each (including a 10,000 draw burn-in period.)

For the policy rule, the estimates are largely in line with Del Negro et al. (2017). The posterior mean for the interest-rate smoothing parameter,  $\rho_R$ , is near 0.7, while the posterior means for the policy-rate responses to inflation and output growth,  $\gamma_\pi$  and  $\gamma_g$ , are about 1.7 and 0.7, respectively. The policy rule also involves a smaller but statistically significant response of the short-term nominal rate to the output gap, as the posterior mean of  $\gamma_x$  equals 0.14.

The posterior mean of the adjustment cost parameter for investment,  $\varphi_I$ , is 3.7 with a 90 percent credible interval covering values from 2.2 to 5.2. This mean estimate is above the value of 3 estimated by Justiniano, Primiceri, and Tambalotti (2011), but below that of Smets and Wouters (2007). The posterior mean of the habit persistence parameter,  $\gamma$ , is equal to 0.7, and the 90 percent credible interval—covering values between 0.63 to 0.76—is relatively tight. The parameter governing the elasticity of labor supply,  $\sigma_L$ , has a mean of 2. The posterior mean of the elasticity of the rental cost of capital with respect to the utilization,  $\sigma_a$ , is 5.3—similar to the estimates of Altig et al. (2011) and Justiniano, Primiceri, and Tambalotti (2011).

The posterior mean of the price adjustment cost parameter,  $\varphi_p$ , is around 100 with a weight on lagged indexation,  $1 - a$ , equal to 0.56, implying a linearized slope coefficient for the New Keynesian Phillips curve of 0.07. As shown in the online Appendix, such a slope coefficient corresponds to a frequency of price changes of slightly more than one year in a model with Calvo contracts. Nominal wages are estimated to be extremely rigid but, as shown in the online Appendix, less so than in Justiniano, Primiceri, and Tambalotti (2011).

While the discussion so far has emphasized that most of the estimates are similar to those in the literature, this is not the case for the shock parameters. Most strikingly, the mean of the standard deviation of the innovation to this shock,  $\sigma_\mu$ , is about one-half of the value found by Justiniano, Primiceri, and Tambalotti (2011). This lower estimate primarily reflects our use of the nonlinear model rather than its linear

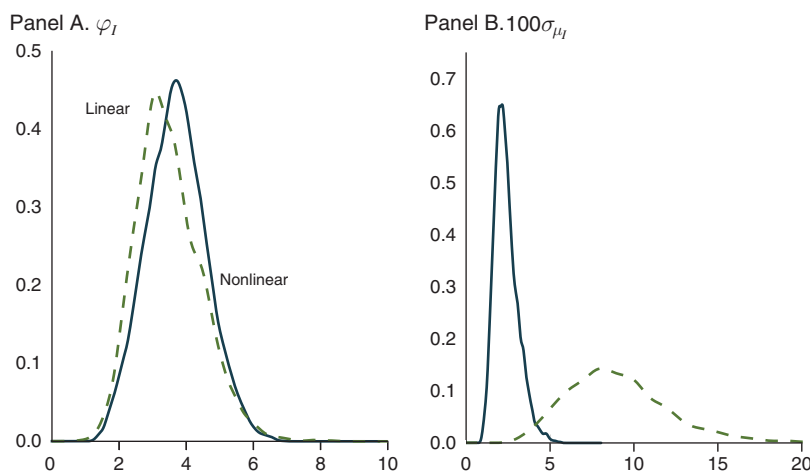


FIGURE 1. COMPARISON OF THE POSTERIOR ESTIMATES: NONLINEAR VERSUS LINEAR

*Note:* Figure shows the posterior density functions (kernel density estimate) for  $\varphi_I$  and  $100\sigma_{\mu_I}$  under the nonlinear (solid lines) and linearized (dashed lines) versions of the model.

approximation.<sup>9</sup> To highlight this difference, panel B of Figure 1 shows that the posterior distribution of  $\sigma_{\mu_I}$  is considerably below the distribution derived from estimating the linearized version of the model. While the estimate of this parameter is the one that is most substantially affected by estimating the nonlinear model rather than the linear model, there are other notable differences in the estimates as well. The estimated volatilities of the risk premium shock, government spending shock, and technology shocks are all smaller using the nonlinear model, and the parameter estimates of the policy rule differ somewhat as well.<sup>10</sup> Moreover, panel A shows that the estimate of the adjustment cost on investment is higher in the nonlinear model, though the difference is not as dramatic as the estimate of the innovation variance of the MEI shock.

In the online Appendix we show the parameter estimates imposing a lower measurement error ( $m_e = 0.1$ ). Most of the parameter estimates are similar to the baseline case. However, there are a couple of important exceptions. The degree of wage and price rigidities is notably smaller than under the baseline estimates, and the estimated size of the technology shock is much larger.

### B. The Nonlinear Propagation of Shocks

If the linear model approximated the nonlinear dynamics well, the two sets of parameter estimates would be the same. However, Figure 1 demonstrates that some of the parameters from these two versions of the model can differ substantially,

<sup>9</sup>The differences in parameter estimates between the linear and nonlinear versions of the model would be larger if the measurement error were smaller. All other things equal, the estimated structural shocks would be larger in that case, causing the performance of the linear approximation to deteriorate further relative to the nonlinear solution.

<sup>10</sup>See the online Appendix for the estimates based on the linear version of our model.



and in this subsection we show that this result reflects that the linear approximation understates the endogenous propagation of shocks in the model. To do so, we use the mean estimates derived from the nonlinear model to simulate both the linear and nonlinear versions of the model and compare the impulse responses to the economy's two financial shocks.

In order to highlight the difference between the linear and nonlinear solutions, Figure 2 shows the effects of a one-standard-deviation increase in  $\epsilon_{\eta,t}$ , the innovation to the risk premium shock, using initial conditions in which the economy is moderately away from its nonstochastic steady state.<sup>11</sup> The solid dark lines display the responses using the nonlinear solution, and dashed lines show the responses using the linear solution. By raising risky spreads, this shock reduces economic activity and inflation and also puts downward pressure on the nominal interest rate in both model versions. But, the fall in economic activity is notably larger in the nonlinear version of the model, especially for investment, where the difference is substantial. Because this larger fall in investment persists and because it also reduces the stock of capital, there is a larger and more persistent fall in output, consumption, and hours worked in the nonlinear version of the model. In contrast, the price of installed capital falls by less in the nonlinear version of the model than in the linear version. Overall, Figure 2 indicates that the linear solution poorly approximates the stronger propagation inherent in the nonlinear dynamics even for moderately sized innovations in the shock.<sup>12</sup> Of course, these differences are magnified for larger shocks.

The stronger propagation inherent in the nonlinear dynamics is not driven by the ZLB but by the investment adjustment costs, which imply that optimal investment satisfies

$$(18) \quad 1 + q_t \mu_t [S_t + \varphi_I \Delta_{I,t}] = q_t \mu_t + \varphi_I E_t \left\{ q_{t+1} \mu_{t+1} m_{t+1} \Delta_{I,t+1} \frac{I_{t+1}}{G_Z I_t} \right\},$$

where  $S_t$  is defined in equation (9),  $m_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$  denotes the stochastic discount factor,  $\Delta_{I,t} = \frac{I_t}{G_Z I_{t-1}} \left( \frac{I_t}{G_Z I_{t-1}} - 1 \right)$ , and  $q_t$  denotes the price of newly installed capital. The left-hand side of equation (18) reflects the cost of increasing investment by an extra unit and the right-hand side reflects the benefit. On the cost side, increasing investment in the current period means that the household forgoes consumption today and also incurs costs to transform the final good into new capital. On the benefit side, the extra investment boosts the amount of installed capital and the value to doing so reflects both the price of the installed capital,  $q_t$ , and the efficiency of the extra investment,  $\mu_t$ . The benefits also reflect that raising investment today reduces the burden associated with incurring the adjustment costs in the future.

<sup>11</sup> In particular, we choose initial conditions for the risk premium shock ( $\eta_0$ ) so that it is one standard deviation above its unconditional mean while the lagged values of the other state variables are equal to their nonstochastic steady-state values.

<sup>12</sup> Panel H compares the approximation error associated with the Euler equation that determines the relationship between the price of investment and a household's optimal supply of investment, equation (18) along the shocked path. This error is about ten times larger using the linear solution than the nonlinear solution.

