## Value Conversion in IL1 after Lambda Hoisting

Given the closure-conversion and then hoisted restricted language IL1 detailed below

$$\begin{split} v &::= n \mid x \mid \mathsf{halt} \\ e &::= v \mid v_0 + v_1 \mid (v_0, \cdots, v_n) \mid \pi_n v \\ c &::= \mathsf{let} \, x = e \, \mathsf{in} \, c \mid v_0 \, v_1 \, v_2 \mid v_0 \, v_1 \end{split}$$

we want to "lower" numbers and halt into expressions (as Val used in bindings only) and leave only variables as values, so in effect our language will now look like

$$\begin{split} v &::= x \\ e &::= v \mid \mathsf{val}(n) \mid \mathsf{val}(\mathsf{halt})v_0 + v_1 \mid (v_0, \cdots, v_n) \mid \pi_n v \\ c &::= \mathsf{let} \ x = e \, \mathsf{in} \ c \mid v_0 \ v_1 \ v_2 \mid v_0 \ v_1 \end{split}$$

We want to define a set of "lowering" translation  $\mathcal{LV}[v]$ ,  $\mathcal{LE}[e]$ ,  $\mathcal{LC}[c]$  that binds all non-variable values (integers and halts) into their own variables. Therefore, we need to have both the value and the expressions translation be able to be abstracted as bindings.

$$\begin{split} & \mathcal{LV}[\![v]\!] : (\mathsf{var} \times e) \mathsf{list} \times \mathsf{var} \\ & \mathcal{LE}[\![e]\!] : (\mathsf{var} \times e) \mathsf{list} \times \mathsf{e} \\ & \mathcal{LC}[\![c]\!] : c \end{split}$$

We will use the notation

$$let x_i = e_i in x$$

to mean

$$([(x_i,e_i);\cdots(x_n,e_n)],x)$$

and respective syntax sugary for expressions.

## 1 Values

$$\begin{split} \mathcal{LV}[\![n]\!] &= \mathsf{let}\, x = n \, \mathsf{in}\, x \\ \mathcal{LV}[\![x]\!] &= ([],x) \\ \mathcal{LV}[\![\mathsf{halt}]\!] &= \mathsf{let}\, x = \mathsf{halt}\, \mathsf{in}\, x \end{split}$$

## 2 Expressions

Notice immediately that we only ever translate e in the context of let expressions, so we don't have to translate the case of just a value.

$$\begin{split} \mathcal{L}\mathcal{E}[\![v_0+v_1]\!] &= \mathsf{let}(l_0,x_0) = \mathcal{L}\mathcal{V}[\![v_0]\!] \, \mathsf{in} \, \mathsf{let}(l_1,x_1) = \mathcal{L}\mathcal{V}[\![v_1]\!] \, \mathsf{in} \, (l_0 \cup l_1,x_0+x_1) \\ \mathcal{L}\mathcal{E}[\![(v_0,\cdots,v_n)]\!] &= \mathsf{let}(l_0,x_0) = \mathcal{L}\mathcal{V}[\![v_0]\!] \, \mathsf{in} \cdots \mathsf{let}(l_n,x_n) = \mathcal{L}\mathcal{V}[\![v_n]\!] \, \mathsf{in} \, (\bigcup l_k,(x_1,\cdots,x_n)) \\ \mathcal{L}\mathcal{E}[\![\pi_n v]\!] &= \mathsf{let}(l,x) = \mathcal{L}\mathcal{V}[\![v]\!] \, \mathsf{in} \, (l,\pi_n x) \end{split}$$

## 3 Commands

$$\mathcal{LC}[\![\![\mathsf{let}\,x=e\,\mathsf{in}\,c]\!] = \mathsf{let}(x_i=e_i,e') = \mathcal{LE}[\![\![e]\!]\!] \,\mathsf{in} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \\ \mathsf{this} \;\mathsf{is} \;\mathsf{the}\;\mathsf{actual}\;\mathsf{command}\;\mathsf{returned} \\ \mathcal{LC}[\![\![v_0\,\,v_1]\!]\!] = \mathsf{let}(x_i=e_i,x) = \mathcal{LV}[\![\![v_0]\!]\!], \\ (y_j=e_j',y) = \mathcal{LV}[\![\![v_1]\!]\!] \,\mathsf{in} \qquad \mathsf{let}\,x_i=e_i,y_j=e_j'\,\mathsf{in}\,x\;y$$

and without loss of generality, the same applies to the 3 argument case.