# Money Creation and Banking: Theory and Evidence*

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**Abstract**

This paper studies the role of banks' money creation in the monetary transmission. I develop a monetary-search model in which the demand for monetary base and the money multiplier are endogenously determined through the banks' money creation channel. I evaluate the theory by matching it with data, and the calibrated model can account for the evolution of the quantity of reserves, excess reserves, and the money multiplier from 1968 to 2015

**JEL Classification Codes:** E42, E51

**Keywords:** Money, Credit, Interest on Reserves, Banking, Monetary Policy

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> [A] model of the banking system in which currency, reserves, and deposits play distinct roles ... seems essential if one wants to consider policies like reserves requirements, interest on deposits, and other measures that affect different components of the money stock differently. **Lucas (2000)**

# 1 Introduction

This paper develops a theory of money and banking that articulates banks' role in money creation and its interaction with credit. The key focal points are the endogenous determination of demand for base money through banking, and the central bank's control of base money to peg the short-term interest rate, which in turn influences the bank's money creation activity and other macroeconomic variables.

As pointed about by Romer (2000), the central bank conducts the monetary policy through the intervention in the market for base money. However, most of the leading models of monetary policy analysis do not consider the transmission mechanism via the monetary base market. For example, the New Keynesian models abstract from the money market mechanism. In the New Monetarist framework, where monetary frictions are explicit, most models focus on monetary aggregates rather than the transmission mechanism from the market for monetary base to the monetary aggregate through the banking sector. Rather than abstract from the key mechanism, this paper revisits the issue of money creation to understand the role of banking in monetary transmission and account for a number of observations in a unified framework.

Specifically, Section 2 provides four empirical findings from US data, which guide the modeling.

1. The reserves are not independent of interest rates during 2009-2015. The reserves and the banks' money creation activities are closely related to the interest rates.
2. The excess reserves to deposit ratio had been close to zero until 2007. The excess reserve ratio skyrocketed as the Fed introduced the interest on reserve.
3. The required reserve ratio does not have a negative relationship with the M1 money multiplier, and there were two structural breaks in the evolution of the M1 money multiplier: 1992 and 2008.
4. Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping, stable M1 money demand.

This paper advances a theory of money and banking, which accounts for all four observations mentioned above as well as the money creation process in the US economy.

This paper builds a monetary banking model based on Lagos and Wright (2005) to understand the monetary transmission. The model features the explicit structure of monetary exchange and the role of financial intermediations in money creation. Agents can trade by using cash, transaction deposit, and unsecured credit (e.g., credit card). The banks create deposits by making loans, which can be either transaction deposits or non-transaction deposits. However, the creation of transaction deposits is constrained by reserve requirements. The bank's demand for reserves, whether banks hold excess reserves, and the money multiplier are endogenously determined, with the central bank's monetary policy and credit conditions of the economy playing crucial roles.

When banks hold excess reserves, the bank's reserve requirement constraint does not bind and a change in reserve requirement does not change the money multiplier. Instead, the money multiplier is determined by the nominal interest rate and the interest on reserves. Lowering the nominal interest rate increases reserves, but the banks do not create deposit money proportionally, which lowers the money multiplier. A higher interest on reserves decreases the money multiplier because the bank has more incentive to hold reserves and less incentive to deposit money. The interest on reserves and the nominal interest rate play distinct roles and they jointly determine the quantity of reserves.

Unsecured credit can substitute for other means of payment as in Gu, Mattesini and Wright (2016).[1] Better credit conditions lower transaction deposit balances as credit can substitute for money. This decrease in deposit balances leads to a lower money multiplier, regardless of whether banks hold excess reserves or not. When banks hold excess reserves, the money multiplier decreases as the interest on reserves decreases, but it increases when short-term policy rates increase. The model also generates a positive pass-through from short-term policy rates to nominal lending rates.

Next, I quantify the model by calibrating the model and asking how it accounts for observations in Section 2 and the money creation process. Given monetary policy behaved as it did, how well can the model account for the behavior of reserves, excess reserves, and money multiplier? The analysis shows the model can explain the historical evolution of the money creation process including all the observations from Section 2. Consistent with data, the model generates zero excess reserves between 1968 and 2007 as well as massive increases in excess reserves after 2008. The model gives the

[1] By modeling unsecured credit with an exogenous credit limit, this paper follows Gu et al. (2016). For other approaches to introducing credit to the monetary economy, see Sanches and Williamson (2010), Lotz and Zhang (2016) and Williamson (2016).

counterfactual reserves to output ratio which closely tracks its actual behavior from 1968 to 2015. It also generates drops in money multiplier during the 1990s and 2000s without excess reserves and its more drastic drops after 2008 were accompanied by the massive increase in excess reserves. The quantitative exercise shows that dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves whereas the decrease in the money multiplier before 2008 is driven by better credit conditions.

Contrary to previous approaches that assumed no changes in the short-term interest rate during 2009-2015 (so-called zero lower bound period) and merely focused on the quantity of reserves, this study confirms that the short-term interest rate did change during that period, and its movements directly reflected in the quantity of reserves.

**Related Literature** This paper contributes to three strands of literature. First, it contributes to the growing literature on unconventional monetary policy and bank reserves. Many models of unconventional monetary policy take the zero lower bound constraint as a given and focus on the effect of asset purchases by issuing reserves, which is assumed to be independent of short-term interest rates. (Curdia and Woodford, 2011, Gertler and Karadi, 2011 and Lee, 2021). By focusing on which assets the central bank purchases, some recent works (e.g., Williamson, 2016 and Bhattarai, Eggertsson and Gafarov, 2015) study quantitative easing as reducing maturity of government debt. They take ZLB bound constraint seriously and treat bank reserves and short-term bonds as perfect substitutes under ZLB. Given this ZLB constraint, they focus on the effect of maturity transformation. Whereas these models assume independence of reserves to short-term interest rates, this paper shows that this independence assumption does not hold in the data. In contrast to the conventional approach to unconventional monetary policy, the quantitative analysis shows that the changes in reserves correspond to the changes in short-term interest rates. Rather than relying on the assumption of independence, this paper articulates an explicit determination mechanism that determines the demand for reserves and the money multiplier.

