Quadruped Bounding Gait Using LQR Feedback

16-868: Biomechanics

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***Abstract*—Natural stability and dynamic capability of quadruped systems have inspired academia and industry alike in developing a quadruped robot and controller. Among many different gait patterns accessible for quadruped, bounding gait—which is similar to a bouncing rabbit or galloping horse—is an effective gait pattern. Although this pattern was achieved using hardware with heuristic controllers, there wasn't any recreation of this behavior in spring loaded inverted pendulum (SLIP) model. Using a block mass on massless springs with parameters same as Minitaur's hardware parameters, quadruped bounding gait will be simulated. Due to its inherent instability, feedback control for legs' angle of attack is implemented to recreate the bounding gait. The system is linearized around quasi-equilibrium point and linear quadratic regulator (LQR) is used for feedback control. Effectiveness of feedback control is shown by analyzing the state space orbit's basin of attraction.**

***Index Terms*—quadruped, bounding gait, LQR controller, spring loaded inverted pendulum, SLIP**

I. Introduction

A. Motivation

The concept of natural dynamics can be found in many scientific domain that is dealing with dynamical processes. For industrial robots, natural dynamics have been considered an undesirable side-effect that degrades kinematic precision. To reduce this impact, traditional robots are built as stiff as possible. [1] In controlled environment with deterministic surroundings, stiff robots can show great performance. On the other hand, in unknown environments, uncertainty prevails. Compliance is more desired than precision. Traditional stiff robots capabilities are limited in such an environment. Animals utilizes passive elastic structures such as tendons and muscles to store energy. A substantial part of motions emerges naturally. Therefore, approximating quadruped gait with simple SLIP allows more effective control of quadruped locomotion. [2]

B. Goal

Natural stability of quadruped—its ability to stand without control—and dynamic capabilities brought authors attention to quadruped locomotion. Quadruped locomotion can be achieved with many—more than fifty—different gait patterns. [3] Not all patterns are naturally stable in SLIP model, where legs are approximated using springs. David Remy had shown that two two-beat gait and two four-beat gait arise naturally from a quadruped spring mass model: pace, trot, single foot lateral, and single foot diagonal. [1] Among other gait patterns, authors will try to reproduce a bounding gait in SLIP, where two front legs and two hind legs are coupled during locomotion. Because bounding gait (which is similar to galloping) is more effective for high speed four-legged locomotion, [4] it is relevant and necessary for researches regarding quadruped locomotion. There haven't been any approaches using SLIP to reproduce bounding gait in academia. Some have approached the problem in a more heuristic method using feedback control to ensure touch down angles. [5] Other robots like MIT Cheetah also obtains bounding gait using a heuristic controller. [6] In this paper, authors will recreate bounding motion with spring mass model. The goal is to obtain a working SLIP and apply it to Minitaur Robot. [7]

II. Model

A. Simulation model

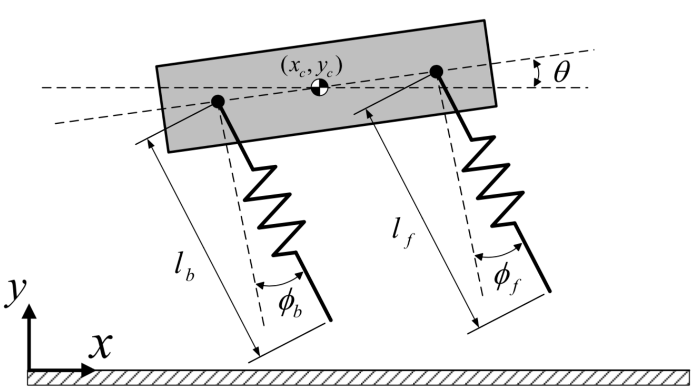


Figure 1: Quadruped model [8]

A quadruped SLIP model developed in Simulink [9] by Joe Norby's group was used as the initial starting point. Each leg would switch between flight phase and stance phase.

1) Assumptions: The SLIP model is constrained in a 2D space. A bounding gait requires that the front legs and the back legs are coupled. Because the motion of Minitaur's leg is constraint to a 2D movement, there should be no roll movements. It is therefore reasonable to reduce our model to a 2D space. Minitaur is modeled as a block mass on massless springs. It weights 6 kg in total, and the leg actuators, which make up most of Minitaur's weight, are located on the body. It is reasonable to implement it as a mass body on massless legs.

