Class: CS-325

Term: Fall 2017

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Homework: #6

Problem 1: (7 points)

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

We can compute the shortest path from s to t in a weight directed graph by solving.

max dt

subject to

ds = 0

dv - du <= w(u,v) for all (u,v) ∈ E

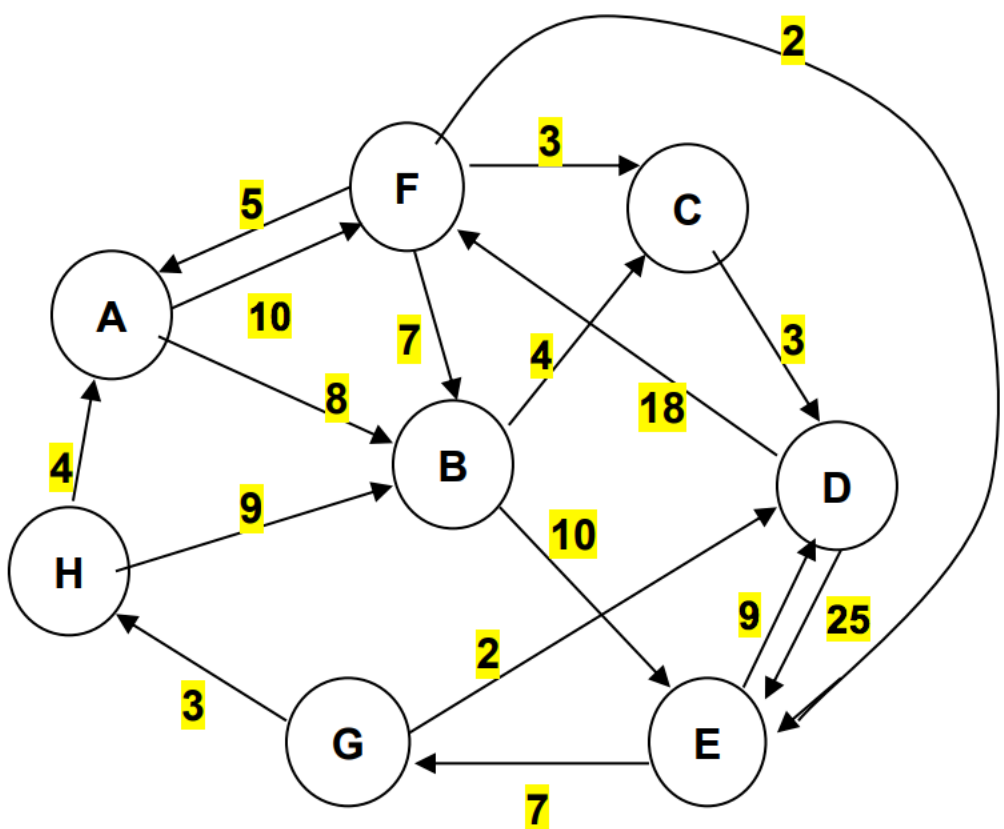
We can compute the single-source by changing the objective function to

max $\sum v \in V\, d$v

Use linear programming to answer the questions below. Submit a copy of the LP ode and output.

a) Find the distance of the shortest path from G to C in the graph below.

b) Find the distances of the shortest paths from G to all other vertices.



Answer:

