CS 157 Homework 3

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Problem 3

1

a) 0
b) 0
c) 2
d) 6
e) 8
f) 10
g) 10
h) 8

2

Consider the sequence $X_1 \cdots X_n$ (from 1 to n) instead of $X_0 \cdots X_{n-1}$ (from 0 to n-1)

- Let $T(i,j)$ be $n \times n$ matrix.

where
$$T(i,j) = \begin{cases} 0, & \text{if } i \geq j \\ \text{the length of the longest palincomplementary subsequence of } X_i X_{i+1} \cdots X_j, & \text{if } i < j \end{cases}$$

- The recurrence relation is

$$T(i,j) = \begin{cases} \max\{T(i,j-1),\ T(i+1,j)\} & \text{if } X_i \neq \overline{X_j} \\ T(i+1,j-1) + 2 & \text{if } X_i = \overline{X_j} \end{cases}$$

- The order of filling T is as follows,

for $k = 1$ to $n-1$
  for j, i such that $i < j$ and $j - i = k$.
    fill $T(i,j)$

pseudocode on input : $x_1 \ldots x_n$.

```
∀ i ≥ j , T(i, j) ← 0.
for K = 1 to n-1
    for i, j such that i < j and j - i = K
        if x_i == x̄_j , T(i, j) ← T(i+1, j-1) + 2.
        else , T(i, j) ← max{ T(i+1, j) , T(i, j+1) }.
return T(1, n).
```

3

base case: $T(i,j) = 0$ if $i \geq j$.

indeed, if $i > j$, then one cannot define ~~sequence~~.

if $i = j$, then we have a sequence of size 1, which has an empty LPS since $x \neq \overline{x}$.

recursion:

$$T(i,j) = \begin{cases} \max\{ T(i+1, j), T(i, j+1) \} & \text{if } x_i \neq \overline{x_j} \\ T(i+1, j-1) + 2 & \text{if } x_i = \overline{x_j} \end{cases}$$

↳ **prf**:

if $x_i \neq \overline{x_j}$, then, x_i and x_j cannot both be part of the LPS.

if so, then $x_i \ldots x_j \neq \overline{x_j} \ldots \overline{x_i}$; contradiction.

Then, x_i or x_j is not present in LPS.

Thus $LPS(x_i \ldots x_j) = \underset{\text{length of LPS.}}{\max} \{ LPS(x_{i+1} \ldots x_j), LPS(x_i \ldots x_{j-1}) \}$.

if $x_i = \overline{x_j}$, then, LPS of $x_i \ldots x_j$ is

$T' := \underset{\text{length}}{\max} \{ LPS(x_{i+1} \ldots x_j), LPS(x_i \ldots x_{j-1}), (x_i, LPS(x_{i+1} \ldots x_{j-1}), x_j) \}$.

Note that $\max\{T(i+1, j), T(i, j-1)\} \leq T(i+1, j-1) + 2$.

Then, length of T' = length of $(x_i, LSP(x_{i+1}, \ldots x_{j-1}), x_j)$

$\therefore T(i,j) = T(i+1, I-1) + 2$

Problem 4

1

a) 31

b) 6

c) 10

2

Let n be the number of coins that Maria has (i.e, $n = |A|$).

- Let T be $n \times S$ matrix
  where $T(i, B)$ = the number of ways to form B with $A[1 \dots i]$.

- The order of filling T is downward from left to right. In other words,

  for $B = 1$ to S.
    for $i = 1$ to n.
      fill $T(i, B)$.

- Let $T(i, B)$ $\forall$ i or B out of bounds be 0. (ex, $T(-1, -100) = T(1, -1) = T(-1, 1) = 0$)
  Then,

$$T(i, B) = \begin{cases} T(i-1, B - A[i]) + T(i-1, B) + 1 & \text{if } A[i] = B \\ T(i-1, B - A[i]) + T(i-1, B) & \text{if } A[i] \neq B \end{cases}$$

• Let $T(i, B)$ $\forall$ i or B out of bounds be 0.

pseudocode on input $(A[1..n], S)$.

```
for B = 1 to S.
    for i = 1 to n.
        if A[i] == B.
            T(i, B) <- T(i-1, B-A[i]) + T(i-1, B) + 1.
        else
            T(i, B) <- T(i-1, B-A[i]) + T(i-1, B)
return T(n, S).
```

3

The algorithm has $n \cdot S$ iterations of constant operation.

Thus, the runtime is $O(nS)$.