Second, this paper contributes to a large literature on inside money and money creation. The difference between inside money and outside money has been highlighted since Sargent and Wallace (1982) and Freeman and Huffman (1991). Previous works capturing the explicit role of reserve requirements include Freeman (1987), Haslag and Young (1998), and Freeman and Kydland (2000). Freeman (1987) and Haslag and Young (1998) study the impact of money creation and the reserve requirements on seigniorage revenue. Freeman and Kydland (2000) develop a tractable model of endogenous money multiplier. They show that the money-output correlation can be explained by the endogenous money supply resulting from households' choices in response to the business cycle. Recent advances in monetary economics based on search-theoretic framework provide a deeper understanding on banking and inside money. For example, Gu, Mattesini, Monnet and Wright (2013) study the environment where banking arises endogenously, and they show that banking can improve the economy by facilitating trade using inside money. Andolfatto, Berentsen and Martin (2020) integrate a model of bank and financial markets by Diamond (1997) with the Lagos and Wright (2005) framework and deliver a model where the fractional reserve banking arises in the equilibrium. This paper contributes to the literature by constructing a search model of money and banking which can explain the money creation process observed from data.

Third, this work relates to the literature that studies money and credit explicitly. Gu et al. (2016) show that if money is essential, the nonmonetary credit is irrelevant. Changes in credit conditions only crowd out real balances. This neutrality result can be overturned if one introduces the costly credit (Bethune, Choi and Wright, 2020 Wang, Wright and Liu, 2020). Han and He (2019) show that in an economy in which both monetary credit and nonmonetary credit are used, credit is relevant even when money is essential. In this paper, there are fiat money costly monetary credit (deposit money), and nonmonetary credit (unsecured credit), and the neutrality result does not hold. This is because there exists monetary credit and the bank's intermediation of monetary credit is costly.

This paper is organized as follows. Section 2 provides motivating evidence. Section 3 constructs the search-theoretic monetary model of money creation. Section 4 calibrates the model to quantify the theory. Section 5 concludes.

## 2 Motivating Evidence

This section presents a list of observations about money creation and money demand, which motivates the theoretical framework developed in the next sections.

**Observation 1.** The reserves are not independent of interest rates during 2009-2015. The interest rates are closely related to the quantity of reserves and the bank's money creation activity.

The left panel of Figure 1 shows the reserves to deposit ratio and the spread between the federal funds rate and the interest on reserves during 2009-2015, which is often called the zero lower bound (ZLB) period. Their opposite movements are evident. This contradicts many monetary models of unconventional monetary policies in the post-2008 era, which assume the independence of the quantity of reserves from interest rate management in an ample-reserves regime. (e.g., Bech and Klee, 2011; Curdia and Woodford, 2011; Kashyap and Stein, 2012; Cochrane, 2014; Ennis, 2018; Piazzesi, Rogers and Schneider, 2019).

The assumption of independence is based on the idea that the interest rate paid on reserves sets a floor for the short-term policy rate. When the target policy rate reaches the interest rate on reserves, money is "divorced" from interest rate management, allowing the central bank to determine reserves independently of the interest rate. However, in contrast to this notion, interest on reserves has been the upper bound of target policy rates. Appendix A provides a further discussion on interest on reserves and the overnight reverse repurchase facility.