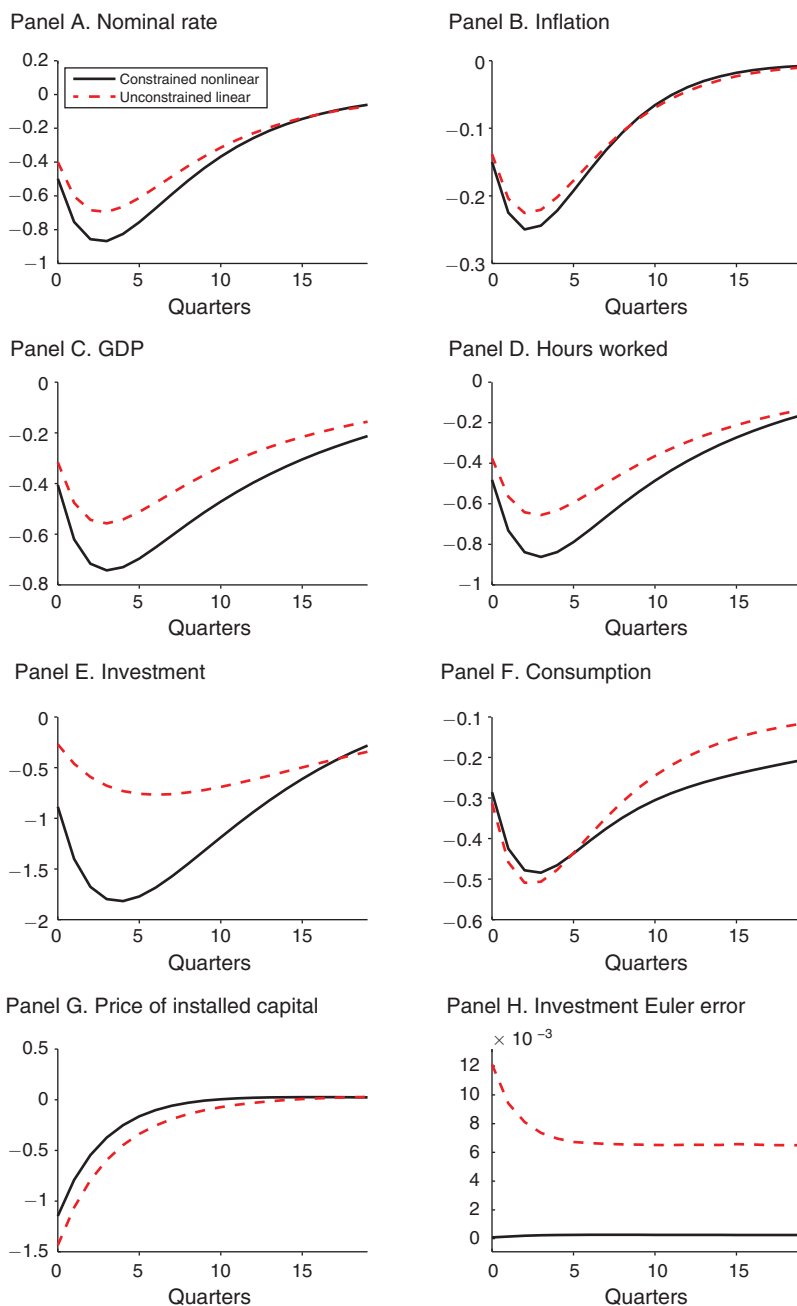


FIGURE 2. RESPONSE TO AN EXOGENOUS INCREASE IN THE RISK PREMIUM

*Notes:* Figure reports the difference between the mean values of a shocked path, which introduces a one-standard-deviation innovation in the risk premium shock at date 1,  $\epsilon_{\eta,1}$ , and the mean values of a path in which this innovation does not occur. Both paths are constructed using the posterior mean parameter values and initial conditions such that  $\ln(\eta_0)$  is one standard deviation above its nonstochastic steady state, and all other variables are at their nonstochastic steady states. The dashed line shows the responses for the unconstrained linear model, while the solid line shows the responses for the constrained nonlinear model. Panel H with the investment Euler error shows the mean absolute values of the errors along the shocked path at each date.

Although the quadratic nature of the adjustment costs make it difficult to see, equation (18) implies the supply of investment is an increasing function of  $q_t$ . All else equal, a reduction in the stochastic discount factor, either due to an increase in the real policy rate or the risky spread, tends to lower the supply of investment because the future benefits fall. This effect, however, is absent from the linearized counterpart of equation (18), because the linearized equation is approximated around the nonstochastic steady state where this effect is negligible. However, away from there, this effect will not necessarily be inconsequential, as Figure 2 demonstrates. In that case, the increase in the risky rate leads to a reduction in investment supply that contributes to a slump in investment. Furthermore, this shift in investment supply does not have to be large, because the elasticity of investment demand with respect to  $q_t$  is high, meaning that relatively small changes in investment supply can induce relatively large movements in the level of investment.<sup>13</sup> The linearized dynamics neglect the shift in investment supply, and hence tend to understate the drop in investment.

The linear dynamics also poorly capture the propagation of the MEI shock, whose estimated volatility is much smaller once we account for the stronger nonlinear propagation. Figure 3 shows the effects of a one-standard-deviation reduction in the innovation to the marginal efficiency of investment,  $\epsilon_{\mu,t}$ . This shock also produces a much larger fall in investment in the nonlinear version of the model than in the linear version. This lower investment in turn induces a larger fall in the stock of capital, which helps engender more persistent declines in output and hours worked in the nonlinear version of the model.

What are the implications of the stronger propagation of the nonlinear dynamics for the business cycle? As shown in Table 2, the nonlinear version of the model relies on its stronger internal propagation mechanism to generate output volatility in line with the observed data. To see this, column 3 of Table 2 (Nonlinear Constrained) shows the volatility of output, consumption, investment, and hours at business-cycle frequencies simulating the nonlinear model using its parameter estimates. The standard deviations of output and consumption are near their empirical counterparts, while the volatilities of investment and hours are only somewhat lower than their empirical counterparts. If instead we used these same parameter estimates and simulated the linearized dynamics of the model, the linearized version of the model would substantially understate the volatility of investment, as shown in the final column. This reflects that, if the linearized model has to rely on the parameter estimates coming from the nonlinear model, the estimate of the volatility of the MEI shock is too small given its weaker endogenous propagation. Thus, it fails to generate enough volatility in output, hours, and especially in investment. For the linearized model to generate a realistic volatility of investment, it must rely on a much larger exogenous MEI shock, because it poorly approximates the nonlinear model's dynamics. Hence, to reliably disentangle the endogenous and exogenous sources of business-cycle fluctuations, one should estimate the nonlinear model rather than its linear approximation.

<sup>13</sup>The investment-demand relationship can be derived from the Euler equation for the capital stock which relates  $q_t$  to the rental rate of capital. The rental rate of capital, in turn, can be expressed to depend on investment using the capital accumulation equation. See the online technical Appendix for the details of this equation.