2) Parameters: To closely resemble Minitaur, same hardware parameters such as mass, body and leg length, body rotational inertia are implemented into simulink model. The physical model is shown in Figure 1.

For a SLIP model, spring coefficient k is very important, it would affect what kind of gait the model generates. Since Minitaur does not have spring loaded, k is chosen according to papers. Dimensionless relative stiffness k_{ref} is defined as:

$$k_{ref} = \frac{\frac{F}{mg}}{\frac{\Delta l}{l}} \text{ [10]}$$

For animal trotters and hoppers, this value is found to be in the range of $[15.4, 27.2]$. The stiffness of our model is chosen accordingly, having its k_{ref} at a value of 17.33.

B. Method

As the previous section mentioned, bounding gait is not naturally found. For the non-linear Minitaur system, the following approach is used to find the bounding gait controller. First, a quasi-equilibrium point, around which Minitaur could perform a periodic bounding gait, is identified. Then, the system is linearized around this system by Jacobian analysis. Finally, an LQR approach is used to optimize the controller.

1) State Analysis: To fully describe a quadruped motion in 2D, 6 states are needed, which are body position x, y, body orientation θ, translational speed $\dot{x}$, $\dot{y}$ and rotational speed $\dot{\theta}$. A periodic gait requires the next states to match the previous states in a cycle, which satisfies:

$$x_{k+1} = P(x_k)$$

Three states are eliminated in our analysis. Motions are analyzed at every apex point, where $\dot{y}$ is always 0. Minitaur keeps moving forward in every step, therefore x is not relevant. Lastly, in an energy-conserved system, when potential energy is specified by y, kinetic energy is specified by $\dot{x}$, $\dot{\theta}$ which specifies rotational energy can be automatically determined.

The controller that helps Minitaur return to a periodic bounding gait will have the states mentioned above as outputs. To allow Minitaur to return to the same states every cycle, angles of attack will be specified to satisfy the periodicity requirements. Front leg's angle of attack ϕ_f and back leg's angle of attack ϕ_b will therefore be our control variables.

2) Jacobian Analysis: After identifying an equilibrium point for bounding gait motion, Jacobian analysis was utilized to stabilize the system. A system can be represented as the following:

$$x_{k+1} = Ax_k + Bu$$

$$\delta_{x_{k+1}} = f(\bar{x} + \delta_{x_{k(t)}}, \bar{u} + \delta_u(t))$$

where x represents the states and u represents the inputs. Jacobian analysis can be done by perturbing the system from the equilibrium point by δ. A self-stabilizing equilibrium point would have all of its A matrix's Jacobian eigenvalues within an unit circle. However, for Minitaur simulation, the magnitude of the eigenvalues of the quasi-equilibrium point are close 1, but not less than 1. (This corresponds to the fact that bounding gait was not found as a natural gait pattern.) That is why a controller is needed to stabalize the system for a periodic gait. With a feedback control loop, $u = -Kx$. And the goal is to adjust K so as the eigenvalues of the matrix $(A - BK)$ will be within an unit circle.

3) LQR controller implementation: The controller gain K was found by utilizing LQR optimization. Given the Minitaur system, a discrete approach was chosen. Identity matrix was given to both Q and R for solving the algebraic Ricatti equation.

III. Results

An efficient dynamic system should take advantage of the natural properties of the system and thus reduce the power requirement of actuators. This paper achieved a stable, periodic bounding gait—which is naturally unstable—using the LQR controller around quasi-equilibrium point by measuring the state at every apex and decide the desired landing angle of attack of each leg. The actuators only need to maintain the angle when landing and natural dynamics will do the rest of the works.

A. Steady states orbits

In steady gaits, all states of the system follow a periodic orbit in state space [1]. Figure 2 shows the resulting trajectory in state space with LQR control. When starting with different initial conditions, the robot's states converge to a close orbit in state space and follows it thereafter. The blue contour is the convergence zone where states follow. The five red points are the boundary of the convergence: the largest tolerance when varying one initial state while keeping others unchanged. In this case, the acceptable range of perturbation for each state is:

$$\Delta y = [-0.094, 0] \ (m)$$
$$\Delta\theta = [-8.355, 26.924] \ (deg)$$
$$\Delta\dot{x} = [-0.223, 0] \ (m/s)$$