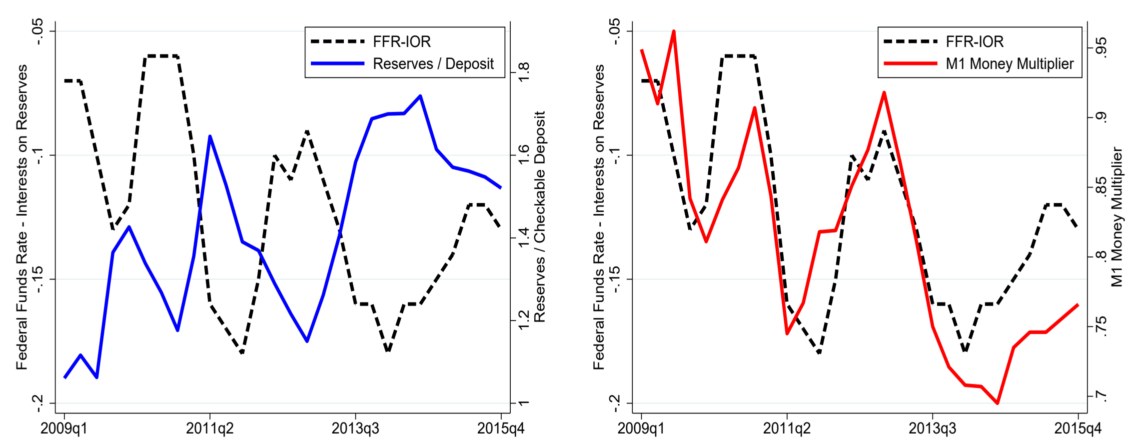


**Figure 1:** US excess reserves and M1 multiplier in the post-2008 period

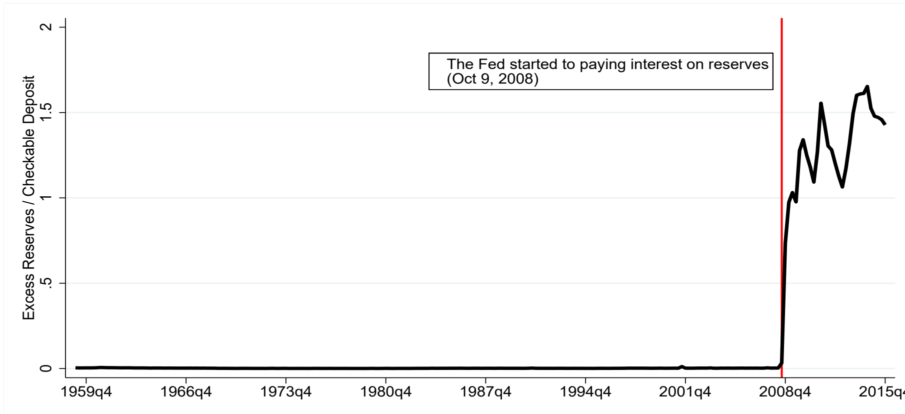


**Figure 2:** Excess reserves ratio

In addition, the right panel of Figure 1 shows that the M1 money multiplier moves together with the interest rate spread. This implies that the monetary policy during the post-2008 period is closely related to the bank's money creation activity, which was not given much attention in many monetary models of the last few decades.

**Observation 2.** The excess reserves to deposit ratio had been zero until 2007. The excess reserve ratio skyrocketed as the Fed introduced the interest on reserve.

Figure 2 plots the excess reserves to deposit ratio from 1959Q1 to 2015Q4. Before the 2008 Great Recession, the ratio remained at zero. However, it rose drastically after the recession and exceeded the value of 1, implying that banks have held more excess reserves than the amount of checking accounts they issued. Figure 2 also shows that the dramatic increase in excess reserves coincided with the Fed's introduction of the interest on reserves, as pointed out by Nakamura (2018).

**Observation 3.** The required reserve ratio does not have a negative relationship with the money multiplier, and there were two structural breaks in the evolution of the money multiplier: 1992 and 2008.

The top-left panel of Figure 3 plots the US M1 multiplier over time. Whereas the money multiplier decreased drastically since 2008, the US M1 money multiplier already had been declining since the early 1990s. An increase in required reserves did not accompany this decrease in the money multiplier. The bottom-left panel of Figure 3 plots the M1 multiplier against the required reserves.[2] There is no negative

[2] The required reserve ratio presented in Figure 3 is computed by (Required Reserves)/(Total

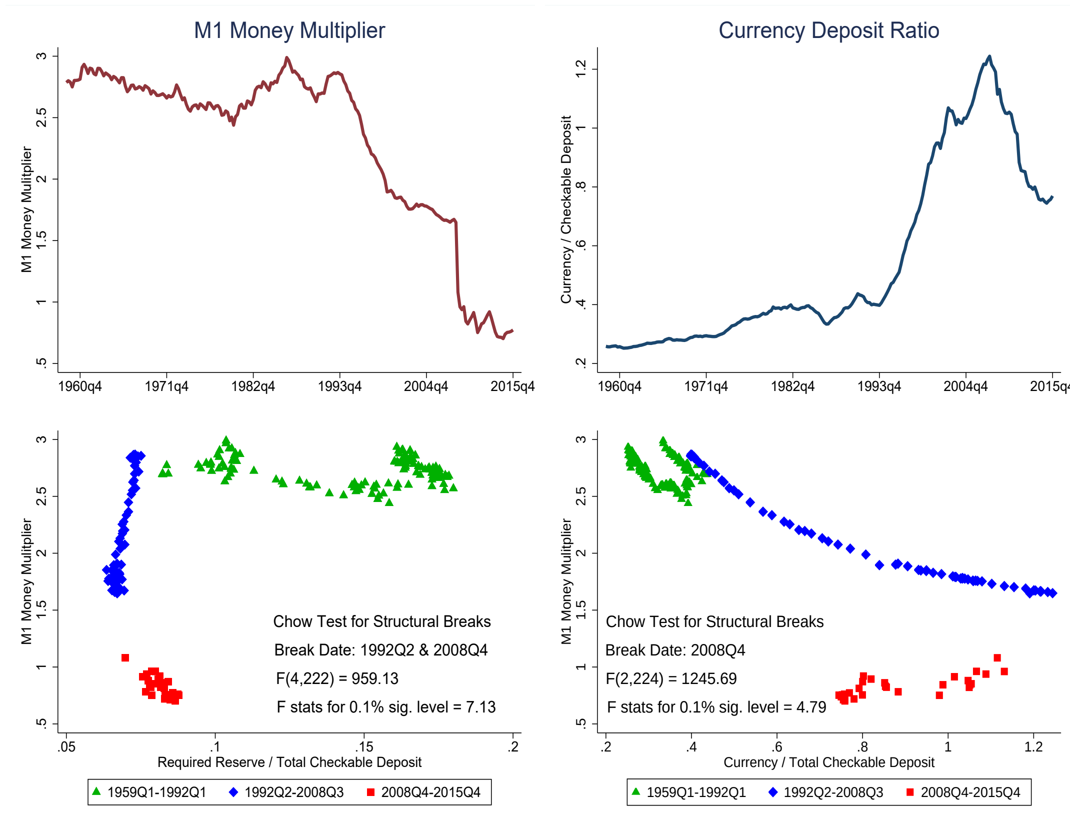


**Figure 3:** Money multiplier, currency/deposit ratio and required reserve ratio

Chow tests for structural breaks are implemented. The bottom-left panel reports a test statistic with the null hypothesis of no structural breaks in 1992Q2 and 2008Q4 and the bottom-right panel reports a test statistic with the null hypothesis of no structural break in 2008Q4. Sample periods are 1959Q1-2015Q4. Appendix C.1 contains details of the Chow tests.

relationship, which differs from the textbook explanation of the money multiplier.

The left panels of Figure 3 show that the US M1 money multiplier has been decreasing since 1992. However, the declining trends before and after 2008 are different. As the top-right panel of Figure 3 shows, the decline during 1992-2007 is accompanied by a huge increase in the ratio of currency to deposit, whereas the decline after 2008 is accompanied by a huge drop in the ratio of currency to deposit. It is also worth noting that the M1 multiplier has been lower than 1 since 2009 which contradicts to the textbook theory of money creation.