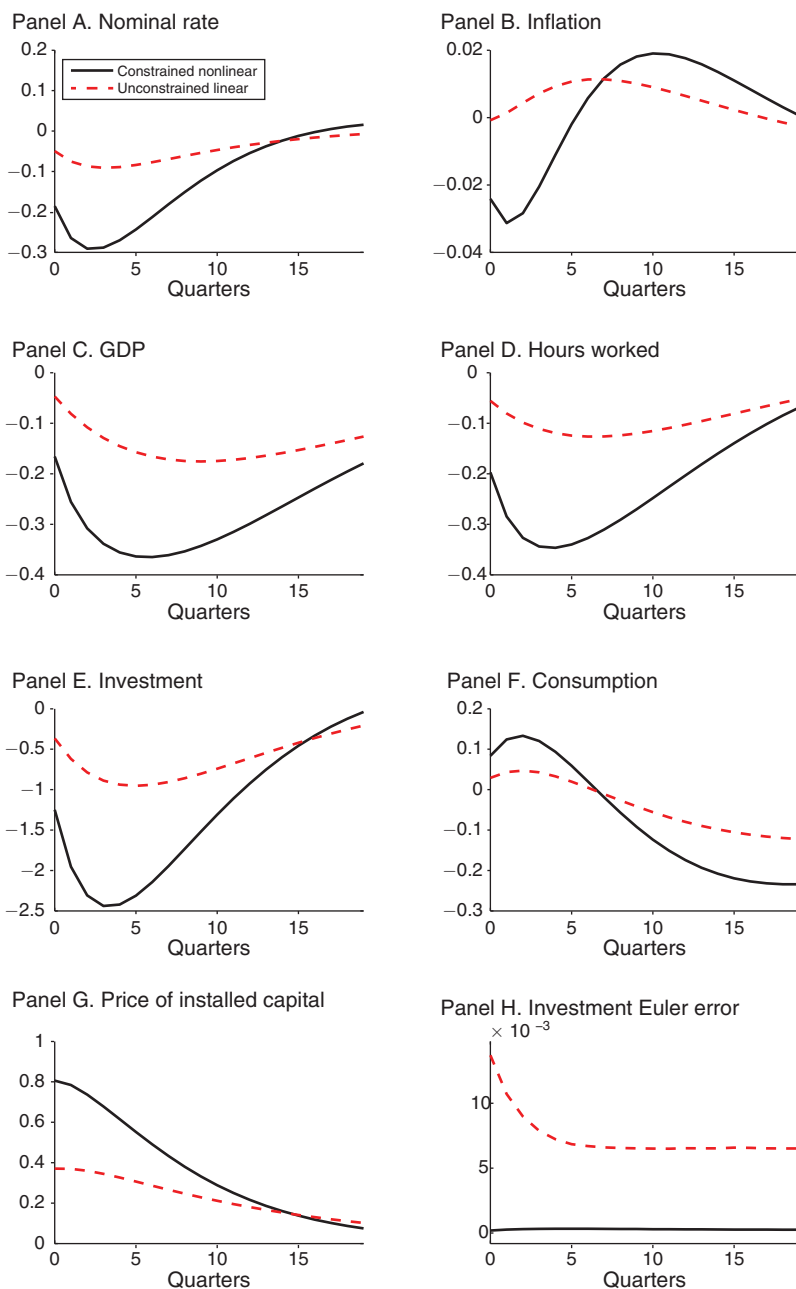


FIGURE 3. RESPONSE TO A FALL IN INVESTMENT EFFICIENCY

*Notes:* Figure reports the difference between the mean values of a shocked path, which introduces a one-standard-deviation innovation in the MEI shock at date 1,  $\epsilon_{\mu,1}$ , and the mean values of a path in which this innovation does not occur. Both paths are constructed using the posterior mean parameter values and initial conditions such that  $\ln(\mu_0)$  is one standard deviation above its nonstochastic steady state, and all other variables are at their nonstochastic steady states. The dashed line shows the responses for the unconstrained linear model, while the solid line shows the responses for the constrained nonlinear model. Panel H with the investment Euler error shows the mean absolute values of the errors along the shocked path at each date.

TABLE 2—STANDARD DEVIATIONS OF AGGREGATE VARIABLES  
AT BUSINESS-CYCLE FREQUENCIES

Variable	Data	Model versions		
		Nonlinear Constrained	Nonlinear Unconstrained	Linear
Output	1.13	1.15	1.10	0.88
Consumption	0.78	0.77	0.75	0.81
Investment	4.96	4.01	3.92	1.59
Hours	1.80	1.34	1.29	1.04

*Notes:* The model series are constructed from 1,000 draws from the posterior distribution. For each parameter draw, a data series with 125 observations is simulated, and the HP-filter is applied to the series with a smoothing parameter of 1,600. Simulations for all three model versions use the parameters estimated using the nonlinear constrained version of the model.

### C. Time Series Properties of the Model

Figure 4 shows the implications of the estimated parameters by comparing the observed data on output growth, investment growth, consumption growth, inflation, and the nominal interest rate with the smoothed estimates produced by the model under the baseline value of the measurement error ( $m_e = 0.25$ ). The figure also shows the 68 percent credible interval around these values.<sup>14</sup>

The model generally tracks the fluctuations in output, consumption, and investment growth, and generates contractions in these variables in all three of the recessions included in our sample period. For the Great Recession, the model accounts for the sharp falls in output and investment that occurred and modestly understates the fall in consumption. In addition, the model mimics well the slow growth in the variables that characterized the subsequent recovery. The model captures most of the low and medium frequency variation in quarterly inflation, displayed in panel D, though some of the high-frequency movements remain unexplained. For the Great Recession, the model accounts for a substantial part of the fall in inflation as well as its rebound.

Panel E shows that the smoothed values for the nominal interest rate are close to the observed values and panel F displays the central bank's notional or desired path for the nominal interest rate, which provides a measure of the severity of the lower-bound constraint on actual monetary policy. From 2009 onward, the notional interest rate is well below zero. It falls to about  $-5.5$  percent in the first half of 2009, gradually moves up to about  $-1$  percent at the end of 2012, and then hovers around that level through 2014:IV.

Figure 5 displays the (smoothed) estimates of output growth and inflation for the model with baseline measurement error ( $m_e = 0.25$ ) and compares them to the corresponding estimates from the model version with lower measurement error ( $m_e = 0.1$ ). Although the two versions of the model produce similar estimates of

<sup>14</sup>To construct these series, we proceed as follows. For each draw  $\theta \sim p(\theta | Y_{1:T})$ , we sample from  $p(S_{1:T} | Y_{1:T}, \theta)$  using the particle filter and report  $h(S_{1:T})$  for the appropriate function  $h(\cdot)$ . The credible intervals include both uncertainty surrounding  $\theta$  as well as uncertainty surrounding  $S_{1:T}$ .

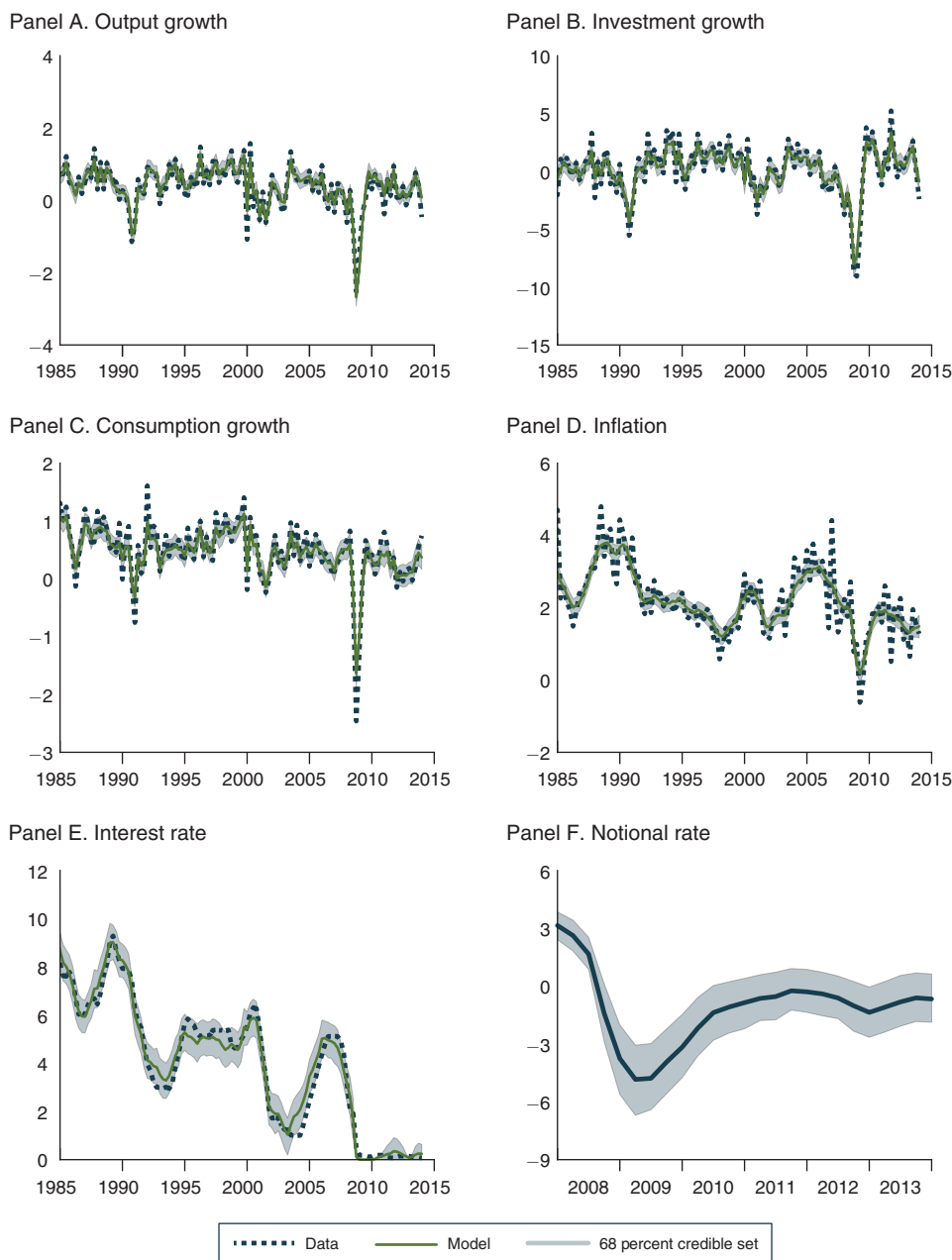


FIGURE 4. SMOOTHED ESTIMATES OF MODEL OBJECTS

*Note:* Figure shows the time series of the means (solid lines) and 68 percent bands (shaded regions) of the smoothed distributions of model variables, as well as their data counterparts over the estimation period. Panel F shows estimates of the shadow (notional) rate from 2008:I onward.

