1. Introduction

Financial and economic crises are well documented in the literature. The 17th century witnessed the boom and bust of the first acknowledged bubble, which took place in a flowered Netherlands: the speculative fever over tulips that swept up in the Dutch society in mid-1600s (nowadays known as “Tulipomania”) raised up flower bulbs prices and spawned a subsequent crash. Three centuries after the episode, a major financial bubble led to the Great Depression (1929—1939), one of the longest-lasting economic turndowns in the history up to date.

The deepest wound, however, was yet-to-come in 2007—2009, when a housing market boom and bust dragged the worldwide economic into a tough recession. Many financial and non-financial companies went into bankruptcy (including the too-big-too-fail Lehman Brothers), thousands of investors were wiping out the market and, more sadly, millions of people lost their jobs or homes.

Each of these financial bubbles has raised questions against the prediction power of traditional economic models, whose main assumption was the rationality of economic agents. Various researchers have argued that the blame of dysfunctional financial markets lies with a higher degree of (intrinsic-to-all-markets) “moral hazard” (Ericson and Doyle, 2003), while others highlighted specific macro and micro-level factors that led to determined financial bubble. These explanations line up with the traditional theory and, deep down, keep strings attached with the tenet of rationality.

On the other hand, other authors would take a more radical and disruptive path while trying to explain economic agents’ behaviors in dysfunctional markets. They would let go of the rationality assumption and argue that emotional factors blurred agents minds and let them take poor economic decisions.

Last but not least, there’s a third and increasing line of argument (the one we shall pursue in this article), which takes into account the concept of bounded rationality. This concept was first introduced by Simon (1957) and, in the context of financial bubbles, may imply that the financial agents did not seek for all the necessary information to take the best decisions or were unprepared to deal with the complexity of the financial products.

Searching, selecting and interpretating financial information are costly to consumers and it may lead them to take economic decisions relying on inaccurate information. It seems like a rational choice since consumers weigh the trade-off between (i) the cost of seeking information and educating themselves and; (ii) losing their investments if the available information is not trust-worthy. This choice, however, may leave a breach to predatory behaviors, where sellers would take advantage of cognitively limited buyers.

One of the first frameworks addressing predatory behaviors in dysfunctional financial markets were proposed by Mesly et al. (2019). Huck et al. (2020) and Mesly et al. (2020) extended their work with insightful aggregations, such as combining the Lotka Volterra's model, borrowed by Biology, with the traditional demand-supply theory.

The main purpose of this article is to develop a satisfactory explanation for predatory behaviors in financial market and how these behaviors may lead the economic system into a crash even though just a small portion of agents acts in a predatory way. To achieve this goal, we take Mesly et al. (2020) theorical framework, which uses Lotka Volterra's equations, as a starting point. The model is presented right in the section below.

2. A framework on financial predation in economic crises

2.1. A glance at Lotka Volterra's traditional model

In ecology, a classic application of Lotka Volterra's predator-prey model lies in the study of Canadian lynx and snowshoe hare populations (Elton and Nicholson, 1942). The interaction between these two species may provide us a suitable backdrop to introduce the mathematical framework of LV model, which we shall present throughout this chapter.

Our hypothetical ecosystem is comprised of lynx and hare populations that cohabitate a meadow. The grassland serves as food supply to the herbivorous population of hare. On the other hand, lynx's diet is almost entirely made up of snowshoe hares and, therefore, its food supply is closely related to the size of prey population. Our main goal is to describe how the lynx and hare populations' sizes changes over time and how these two rates are intertwined. The following equations describe the predation process proposed by Lotka and Volterra:

$$\frac{dx}{dt} = rx - \alpha xy, for\ prey$$

$$\frac{dy}{dt} = \alpha\beta - vy, for\ predators$$

The path of both populations over time follows this set of differential equations. Prey population size (equation 1) depends on its intrinsic growth rate (i.e., its birth rate) and the initial number of preys.

Parameters	***Interpretation***
r	Prey birth rate;
v	Predator death rate;
α	Predation rate;
β	Conversion efficiency.

Table 1. Interpretation of each parameter on Lotka-Volterra's traditional model.

2.2. Adapting predator-prey model to a financial-crisis environment

Lotka Volterra's predator-prey model was found to be useful in many other fields beyond Ecology. There's a meaningful – and yet timid – literature regarding the use of LV equations in economic theory, which starts with Goodwin's model for the relationship between wage and employment growth and stretches over models that describe the cyclic behavior of deposits and loans. In this article, we present an adapted Lotka Volterra's predator-prey model which depicts predatory behavior between economic agents in the context of a financial crisis. We specifically develop this framework from the 2007-2009 financial crisis perspective; therefore, we focus on the housing market dynamics and characters.

