On the Instability of Fractional Reserve Banking

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Introduction

Is fractional reserve banking particularly unstable?

- Yes:
  - Peel's Banking Act of 1844
  - Chicago plan of banking reform with 100% reserve requirement
  - Friedman (1959) supported the Chicago plan.
  - An important cause of boom-bust cycles:
    Fisher (1935), Von Mises (1953), Minsky (1957), and Minsky (1970):
- No:
  - Becker (1956)
  - Adam Smith's the Wealth of Nations (Book II, chapter 2)
- Sargent (2011) summaries the historical debate on this.
- Still on going debate: Switzerland's national referendum of 100% reserve banking in 2018.

This Paper

- Focuses on the instability as endogenous cycles (self-fulfilling prophecy)
  - not focusing on banking panic or bank run.
- Constructs a search-theoretic monetary model of fractional banking by extending Berentsen et al. (2007, JET)
- An economy is more prone to exhibit cyclic, and chaotic dynamics under lower reserve requirements
  - Different from the argument that fluctuations due to exogenous shocks can be amplified by fractional reserve banking.
  - The endogenous cycle arises even if we shut down the stochastic component of the economy

Literature

- Money, credit and banking in the search model:
  Berentsen et al. (2007), Lotz & Zhang (2016), Gu et al. (2016)

- Fractional reserve banking:
  Freeman & Huffman (1991), Freeman & Kydland (2000), Chari & Phelan (2014), Andolfatto et al. (2020)

- Endogenous fluctuations, chaotic dynamics, and indeterminacy:
  Baumol & Benhabib (1989), Azariadis (1993), Benhabib & Farmer (1999) Gu et al. (2013), Gu et al. (2019)

Model

Environment

- Time, goods
- Agents, banks, and the central bank
- Preferences

Environment

- Time, goods
    1. $t = 0, 1, 2..., \infty$
    2. Each period has three subperiods:
       - Centralized Settlement Market (CM)
       - Centralized Financial Market (FM)
       - Decentralized Goods Market (DM): bilateral trade, subject to anonymity, limited commitment
    3. Perishable DM/CM goods.
- Agents, banks, and the central bank
- Preferences

Environment

- Time, goods
- Agents, banks, and the central bank
  1. Agents: measure 1; maximize life time utility;
     with prob σ, buyer, with prob $1-\sigma$, seller in the DM;
     DM types are realized in the FM.
  2. Banks accept deposit and lend loan.
  3. The central bank control money supply M_t via lump-sum tax/transfer. Let γ money growth rate.
- Preferences

Environment

- Time, goods
- Agents, banks, and the central bank
- Preferences
$$U(X) - H + u(q) - c(q)$$
  - CM consumption X; CM disutility for production H; DM consumption q; discount factor: β
  - efficient DM consumption, q^* solves $u'(q^*) = c'(q^*)$.

Timeline

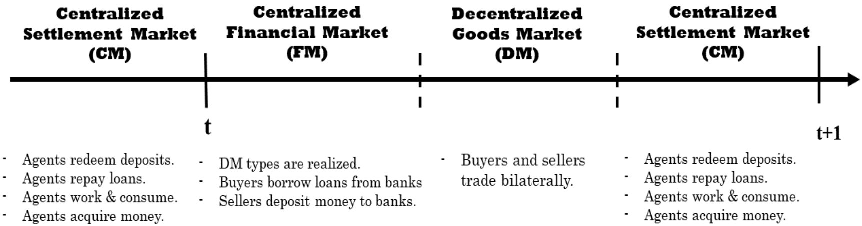

Figure 1: Timeline

CM Problem

$$
\begin{aligned}
W_t(m_t, d_t, \ell_t) = &\max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta G_{t+1}(\hat{m}_{t+1}) \\
\text{s.t. } \phi_t \hat{m}_{t+1} + X_t = &H_t + T_t + \phi_t m_t + (1 + i_{d,t})\phi_t d_t - (1 + i_{l,t})\phi_t \ell_t
\end{aligned} \tag{1}
$$

- Standard results: $W_t(m_t, d_t, \ell_t)$ is linear in m_t, d_t, and ℓ_t
- FOC for $\hat{m}_{t+1}$:

$$
\phi_t = \beta G'_{t+1}(\hat{m}_{t+1}) \tag{2}
$$

FM Problem

- Types are realized at the FM.

$$G_t(m) = \sigma G_{b,t}(m) + (1-\sigma) G_{s,t}(m) \tag{3}$$

- Type-j agent solves the following problem

$$G_{j,t}(m) = \max_{d_{j,t}, \ell_{j,t}} V_{j,t}(m - d_{j,t} + \ell_{j,t}, d_{j,t}, \ell_{j,t}) \quad \text{s.t} \quad d_{j,t} \leq m \tag{4}$$

where $j \in \{b, s\}$

- FOCs are:

$$\frac{\partial V_{j,t}}{\partial \ell_{j,t}} \leq 0 \tag{5}$$

$$\frac{\partial V_{j,t}}{\partial d_{j,t}} - \lambda_d \leq 0 \tag{6}$$

where λ_d is the Lagrange multiplier for $d_{j,t} \leq m$.

