Value Conversion in IL1 after Lambda Hoisting

Given the closure-conversion and then hoisted restricted language IL1 detailed below

$$\begin{split} v &::= n \mid x \mid \mathsf{halt} \\ e &::= v \mid v_0 + v_1 \mid (v_0, \cdots, v_n) \mid \pi_n v \\ c &::= \mathsf{let} \, x = e \, \mathsf{in} \, c \mid v_0 \, v_1 \, v_2 \mid v_0 \, v_1 \end{split}$$

we want to "lift" numbers and halt into expressions (as Val used in bindings only) and leave only variables as values, so in effect our language will now look like

$$\begin{split} v &::= x \\ e &::= v \mid \mathsf{val}(n) \mid \mathsf{val}(\mathsf{halt})v_0 + v_1 \mid (v_0, \cdots, v_n) \mid \pi_n v \\ c &::= \mathsf{let}\, x = e \,\mathsf{in}\, c \mid v_0 \,\, v_1 \,\, v_2 \mid v_0 \,\, v_1 \end{split}$$

We want to define a set of "lowering" translation $\mathcal{LV}[v]$, $\mathcal{LE}[e]$, $\mathcal{LC}[c]$ that binds all non-variable values (integers and halts) into their own variables. Therefore, we need to have both the value and the expressions translation be able to be abstracted as bindings.

$$\begin{split} \mathcal{LV}[\![v]\!] : (\mathsf{var} \times e) \mathsf{list} \times \mathsf{var} \\ \mathcal{LE}[\![e]\!] : (\mathsf{var} \times e) \mathsf{list} \times \mathsf{e} \\ \mathcal{LC}[\![c]\!] : c \end{split}$$

We will use the notation

$$let x_i = e_i in x$$

to mean

$$([(x_i,e_i);\cdots(x_n,e_n)],x)$$

and respective syntax sugary for expressions.

1 Values

$$\begin{split} \mathcal{LV}[\![n]\!] &= \mathsf{let}\, x = n \, \mathsf{in}\, x \\ \mathcal{LV}[\![x]\!] &= ([],x) \\ \mathcal{LV}[\![\mathsf{halt}]\!] &= \mathsf{let}\, x = \mathsf{halt}\, \mathsf{in}\, x \end{split}$$

2 Expressions

Notice immediately that we only ever translate e in the context of let expressions, so we don't have to translate the case of just a value.

$$\begin{split} \mathcal{L}\mathcal{E}[\![v_0+v_1]\!] &= \mathsf{let}(l_0,x_0) = \mathcal{L}\mathcal{V}[\![v_0]\!] \, \mathsf{in} \, \mathsf{let}(l_1,x_1) = \mathcal{L}\mathcal{V}[\![v_1]\!] \, \mathsf{in} \, (l_0 \cup l_1,x_0+x_1) \\ \mathcal{L}\mathcal{E}[\![(v_0,\cdots,v_n)]\!] &= \mathsf{let}(l_0,x_0) = \mathcal{L}\mathcal{V}[\![v_0]\!] \, \mathsf{in} \cdots \mathsf{let}(l_n,x_n) = \mathcal{L}\mathcal{V}[\![v_n]\!] \, \mathsf{in} \, (\bigcup l_k,(x_1,\cdots,x_n)) \\ \mathcal{L}\mathcal{E}[\![\pi_n v]\!] &= \mathsf{let}(l,x) = \mathcal{L}\mathcal{V}[\![v]\!] \, \mathsf{in} \, (l,\pi_n x) \end{split}$$

3 Commands

$$\mathcal{LC}[\![\![\text{let } x = e \text{ in } c]\!] = \text{let}(x_i = e_i, e') = \mathcal{LE}[\![\![e]\!]\!] \text{ in } \underbrace{\text{let } x_i = e_i, x = e' \text{ in } \mathcal{LC}[\![\![c]\!]\!]}_{\text{this is the actual command returned}}$$

$$\mathcal{LC}[\![\![v_0 \ v_1]\!]\!] = \text{let}(x_i = e_i, x) = \mathcal{LV}[\![\![v_0]\!]\!], (y_j = e'_j, y) = \mathcal{LV}[\![\![v_1]\!]\!] \text{ in } \text{let } x_i = e_i, y_j = e'_j \text{ in } x \text{ } y$$