CSCI 510 **Models of Computation**

HW3: You Oughta Know

Due: Sep 29, 2010

Include your *full name*, *CS login*, and the problem number(s) on each piece of paper you hand in, and please staple your pages together before handing in.

While collaboration is encouraged in this class, please remember not to take away notes from collaboration sessions other than your scheduled lab section.

Problem 1

A palindrome is a string W with the property that $W = W^R$, where W^R is the *reverse* of W. For example, 010 and 0110 are both palindromes.

Describe in detail a Turing machine that recognizes the language of palindromes where the input alphabet is $\{0, 1\}$. The description of your Turing Machine should be in the form of a diagram of the states and transitions. Be sure to specify the tape alphabet.

The following questions are lab problems. Please remember to prepare a solution for your assigned problem before going to your lab section.

Lab Problem 1

Consider the Boolean function

$$f(x_1, x_2, \ldots, x_n) = x_1 \wedge x_2 \wedge \cdots \wedge x_n.$$

The number of gates required to construct a circuit that computes this function for a fixed n is dependent on the number of inputs allowed to each AND gate in the circuit.

a. Show that the number of 2-input AND gates required to construct a circuit that computes f is $n - 1$ and the depth of such a circuit is at least $\lceil \log_2 n \rceil$.

b. Find a similar formula for the size and depth of a circuit constructed using r-input AND gates for an arbitrary (but fixed) r.

Lab Problem 2

A standard Turing machine has a single tape that has a left end but extends infinitely to the right. In this problem you will examine two variants of the standard Turing machine and prove that they are all equivalent.

a. Show that a Turing machine with a single doubly-infinite tape (that is, a tape that extends infinitely in both directions) can be simulated by a standard TM and vice versa. This shows that the two are equivalent models of computation.

b. Now consider a Turing machine with multiple tapes, each of which has a left end but extends infinitely to the right. Each tape has its own head, and the heads of different tapes can be in different places at the same time. Prove that, for an arbitrary but fixed n, a standard TM can simulate an n-tape TM and vice versa.

Lab Problem 3

In class you saw two types of bit shifting:

- Logical shifting, where the bits are shifted and then the open slots are filled with 0s. So if we logically shift `10101110` left by two bits, we get `10111000`.
- Cyclic shifting, where the bits that are shifted off one end wrap around. So, if we cyclically shift `10101110` left by two bits, we get `10111010`.

a. Reduce logical shifting to cyclic shifting. That is, assuming you have a machine M that can perform cyclic bit shifting, construct a machine, using M as a subroutine, that performs logical bit shifting.

b. Reduce cyclic shifting to logical shifting.

Note: You may not explicitly inspect or change individual bits for either of these reductions.