The bottom-left panel of Figure 3 displays two structural breaks in the relationship

Checkable Deposits). The legal reserve requirement for net transaction accounts was 10% from April 2, 1992, to March 25, 2020, but some banks are imposed upon by lower requirements or exempt depending on the size of their liabilities. These criteria changed 27 times from the 1st quarter of 1992 to the last quarter of 2019. From March 2020, all the required reserve ratios have become zero. See Feinman (1993) and https://www.federalreserve.gov/monetarypolicy/reservereq.htm for more details on the historical evolution of the reserve requirement policy of the United States.

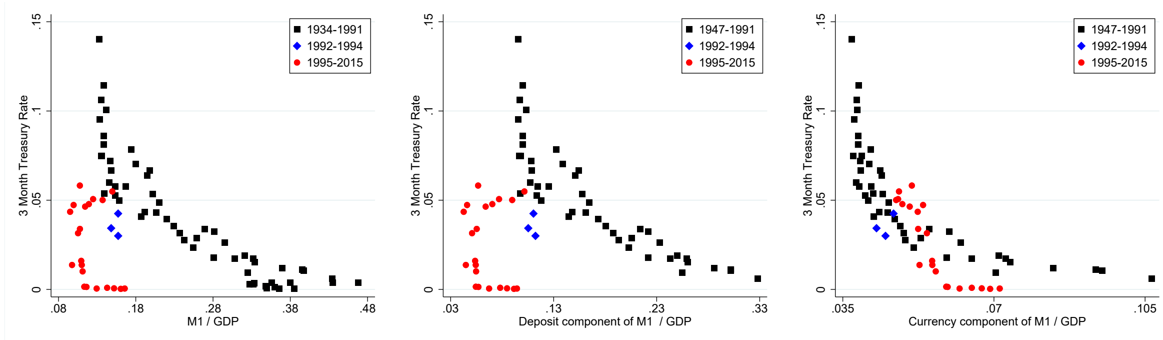


**Figure 4:** US Money demand for M1 and its components

between the M1 multiplier and the required reserve ratio: one in 1992Q3 and another in 2008Q4. In addition, the top-left panel of Figure 3 shows a structural break in the relationship between the M1 multiplier and the currency deposit ratio, which occurred in 2008Q4. The Chow tests for these breaks are described in more detail in Appendix C.2. The structural break of 2008Q4 coincided with the dramatic increase in excess reserves, which is illustrated in Fact 2.

**Observation 4.** Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping, stable M1 money demand.

Observation 3, discussed earlier, identified two structural breaks in 1992 and 2008. To gain a better understanding of the 1992 break, M1 can be decomposed into its deposit and currency components, as illustrated in Figure 4. The figure plots the ratio of M1 and its components to GDP against the 3-month Treasury Bill rate, revealing a breakdown in M1 around 1992 that coincided with the structural break observed in Figure 3.[3] As noted by Lucas and Nicolini (2015), this breakdown was caused by the deposit component rather than the currency component, which displays stable downward-sloping demand.

If an increased availability of consumer credit crowds out deposits but not cash, which implies that once one account for the substitution effect of the newly available consumer credit, there should still be a negative relationship between the real money balance and the interest rate. Following Cagan (1956), and Ireland (2009), I relate the natural logarithm of $m$, the ratio of money balances to income, to the short-term

[3]One may think this is due to the introduction of retail sweep accounts and ATS in the 1990s. However, ATS was introduced in 1994 and the break of M1 occurred in 1992 which was before 1994. In addition to that, using sweep adjusted M1, Kejriwal, Perron and Yu (2022) also found a similar structural break. See Appendix B.3 for more discussion.

**Table 1:** Cointegration regressions and tests

| Dependent Variable: | $ln(m_t)$ | | $ln(d_t)$ | |
|---|---|---|---|---|
| | OLS | CCR | OLS | CCR |
| | (1) | (2) | (3) | (4) |
| $r_t$ | 1.600*** | −2.752** | 4.928*** | −4.990* |
| | (0.419) | (1.171) | (0.944) | (2.647) |
| $ln(uc_t)$ | | −0.282*** | | −0.583*** |
| | | (0.098) | | (0.124) |
| $adjR^2$ | 0.109 | 0.981 | 0.230 | 0.959 |
| Observation | 112 | 112 | 112 | 112 |
| Johansen $r = 0$ | 15.031 | 50.151 | 10.348 | 53.881 |
| Johansen $r = 1$ | 0.034 | 9.842 | 0.563 | 10.538 |

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in columns (1)-(4). Appendix C.2 contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

nominal interest rate, denoted by $r$. I also regress $r$ on the natural logarithm of $d$, the ratio of deposit balances to income.

$$ln(m_t) = \beta_0 + \beta_1 r_t + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \epsilon_t$$

In addition to the above specifications, to capture the impact of the improved availability of consumer credit that can substitute the deposit, I add a logarithm of $uc$, the ratio of unsecured credit to income as another regressor as follows.[4]

$$ln(m_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t$$

I focus on the post-1980 period, until the arrival of the Great Recession. In Table 1, columns (1) and (3) report the estimates without unsecured credit, and columns (2) and (4) report the estimates with unsecured credit. The Johansen tests in columns (1) and (3) fail to reject the null hypothesis of no cointegration, which confirms the apparent breakdowns from Figure 4, and ordinary least squares (OLS) estimates from columns (1) and (3) both report positive coefficients on $r_t$ that contradict the conventional notion of money demand: the stable downward-sloping relationship between real balances and interest rates. In columns (2) and (4), however, the Johansen tests reject their null of no cointegration at a 99 percent confidence level, suggesting there exists a stable relationship between real money balances, interest rates, and real balances of unsecured credit. To estimate the cointegration relationship, I implement the

[4]Following to Krueger and Perri (2006), I use revolving consumer credit.

canonical cointegrating regression, proposed by Park (1992), in columns (2) and (4). The estimated coefficients on $r_t$ and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once one accounts for the substitution effect of consumer credit, there still exists a stable negative relationship between real money balances and the interest rates. This substitution effect is a potential explanation for the decline of the money multiplier during 1992Q2-2008Q3.