DM trade

- In the DM, a buyer meets a seller with probability α and a seller meets a buyer with probability α_s.

- The buyer's DM value function

$$V_{b,t}(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}) = \alpha[u(q_t) - p_t] + W(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t})$$

where $p_t \leq m_t - d_{b,t} + \ell_{b,t}$.

- The seller's DM value function

$$V_{s,t}(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}) = \alpha_s[p_t - c(q_t)] + W_t(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t})$$

DM trade

- A general trading mechanism $p = v(q)$, where $p \leq z$ and $v'(q) > 0$. (Gu & Wright 2016).
  (This includes Nash bargaining and Kalai bargaining)
  general trading mechanism

- Let p^* be a payment to get q^*.

- Terms of trade are given by

$$p = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{if } z \geq p^* \end{cases} \qquad q = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{if } z \geq p^* \end{cases}$$

DM trade

Differentiating $V_{b,t}$ yields

$$\frac{\partial V_{b,t}}{\partial m} = \phi_t[\alpha\lambda(q_t) + 1] \tag{7}$$
$$\frac{\partial V_{b,t}}{\partial \ell} = \phi_t[\alpha\lambda(q_t) - i_l] \tag{8}$$
$$\frac{\partial V_{b,t}}{\partial d} = \phi_t[-\alpha\lambda(q_t) + i_d] \tag{9}$$

where liquidity premium λ is defined as $\lambda(q) \equiv u'(q)/v'(q) - 1$ if $p^* > z$ and $\lambda(q) \equiv 0$ if $z \geq p^*$. Differentiating $V_{s,t}$ yields

$$\frac{\partial V_{s,t}}{\partial m_t} = \phi_t, \qquad \frac{\partial V_{s,t}}{\partial d} = \phi_t i_d, \qquad \frac{\partial V_{s,t}}{\partial \ell} = -\phi_t i_l.$$

Bank's Problem

- A representative bank accepts nominal deposit and lends nominal loan.
- The bank maximizes profit

$$\max_{d,\ell} \quad (i_l \ell - i_d d) \quad s.t. \quad \chi \ell \leq d \tag{10}$$

  subject to reserve requirement, χ
- FOCs are

$$0 = i_l - \lambda_L \tag{11}$$
$$0 = -i_d + \lambda_L / \chi \tag{12}$$

- For $\lambda_L > 0$, we have

$$i_l = \chi i_d$$

Equilibrium

Definition

Given (γ, χ), an equilibrium consists of the sequences of prices $\{\phi_t, i_{l,t}, i_{d,t}\}_{t=0}^{\infty}$, real balances $\{m_t, \ell_{b,t}, \ell_{s,t}, d_{b,t}, d_{s,t}\}_{t=0}^{\infty}$, and allocations $\{q_t, X_t, \ell_t\}_{t=0}^{\infty}$ satisfying the following:

- Agents solve CM and FM problems: (1) and (4)
- A representative bank solves its profit maximization problem: (10)
- Markets clear in every period:
  1. Deposit Market: $\sigma d_{b,t} + (1-\sigma) d_{s,t} = d_t$
  2. Loan Market: $\sigma \ell_{b,t} + (1-\sigma)\ell_{s,t} = \ell_t$
  3. Money Market: $m_t = M_t$

Equilibrium

Given (γ, χ), an equilibrium can be summarizes into the following difference equation:

$$z_t = f(z_{t+1}) \equiv \frac{z_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}) + 1\right] \tag{13}$$

where $1 + i \equiv \gamma/\beta$, $z_t = \phi_t m_t(1-\sigma+\sigma\chi)/\sigma\chi$, and $L(z) \equiv \lambda \circ v^{-1}(z)$ is liquidity premium.

Stationary Equilibrium

- Given $i \in [0, \bar{\iota})$ and $\chi \in (0,1]$ with $\bar{\iota} = \alpha(1-\sigma+\sigma\chi)L(0)/\chi$, an unique stationary monetary equilibrium exists satisfying

$$\chi i = (1-\sigma+\sigma\chi)\alpha L(z_s)$$

  where $z_s = v(q_s)$.