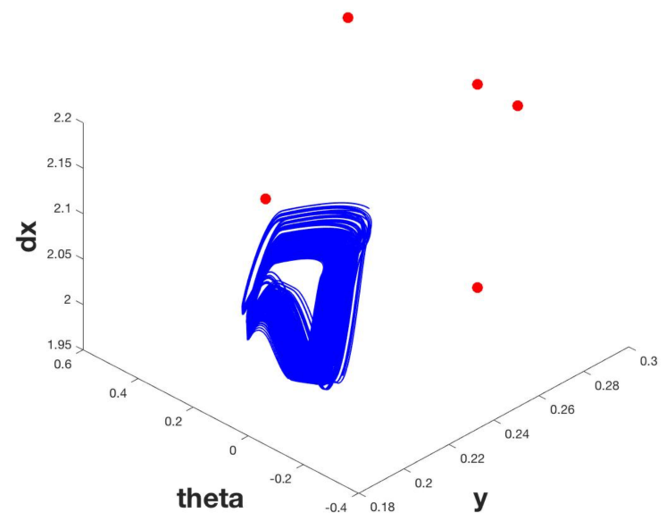


Figure 2: States will converge to a close orbit (blue) when starting with different initial conditions (red points)

B. Orbit's basin of attraction

Given that LQR control allows some level of disturbance, an exhaustive search through 1000 initial conditions is done in order to find out the maximal tolerance of perturbation on initial conditions. Figure 3 is an extended version of Figure 2, demonstrating the point cloud and the boundary of convergence of the initial conditions. The three-view diagram (Figure 4) of the state space provides a clearer view of the basin of attraction and the stable range of initial conditions. That is, the robot will be stable if it starts from any of the points within the boundary.

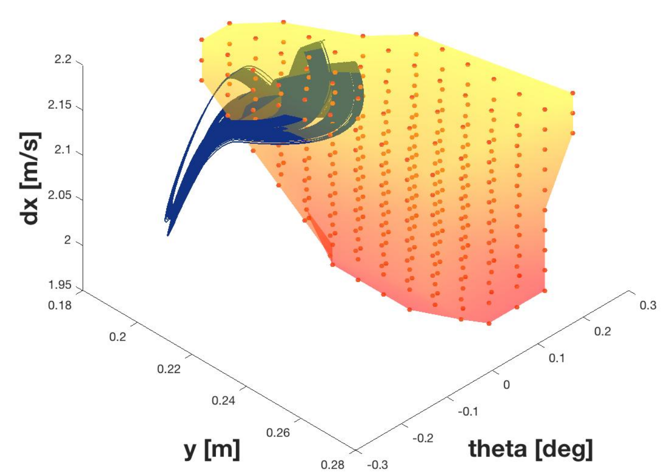
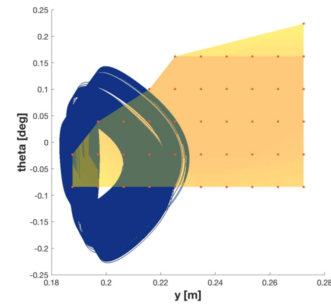
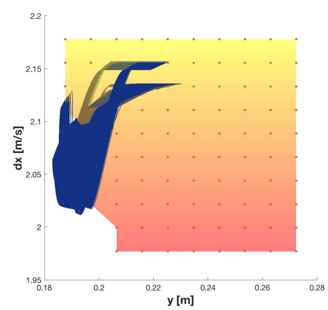


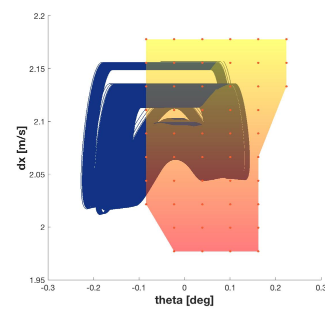
Figure 3: Basin of attraction in state space (orange zone), formed by convergent initial conditions and its boundary



(a) Top view



(b) Front view



(c) Side view

Figure 4: Three-view diagram of basin of attraction

IV. DISCUSSION

Remy has predicted the existence of gait models that can be stabilized in a way that the system draws a certain pattern of orbit in a state space as shown in Figure 5. [1] Despite that, since his interest of research was solely on natural dynamics, his models were limited to discovering simple gaits, assuming at least two legs contacting on the ground with steady phase shifts. [2] In this paper, the LQR control method was implemented such that it successfully achieves reproducing bounding gait, which does not exist in the natural world. Feedback control not only stabilizes the gaits associated with unstable eigenvalues, but also significantly improves the tolerance against the disturbance.