These findings suggest that a desirable monetary model for studying monetary transmission should have the following properties. First, the model should be capable of explaining how the amount of reserves is linked to interest rate management, regardless of whether banks hold excess reserves or not. Second, the model should feature the distinct roles of interest on reserves and nominal interest rates. Third, the model should be able to answer why banks are holding excess reserves, whereas they did not before 2008. Lastly, the model needs to capture the interaction between money and credit. In the following sections, by incorporating these four properties, I develop a theoretical model of the money creation process which is consistent with the above four observations, both qualitatively and quantitatively.

# 3 Model

The model constructed here extends the standard monetary search model (Lagos and Wright, 2005) by introducing fractional reserve banking, unsecured credit, and the standard growth theory with capital accumulation. Time is discrete and two markets convene sequentially in each time period: (1) a frictionless centralized market (CM, hereafter), where agents work, consume, and adjust their balances, following after (2) a decentralized market (DM, hereafter), where buyers and sellers meet and trade bilaterally. The DM trade features imperfect record-keeping and limited commitment. Due to these two frictions, some means of payment are needed in DM trades. Below I describe the economic agents in this economy and the different types of DM meetings.

There is a unit mass of households who discount their utility each period by $\beta$. The preferences of the households for each period are

$$\mathcal{U} = U(C) - \zeta H + u(q) - c(q)$$

where $C$ is the CM consumption, $\zeta > 0$ is a parameter, $H$ is the CM disutility from production, and $q$ is the DM consumption. As standard, assume $U'$, $u'$, $c' > 0$, $U''$, $u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. Consumption goods are perishable. The efficient consumption in the DM is denoted by $q^*$ which solves $u'(q^*) = c'(q^*)$. At the beginning of the DM, households get a preference shock such that they can be either buyers or sellers. A household will be a buyer with probability $\nu$ while with probability $1 - \nu$ a household is a seller.

There is a representative firm. During the CM a young firm is born and becomes old and dies in the next CM. The firm maximizes its profit by producing CM consumption goods by hiring labor and using capital as inputs. The technology is given by $F(K, N)$ where $K$ is the capital input and $N$ is the labor input. The production function $F(K, N)$ satisfies $F_K, F_N > 0$, $F_{KK}, F_{NN} < 0$ and constant returns to scale. The capital is depreciated by $\delta$ every period. Assume that a firm and households are anonymous and a firm cannot commit to honour intertemporal promises. Therefore, a firm needs a medium of exchange to purchase capital goods. In order to acquire funding to pay for investment, firms can borrow loan from banks. There exists a capital producer whose technology can transform CM consumption goods into capital goods with cost $\Phi(\cdot)$.

There are measure $n$ of active banks that is endogenously determined by the free entry condition in the equilibrium. In the CM, the banks make portfolio choices for reserves, loans, transaction deposits, and non-transaction deposits. The banks extend loans by creating deposits that households can use as a means of payment to trade goods in the DM. The loans are paid back with interest rate $i_\ell$. Enforcing repayment is costly. The cost function is described by $\eta(\tilde{\ell})$, where $\tilde{\ell}$ is the loan in real terms, $\eta', \eta'' > 0$, and $\eta(0) = 0$. Managing reserve balances incurs a cost. The cost is represented by a cost function $\gamma(\tilde{r})$, where $\tilde{r}$ is the amount of reserves in real terms, $\gamma', \gamma'' > 0$, and $\gamma(0) = 0$. The reserve earns a nominal interest rate of $i_r$. The bank's transaction deposit creation is constrained by the reserve requirement, i.e., the bank should hold reserves at least $\chi\tilde{d}$, where $\chi$ is the reserve requirement and $\tilde{r}$ is the real balance of reserves.

There are three types of DM meetings that are distinguished by which the payment methods sellers accept. In DM1, there is no record-keeping device, and the seller can only recognize cash. In DM2, the seller accepts transaction deposits and unsecured credit. In DM3, she accepts cash, transaction deposits and unsecured credit. The buyer can trade using unsecured credit with credit limit $\bar{b}$ as the trading is monitored imperfectly.[5]

[5]The acceptance of different means of payment can be endogenized as in Lester, Postlewaite and Wright (2012) or Lotz and Zhang (2016) but here we assume the types of meetings are exogenously

The central bank controls the base money supply $B_t$ in the CM. Let $\mu$ denote the base money growth rate. Then the changes in the real balance of base money can be written as

$$\mu_{t+1} B_t = B_{t+1} - B_t,$$

The base money is held in two ways: (1) $C$ as currency in circulation, i.e., outside money held by households; (2) $R$ as reserves held by banks. Thus,

$$B_t = M_t + R_t.$$

The central bank can control the base money supply in two ways. First, it can conduct a lump-sum transfer or collect a lump-sum tax in the CM. Second, it can increase the money supply by paying interest on reserves, $i_r$. Let $T$ represents a lump-sum transfer (or tax if it is negative). The central bank's constraint is

$$\mu_{t+1}\phi_{t+1} B_t = \phi_{t+1}(B_{t+1} - B_t) = T_{t+1} + i_{r,t+1}\phi_{t+1} R_{t+1},$$

where $\phi$ is the price of money in terms of the CM consumption good.