- Simple examples for $\bar{\iota}$ under the Inada condition $u'(0) = \infty$
  - with the Nash bargaining we have $\bar{\iota} = \infty$
  - with the Kalai bargaining we have $\bar{\iota} = \theta\alpha(1-\sigma+\sigma\chi)/\chi(1-\theta)$

Proposition

In the stationary equilibrium, lowering the nominal interest rate or lowering reserve requirement increases DM consumption.

Cycles

Recall the difference equation (13)

$$z_t = f(z_{t+1}) \equiv \underbrace{\frac{z_{t+1}}{1+i}}_{\text{increasing in } z_{t+1}} \underbrace{\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1})+1\right]}_{\text{decreasing in } z_{t+1}}$$

- $f(z_{t+1})$ is generally nonmonotone.
- If the second term dominates the first term, we can have $f'(\cdot) < -1$ which is a standard condition for the existence of cyclic equilibria
  - If $f'(z_s) < -1$, there is a two-period cycle with $z_1 < z_s < z_2$. (Azariadis 1993)

Proposition

Assume that the buyer makes a take-it-or-leave-it offer to the seller in the DM. Let $-qu''(q)/u' = \eta$ *and* $c(q) = q$*. If* $\chi \in (0, \chi_m)$*, where*

$$\chi_m \equiv \frac{\alpha\eta(1-\sigma)}{\eta(1-\alpha\sigma)+(2-\eta)(1+i)} \tag{14}$$

then $f'(z_s) < -1$*.*

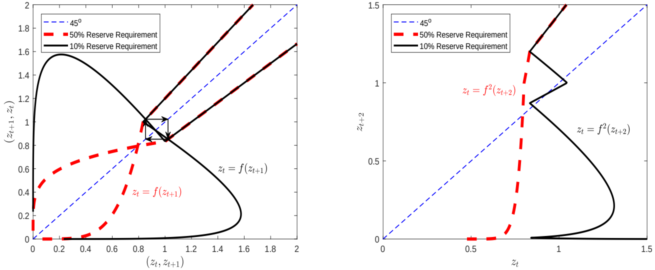

Figure 2: A Two-period Cycle under Fractional Reserve Banking

Proposition (**Three-period Monetary Cycle and Chaos**)

There exists a three-period cycle with $z_1 < z_2 < z_3$ *if* $\chi \in (0, \hat{\chi}_m)$, *where*

$$\hat{\chi}_m \equiv \frac{(1-\sigma)\alpha L\left(\frac{p^*}{1+i}\right)}{(1+i)^3 - 1 - \sigma\alpha L\left(\frac{p^*}{1+i}\right)}$$

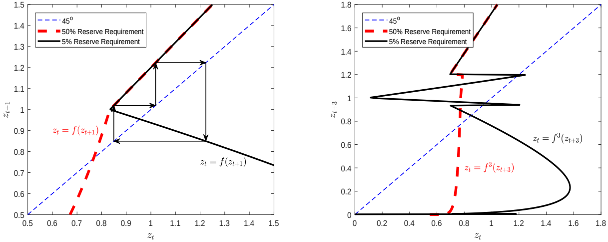

Figure 3: A Three-period Cycle under Fractional Reserve Banking

The existence of three period-cycle implies cycles of all orders as well as chaotic dynamics (see Sharkovskii 1964 and Li & Yorke 1975).

Self-Fulfilling Bubble and Burst Equilibria

- For simplicity, assume followings
  - the buyer makes a take-it-or-leave-it offer to the seller;
  - the DM utility function and the cost function satisfies $-qu''(q)/u'(q) = \eta$ and $c(q) = q$.
- Consider the equilibria that real balance increases above the steady state until certain time, T, and crashes to zero.
  - More specifically, consider a sequence of real balance $\{z_t\}_{t=0}^{\infty}$ with $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > z_s$ (bubble) that crashes to 0 (burst) as $t \to \infty$, where $T \geq 1$ and $z_T > z_0 > 0$.