A. Limitations

While calculating LQR controller does not necessarily need mathematical computation, its preliminary step, finding initial conditions of the three states (y, θ, and $\dot{x}$) may require some intensive work. These initial conditions be quasi-equilibrium points that can lead the model to perform bounding gait for significant amount of time, even without the LQR controller. Limitation lies in the fact that there is no true equilibrium point where model is self stable. Bounding gait without feedback control is extremely sensitive that even a thousandth of difference in each parameter can affect performing time.

B. Improvements

One possible method to reduce the work of initial condition derivation is to use optimization algorithm to

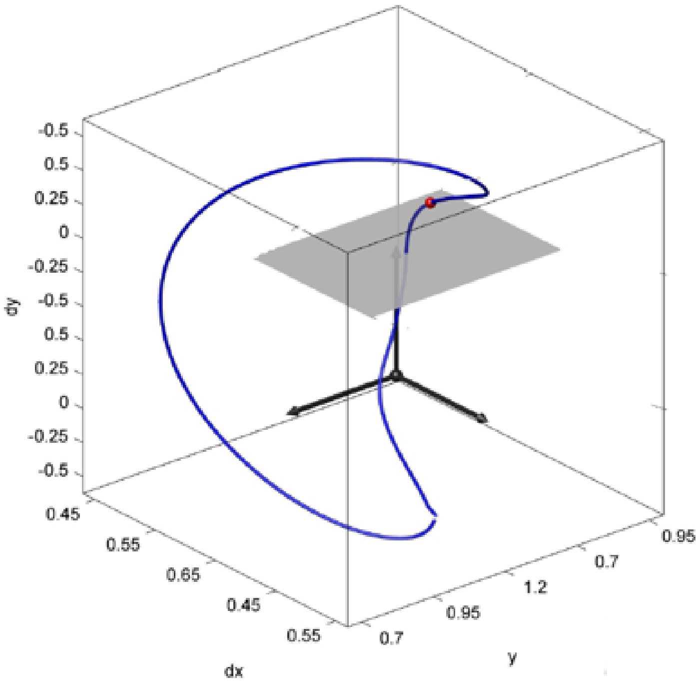


Figure 5: The states of a periodic and steady gait will converge to a periodic orbit in state space [1]

find candidates for equilibrium points. Cost function for optimization in this case is a simulation that returns the difference of each of the three states between initial consecutive apex heights of the model as an output (Δy, $\Delta\theta$, and $\Delta\dot{x}$). Smaller difference indicates that the model is more stable in the beginning, and hence the corresponding points can be good candidates for initial conditions.

Some candidates were found and are shown in Figure 6. This represents the map of evaluation function based on the output of test function such that:

$$f(y, \theta, \dot{x}) = \frac{1}{\Delta y + \Delta\theta + \Delta\dot{x}}$$

The region with high evaluation score (colored yellow) is potential points of convergence. Once the candidates for equilibrium are found, same method can be used to linearize and stabilize around those points. If different bounds of states are chosen, different basin of attraction may be discovered.

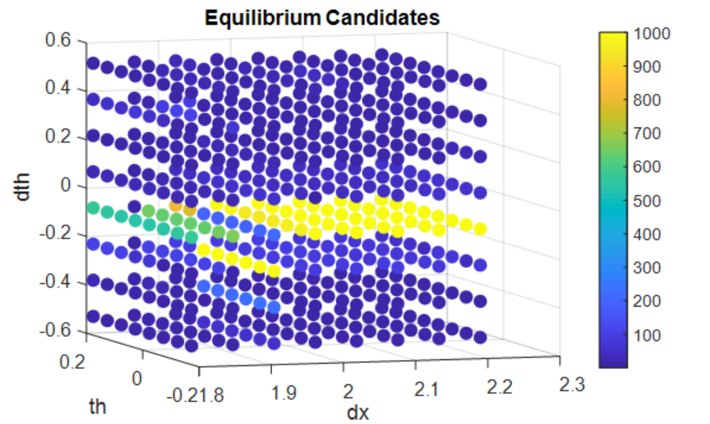


Figure 6: Mapping of Evaluation Function for Potential Equilibrium Candidates

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