HW5: Livin' on a Prayer

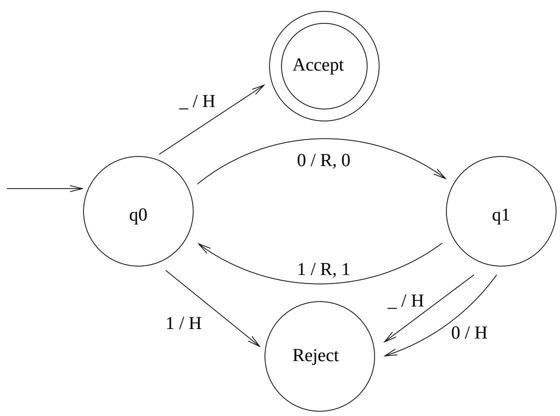
Due: Oct 13, 2010

Include your *full name*, *CS login*, and the problem number(s) on each piece of paper you hand in, and please staple your pages together before handing in.

While collaboration is encouraged in this class, please remember not to take away notes from collaboration sessions other than your scheduled lab section.

Problem 1

In this problem you will construct a circuit to compute the same function as a Turing Machine with a fixed-length tape. Consider the TM whose tape has 5 spaces and whose control unit is as follows:



Note that if the TM tries to move right off the end of the tape, it instead stays where it is (and, in this case, will likely reject soon after).

Construct a circuit that returns 1 whenever the above TM would accept and 0 whenever it would reject. The circuit will always take input from 5 tape cells regardless of whether or not they are blank. For this problem each cell on the tape is represented by 2 bits of input to the circuit: `00` for 0, `01` for 1, and `10` for blank (`11` will never appear).

The following questions are lab problems. Please remember to prepare a solution for your assigned problem before going to your lab section.

Note: We will not ask you to reduce from an NP-complete problem you have not seen without telling you what to reduce from!

Lab Problem 1

A riot has broken out at the a rock concert! Dream Theater played Panic Attack and people started panicking. Everyone started yelling at a bunch of people, only making those other people more panicked. The real problem occurred when they got into loops: one person screaming eventually caused someone else to scream at the original person. Eventually the noise became deafening and the band stopped playing. People who weren't being yelled at were able to calm down, but the loops just got worse.

Thankfully, the police arrived on the scene to stop the commotion. However, they only brought a few squad cars, and they only have enough space to take k people with them. They need to determine if they can find k people such that, if they take those people to the station, there won't be any loops and everyone will be able to calm down.

To model the problem, they represent the crowd as a **directed** graph. Each node represents a person, and an edge from x_1 to x_2 means x_1 is yelling at x_2. A feedback loop is represented by a *cycle* in the graph. If the police can eliminate all cycles in the graph by removing k or fewer vertices, then the riots will stop, people will calm down, and the concert can continue. In other words, the police need to decide the language below, where $G = (V, E)$.

$$
\begin{aligned}
\textsc{RiotControl} = \{\langle G, k\rangle \quad | \quad & \exists V' \subseteq V \text{ such that} \\
& |V'| \leq k, \text{ and} \\
& E' = \{(u, v) \in E \mid u \in V' \text{ or } v \in V'\}, \text{ and} \\
& G' = (V - V', E - E') \text{ is acyclic}\}
\end{aligned}
$$

Prove that RiotControl is NP-Complete.

Lab Problem 2

Your three-person band is just starting to make it big. You've already been on a small tour and signed a deal for two records, but you are finally starting accumulate a large number of fans. As a result, your label has decided to start marketing band merchandise. They've made band shirts, bags, guitar cases, decals, you name it. Since they are for your band, the label has decided to send you some of the merchanise for free to distribute amoung your three band members. In the end, the label sends you n pieces of merchanise. For each item i, $1 \leq i \leq n$, the item has a retail price of v_i, which is a positive integer written in binary. Everyone in the band wants to be fair, so you decide to split the merchanise such that each band member gets the same value of gear.

One can formally model the problem by the following language:

MERCHANDISE $= \{\langle v_1, \ldots, v_n \rangle \mid$ each v_i is a binary integer and there exists a way to split the set of integers into disjoint sets S_1, S_2 and S_3 such that the sum of the elements within each set S_i is equal.$\}$

Prove that MERCHANDISE is NP-complete.

Lab Problem 3

An instance of *family tree* is a directed graph (**digraph**). Each vertex represents a person. An edge (u, v) means that u is a (biological) parent of v (equivalently, that v is a child of u). Each vertex has either two parents, or none. The graph must be acyclic, because one cannot be one's own ancestor. Finally, although the vertices of a family tree are not labelled with the biological sexes of the people they represent, people nevertheless have biological sexes, so there exists some assignment of biological sexes to the vertices such that every child has parents of distinct biological sexes.

In this problem, we define the language of Yes instances FAMILY-TREE(g) that consists of digraphs representing valid family trees with g biological sexes.

(a) FAMILY-TREE(2) is either NP-complete or is in P. If it is NP-complete, give a reduction from k-COLOR. If it is in P, give an algorithm that decides it.

(b) For $g \geq 3$, FAMILY-TREE(g) is either NP-complete or is in P. If it is NP-complete, give a reduction from k-COLOR. If it is in P, give an algorithm that decides it.

Remember: even though there are g biological sexes, each child has *exactly* two parents of different biological sexes, and the input doesn't specify the biological sex of any vertices.

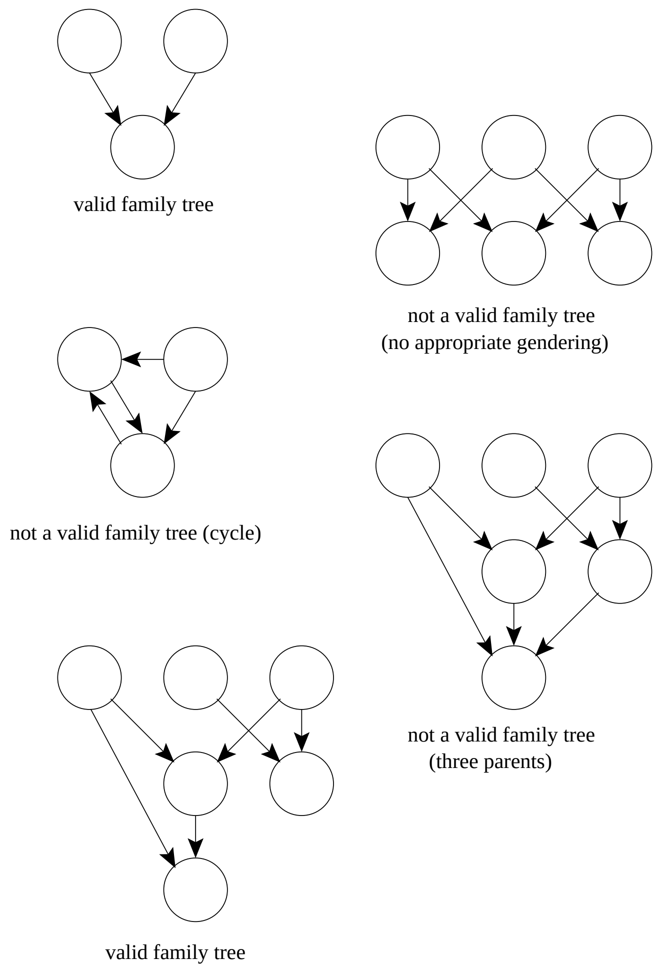


Figure 1: Examples of members and non-members of FAMILY-TREE(2).