Causal Mechanisms

Exercise

Stat286/Gov2002

Question 1

In the lecture, we have decomposed the total effect into the sum of natural direct and indirect effects:

$$Y_i(1, M_i(1)) - Y_i(0, M_i(0)) \;=\; \{Y_i(1, M_i(0)) - Y_i(0, M_i(0))\} + \{Y_i(1, M_i(1)) - Y_i(1, M_i(0))\}.$$

(a) Assume a binary mediator, i.e., $M_i \in \{0, 1\}$, for simplicity. Prove the following alternative decomposition.

$$\begin{aligned} Y_i(1, M_i(1)) - Y_i(0, M_i(0)) &= \{Y_i(1, M_i(0)) - Y_i(0, M_i(0))\} + \{Y_i(0, M_i(1)) - Y_i(0, M_i(0))\} \\ &\quad + \{Y_i(1, 1) - Y_i(1, 0) - Y_i(0, 1) + Y_i(0, 0)\}\{M_i(1) - M_i(0)\} \end{aligned}$$

Give an interpretation of each term and the whole decomposition.

(b) Continue to assume a binary mediator for simplicity. Prove the following yet another alternative decomposition and give an interpretation of each term.

$$\begin{aligned} Y_i(1, M_i(1)) - Y_i(0, M_i(0)) &= \{Y_i(1, 0) - Y_i(0, 0)\} + \{Y_i(1, 1) - Y_i(1, 0) - Y_i(0, 1) + Y_i(0, 0)\}M_i(0) \\ &\quad + \{Y_i(1, 1) - Y_i(1, 0) - Y_i(0, 1) + Y_i(0, 0)\}\{M_i(1) - M_i(0)\} \\ &\quad + \{Y_i(0, 1) - Y_i(0, 0)\}\{M_i(1) - M_i(0)\} \end{aligned}$$

In particular, show that $\{Y_i(0, 1) - Y_i(0, 0)\}\{M_i(1) - M_i(0)\} = Y_i(0, M_i(1)) - Y_i(0, M_i(0))$. Give an interpretation of each term and the whole decomposition.

Question 2

In this question, we reanalyze the data used in this module's review question.

Any researcher who is conducting a mediation analysis should be concerned about the possible presence of unmeasured confounders (both pre-treatment and post-treatment). Here, we consider the impact of unmeasured pretreatment confounders based on the following models.

$$\begin{aligned} \Pr(M_i \leq m \mid T_i, \mathbf{X}_i, U_i) &= \operatorname{logit}^{-1}(\alpha_{1m} + \beta_1 T_i + \boldsymbol{\xi}_1^\top \mathbf{X}_i + \lambda_1 U_i) \text{ for } m = 1, 2, 3 \\ \Pr(Y_i = 1 \mid T_i, M_i, \mathbf{X}_i, U_i) &= \operatorname{logit}^{-1}\Big(\alpha_2 + \beta_2 T_i + \sum_{m=2}^{4} \big(\gamma_m M_{im} + \delta_m T_i M_{im}\big) + \boldsymbol{\xi}_2^\top \mathbf{X}_i + \lambda_2 U_i\Big) \end{aligned}$$

where U_i is the unobserved pre-treatment confounder. For the sake of simplicity, assume that U_i follows a Bernoulli distribution with success probability 0.5.

(a) Derive the bias of the identification formula obtained in Question 1(a) under this alternative setting.

(b) Plot the estimated bias as a function of λ_0 and λ_1. Give a brief interpretation of the result.