Our adapted model starts with a representative economy where there are two economic agents: (i) sellers of subprime housing mortgages; and (ii) potential house buyers. The relationship between sellers and buyers of subprime mortgages may present itself as a predatory interaction, which obeys Lotka Volterra's equations. As presented in prior section 2.1, LV equations take in the following form:

$$\frac{dx}{dt} = rx - \alpha xy, for\ prey$$

$$\frac{dy}{dt} = \alpha\beta - vy, for\ predators$$

In the context of our subprime-infected housing market, sellers act as predators, while buyers may find themselves into prey positions. As such, $\frac{dx}{dt}$ ($\frac{dy}{dt}$) represents the changes in the number of sellers (predators) and buyers (prey) over time. Reinterpreting the coefficients in an economic sense, r is the rate at which new potential buyers hit the market, while v is the rate at which sellers leave it. The parameter α measures the probability of a prey getting caught by a predator or, in a more financial language, the probability of a buyer encountering and purchasing a subprime mortgage from a predatory seller. Finally, the parameter β equals the rate at which sold mortgages attract new sellers to the housing market, which, from a buyer's point of view, can be a proxy for the risk of purchasing a mortgage. All four parameters may (and should be) calibrated with real-world financial data.

Parameters	***Interpretation***
r	Rate at which new potential buyers hit the market;
v	Rate at which sellers leave the market;
α	Probability of a buyer purchasing a subprime mortgage from a seller;
β	Rate at which sold mortgages attract new sellers to the market.

Table 2. Interpretation of each LV parameter on our financial-predation model.

From a model engineering perspective, our model is as simple as it gets: a pair of first-order differential equations, with four known parameters (r, v, α, β) and two variables (x, y). A more interesting discussion lays under which kind of circumstances predatory behaviors may prevail in financial markets. This is exactly the topic we would like to address in the subsection that follows.

2.3. On the predatory behavior in financial market

The traditional theory on consumer behavior takes rationality as main assumption. In times of crises, however, conventional economic models may fail to provide a satisfactory explanation on consumers' and sellers' behaviors. We do not intend to let go of the rationality tenet, but otherwise suggest that consumers operate under *bounded* rationality in dysfunctional financial markets.

2.4. Prey-predator dynamics in the light of economic cycle

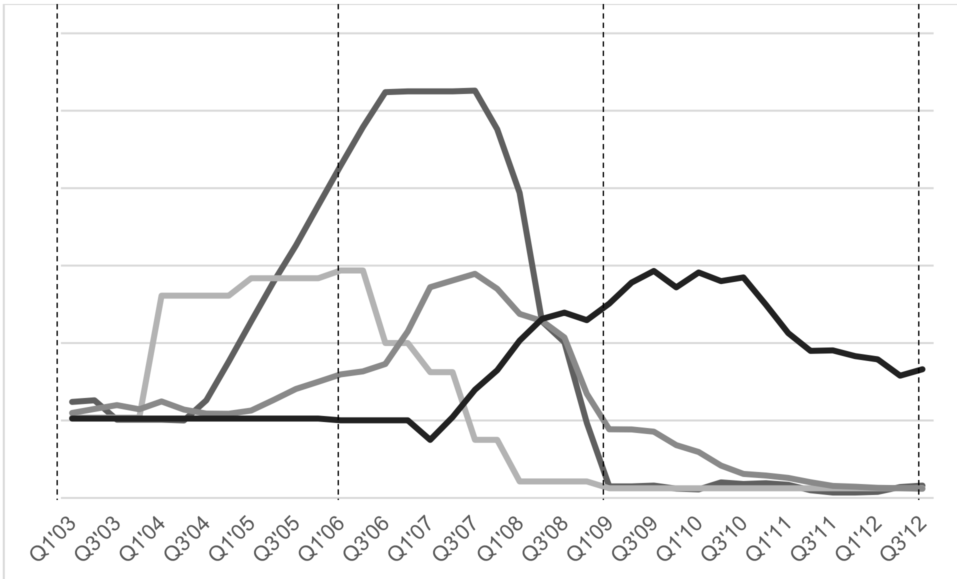
3. Empirical essay on financial predation

3.1. Finding proxies for each variable

In this section, we shall present an empirical illustration of our framework developed in the previous chapter. We start by outlining which data taken from subprime mortgage crisis are used as proxies for each one of our model's variables. For prey, for example, we've used the total number of foreclosures, adjusted by the average rate of foreclosures before the referred crisis. Note that this might underestimate the number of potential buyers, since not every On its turn, for predators, we compiled the ratio between shadow and traditional banking liabilities. For toxic products, we've taken the percentage of subprime on total mortgages sold in as a proxy. Finally, for regulations, we've turned to basic interest rate, which shall be our best way to describe Federal Reserve's (Feds) efforts to regulate the market. Table 1 sums up all of our model's variables and their respective proxies:

Variable	***Proxies***
Prey	Total number of foreclosures, adjusted by the average rate of foreclosures before the crisis;
Predator	Ratio between shadow and traditional banking liabilities;
Toxic products	Percentage of subprime on total mortgages sold;
Regulations	Basic interest rate.

Empirical data applied to this modelling exercise has been taken from 2003 and 2012, a time slot on which all four phrases of our model (Q1, Q2, Q3, and Q4) have been observed. In Figure 1, we've plotted our market data over time and outlined each one of the four phrases.