output (as well as consumption and investment), the version with lower measurement error does a better job fitting the high frequency movements in inflation. This result is due to the more prominent role of technology shocks and the lower degree

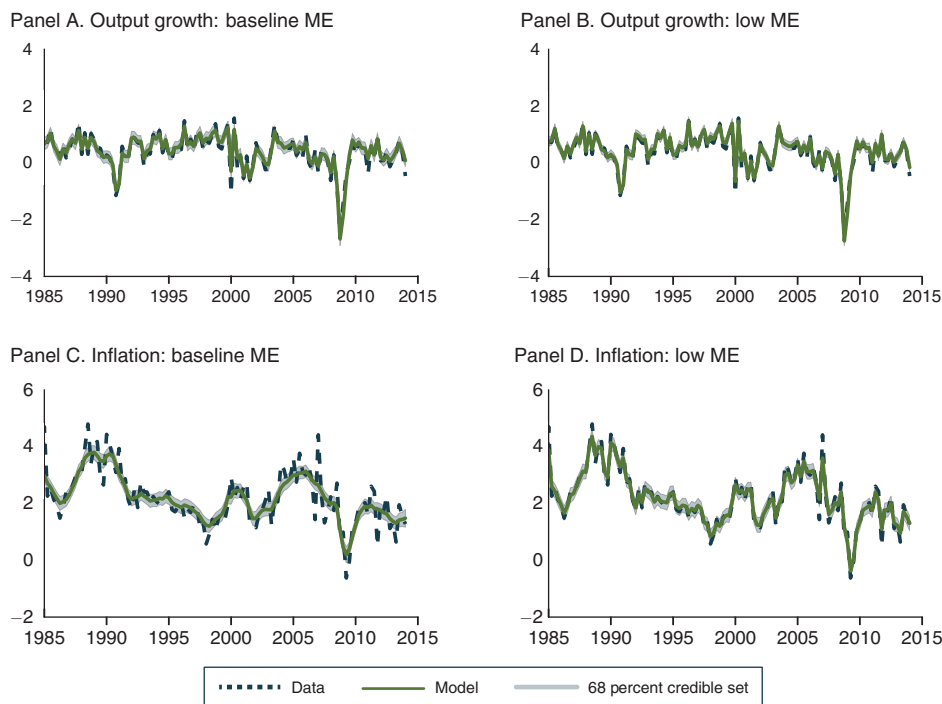


FIGURE 5. MODEL OBJECTS FOR DIFFERENT VALUES OF MEASUREMENT ERROR

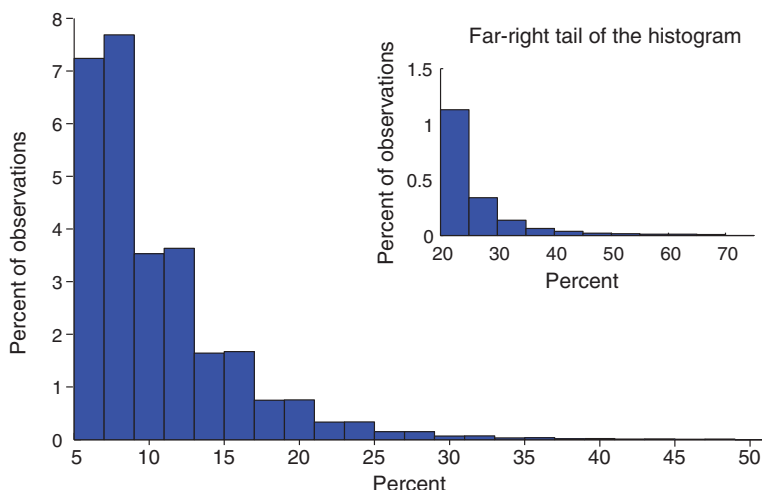
*Notes:* The four panels show the time series of the means (solid lines) and 68 percent bands (shaded regions) of the smoothed distributions of model variables, as well as their data counterparts over the estimation period. Baseline ME corresponds to the estimated model in which  $m_e = 0.25$ , and low ME corresponds to  $m_e = 0.1$ .

of price stickiness that produce larger and more transitory movements in inflation when  $m_e = 0.1$ .

In Figure 6, we examine the estimated model's implications for the probability of hitting the lower bound and the duration of a lower bound spell.<sup>15</sup> Panel A displays the distribution of the probability of the nominal rate being at the zero lower bound by simulating data from the model using parameter draws from the posterior distribution. On average, the model implies that there is about a 4 percent probability of the nominal rate being at the ZLB. In comparison, in our sample from 1983:I–2014:I, the nominal rate was at the zero lower bound in the last 21 quarters of the 125 observations, yielding a probability near 17 percent. Although this is considerably larger than the mean estimate derived from the model, panel A of Figure 6 also shows that estimates of this probability are disperse and that the distribution has a long right tail: about 15 percent of the draws have a probability of being at the ZLB more than 10 percent of the time and about 3 percent of the draws have a probability of being there more than 17 percent of the time.

<sup>15</sup>For each draw of parameters, we simulate 1,000 datasets each with 125 observations. We used 1,000 draws of parameters from the posterior distribution.



Panel A. Right tail of sample histogram of  $\Pr(0 \leq R_t \leq \bar{R})$ 

Panel B. Right tail of histogram of duration of ZLB spells

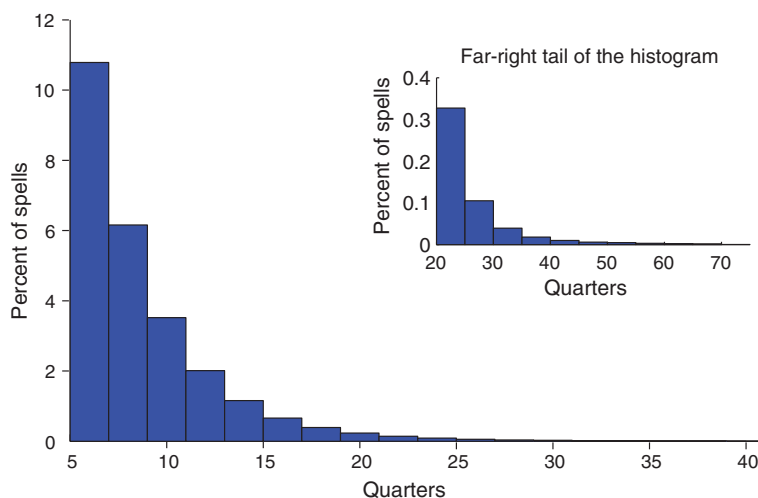


FIGURE 6. DISTRIBUTION OF THE PROBABILITY AND DURATION OF BEING AT THE ZERO LOWER BOUND

*Notes:* We draw 1,000 times from our posterior distribution. For each draw, we simulate the economy for 1,000,000 time periods. For panel A, we break the 1,000,000 time periods into bins of 125 time periods, compute the fraction of time the economy is at the ZLB, and report that distribution. For these simulations, panel B computes the duration of ZLB spells using each of the 1,000,000 time periods.

Panel B of Figure 6 displays the distribution of the duration of lower bound episodes. The average duration for a lower bound spell is just over 3.5 quarters and the median is 2. The distribution for the duration of a spell is skewed to the right and also has a long right tail. The ZLB spell at the end of our sample lasted 21 quarters, and spells with durations of 21 quarters or longer occur account for about 1/2 percent of the total number of lower bounds spells. Accordingly, our model implies that the long ZLB spell in the United States was very unlikely, and from the perspective

of households and firms in the model, would have been difficult to predict *ex ante*. The result is in line with the view that the financial crisis and its effects on the macroeconomy were difficult to foresee.

#### IV. What Accounts for the Great Recession?

Before quantifying how much the zero lower bound contributed to the Great Recession, we examine the estimated path of the shocks and their relative contributions to the Great Recession.

##### A. The Path of the Estimated Shocks

Figure 7 displays the smoothed estimates of the risk premium and the marginal efficiency of investment shocks.<sup>16</sup> The vertical axes in these panels display the magnitudes of the risk premium and MEI shocks relative to their standard deviations, and thus, for example, a value of 2 denotes that the size of the shock is two standard deviations above its mean. Panel C of Figure 7 shows the innovations ( $\epsilon_{\eta,t}$ ) to the risk premium shock ( $\eta_t$ ) and panel D displays the equity premium implied by the model.

At the beginning of 2008, the risk premium shock was already elevated and then there was a four-standard-deviation innovation in this shock when the financial crisis began. Although such an innovation is extremely unlikely, the financial crisis itself was an unprecedented event that was followed by the largest contraction in US economic growth since the Great Depression.<sup>17</sup> The model also requires a sequence of positive innovations in the risk premium shock from 2012:I to 2013:II. However, these innovations are small: less than one standard deviation away from their zero mean.

