CSCI 510 **Models of Computation**

HW7: Pump It Up

Due: Nov 3, 2010

Include your *full name*, *CS login*, and the problem number(s) on each piece of paper you hand in, and please staple your pages together before handing in.

While collaboration is encouraged in this class, please remember not to take away notes from collaboration sessions other than your scheduled lab section.

Problem 1

Consider the language

$$L = \{0^m 1^n : m \neq n\}$$

Prove that L is not regular.

Problem 2

A string x is a ***prefix*** of string y if there exists a string z such that $xz = y$. A string x is a ***proper prefix*** of y if it is also true that $x \neq y$. Define the operation NotPrefix on a language A as follows:

$$\textsc{NotPrefix}(A) = \{w \in A \mid w \text{ is not the proper prefix of any string in } A\}$$

Show that the class of regular languages is closed under NotPrefix. That is to say, if A is a regular language, so is NotPrefix(A).

The following questions are lab problems. Please remember to prepare a solution for your assigned problem before going to your lab section.

Lab Problem 1

Show that the following two languages are not regular:

a. $L_a = \{a^{pq} : p \text{ and } q \text{ are both prime and } p \neq q\}$

b. $L_b = \{w : w \text{ is a palindrome}\}$

Lab Problem 2

Sometimes you see people who have the whole "rock star look" – leather jacket, dark sunglasses, metal belt, wild hair. Then you find out that they've never played a single bass groove, and wouldn't know a snare drum from a hi-hat. They're not rockers at all, they just happen to dress that way. Well, the same thing happens with languages.

Consider the language $F = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

a. Show that F is not regular.

b. Show that F looks like a regular language when the pumping lemma is applied. In other words, give a pumping length p and demonstrate that, for all strings $w \in F$, we can write $w = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in F$ for all $i \geq 0$.

c. Explain why parts (a) and (b) do not contradict the pumping lemma.

Lab Problem 3

Let L_A and L_B be regular languages over the same alphabet Σ. Prove or disprove that

$$L_{AB} = \{a_1 b_1 a_2 b_2 \ldots a_i b_i \mid a = a_1 a_2 \ldots a_i \in L_A,\ b = b_1 b_2 \ldots b_i \in L_B\}$$

is a regular language.

Note: It may be the case that there do not exist $a \in L_A$ and $b \in L_B$ such that $|a| = |b|$. In this case, $L_{AB} = \emptyset$.