## 3.1 The CM Problem

Let $W(m, d, s, b)$ denote the CM value function where $m$ is the cash holding, $d$ is the transaction deposit balance, $s$ is the non-transaction deposit balance, and $b > 0$ is the unsecured debt owed to the seller from the previous DM (or unsecured loans to buyer from the previous DM, if $b < 0$). All the state variables are in unit of the current CM consumption good. The CM problem is

$$\begin{aligned} W(m_t, d_t, s_t, b_t) = & \max_{C_t, H_t, \hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}} U(C_t) - \zeta H_t + \beta V(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \\ \text{s.t. } & \frac{\phi_t}{\phi_{t+1}}(\hat{m}_{t+1} + \hat{d}_{t+1} + \hat{s}_{t+1}) + C_t = w_t H_t + m_t + (1 + i_{dt}) d_t + (1 + i_{st}) s_t - b_t + T_t \end{aligned}$$

where $\hat{m}_{t+1}$, $\hat{d}_{t+1}$, and $\hat{s}_{t+1}$ is the cash holding, transaction deposit balance, and non-transaction deposit balance, respectively, carried to the next DM. The first-order con-

given. Lester et al. (2012) endogenize the meeting types by allowing sellers' costly *ex ante* choice to acquire the technology for recognizing the certain type of assets. Similarly, Lotz and Zhang (2016) study the environment with costly record-keeping technology where sellers must invest in a record-keeping technology to accept credit.

ditions (FOCs) are

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t}+\beta V_m(\hat{m}_{t+1},\hat{d}_{t+1},\hat{s}_{t+1})\leq 0, = \text{ if } \hat{m}_{t+1}>0 \tag{1}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t}+\beta V_d(\hat{m}_{t+1},\hat{d}_{t+1},\hat{s}_{t+1})\leq 0, = \text{ if } \hat{d}_{t+1}>0 \tag{2}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t}+\beta V_s(\hat{m}_{t+1},\hat{d}_{t+1},\hat{s}_{t+1})\leq 0, = \text{ if } \hat{s}_{t+1}>0 \tag{3}$$

$$-\frac{\zeta}{w_t}+U'(C_t)=0. \tag{4}$$

The first term on the left-hand side (LHS) of equation (1) is the marginal cost of acquiring cash. The second term is the discounted marginal value of carrying cash to the following DM. Therefore, the choice of $\hat{m}_{t+1}>0$ equates the marginal cost and the marginal return on cash. A similar interpretation applies to equations (2) and (3) for the decision on deposits. The envelope conditions for $W(m,d,s,b)$ are

$$W_d(m_t,d_t,s_t,b_t)=(1+i_{dt})\frac{\zeta}{w_t},\quad W_m(m_t,d_t,s_t,b_t)=\frac{\zeta}{w_t}$$
$$W_s(m_t,d_t,s_t,b_t)=(1+i_{st})\frac{\zeta}{w_t},\quad W_b(m_t,d_t,s_t,b_t)=-\frac{\zeta}{w_t}$$

which implies $W(m_t,d_t,s_t,b_t)$ is linear. This linearity allows us to write

$$W(m_t,d_t,s_t,b_t)=\frac{\zeta}{w_t}\left\{m_t+(1+i_{dt})d_t+(1+i_{st})s_t\right\}-\frac{\zeta}{w_t}b_t+W(0,0,0,0,0).$$

## 3.2 The DM Problem

In the DM, the buyer and seller trade bilaterally. Let $q_j$ and $p_j$ be the DM consumption and payment in a type-$j$ DM meeting. The bilateral trade is characterized by $(p_j,q_j)$. This trade is subject to $p_j\leq z_j$ where $z_j$ is the total liquidity of the buyer in a type-$j$ meeting. The liquidity position for each type of buyer is

$$z_1=m \tag{5}$$

$$z_2=d(1+i_d)+\bar{b} \tag{6}$$

$$z_3=m+d(1+i_d)+\bar{b} \tag{7}$$

The DM terms of trade is determined by Kalai (1977)'s proportional bargaining. Kalai bargaining solves the following problem:

$$\max_{p,q} u(q) - \frac{\zeta}{w}p \quad s.t \quad u(q) - \frac{\zeta}{w}p = \theta\left[u(q) - c(q)\right] \text{ and } p_j \leq z_j$$

where $\theta \in [0, 1]$ denotes the buyers' bargaining power. The payment, $p$, can be expressed as $p = v(q)w/\zeta = \{(1-\theta)u(q) + \theta c(q)\}w/\zeta$. Define *liquidity premium*, $\lambda(q)$, as follows:

$$\lambda(q) = \frac{u'(q)}{v'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{(1-\theta)u'(q) + \theta c'(q)}$$

where $\lambda(q) > 0$ for $q < q^*$ and $\lambda(q^*) = 0$ with $\lambda'(q) < 0$ for $q \in [0, q^*)$. When $z_j \geq p^*$, the buyer has sufficient liquidity to purchase efficient DM output $q^*$. In this case, the payment to the seller is $p^* = v(q^*)$.

The value function of an household at the beginning of DM is

$$V(m, d, s) = \nu V^B(m, d, s) + (1-\nu)V^S(m, d, s)$$

where $V^B(m, d, s)$ and $V^S(m, d, s)$ denotes the value function for a buyer and a seller, respectively. By using the linearity of $W$, we can write a DM value function for a seller as follows:

$$V^S(m, d, s) = \sum_{j=1}^{3}\left\{\frac{\sigma_j \nu}{1-\nu}\left[\frac{\zeta}{w}p_j - c(q_j)\right]\right\} + W(m, d, s, 0)$$

and the value function of a buyer in the DM is

$$V^B(m, d, s) = \sum_{j=1}^{3}\left\{\sigma_j\left[u(q_j) - \frac{\zeta}{w}p_j\right]\right\} + W(m, d, s, 0)$$

where $p_j \leq z_j$. The third term on the right-hand side (RHS) is the continuation value when there is no trade. The rest of the RHS is the surplus from the DM trade. The DM payments are constrained by $p_j \leq z_j$. For compact notation, define inflation rate as $\pi_{t+1} \equiv \phi_t/\phi_{t+1} - 1$. Assuming interior, differentiating $V$ and substituting its derivatives into the FOCs from the CM problem yields

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_1\lambda(q_{1,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})] \tag{8}$$

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_2\lambda(q_{2,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})](1 + i_{d,t+1}) \tag{9}$$

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})(1 + i_{s,t+1}) \tag{10}$$

where $q_{j,t+1} = \min\{q^*, v^{-1}(z_{j,t+1})\}$ and $\lambda(q^*) = 0$.