Because the risk premium shock plays an important role in our analysis, it is interesting to compare it to movements in empirical measures of credit spreads during the financial crisis. Panel A of Figure 7 also compares the estimated risk premium shock to the interest rate differential between BAA corporate bonds and ten-year Treasury bonds and a measure of credit spreads constructed by Gilchrist and Zakrajšek (2012), who abstract from the effect of the expected loss stemming from default on spreads and attempt to isolate the credit risk premium. Since the units of the estimated risk premium shock are not comparable to those of the other two measures of financial stress, it is more useful to focus on how movements in the risk premium shock correlate with these measures, both of which rose sharply in 2008 and declined thereafter. Although the measure constructed by Gilchrist and Zakrajšek (2012) falls

<sup>16</sup>The government spending and monetary policy shocks contributed less, according to the model. Smoothed estimates of the paths of these shocks are available in the online Appendix. The fact that the government spending shock does not play a large role in the Great Recession does not necessarily mean that fiscal policy was unimportant during the episode. The government spending shock in our analysis is a reduced-form shock affecting the economy's resource constraint and reflects other factors besides changes in government spending such as movements in international trade that we do not have in the model.

<sup>17</sup>To be consistent with the literature, we choose to estimate the model using normally distributed shocks. However, with our methodology, it would be straightforward to allow for distributions with fatter tails or time-varying volatility and formally test different assumptions regarding the shock distributions. However, we viewed such an exercise as beyond the scope of this paper.

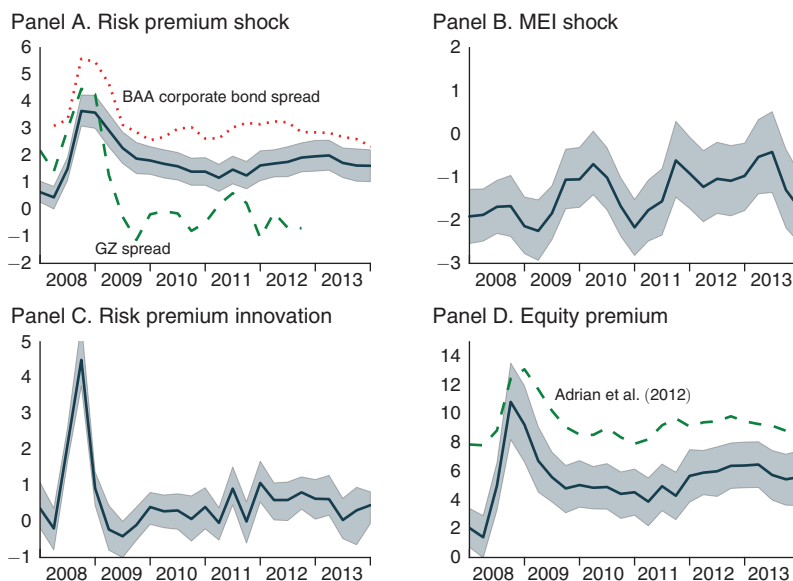


FIGURE 7. PATH OF THE ESTIMATED SHOCKS AND EQUITY PREMIUM DURING THE GREAT RECESSION

Notes: Figure shows the time series of the mean (solid line) and 68 percent bands (shaded region) of the smoothed distributions of  $\ln(\eta_t)$  (panel A),  $\ln(E_t R_{t+1}^k / R_t)$ , the model-implied equity premium (panel D),  $\epsilon_{\eta,t}$  (panel C), and  $\ln(\mu_t)$  (panel B). Both  $\ln(\eta_t)$  and  $\ln(\mu_t)$  are normalized by their unconditional standard deviations. Panel A also includes the standardized estimates of the credit spread from Gilchrist and Zakrajšek (2012) and BAA spread, while panel D shows an estimate of the equity premium, computed as in Adrian, Crump, and Moench (2015).

relatively soon after the financial crisis, the BAA spread remains persistently elevated even after falling in 2009. Similar to the two empirical measures, the model's estimate of the risk premium shock also rose sharply at the end of 2008 and fell in 2009. Even after falling in 2009, like the BAA spread, the risk premium shock remains persistently high. In sum, even though we do not use information on asset prices in our empirical approach, the model's estimated risk premium displays broadly similar behavior as empirical measures of financial stress did at the time.

As can be seen in panel B, low realizations of the MEI shock also contributed to weak activity at the time, as the MEI shock is more than one standard deviation below its mean in 2008 and in the first half of 2009. Thus, the model's two financial shocks are consistent with the view that strains in financial markets triggered the economic slump. In subsequent periods, both shocks begin to move back toward their mean values, but the risk premium shock remains at a high level—two standard deviations above its mean—through the end of 2014. Moreover, the MEI shock's recovery is followed by another fall in 2010 after which it recovers and falls again at the end of 2013.

Panel D displays the model's equity or finance premium, which we measure as the expected excess return on capital,

$$\frac{E_t R_{t+1}^k}{R_t} = \frac{\tilde{r}_{t+1}^k + q_{t+1}(1 - \delta)}{R_t q_t},$$

where  $\tilde{r}_t^k = r_t^k u_t - a(u_t)$  is the user or rental cost of capital adjusted for utilization. The panel also shows an empirical measure of the equity premium from Adrian, Crump, and Moench (2015). The model's estimate of the equity premium peaks at a level of about 11 percent at the end of 2008 and drops in 2009 but remains elevated, averaging about 6 percent through 2013. These movements are broadly consistent with the estimates of Adrian, Crump, and Moench (2015), though our model implies a larger jump in the equity premium at the end of 2008 and a less elevated equity premium overall.

Panel A of Figure 8 shows the difference in the level of technology beginning in 2008:I from the level if it grew at its deterministic rate,  $G_Z$ , for the baseline estimates. Technology or total factor productivity was below trend during the Great Recession and fell further below trend through the end of the sample (2014:I). The dashed and dotted lines show the estimates in Christiano, Eichenbaum, and Trabandt (2015) and Fernald (2015), respectively. Despite our different econometric strategy from these authors, our estimates of total factor productivity are broadly similar to theirs, as total factor productivity, according to these measures, is also estimated to be below trend with the exception of a temporary increase in the measure constructed by Fernald (2015) in 2009.

The large risk premium shock as well as reductions in productivity and the marginal efficiency of investment are all important in accounting for the large drop in activity that occurred during the Great Recession with only a moderate disinflation. The latter two shocks put upward pressure on inflation, offsetting the significant downward pressure on inflation stemming from the risk premium shock. These results are broadly consistent with Christiano, Eichenbaum, and Trabandt (2015) and Del Negro, Giannoni, and Schorfheide (2015) albeit with some differences. In particular, these authors introduce model features that allow financial frictions to break the link between prices and future marginal costs in the NK Philips curve and help mitigate the disinflationary pressure stemming from the risk premium shock.<sup>18</sup>

Figure 8 also compares the smoothed estimates of the technology shocks from 2008–2013 for the baseline value of the measurement error to the case with low measurement error (panel B). For the other shocks, the estimated paths are similar across model versions; however, the technology shock is notably different. For the model with lower measurement error, the level of technology is estimated to have fallen substantially more during the Great Recession and its aftermath: the level of technology at the end of 2013 is about 7 percent below trend, compared to only 3.5 percent below trend under the baseline estimates. Moreover, with lower measurement error, the estimated path of technology shocks lies well below the estimates in Fernald (2015).

Figure 8 highlights another reason for the inclusion of measurement error in the model. Specifically, the model is misspecified to the extent that it omits factors such as oil and commodity price movements that help explain high frequency fluctuations in inflation. Because of such misspecification, the model version with lower measurement error, as suggested by the estimates in Fernald (2015), overstates the role of technology shocks in accounting for the Great Recession; hence, we chose the

<sup>18</sup> For an alternative explanation emphasizing the role of fiscal policy in mitigating the disinflationary pressure, see Bianchi and Melosi (2014).