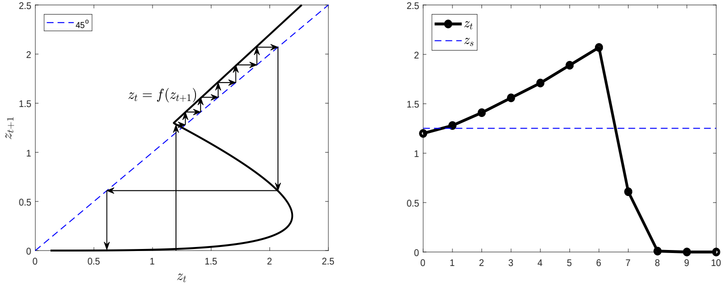

Figure 4: Self-Fulfilling Bubble and Burst Equilibria

Definition (**Self-Fulfilling Bubble and Burst Equilibria**)

For initial level of real balance $z_0 > 0$, a self-fulfilling bubble and burst is a set of sequence $\{z_t\}_{t=0}^{\infty}$ satisfying (15)

$$z_t = \frac{z_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha[u'(z_{t+1})-1]+1\right] \qquad (15)$$

where $0 < z_s < z_T$, $\lim_{t\to\infty} z_t = 0$, $z_T = \max\{z_t\}_{t=0}^{\infty}$ with $T \geq 1$.

Self-Fulfilling Bubble and Burst Equilibria

Proposition (**Existence of Self-Fulfilling Bubble and Burst Equilibria**)

There exist self-fullfilling bubble and burst equilibria, $\{z_t\}_{t=0}^{\infty}$ if

$$0 < \chi < \min\left\{\frac{(1-\sigma)\alpha\eta(1+i)}{(1-\eta)^2 q^* + (1+i)[(1-\eta)(3+i-\eta) - \alpha\sigma\eta]}, \frac{(1-\sigma)\alpha\eta}{2+i(2-\eta)-\alpha\sigma\eta}\right\}$$

- When $z_s > \bar{z}$, where $\bar{z}$ solves $f'(\bar{z}) = 0$, there exist multiple equilibria.
- Then, if $f(\bar{z}) \geq q^*$, the self-fulfilling bubble and burst equilibria exist.

Introducing Unsecured Credit

- Assume the buyer makes a take-it-or-leave-it offer to the seller in the DM and $c(q) = q$

$$V_t^b(m_t + \ell_t, 0, \ell_t) = \alpha[u(q_t) - q_t] + W_t(m_t + \ell_t, 0, \ell_t)$$

  where $q_t = \min\{q^*, \phi_t(m_t + \ell_t) + \bar{b}_t\}$.
- For compact notation, let $w_{t+1} \equiv z_{t+1} + \bar{b}_{t+1}$.
- Given $\bar{b}_t$, solving equilibrium yields

$$z_t = \begin{cases} \dfrac{z_{t+1}}{1+i}\left\{\dfrac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u'(q_{t+1}) - 1\right] + 1\right\} & \text{if } w_{t+1} < q^* \\ \dfrac{z_{t+1}}{1+i} & \text{if } w_{t+1} \geq q^*. \end{cases} \tag{16}$$

where $z_{t+1} = (1-\sigma+\sigma\chi)\phi_{t+1}m_{t+1}/(\sigma\chi)$ appendix

Endogenous Credit Limits

- Credit limit, $\bar{b}_t$, is determined by the incentive condition for voluntary repayment as Kehoe & Levine (1993).
- The buyer is captured with probability μ if she reneges.
- The punishment for a defaulter is permanent exclusion from the DM trade.
- The incentive condition for voluntary repayment is

$$\underbrace{-b_t + W_t(0,0,0)}_{\text{value of honoring debts}} \geq \underbrace{(1-\mu)W_t(0,0,0) + \mu\underline{W}(0,0,0)}_{\text{value of not honoring debts}}.$$

- where the value of autarky is $\underline{W}(0,0,0) = \{U(X^*) - X^* + T\}/(1-\beta)$

Use the incentive condition to get the difference equation of credit limit:

$$\bar{b}_t = \begin{cases} \beta \bar{b}_{t+1} + \dfrac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(w_{t+1}) & \text{if } w_{t+1} < q^* \\ \\ \beta \bar{b}_{t+1} + \dfrac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(q^*) & \text{if } w_{t+1} \geq q^* \end{cases} \tag{17}$$

where $S(w_{t+1}) \equiv [u(w_{t+1}) - w_{t+1}]$ is the buyer's trade surplus and $w_{t+1} = z_{t+1} + \bar{b}_{t+1}$.

The equilibrium can be collapsed into a dynamic system satisfying (16)-(17).

See more on money-credit economy

Cycles with Unsecured Credit

For compact notation, let $\iota \equiv \max\{i, r\}$ where $r = 1/\beta - 1$.