a) The shortest distance of the shortest path from G to C in the graph above is: 16
See code and output below.

```
max dc
ST      dg = 0
        dg - de <= 7
        dh - dg <= 3
        da - dh <= 4
        da - df <= 5
        df - da <= 10
        db - da <= 8
        db - dh <= 9
        de - db <= 10
        dd - de <= 9
        de - dd <= 25
        dd - dc <= 3
        dc - db <= 4
        db - df <= 7
        dd - dg <= 2
        df - dd <= 18
        de - df <= 2
end
```

```
LP OPTIMUM FOUND AT STEP      6

        OBJECTIVE FUNCTION VALUE

        1)      16.00000

  VARIABLE        VALUE          REDUCED COST
        DC        16.000000          0.000000
        DG         0.000000          0.000000
        DE         0.000000          0.000000
        DH         3.000000          0.000000
        DA         4.000000          0.000000
        DF         5.000000          0.000000
        DB        12.000000          0.000000
        DD         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         7.000000          0.000000
        4)         0.000000          1.000000
        5)         3.000000          0.000000
        6)         6.000000          0.000000
        7)         9.000000          0.000000
        8)         0.000000          0.000000
        9)         0.000000          1.000000
       10)        22.000000          0.000000
       11)         9.000000          0.000000
       12)        25.000000          0.000000
       13)        19.000000          0.000000
       14)         0.000000          1.000000
       15)         0.000000          0.000000
       16)         2.000000          0.000000
       17)        13.000000          0.000000
       18)         7.000000          0.000000

NO. ITERATIONS=       6
```

b) The distances of the shortest paths from G to all other vertices is as follows:

g -> a = 7

g -> b = 12

g -> c = 16

g -> d = 2

g -> e = 19

g -> f = 17

g -> h = 3

See code and output below.

```
max da + db + dc + dd + de + df + dh
ST      dg = 0
        dg - de <= 7
        dh - dg <= 3
        da - dh <= 4
        da - df <= 5
        df - da <= 10
        db - da <= 8
        db - dh <= 9
        de - db <= 10
        dd - de <= 9
        de - dd <= 25
        dd - dc <= 3
        dc - db <= 4
        db - df <= 7
        dd - dg <= 2
        df - dd <= 18
        de - df <= 2
end
```

```
LP OPTIMUM FOUND AT STEP      5

        OBJECTIVE FUNCTION VALUE

        1)      76.00000

  VARIABLE        VALUE          REDUCED COST
        DA         7.000000          0.000000
        DB        12.000000          0.000000
        DC        16.000000          0.000000
        DD         2.000000          0.000000
        DE        19.000000          0.000000
        DF        17.000000          0.000000
        DH         3.000000          0.000000
        DG         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          7.000000
        3)        26.000000          0.000000
        4)         0.000000          6.000000
        5)         0.000000          3.000000
        6)        15.000000          0.000000
        7)         0.000000          2.000000
        8)         3.000000          0.000000
        9)         0.000000          2.000000
       10)         3.000000          0.000000
       11)        26.000000          0.000000
       12)         8.000000          0.000000
       13)        17.000000          0.000000
       14)         0.000000          1.000000
       15)        12.000000          0.000000
       16)         0.000000          1.000000
       17)         3.000000          0.000000
       18)         0.000000          1.000000

NO. ITERATIONS=       5
```

Problem 2: (7 points)

Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

type	selling price	labor	material	profit per tie
silk s	6.7	0.75	2.5	3.45
poly p	3.55	0.75	0.48	2.32
blend1 b	4.31	0.75	0.75	2.81
blend2 c	4.81	0.75	0.81	3.25

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal numbers of ties of each type to maximize profit? Include a copy of the code and output.

Answer:

The optimal numbers of ties of each type to maximize profit is as follows:

Silk: 7,000

Polyester: 13,625

Blend 1: 13,100

Blend 2: 8,500

The optimal solution (maximum profit) is $120,196.00

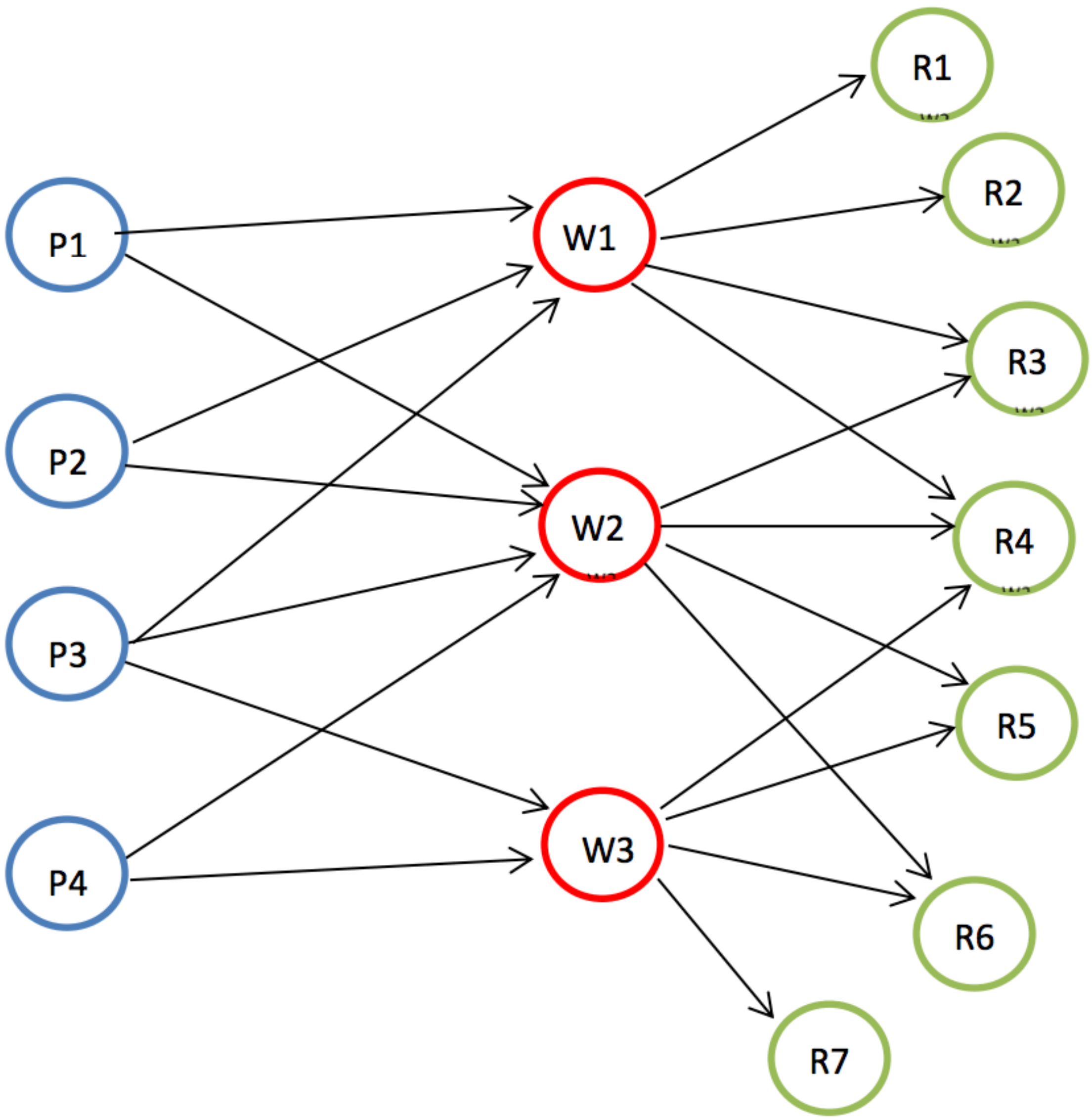
See code and output below.

```
max 3.45s + 2.32p + 2.81b + 3.25c
ST
        0.125s <= 1000
        0.08p + 0.05b + 0.03c <= 2000
        0.05b + 0.07c <= 1250

        s >= 6000
        s <= 7000
        p >= 1000
        p <= 14000
        b >= 13000
        b <= 16000
        c >= 6000
        c <= 8500
END
```

```
LP OPTIMUM FOUND AT STEP      4

       OBJECTIVE FUNCTION VALUE

       1)      120196.0

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000          0.000000
        P     13625.000000          0.000000
        B     13100.000000          0.000000
        C      8500.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)       125.000000          0.000000
       3)         0.000000         29.000000
       4)         0.000000         27.200001
       5)      1000.000000          0.000000
       6)         0.000000          3.450000
       7)     12625.000000          0.000000
       8)       375.000000          0.000000
       9)       100.000000          0.000000
      10)      2900.000000          0.000000
      11)      2500.000000          0.000000
      12)         0.000000          0.476000

NO. ITERATIONS=       4
```