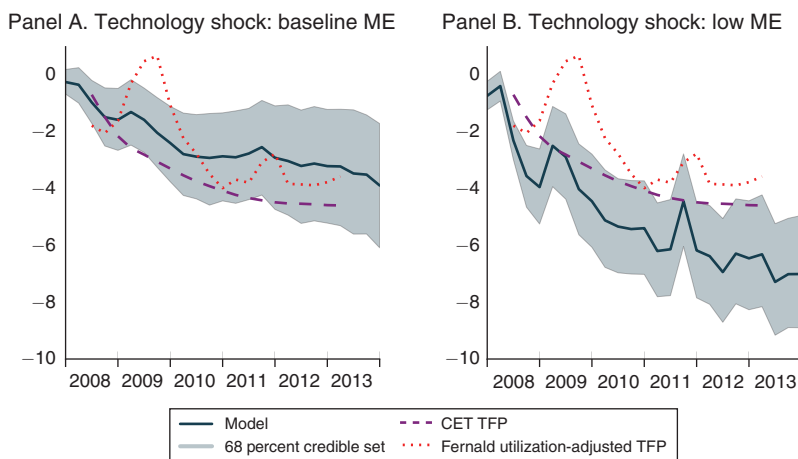


FIGURE 8. PATH OF THE ESTIMATED TECHNOLOGY SHOCK

*Notes:* The two panels show the time series of the mean (solid line) and 68 percent bands (shaded regions) of the smoothed distribution of the difference between the smoothed estimate of  $\ln(Z_t)$  and its forecasted trajectory in 2007:IV under alternative assumptions for the measurement error. The panels also show the utilization-adjusted estimate of TFP (dotted line) of Fernald (2014) and the calibrated TFP measure (dashed line) used in Christiano, Eichenbaum, and Trabandt (2015), both taken from Christiano, Eichenbaum, and Trabandt (2015). Baseline ME corresponds to the estimated model in which  $m_e = 0.25$ , and low ME corresponds to  $m_e = 0.1$ .

model version with higher measurement error as the baseline with the implication that there is greater uncertainty surrounding our estimates of the cost of the ZLB.<sup>19</sup>

### B. The Contribution of the Estimated Shocks to the Great Recession

Figure 9 shows how much of the model's fit is attributable to individual shocks for output growth, investment growth, consumption growth, inflation, and both the observed and the notional nominal interest rates in the baseline model. More specifically, it displays the model's dynamics if only one of the estimated shocks were present during the Great Recession and compares this path to the smoothed values that are generated using all five shocks (the solid line denoted as the baseline in the figure).<sup>20</sup> A clear picture emerges from the figure: the large contraction in output, consumption and investment growth is mostly explained by the risk premium shock. This shock also generates a somewhat larger fall in inflation than under the baseline path. However, as shown in panel D, these disinflationary effects are offset by the upward pressure on inflation induced by the fall in the level of technology.

<sup>19</sup>For a related approach in addressing model misspecification, see Canova (2014), who proposes a flexible, nonstructural link between a DSGE model and the data. It would be preferable, of course, to incorporate features into the structural model that allow it to capture the high-frequency movements in inflation and we leave this task to future research.

<sup>20</sup>We compute the counterfactual with only one shock present as follows. Starting in 2007:IV, we take the estimated state as an initial value, and simulate the economy forward feeding in the smoothed values of one of the shock processes, assuming that the variances of the other shocks are set to zero.

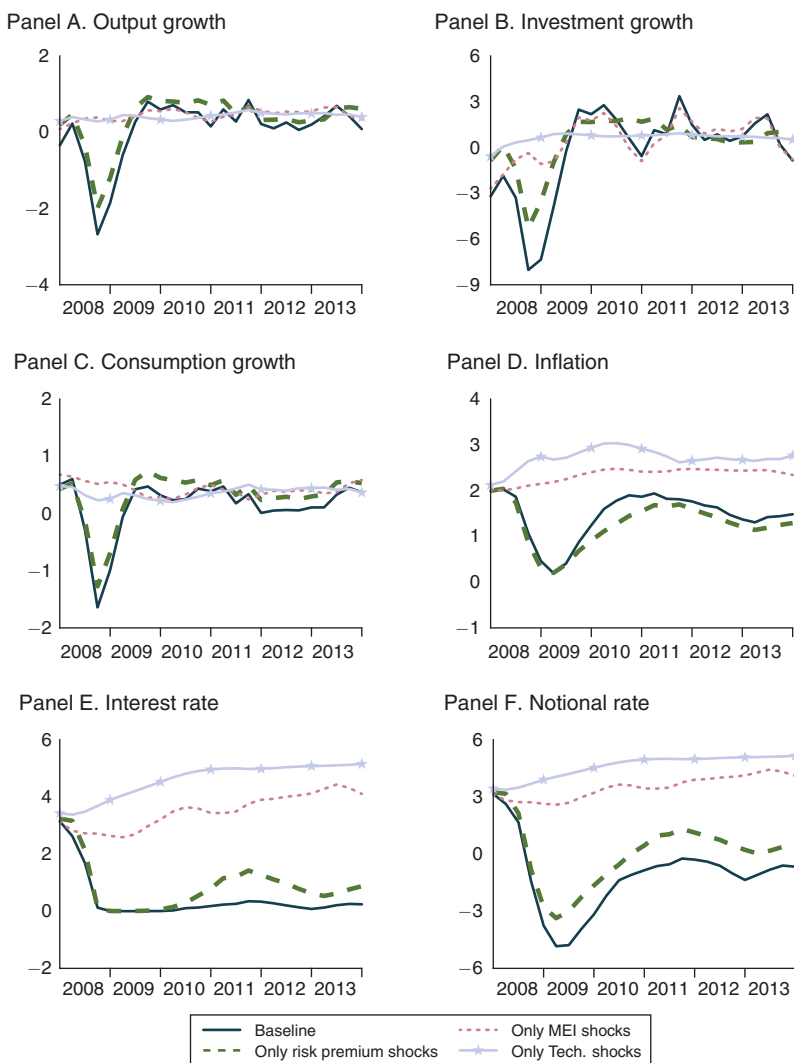


FIGURE 9. CONTRIBUTION OF THE ESTIMATED SHOCKS TO THE GREAT RECESSION

*Notes:* Figure shows counterfactual trajectories of output growth (panel A), investment growth (panel B), consumption growth (panel C), inflation (panel D), the interest rate (panel E), and the notional rate (panel F). The trajectories are computed using smoothed estimates of the states in 2007:IV as initial conditions and the smoothed shock estimates from 2008:I to 2014:I for only liquidity shocks (dashed lines), only MEI shocks (dotted line), only technology shocks (line with triangles), and all of the structural shocks (solid line).

The risk premium shock is also largely responsible for driving the nominal rate to the zero lower bound. By reducing both output growth and inflation, this shock pushes the notional interest rate well below zero (to about  $-5.5$  percent), causing the interest rate to hit and then stay at the lower bound. A decline in the MEI shock in 2011 helped put downward pressure on the nominal interest rate in 2011 and 2012 and contributed significantly to the sharp decline in investment in late 2008.

While the technology shock plays an important role in moderating the decline in inflation during the ZLB episode, the model estimates imply that monetary policy

shocks were not important during the ZLB episode.<sup>21</sup> However, this does not necessarily imply that monetary policy is unimportant, because the systematic part of the rule and the lower bound constraint are crucial determinants of the model's dynamics.

The online Appendix presents the shock decomposition for the model version with lower measurement error. In that version, the risk premium shocks still play the predominant role in generating the large contraction in output but now technology shocks contribute noticeably to the declines in output and consumption growth at the end of 2008. Moreover, there is more upward pressure on inflation emanating from the technology shock in this version of the model.

## V. How Costly Was the Interest-Rate Lower Bound?

### A. *The Contribution of the Lower Bound to the Great Recession*

To determine the role of the zero lower bound constraint during the Great Recession, we compare the estimated model outcomes (in which the ZLB constraint is imposed) to the outcomes in a hypothetical scenario in which monetary policy is free to adjust the nominal interest rate in an unconstrained manner.<sup>22</sup> Figure 10 compares the levels of output and the price level in these two scenarios under the posterior mean estimates of the baseline model. Panel A shows that in 2009:II, output was about 6 percent below its level in 2007:IV. If monetary policy could have cut the nominal interest rate more aggressively, it would have helped to offset the contractionary effects of the shocks and output would have fallen by only 4 percent; thus, the zero lower bound accounted for about 30 percent of the sharp drop in output that occurred in 2009.<sup>23</sup>

While the ZLB contributed significantly to the sharp fall in economic activity in 2009, it was an even more important factor in holding back activity during the subsequent recovery. Figure 10 shows that output did not recover back to its 2007:IV, pre-recessionary level until 2012:IV, five years later.<sup>24</sup> During those five years, the average level of output was 2.4 percent below its level in 2007:IV. In comparison, average output over those years would have only been about 1.1 percent lower in the unconstrained scenario, as output would have been above its pre-recessionary level about a year earlier than in the estimated, constrained scenario. Thus, these estimates imply that the presence of the ZLB accounted for a little over 50 percent of the lower output over the 2008–2012 period.