### 3.3 The Bank's Problem

A bank maximizes its profit by choosing $\{\tilde{\ell}, \tilde{r}, \tilde{d}, \tilde{s}\}$ subject to its balance sheet identity constraint and reserve requirement constraint, where $\tilde{\ell}$ is lending, $\tilde{r}$ is reserves, $\tilde{d}$ the transaction deposit issuance, and $\tilde{s}$ the non-transaction deposit issuance, respectively, denoted in real terms. Since banking is competitive, a bank takes as given the interest rates: lending rate $i_\ell$, transaction deposit rate $i_d$, non-transaction deposit rate $i_s$. Interest on reserves $i_r$ is determined by the central bank.

$$\max_{\tilde{r},\tilde{d},\tilde{\ell},\tilde{s}} \quad (1+i_\ell)\tilde{\ell} + (1+i_r)\tilde{r} - (1+i_d)\tilde{d} - (1+i_s)\tilde{s} - \gamma(\tilde{r}) - \eta(\tilde{\ell}) \tag{11}$$
$$\text{subject to} \quad \tilde{\ell} + \tilde{r} = \tilde{d} + \tilde{s} \quad \text{and} \quad \tilde{r} \geq \chi\tilde{d}$$

In the first constraint, balance sheet identity, the LHS is assets, which include reserves and loans, and the RHS is liabilities, which include transaction deposits and non-transaction deposits. Let $\lambda_\chi$ denote the Lagrange multiplier for the reserve requirement constraint. The FOCs for the bank's problem can be written as

$$0 = i_\ell - i_s - \eta'(\ell) \tag{12}$$
$$0 = i_r - i_s - \gamma'(\tilde{r}) + \lambda_\chi \tag{13}$$
$$0 = i_s - i_d - \lambda_\chi\chi. \tag{14}$$

The bank's *ex post* profit equals to the entry cost, $\kappa$

$$(1+i_\ell)\tilde{\ell} + (1+i_r)\tilde{r} - (1+i_d)\tilde{d} - (1+i_s)\tilde{s} - \gamma(\tilde{r}) - \eta(\tilde{\ell}) = \kappa. \tag{15}$$

Suppose there are active banks i.e., $n > 0$. Consider two cases. In the first case, the reserve requirement constraint is binding, i.e., $\lambda_\chi > 0$. In the second case, the reserve requirement constraint is loose, i.e., $\lambda_\chi = 0$. We call the first case a "scarce-reserves case," and the second an "ample-reserves case."

**The Scarce-Reserves Case** The bank does not have enough reserves. It needs to acquire reserves to issue more transaction deposit, which implies a binding constraint.

With $\lambda_\chi > 0$, the bank's FOCs (12)-(14) give

$$i_d = (1-\chi)i_s + \chi i_r - \chi\gamma'(\tilde{r}) \tag{16}$$

$$i_\ell = i_s + \eta'(\tilde{\ell}). \tag{17}$$

**The Ample-Reserves Case** The bank has sufficient reserves. Its lending constraint does not bind. Then the three FOCs for the bank's problem are

$$i_d = i_s \tag{18}$$

$$i_r = i_s + \gamma'(\tilde{r}) \tag{19}$$

$$i_\ell = i_s + \eta'(\tilde{\ell}). \tag{20}$$

## 3.4 The Firm and Capital Producer

A representative firm maximizes its profit by producing CM consumption goods and using its capital $K$ and hiring labor $N$ as inputs. In the CM of $t-1$, the firm borrows funds $L_t$ from banks, and purchases capital goods $K_t$ using the funds. In the CM of $t$, the firm produces by hiring. The firm's problem can be written as follows:

$$\max_{N_t,K_t,L_t} L_t - Q_{t-1}K_t + \beta\left[F(K_t,N_t) - w_tN_t + Q_t(1-\delta)K_t - (1+i_{\ell,t})\frac{L_t}{1+\pi_t}\right]$$

subject to $L_t = Q_{t-1}K_t$, where $Q_t$ is price of capital in terms of CM consumption good at period $t$. The firm's problem gives

$$F_N(K_t,N_t) = w_t, \quad F_K(K_t,N_t) = Q_{t-1}\frac{1+i_{\ell,t}}{1+\pi_t} - Q_t(1-\delta) \tag{21}$$

A firm purchases capital from perfectly competitive capital producing firms at the end of period $t-1$. This capital is used in production at $t$ and its undepreciated $(1-\delta)K_t$ part is resold to a capital producer once the production is over. Capital law of motion is given as:

$$K_{t+1} = (1-\delta)K_t + I_t$$

A capital producer can transform CM consumption goods into capital goods with cost $\Phi(I_t)$. Formally, a capital producer solves the following profit-maximization problem:

$$\max_{I_t} Q_tI_t - \Phi(I_t)$$

which gives $Q_t = \Phi'(I_t)$. Assuming a linear production technology, $\Phi(I_t) = I_t$, gives $Q_t = 1$ for all $t$. Given above results, define the real lending rate:

$$\rho_t \equiv \frac{1 + i_{\ell,t}}{1 + \pi_t} - 1$$

Then we have the following equilibrium condition for the real lending rate and the marginal product of capital as follows:

$$F_K(K_t, N_t) = \rho_t + \delta \tag{22}$$

## 3.5 Equilibrium

In the equilibrium, the resource constraint for CM consumption goods and labor market clearing condition are satisfied.