Proposition (**Monetary Cycles with Unsecured Credit**)

There exist two period cycles of money and credit with $w_1 < q^* < w_2$ if $\chi \in (0, \chi_c)$, where $w_j = z_j + \bar{b}_j$ and

$$\chi_c \equiv \frac{(1-\sigma)\alpha\left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}{(1+i)^2 - 1 - \sigma\alpha\left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}.$$

There exist three period cycles of money and credit with $w_1 < q^* < w_2 < w_3$, if $\chi \in (0, \hat{\chi}_c)$, where

$$\hat{\chi}_c \equiv \frac{(1-\sigma)\alpha\left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}{(1+i)^3 - 1 - \sigma\alpha\left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}.$$

Other Applications

Sunspot cycles

- Stochastic cycles which are independent from the fundamental.

Sunspot cycles

Empirical Evaluation

- Negative association between required reserve ratio and volatility of real balance of inside money.

See empirical evaluation

Conclusion

- Lowering reserve requirement induce instability: more prone to exhibit endogenous cyclic, chaotic, and stochastic dynamics
- This result holds in the extended model with credit.
- Lowering the reserve requirement increases the welfare at the steady state.

Appendix

Trade Mechanisms

A general trading mechanism Γ mapping the buyer's z_b into pair (p, q) with it feasibility constraint $p \leq z_b$

- **Axiom 1**: (Feasibility)
  $\forall z,\ 0 \leq \Gamma_p(z) \leq z, 0 \leq \Gamma_q(z)$.
- **Axiom 2**: (Individual Rationality)
  $\forall z,\ u \circ \Gamma_q(z) \geq \Gamma_p(z)$ and $\Gamma_p(z) \geq c \circ \Gamma_q(z)$
- **Axiom 3**: (Monotonicity)
  $\Gamma_p(z_2) > \Gamma_p(z_2) \Leftrightarrow \Gamma_q(z_2) > \Gamma_q(z_2)$
- **Axiom 4**: (Bilateral Efficiency)
  $\forall z,\ (p', q')$ with $p' \geq z$ such that
  $u(q') - p' \leq u \circ \Gamma_q(z) - \Gamma_p(z)$ and
  $p' - c(q') \geq \Gamma_p(z) - c \circ \Gamma_q(z)$

Trade Mechanisms

- Let $p^* = \inf\{\hat{z}_b : \Gamma_p(\hat{z}_b) = q^*\}$ be a payment to get q^*.
- Gu & Wright (2016) show that Any Γ satisfying Axioms 1-4 takes the following form

$$\Gamma_p(z) = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{otherwise} \end{cases} \qquad \Gamma_q(z) = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{otherwise} \end{cases}$$

  where v is some strictly increasing function with $v(0) = 0$ and $v(q^*) = p^*$

DM trade

Sunspot Cycles

- Consider a Markov sunspot variable $S \in \{1, 2\}$. This sunspot variable is not related with fundamentals.
- Let $Pr(S_{t+1} = 1 | S_t = 1) = \zeta_1$, $Pr(S_{t+1} = 2 | S_t = 2) = \zeta_2$
- The sunspot is realized in the CM.
- CM value function is written as

$$
\begin{aligned}
W_t^S(m_t, d_t, \ell_t) = \max_{X_t, H_t, \hat{m}_{t+1}} & \; U(X_t) - H_t \\
& + \beta \left[\zeta_s G_{t+1}^S(\hat{m}_{t+1}) + (1 - \zeta_s) G_{t+1}^{-S}(\hat{m}_{t+1}) \right]
\end{aligned}
$$

s.t. $\phi_t^S \hat{m}_{t+1} + X_t = H_t + T_t + \phi_t^S m_t + (1 + i_{d,t}) \phi_t^S d_t - (1 + i_{l,t}) \phi_t^S \ell_t.$

- The first order condition can be written as

$$
-\phi_t^S + \beta \zeta_s G_{t+1}'^S(\hat{m}_{t+1}) + \beta (1 - \zeta_s) G_{t+1}'^{-S}(\hat{m}_{t+1}) = 0. \qquad (18)
$$

$$G'^{S}_{t+1}(m^{S}_{t+1}) = \phi^{S}_{t+1}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{S}_{t+1}) + 1\right] \tag{19}$$

Substituting (19) into (18) and multiplying $(1-\sigma+\sigma\chi)m_{t+1}/(\sigma\chi)$ to the both sides yield

$$\begin{aligned} z^{S}_{t} =& \frac{\zeta_s z^{S}_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{S}_{t+1}) + 1\right] \\ &+ \frac{(1-\zeta_s) z^{-S}_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{-S}_{t+1}) + 1\right] \\ =& \zeta_s f(z^{S}_{t+1}) + (1-\zeta_s) f(z^{-S}_{t+1}) \end{aligned} \tag{20}$$

Sunspot Cycles

Definition (**Proper Sunspot Equilibrium**)

A proper sunspot equilibrium consists of the sequences of real balances $\{z_t^S\}_{t=0,S=1,2}^{\infty}$, where z_1 is not equal to z_2, and probabilities (ζ_1, ζ_2), solving (20) for all t.