Problem 3: (7 points)

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (pi) must be shipped to a Warehouse (wj) before being shipped to the Retailer (rk). Each Plant will have an associated supply (si) and each Retailer will have a demand (dk). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (pi)to warehouse (wj) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (wj)to a retailer (rk) have costs associated denoted cw(j,k). The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	$10	$15	X
P2	$11	$8	X
P3	$13	$8	$9
P4	X	$14	$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	$5	$6	$7	$10	X	X	X
W2	X	X	$12	$8	$10	$14	X
W3	X	X	X	$14	$12	$12	$6

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

The tables below give the capacity of each plant (supply) and demand for each retailer (per week).

Your goal is to determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost. Include a copy of the code and output.

Answer:

Below is a list of the optimal shipping routes along with their optimal number of refrigerators to be shipped on each route:

P1 -> W1 = 150

P1 -> W2 = 0

P2 -> W1 = 200

P2 -> W2 = 250

P3 -> W1 = 0

P3 -> W2 = 150

P3 -> W3 = 100

P4 -> W2 = 0

P4 -> W3 = 150

W1 -> R1 = 100

W1 -> R2 = 150

W1 -> R3 = 100

W1 -> R4 = 0

W2 -> R3 = 0

W2 -> R4 = 200

W2 -> R5 = 200

W2 -> R6 = 0

W3 -> R4 = 0

W3 -> R5 = 0

W3 -> R6 = 150

W3 -> R7 = 100

The minimized shipping costs is $17,100.00

See code and output below.

```
min     10S11 + 15S12 + 11S21 + 8S22 + 13S31 + 8S32 + 9S33 +
        14S42 + 8S43 + 5D11 + 6D12 + 7D13 + 10D14 + 12D23 +
        8D24 + 10D25 + 14D26 + 14D34 + 12D35 + 12D36 + 6D37
ST
        S11 + S12 <= 150
        S21 + S22 <= 450
        S31 + S32 + S33 <= 250
        S42 + S43 <= 150

        D11 >= 100
        D12 >= 150
        D13 + D23 >= 100
        D14 + D24 + D34 >= 200
        D25 + D35 >= 200
        D26 + D36 >= 150
        D37 >= 100

        S11 + S21 + S31 - D11 - D12 - D13 - D14 = 0
        S12 + S22 + S32 + S42 - D23 - D24 - D25 - D26 = 0
        S33 + S43 - D34 - D35 - D36 - D37 = 0

        S11 >= 0
        S12 >= 0
        S21 >= 0
        S22 >= 0
        S31 >= 0
        S32 >= 0
        S33 >= 0
        S42 >= 0
        S43 >= 0

        D11 >= 0
        D12 >= 0
        D13 >= 0
        D14 >= 0
        D23 >= 0
        D24 >= 0
        D25 >= 0
        D26 >= 0
        D34 >= 0
        D35 >= 0
        D36 >= 0
        D37 >= 0
END
```

LP OPTIMUM FOUND AT STEP 13

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE	VALUE	REDUCED COST
S11	150.000000	0.000000
S12	0.000000	8.000000
S21	200.000000	0.000000
S22	250.000000	0.000000
S31	0.000000	2.000000
S32	150.000000	0.000000
S33	100.000000	0.000000
S42	0.000000	7.000000
S43	150.000000	0.000000
D11	100.000000	0.000000
D12	150.000000	0.000000
D13	100.000000	0.000000
D14	0.000000	5.000000
D23	0.000000	2.000000
D24	200.000000	0.000000
D25	200.000000	0.000000
D26	0.000000	1.000000
D34	0.000000	7.000000
D35	0.000000	3.000000
D36	150.000000	0.000000
D37	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	-11.000000
14)	0.000000	-8.000000
15)	0.000000	-9.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000