Panel B of Figure 10 shows that the ZLB constraint also contributed to a lower price level. Inflation averaged 1.45 percent per year from 2008:I until the end of

<sup>21</sup> Monetary policy shocks are still identified at the lower bound, because these shocks can persistently lower the notional interest rate. As a result, agents anticipate that the actual nominal rate will be at the lower bound for longer, an expectation that affects current outcomes.

<sup>22</sup> The hypothetical scenario uses the same estimated initial conditions and time series for the shocks as the estimated model in which the constraint is imposed. These values are then used to simulate the model ignoring the ZLB constraint on the estimated rule.

<sup>23</sup> This is the mean estimate, and the 68 percent interval around it is wide, covering values between 6 to 46 percent.

<sup>24</sup> The baseline path does not exactly match the observed output series because of measurement error and recovers two quarters earlier than observed output.



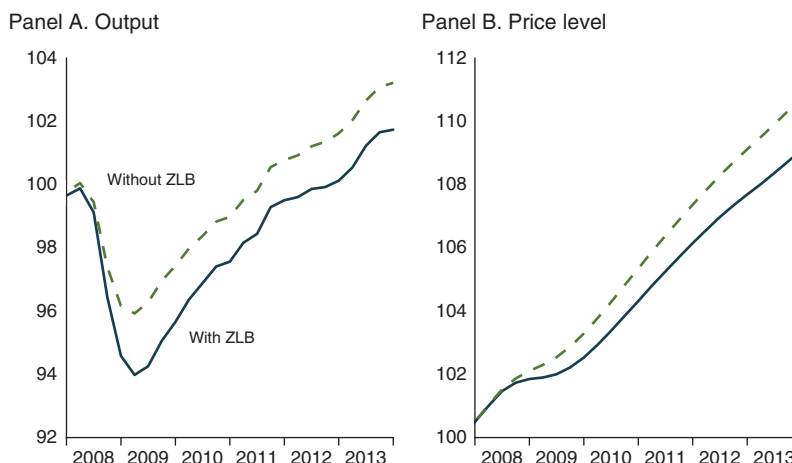


FIGURE 10. CONTRIBUTION OF THE ZERO LOWER BOUND TO THE GREAT RECESSION

*Notes:* Figure shows the posterior mean trajectories of the levels of output and prices. The solid lines show the effects imposing the ZLB constraint and the dashed lines show the effects without this constraint. The simulations use the smoothed estimates of the states in 2007:IV as initial conditions and the smoothed shock estimates from 2008:I to 2014:I to construct the paths. Each series is normalized to 100 in 2007:IV.

2012, below the Federal Reserve's inflation target. Absent the zero lower bound constraint, the mean estimates indicate that inflation would have averaged around 1.75 percent per year, and hence was 0.3 percentage points lower, on average, over the 2008–2012 period. Overall, our results suggest that the interest-rate lower bound was a significant constraint on monetary policy that exacerbated the recession, inhibited the recovery, and contributed to inflation outcomes below the Federal Reserve's inflation target.

Figure 11 displays the difference in outcomes from the estimated, constrained scenario for output, investment, consumption, and the notional rate. For each variable, the solid line shows the point estimate of the effect of ZLB constraint and the shaded region displays the 68 percent credible region. As emphasized above, output is about 1.2 percent lower, on average, over the 2008–2012 period because of the constraint. Consumption would have been about 1 percent higher in absence of the constraint, and investment would have been 4 percent higher. Figure 11 also highlights an important caveat to interpreting these results: the estimates are subject to considerable uncertainty as the 68 percent credible region does not exclude the possibility that the estimated effects of the ZLB constraint were much smaller or much larger.

Figure 11 also shows the point estimates of the effect of the ZLB constraint in the model version with lower measurement error. The ZLB constraint accounts for about 15 percent of the sharp drop in output that occurred in 2009 and about 20 percent of the lower level of output over the 2009–2012 period. These smaller effects are driven by the more prominent role of technology shocks in accounting for the fall in output over this period. In particular, there is less of a role for monetary policy to stabilize the fall in output if more of its decline is driven by technology shocks.

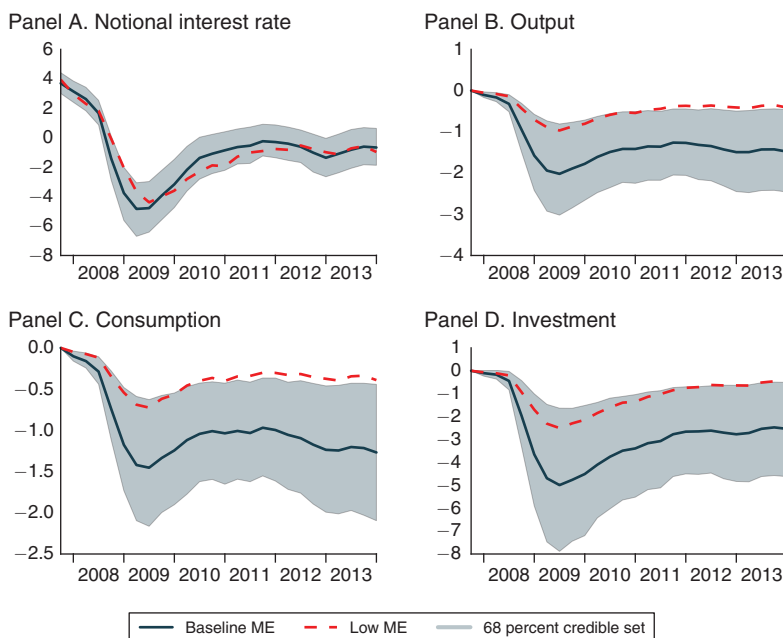


FIGURE 11. EFFECT OF THE LOWER BOUND ON AGGREGATE SPENDING

*Notes:* Figure shows the time series of the mean (solid line) and 68 percent bands (shaded region) of the distribution of differences for the level of output, consumption, and investment simulated with and without the zero lower bound, respectively. Panel A shows the mean estimated path of the notional rate along with 68 percent bands. The dashed lines correspond to the mean estimates of the effect of the ZLB constraint in the model with lower measurement error ( $m_e = 0.1$ )

Hence, the zero lower bound is a less important constraint on monetary policy in this version of the model; however, as discussed earlier, there is evidence that this model version may overstate the role of technology shocks during this period and thus may understate the cost of the ZLB constraint.

#### B. The Contribution of the Lower Bound to the 2003–2004 Deflationary Scare

Our methodology can be used to examine episodes in which the realized nominal rate did not hit the lower bound but economic behavior was affected by the prospect that policy could become constrained. One such episode in which uncertainty about the zero lower bound may have affected economic behavior was in 2003 when inflation was low and the Federal Open Market Committee (FOMC) was concerned about the possibility of further disinflation. This view was summarized by Alan Greenspan, the Chair of the FOMC at the time:

*...[W]e face new challenges in maintaining price stability, specifically to prevent inflation from falling too low... [T]here is an especially pernicious, albeit remote, scenario in which inflation turns negative... engendering a corrosive deflationary spiral...*

—Greenspan (2003)

Although this deflationary scenario never materialized and policy rates never reached the lower bound, this risk was an important consideration in 2003, and we use the estimated model to investigate the effect of the ZLB during this episode. In the model, this constraint has effects on inflation and economic activity, even if it never binds, because agents in the model take into account the entire probability distribution of future outcomes, including those in which the nominal interest rate obtains the zero lower bound, in making their current decisions. Accordingly, we follow the same approach as we used for the Great Recession and compare outcomes in the estimated, constrained model to the hypothetical scenario in which policy is unconstrained.

Panel A in Figure 12 shows the mean trajectories of output with and without the ZLB constraint, and panel B shows the uncertainty faced by the agents about the future likelihood of being at the constraint. It shows that at the end of 2002, taking into account the economy's current state and the estimated policy rule, private sector agents believed there was a 12 percent chance that the nominal interest rate would fall to its lower bound during 2003. Because scenarios in which the ZLB binds also imply higher future real policy rates than if monetary policy could act in an unconstrained fashion, these scenarios are also characterized by downward shifts in the distributions of outcomes in aggregate spending and output; accordingly, the estimated mean outcome for output is lower in the constrained case than the unconstrained case. Figure 12 shows that the effects of this uncertainty on the estimated mean outcomes are relatively small: the estimated level of output is about 0.2 percent lower throughout 2003 and 2004 than in the unconstrained case. Still, the effect is a persistent one, and the results highlight that uncertainty about the course of monetary policy did have a tangible economic impact during this episode, an effect that is omitted *a priori* using the standard approach to estimate DSGE models.