Using the textbook treatment from Azariadis (1993), it is straightforward to show that if $f'(z_s) < -1$, there exist (ζ_1, ζ_2), $\zeta_1 + \zeta_2 < 1$, such that the economy has a proper sunspot equilibrium in the neighborhood of z_s.

Back to other applications

Equilibrium

The equilibrium can be collapsed in to a dynamic system satisfying (21)-(22).

$$z_t = \begin{cases} \frac{z_{t+1}}{1+i}\left\{\frac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u'(w_{t+1})-1\right]+1\right\} & \text{if } w_{t+1} < q^* \\ \frac{z_{t+1}}{1+i} & \text{if } w_{t+1} \geq q^*. \end{cases} \tag{21}$$

$$\bar{b}_t = \begin{cases} \beta\bar{b}_{t+1} + \frac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(w_{t+1}) & \text{if } w_{t+1} < q^* \\ \beta\bar{b}_{t+1} + \frac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(q^*) & \text{if } w_{t+1} \geq q^* \end{cases} \tag{22}$$

where $z_{t+1} = (1-\sigma+\sigma\chi)\phi_{t+1}m_{t+1}/(\sigma\chi)$, $w_{t+1} = z_{t+1} + \bar{b}_{t+1}$, and $S(z_{t+1}+\bar{b}_{t+1}) \equiv [u(z_{t+1}+\bar{b}_{t+1}) - z_{t+1} - \bar{b}_{t+1}]$.

Stationary Equilibrium

Let $r = 1/\beta - 1$. The debt limit at the stationary equilibrium, $\bar{b}$, is a fixed point satisfying $\bar{b} = \Omega(\bar{b})$ where

$$\Omega(\bar{b}) = \begin{cases} \frac{\mu\sigma\alpha}{r}[u(\tilde{q}) - \tilde{q}] - \frac{i\mu\sigma\chi}{1-\sigma+\sigma\chi}[\tilde{q} - \bar{b}] & \text{if } \tilde{q} > \bar{b} \geq 0 \\ \frac{\mu\sigma\alpha}{r}[u(\bar{b}) - \bar{b}] & \text{if } q^* > \bar{b} \geq \tilde{q} \\ \frac{\mu\sigma\alpha}{r}[u(q^*) - q^*] & \text{if } \bar{b} \geq q^* \end{cases} \tag{23}$$

where $\tilde{q}$ solves $u'(\tilde{q}) = 1 + i\chi/[\alpha(1-\sigma+\sigma\chi)]$.
Money and credit coexist if and only if $0 < \mu < \min\{1, \tilde{\mu}\}$, where

$$\tilde{\mu} \equiv \sigma \left\{ i\chi[(1-\sigma+\sigma\chi)/\tilde{q} - 1] + (\alpha/r)(1-\sigma+\sigma\chi)^2[u(\tilde{q})/\tilde{q} - 1] \right\}$$

since they coexist when $\bar{b} < \tilde{q}$. The DM consumption is decreasing in i in the monetary equilibrium.

Back

Empirical Appendix

Table 1: Effect of Required Reserve Ratio

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: σ_t^{Roll}	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
χ	-0.283***	-0.245***	-0.267***	-0.221***	-0.306***	-0.227***	-0.307***	-0.220***
	(0.027)	(0.002)	(0.027)	(0.003)	(0.029)	(0.004)	(0.027)	(0.005)
`ffr`		-0.109***		-0.125***		-0.187***		-0.207***
		(0.002)		(0.003)		(0.004)		(0.004)
Constant	0.074***	0.074***	0.070***	0.071***	0.074***	0.075***	0.073***	0.073***
	(0.003)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)
Obs.	49	49	49	49	49	49	49	49
$adjR^2$	0.700	0.621	0.728	0.648	0.740	0.650	0.764	0.665
$\lambda_{trace}(r=0)$	**9.807**	35.688	**9.120**	35.145	**9.109**	35.367	**8.593**	35.028
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	3.324	**10.682**	2.839	**10.065**	2.723	**9.894**	2.417	**9.345**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), (3), (5) and (7), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2), (4), (6), and (8), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; χ denotes the required reserve ratio, `ffr` denotes federal funds rates and σ_t^{Roll} denotes the cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Empirical Appendix

