HW9: Round and Round

Due: Dec 1, 2010

Include your *full name*, *CS login*, and the problem number(s) on each piece of paper you hand in, and please staple your pages together before handing in.

While collaboration is encouraged in this class, please remember not to take away notes from collaboration sessions other than your scheduled lab section.

Problem 1

In the decision problem SAT, we are given a list of logical clauses consisting of one or more possibly negated literals, all OR'd together; we say that such a list of clauses is in SAT if some assignment of truth values satisfies every single clause. In the optimization version, MAX SAT, the goal is to find some assignment of truth values that satisfies the maximum possible number of clauses, even if it is not possible to satisfy every single clause.

Give a polynomial time 1/2-approximation algorithm for MAX SAT, and show that it satisfies at least 1/2 as many clauses in the list as it is possible to satisfy.

Problem 2

For each of the following languages, explain why it is decidable, undecidable but Turing recognizable, or unrecognizable.

Note: "Turing recognizable" is the same as "recursively enumerable."

a. $L = \{\langle M \rangle \mid \texttt{guitar} \in L(M)\}$.

b. $L = \{\langle M_1, M_2 \rangle \mid L(M_1) = L(M_2)\}$.

c. $L = \{\langle D \rangle \mid D \text{ is an FSM that accepts } \emptyset\}$.

d. $L = \{\langle D \rangle \mid D \text{ is an FSM that accepts } \Sigma^*\}$.

The following questions are lab problems. Please remember to prepare a solution for your assigned problem before going to your lab section.

Lab Problem 1

A Turing Machine is **self-terminating** if it halts when given its own description as input.

a. Prove that the language $L_{NST} = \{\langle M \rangle \mid M \text{ is not self-terminating}\}$ is undecidable. You may not use Rice's Theorem.

b. Prove that the language L_{NST} is not Turing recognizable. In your proof you should show that the language A_{TM} defined in the last homework is reducible to the complement of L_{NST} (which you may refer to as L_{ST} for simplicity).

Lab Problem 2

Explain why each of the following languages is decidable:

a. $L = \{\langle M, w \rangle \mid \text{when } M \text{ is run on } w\text{, it only moves left}\}$

b. $L = \{\langle M, w \rangle \mid \text{when } M \text{ is run on } w\text{, it only moves right}\}$

c. $L = \{\langle M, w \rangle \mid \text{when } M \text{ is run on } w\text{, the head reverses direction at least once}\}$

Note: When a Turing machine moves left at the start of the tape, it just stays at the start of the tape.

Hint: Consider how you would detect an infinite loop, given that you haven't changed direction.

Lab Problem 3

Consider the following language:

$$L = \{\langle M, k \rangle \mid \; M \text{ is a TM that halts on at least } k \text{ inputs}\}$$

Is this language Turing-recognizable? If so, is it decidable? Prove your answers.

Hint: You may want to use non-determinism.