NO. ITERATIONS= 13

Problem 4: (9 points)

Veronica the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients:

Tomato, Lettuce, Spinach, Carrot, Smoked Tofu, Sunflower Seeds, Chickpeas, Oil

Each salad must contain:

- At least 15 grams of protein
- At least 2 and at most 8 grams of fat
- At least 4 grams of carbohydrates
- At most 200 milligrams of sodium
- At least 40% leafy greens by mass.

The nutritional contents of these ingredients (per 100 grams) and cost are

Ingredient	Energy (Cal)	Protein (grams)	Fat (grams)	Carbohydrate (grams)	Sodium (mg)	Cost (100g)
Tomato	21	0.85	0.33	4.64	9.00	$1.00
Lettuce	16	1.62	0.20	2.37	28.00	$0.75
Spinach	40	2.86	0.39	3.63	65.00	$0.50
Carrot	41	0.93	0.24	9.58	69.00	$0.50
Sunflower Seeds	585	23.4	48.7	15.00	3.80	$0.45
Smoked Tofu	120	16.00	5.00	3.00	120.00	$2.15
Chickpeas	164	9.00	2.6	27.0	78.00	$0.95
Oil	884	0	100.00	0	0	$2.00

Part A: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What is the cost of the low calorie salad?

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. How many calories are in the low cost salad? Include a copy of the code/ file with the HW.

Answer:

Part A:

The combination of ingredients that minimizes calories but meets all nutritional requirements is as follows:

Lettuce = 0.585

Smoked Tofu = 0.878

The cost of the low calorie salad is $2.33 and the calorie count is 114.75.

See code and output below.

```
min 21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O
ST
       .85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15
       .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O >= 2
       .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O <= 8
       4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4
       9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200
       .6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .4O >= 0
END
```

```
 LP OPTIMUM FOUND AT STEP      12

        OBJECTIVE FUNCTION VALUE

        1)      114.7541

  VARIABLE        VALUE          REDUCED COST
         T         0.000000         16.901640
         L         0.585480          0.000000
         S         0.000000         14.513662
         C         0.000000         36.289616
        SS         0.000000        408.387970
        ST         0.878220          0.000000
        CP         0.000000         97.551910
         O         0.000000        886.404358


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.650273
        3)         2.508197          0.000000
        4)         3.491803          0.000000
        5)         0.022248          0.000000
        6)        78.220139          0.000000
        7)         0.000000         -6.010929

 NO. ITERATIONS=      12
```

Part B:

The combination of ingredients that minimizes cost is as follows:

Spinach = 0.832

Sunflower Seeds = .096

Chickpeas = 1.152

The number of calories in the low cost salad is 278.49 and the price is $1.55.

See code and output below.

```
min 1T + .75L + .5S + .5C + .45SS + 2.15ST + .95CP + 2O
ST
      .85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15
      .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O >= 2
      .33T + .2L + .39S + .24C + 48.7SS + 5ST + 2.6CP + 100O <= 8
      4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4
      9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200
      .6L + .6S - .4T - .4C - .4SS - .4ST - .4CP - .4O >= 0
END
```

```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      1.554133

 VARIABLE        VALUE          REDUCED COST
        T         0.000000          1.002081
        L         0.000000          0.402912
        S         0.832298          0.000000
        C         0.000000          0.486914
       SS         0.096083          0.000000
       ST         0.000000          0.405609
       CP         1.152364          0.000000
        O         0.000000          7.281258


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -0.131261
       3)         6.000000          0.000000
       4)         0.000000          0.051847
       5)        31.576324          0.000000
       6)        55.651089          0.000000
       7)         0.000000         -0.241358

 NO. ITERATIONS=       3
```