Table 2: Unit Root Tests

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`		-6.766	-1.704	-2.362
χ		-1.492	-1.173	-1.341
σ_t^{Roll}	(CPI)	-4.708	-2.191	-2.090
σ_t^{Roll}	(Core CPI)	-4.681	-2.189	-1.978
σ_t^{Roll}	(PCE)	-4.329	-2.038	-2.047
σ_t^{Roll}	(Core PCE)	-4.076	-1.954	-1.930
Δ`ffr`		-28.373***	-5.061***	-6.357***
$\Delta\chi$		-31.818***	-4.802***	-3.693***
$\Delta\sigma_t^{Roll}$	(CPI)	-24.905***	-3.416**	-2.942**
$\Delta\sigma_t^{Roll}$	(Core CPI)	-24.758***	-3.509**	-2.942**
$\Delta\sigma_t^{Roll}$	(PCE)	-23.691***	-3.330**	-2.842*
$\Delta\sigma_t^{Roll}$	(Core PCE)	-22.826***	-3.296**	-2.768*

Note: `ffr` denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Empirical Appendix

Table 3: Effect of Required Reserve Ratio:Robustness Check (Quarterly)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent	OLS	CCR	OLS	CCR	OLS	CCR	OLS	CCR
variable: σ_t^{Roll}	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
χ	-0.282***	-0.452***	-0.266***	-0.400***	-0.305***	-0.485***	-0.306***	-0.476***
	(0.016)	(0.001)	(0.014)	(0.003)	(0.015)	(0.000)	(0.014)	(0.006)
ffr		-0.050***		-0.058***		-0.015***		-0.047***
		(0.000)		(0.002)		(0.000)		(0.005)
Constant	0.074***	0.085***	0.070***	0.079***	0.074***	0.089***	0.073***	0.086***
	(0.002)	(0.000)	(0.002)	(0.000)	(0.002)	(0.000)	(0.002)	(0.001)
Obs.	196	196	196	196	196	196	196	196
$adjR^2$	0.696	0.240	0.725	0.263	0.737	0.222	0.761	0.268
$\lambda_{trace}(r=0)$	**9.496**	31.950	**11.045**	33.808	**10.930**	34.481	**12.103**	35.951
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	1.677	**11.162**	1.959	**12.266**	1.938	**12.094**	1.887	**12.485**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; ffr denotes federal funds rates and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Empirical Appendix

Table 4: Unit Root Tests:Robustness Check (Quarterly)

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`		-8.611	-1.956	-2.183
χ		-1.335	-1.145	-1.199
σ_t^{Roll}	(CPI)	-4.320	-2.062	-1.554
σ_t^{Roll}	(Core CPI)	-4.388	-2.201	-1.924
σ_t^{Roll}	(PCE)	-3.822	-1.946	-1.868
σ_t^{Roll}	(Core PCE)	-3.565	-1.928	-2.023
Δ`ffr`		-139.701***	-10.792***	-10.288***
$\Delta\chi$		-163.796***	-12.272***	-9.909***
$\Delta\sigma_t^{Roll}$	(CPI)	-23.132***	-2.604*	-3.576***
$\Delta\sigma_t^{Roll}$	(Core CPI)	-30.423***	-3.544***	-4.894***
$\Delta\sigma_t^{Roll}$	(PCE)	-24.507***	-2.874*	-4.362***
$\Delta\sigma_t^{Roll}$	(Core PCE)	-28.054***	-3.373**	-5.138***

Note: `ffr` denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Empirical Appendix

Table 5: Effect of Required Reserve Ratio:Robustness Check (pre-2008)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: σ_t^{Roll}	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
χ	-0.266***	-0.297***	-0.266***	-0.268***	-0.307***	-0.288***	-0.305***	-0.277***
	(0.030)	(0.001)	(0.030)	(0.001)	(0.032)	(0.002)	(0.029)	(0.002)
`ffr`		-0.107***		-0.124***		-0.189***		-0.210***
		(0.001)		(0.001)		(0.002)		(0.002)
Constant	0.070***	0.080***	0.070***	0.076***	0.074***	0.082***	0.072***	0.080***
	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.002)
Obs.	43	43	43	43	43	43	43	43
$adjR^2$	0.727	0.659	0.727	0.710	0.739	0.708	0.759	0.734
$\lambda_{trace}(r=0)$	**8.373**	32.228	**7.438**	31.299	**7.661**	31.867	**6.897**	31.250
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	1.504	**9.554**	1.125	**8.428**	1.146	**8.603**	0.938	**7.693**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; `ffr` denotes federal funds rates and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Empirical Appendix

