CS157 Problem Set 1

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1) 4 → 3 → 2 → 1 → 5.

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2) Alg.

① Sort by the average volume per page in descending order.

② The priority is in the descending magnitude in the sorted array.

Implementation

- Let A be an array with elements (i, P_i, V_i, a_i) for $i = 1 \ldots n$.
- For $i \leftarrow 1$ to n,

$$a_i \leftarrow \frac{V_i}{P_i}$$

- Sort the array A in descending order by a_i.

☆ Printing priority is the order of A.

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3) We prove that taking the maximum $\frac{V_i}{P_i}$ minimizes the total volume at each step.

① Claim; if $\frac{V_i}{P_i} < \frac{V_j}{P_j}$, then printing j^{th} student's papers first minimizes the total volume.

↳ pf: $\exists$ two choices in this case.

ⓐ $V_i(P_i + P_j) + V_j P_j$ ← printing j^{th} student's papers first.

ⓑ $V_j(P_i + P_j) + V_i P_i$ ← printing i^{th} student's papers first.

Consider the difference of ⓐ and ⓑ

$$\underbrace{V_i(P_i + P_j) + V_j P_j}_{ⓐ} - \underbrace{V_j(P_i + P_j) + V_i P_j}_{ⓑ}$$

$$= V_i P_j - V_j P_i$$

Now, since $\frac{V_i}{P_i} < \frac{V_j}{P_j} \Rightarrow V_i P_j < V_j P_i$

ⓐ − ⓑ < 0.

Then, ⓐ < ⓑ.

Thus, printing j^{th} student's paper first minimizes the total volume.

② Claim; If $\frac{V_i}{P_i} < \frac{V_j}{P_j} < \frac{V_k}{P_k}$, then printing k^{th} papers first minimizes the total volume at the first step.

By the claim ①, we can easily see that printing k^{th} papers first minimizes the total volume at the first step.

By the claim ②, $\exists$ a transitive property for the relation $\frac{V_i}{P_i} < \frac{V_j}{P_j}$ ~ printing j^{th} student's papers first minimizes the total volume at the current step.

Then at the n^{th} step, we have minimized the total volume.

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4) The for loop has n iterations of constant operations.

↳ $O(n)$.

The sorting takes $O(n\log(n))$

$\therefore$ The running time of the algorithm is $O(n \log n)$.