Proof of missing a top k $|B_k|$ is the cardinality of k simplification of annexstein $k = 1 \Rightarrow B_2$ is degenerate case
probability of exactly half in a population:

$$2^{-n}\binom{n}{n/2}$$

for nodes N its the probability of this not being even as pigeon hole principle will force one of the nodes to have $> |B_k|/N$

$$\left(2^{-n}\binom{n}{n/2}\right)^N$$

the assymptotic complexity is deminated by c^{-nN} which converges fast!
figuring out what $+1$ means to the deviation under the cardinality distribution of buckets is put off for now...(probably difficult)
now lets consider the bound on $|B_k|/N$ as N increases: This is simple, its power law convergence: $\frac{n}{N}$ so put these together and we have a sandwich.

$$\lim_{N\to\infty} \frac{\left(2^{-n}\binom{n}{n/2}\right)^N}{n/N}$$

using stirling's approximation for the binomial distribution:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

substitute in binomial distribution

$$\binom{n}{n/2} = \frac{n!}{(n/2)!(n-n/2)!} = \frac{n!}{2(n/2)!}$$

$$\approx \frac{\sqrt{2\pi N}\left(\frac{N}{e}\right)^N}{2\sqrt{\pi N}\left(\frac{N/2}{e}\right)^{N/2}}$$

drop out the constants...

$$\approx \Theta\left(\frac{N^N}{N^{N/2}}\right) = \Theta\left(N^{N/2}\right)$$

Plug this into the limit above, some constants can be fixed here to give a tight analysis, but we will stick with this approximation for simplicity.

$$\lim_{N\to\infty} \frac{\left(2^{-N}N^{N/2}\right)^N}{n/N}$$

bottom n is constant, and near 1 lets drop it too

$$\lim_{N\to\infty} \left(2^{-N}N^{N/2}\right)^N$$

so 2^{-N^2} is converging quickly. is this faster than $N^{\frac{N^2}{2}}$ whoa! this grows really fast.

$$\lim_{N\to\infty}\left(2^{-N}N^{N/2}\right)^N \neq c$$