Table 6: Unit Root Tests:Robustness Check (pre-2008)

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
$\texttt{ffr}$		-9.476	-2.258	-2.868**
χ		-0.768	-0.660	-0.877
σ_t^{Roll}	(CPI)	-2.966	-1.738	-1.770
σ_t^{Roll}	(Core CPI)	-2.860	-1.641	-1.495
σ_t^{Roll}	(PCE)	-2.662	-1.515	-1.627
σ_t^{Roll}	(Core PCE)	-2.412	-1.371	-1.400
$\Delta\texttt{ffr}$		-25.378***	-4.773***	-5.833***
$\Delta\chi$		-28.208***	-4.594***	-3.658***
$\Delta\sigma_t^{Roll}$	(CPI)	-25.627***	-4.281***	-3.813***
$\Delta\sigma_t^{Roll}$	(Core CPI)	-25.836***	-4.329***	-3.764***
$\Delta\sigma_t^{Roll}$	(PCE)	-24.420***	-4.101***	-3.594**
$\Delta\sigma_t^{Roll}$	(Core PCE)	-23.848***	-4.034***	-3.464**

Note: $\texttt{ffr}$ denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

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References

Andolfatto, D., Berentsen, A. & Martin, F. M. (2020), 'Money, banking, and financial markets', *The Review of Economic Studies* **87**(5), 2049–2086.

Azariadis, C. (1993), *Intertemporal Macroeconomics*, Blackwell Publishing, Cambridge, MA.

Baumol, W. J. & Benhabib, J. (1989), 'Chaos: Significance, mechanism, and economic applications', *Journal of Economic Perspectives* **3**(1), 77–105.

Benhabib, J. & Farmer, R. E. (1999), Indeterminacy and sunspots in macroeconomics, *in* J. B. Taylor & M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 6, pp. 387–448.

Berentsen, A., Camera, G. & Waller, C. (2007), 'Money, credit and banking', *Journal of Economic Theory* **135**(1), 171–195.

Chari, V. & Phelan, C. (2014), 'On the social usefulness of fractional reserve banking', *Journal of Monetary Economics* **65**, 1–13.

Fisher, I. (1935), *100% money*, Adelphi Company, New York.

Freeman, S. & Huffman, G. W. (1991), 'Inside money, output, and causality', *International Economic Review* pp. 645–667.

Freeman, S. & Kydland, F. E. (2000), 'Monetary aggregates and output', *American Economic Review* **90**(5), 1125–1135.

Gu, C., Mattesini, F., Monnet, C. & Wright, R. (2013), 'Endogenous credit cycles', *Journal of Political Economy* **121**(5), 940–965.

Gu, C., Mattesini, F. & Wright, R. (2016), 'Money and credit redux', *Econometrica* **84**(1), 1–32.

Gu, C., Monnet, C., Nosal, E. & Wright, R. (2019), On the Instability of Banking and Financial Intermediation, Working Papers 1901, Department of Economics, University of Missouri.
URL: *https://ideas.repec.org/p/umc/wpaper/1901.html*

Gu, C. & Wright, R. (2016), 'Monetary mechanisms', *Journal of Economic Theory* **163**, 644–657.

Kehoe, T. J. & Levine, D. K. (1993), 'Debt-constrained asset markets', *The Review of Economic Studies* **60**(4), 865–888.

Li, T.-Y. & Yorke, J. A. (1975), 'Period three implies chaos', *The American Mathematical Monthly* **82**(10), 985–992.

Lotz, S. & Zhang, C. (2016), 'Money and credit as means of payment: A new monetarist approach', *Journal of Economic Theory* **164**, 68–100.

Minsky, H. P. (1957), 'Monetary systems and accelerator models', *American Economic Review* **47**(6), 860–883.

Minsky, H. P. (1970), 'Financial instability revisited: The economics of disaster'. Available from `https://fraser.stlouisfed.org/files/docs/historical/federa%20reserve%20history/discountmech/fininst_minsky.pdf`.

Sargent, T. J. (2011), 'Where to draw lines: Stability versus efficiency', *Economica* **78**(310), 197–214.

Sharkovskii, A. (1964), 'Cycles coexistence of continuous transformation of line in itself', *Ukr. Math. Journal* **26**(1), 61–71.

Von Mises, L. (1953), *The Theory of Money and Credit*, Yale University Press.