3.2. Calibrating the model with the best-fit parameters

We've settled our model's parameters values in a way that minimizes the sum of squared differences between the data and the LV equations' dynamics (Cooper, 2007). Our first step was to set the theoretical model's timescale in order to line up with the timescale of the empirical data. Next, we chose a reasonable set of initial conditions for the number of sellers and buyers of subprime mortgages – i.e., our state variables – when $t = 0$. Finally, we run our model characterized by LV equations over a wide range of parameters values.

For each simulation, we computed the sum of least squares (OLS) between a data point and the corresponding value of the theoretical model for every observation in the data. Then, we summed all these differences. Mathematically, we have:

$$\mathrm{OLS} = \sum_{n=1}^{n_f} (X_n - x_n)^2 + (Y_n - y_n)^2,$$

where n are the observations from the market data, X_n (Y_n) is the number of potential buyers (sellers) of subprime mortgages in the data observation n and x_n (y_n) is the corresponding value from the theoretical model evaluated at observation n. By minimizing OLS, we are in fact minimizing the error between our empirical data and the output of our theorical model.

3.3. Comparing the theoretical model's output against market data

The original market data and the fitted predator-prey model are plotted over time in Figure 3 and 4. Even with all simplifications we've done in our model, we were able to capture a great deal of the data behavior, especially the rise and fall in the number of potential buyers and sellers.

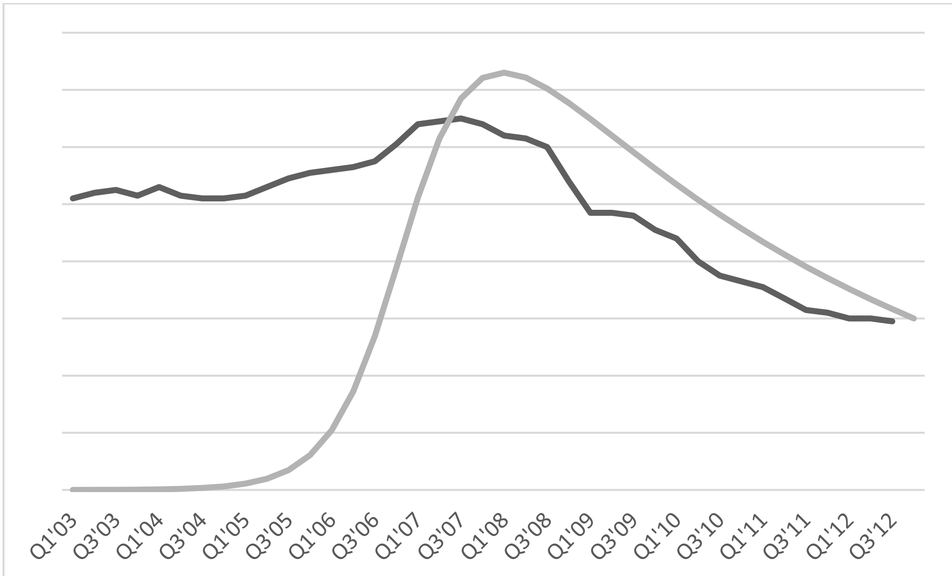


Figure 3. Compares the trajectory of predators (sellers) obtained from: (i) the original market data (darker grey); (ii) the fitted Lotka-Volterra's model (lighter grey).

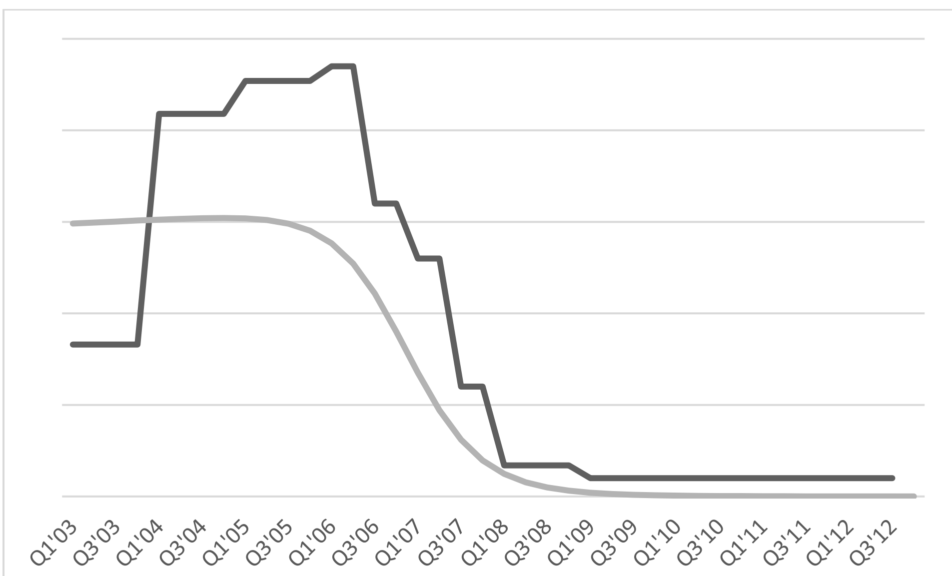


Figure 4. Compares the trajectory of preys (buyers) obtained from: (i) the original market data (darker grey); and (ii) the fitted Lotka-Volterra's model (lighter grey).

5. Conclusion

6. References

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