### *C. Forward Guidance and the Estimated Policy Rule*

In both the 2003 episode and during the Great Recession, forward guidance about the policy rate played a prominent role in FOMC communications. In late 2008, forward-guidance language was reintroduced into the FOMC statement, which indicated that “weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.” This guidance grew more forceful and specific over time, and in late 2012, the FOMC made this forward-guidance more explicitly state-contingent, linking the maintenance of the low funds rate to projected inflation and the level of the unemployment rate. We capture forward guidance through the state-contingent path of rates implied by our estimated interest-rate rule. The presence of the lagged notional rate, in particular, is an important element of a strategy that seeks to maintain future rates at a low level.<sup>25</sup> In particular, when the notional rate is negative, a higher coefficient on the lagged notional rate in the rule, all else equal, increases private sector beliefs that future policy rates will

<sup>25</sup> In light of the evolving nature of the forward-guidance language in FOMC communications throughout the ZLB spell, it remains an open question as to how best to model it. An alternative way of modeling forward guidance is to allow for anticipated shocks in the monetary policy rule. See, for instance, Del Negro, Giannoni, and Patterson (2015). Also, Keen, Richter, and Throckmorton (2016) study the effects of forward guidance modeled in this way in the context of a simple, nonlinear New Keynesian model.

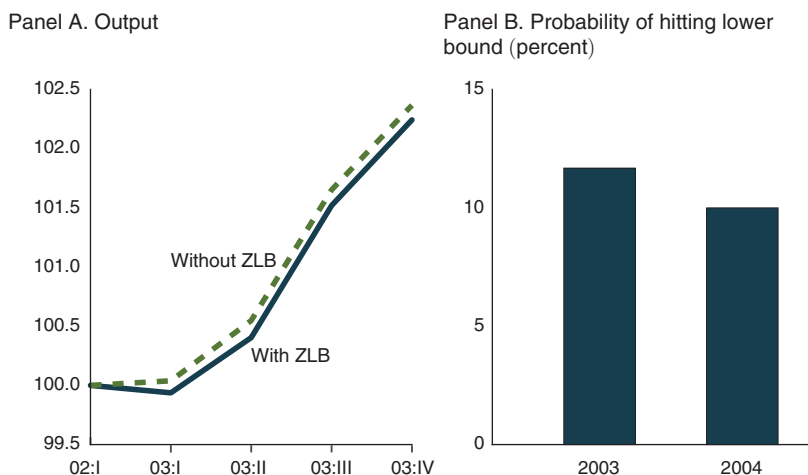


FIGURE 12. THE 2003–2004 EPISODE

Notes: Figure shows the posterior mean trajectories of the levels of output (panel A). The solid line shows the effects imposing the ZLB constraint and the dashed line shows the effects without this constraint. The simulations use the smoothed estimates of the states in 2002:IV as initial conditions and the smoothed shock estimates from 2003:I to 2004:IV to construct the paths. The level of output is normalized to 100 in 2002:IV. Panel B shows the probabilities, conditional on the posterior mean parameter estimates, of hitting the ZLB in 2003 and 2004, given the 2002:IV initial condition (posterior mean of smoothed estimates).

remain at the zero lower bound. Moreover, the further the lagged notional rate falls below zero, the more likely the future policy rate will remain at zero.

To investigate the role of the lagged notional rate in the policy rule, Figure 13 shows a counterfactual simulation of how the economy would have evolved during the Great Recession had policy been governed by an alternative rule that responds to the lagged actual policy rate rather than the lagged notional rate. Because the lagged notional rate fell sharply below zero during the recession and remained negative during the recovery, the estimated rule generates a policy rate path that is more accommodative than the alternative rule. As shown in panel C, the mean estimates for the path of the real interest rate are lower under the estimated rule than the rule that depends on the lagged actual rate. Accordingly, the fall in the mean estimates of output and prices during the Great Recession are smaller for the estimated rule than the alternative rule.

The estimated rule is more accommodative than the alternative rule, because the inclusion of the lagged notional rate generates the belief by private agents that the nominal rate will remain at the zero lower bound longer and under a wider set of circumstances. To highlight this, panel D shows the uncertainty faced by the agents in the model about whether the policy rate one quarter ahead will be at the zero lower bound. In 2010:I, for example, under the estimated rule, private sector agents believed the probability that the nominal rate would *lift off* next quarter was very close to zero. In contrast, in the counterfactual using the alternative rule with lagged actual rate, agents in 2010:I would have believed there was about a 15 percent chance that the policy rate would rise in 2010:II.

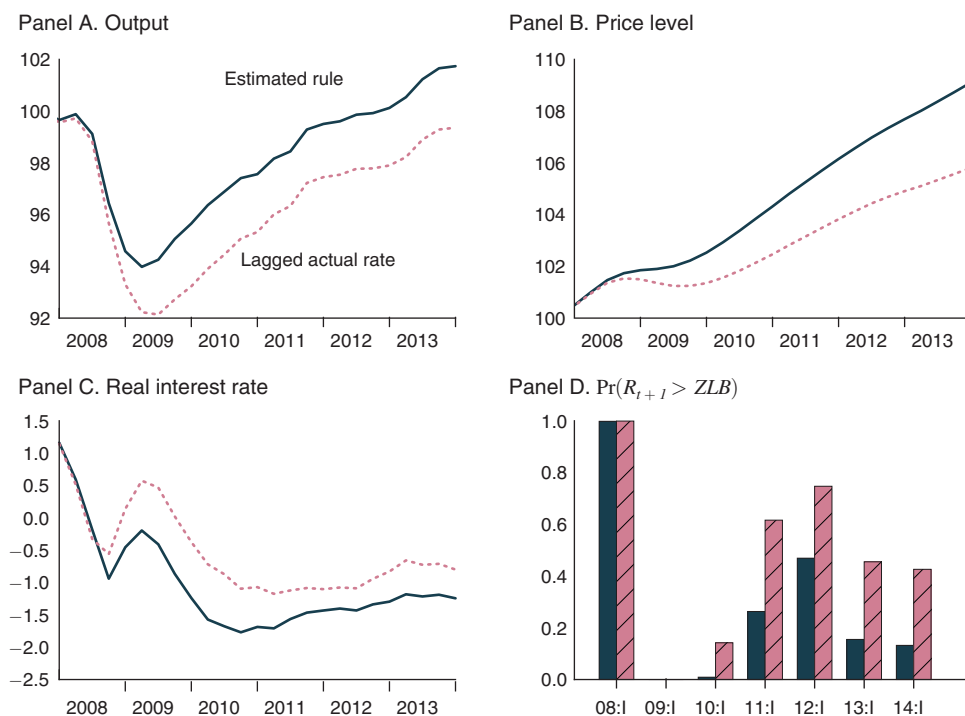


FIGURE 13. EFFECT OF INERTIA IN THE POLICY RULE

*Notes:* Figure shows the posterior mean trajectory of the level of output (panel A), prices (panel B), and the real interest rate (panel C) under the estimated monetary policy rule (solid lines) and the rule with the lagged actual rate (dotted lines). The simulation uses the smoothed estimates of the states in 2007:IV as initial conditions and the smoothed shock estimates from 2008:I to 2014:I to construct the paths. The level of output and prices are normalized to 100 in 2007:IV. Panel D shows the probability at each point in time—according to the agents in the model—that the interest rate next quarter will be away from the ZLB. In computing the probabilities, we use the mean parameter estimates and the mean estimates of the smoothed states.

Under both of these rules, the nominal interest rate is estimated to remain at the zero lower bound through the end of our sample, but the probability of a departure from the ZLB is notably higher under the alternative rule than the estimated rule. In 2014:I, for instance, the agents would have believed that there was about a 40 percent chance of liftoff next quarter under the alternative rule compared to only a 10 percent chance under the estimated rule. Thus, the simulation demonstrates that the inclusion of the lagged notional rate captures the forward-guidance language used by the FOMC at the time by validating private sector beliefs that policy is more likely to remain at the zero lower bound, thereby stimulating the economy.

## VI. Conclusions

In this paper, we estimated a nonlinear DSGE model in which the interest-rate lower bound is occasionally binding. This allowed us to quantify the size and nature of the disturbances that caused the Great Recession as well as pushed the nominal rate to the lower bound in late 2008 and kept it there throughout the economy's long slump. Overall, our results suggest that the interest-rate lower bound was a

significant constraint on monetary policy that exacerbated the recession, inhibited the recovery, and contributed to inflation outcomes below the Federal Reserve's inflation target.

Our results also highlight the importance of estimating the nonlinear model instead of the linear approximation to it. Because the nonlinear model that we estimate has a stronger internal propagation mechanism than its linearized counterpart, we find that there are significant differences in the parameter estimates. In particular, estimates based on the linearized version overstate the importance of the exogenous shocks, and thus the estimation of the nonlinear model is essential in properly quantifying the relative contributions of endogenous and exogenous sources of business-cycle fluctuations.

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