$$C_t + K_{t+1} = F(K_t, N_t) + (1 - \delta)K_t, \text{ and } N_t = H_t \tag{23}$$

The money market clearing conditions are given as below

$$\phi_{t+1}M_{t+1} = m_{t+1}, \quad \phi_{t+1}R_{t+1} = n\tilde{r}_{t+1}, \quad \text{and} \quad B_{t+1} = M_{t+1} + R_{t+1} \tag{24}$$

and market clearing condition for lending and deposits are satisfied.

$$L_{t+1} = n_{t+1}\tilde{\ell}_{t+1}, \quad \text{and} \quad d_{t+1} = n_{t+1}\tilde{d}_{t+1} \tag{25}$$

Given agents' optimal choices and market clearing conditions, we define a monetary equilibrium as follows:

**Definition 1.** *A monetary equilibrium is a sequence of quantities $\{K_t, N_t, m_t, d_t, r_t\}_{t=1}^{\infty}$, prices $\{\phi_t, i_{\ell t}, i_{dt}, w_t, \rho_t\}_{t=1}^{\infty}$, and measures of active banks, $\{n_t\}_{t=1}^{\infty}$ that satisfies:*

1. *The Euler equations (8)-(10), and transversality conditions:*

$$\lim_{t\to\infty} \beta^t K_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t m_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t d_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t s_t = 0$$

2. *Optimality conditions of banks and the firm, (12)-(15) and (21);*
3. *Market clearing (23)-(25), and $\phi_t B_t > 0$;*
4. *Given a sequence for credit condition $\{\bar{b}_t\}_{t=1}^{\infty}$;*
5. *Given a sequence for monetary policy $\{B_t, i_{rt}, \chi_t\}_{t=1}^{\infty}$;*

*6. Given initial capital stock $K_0$ and monetary base stock $B_0$.*

As standard, the interest rate on illiquid nominal bonds $i_t$ is given by the Fisher equation

$$1 + i_t = (1 + \pi_{t+1}) \frac{U'(C_t)}{\beta U'(C_{t+1})} \tag{26}$$

implying $i_t = i_{s,t+1}$.

Similar to Sargent and Wallace (1975) and Gu, Han and Wright (2020), a central bank can peg $i_t$ by letting $B_{t+1}$ evolve endogenously as long as the base money is valued, $\phi_{t+1} B_{t+1} > 0$. Here, monetary policy implementation is different from previous literature. For example, New Keynesian models simply assume that the central bank can determine interest rate. In the other monetary models with fiat money, the central bank implement the monetary policy by controlling aggregate money supply, or by controlling growth rate of aggregate money supply. In this model, the central bank can set interest rates by controlling the supply of base money which eventually influences the supply of monetary aggregate. Here, for the central bank's monetary policy implementation, it is crucial to have monopoly power over the supply of base money which is the sum of reserves and currency in circulation.

Given this environment, we have following results:

**Proposition 1.** *When the central bank does not pay interest on reserves $i_{r,t+1} = 0$, the banks do not hold excess reserves i.e., $\tilde{r}_{t+1} = \chi \tilde{d}_{t+1}$. When the banks hold excess reserves, $\partial \tilde{\ell}_{t+1} / \partial i_t < 0$ and $\partial \tilde{\ell}_{t+1} / \partial i_{r,t+1} > 0$.*

**Proposition 2.** *If $i_t < \bar{\imath} \equiv \max \{\nu(\sigma_1 + \sigma_3)\lambda(0), i_{r,t+1}\}$, then $\phi_{t+1} B_{t+1} > 0$.*

In particular, a direct corollary of Proposition 1 is

**Proposition 3.** *The banks hold excess reserves when $i_{rt} > \hat{\imath}_{rt}$ where the threshold satisfies $\hat{\imath}_{rt} = i_t + \gamma'(\hat{r}_t)$ where $(\hat{r}_t, \hat{\ell}, \hat{n}_t)$ solves*

$$\max \left\{ 0, \frac{\chi \left(p^* - \bar{b}\right)}{\hat{n}_t} \right\} = (1 + i_t)\hat{r}, \quad \eta'(\hat{\ell})\hat{\ell} + (i_r - i)\hat{r} - \gamma(\hat{r}) - \eta(\hat{\ell}) = \kappa.$$

*and (12)-(15), (21), (23)-(25), (26).*

When the central bank does not pays interest on reserves or pays low interest such that $i_{rt} \leq \hat{\imath}_r$, banks only hold required reserves because opportunity cost of holding reserves is larger than its benefit. When the central bank pays interest on reserves high enough such that $i_{rt} > \hat{\imath}_r$, banks always hold positive amount of reserve balances that satisfies $\gamma'(\tilde{r}) = i_r - i$ and $\tilde{r} > \chi_t \tilde{d}_t$. In this case, banks hold a large amount of reserves not due to regulatory requirements but because the benefit of holding reserves outweighs the opportunity cost of holding reserves. Therefore as long as the central bank pays interest with $i_{rt} > \hat{\imath}_r$, there are positive reserve balances. In contrast when $i_{rt} < \hat{\imath}_r$, a high nominal interest rates such that $i_t \geq \nu(\sigma_1 + \sigma_3)\lambda(0)$ results in $\phi_t B_t = 0$. In this case, the opportunity cost of holding currency and transaction deposits is too high for a household to have positive balances. Hereafter, I focus on the case where $i_t < \bar{\imath}$, which guarantees that the central bank has the power to implement its monetary policy.

**Monetary Transmission and Breaking of Neoclassical Dichotomy** In the classical frictionless monetary models, output, and the real interest rate are determined independently of monetary policy. In other words, monetary policy is neutral with respect to those real variables. Here, monetary policy could influence to the real variable such as investment. It is worth to discuss the difference in monetary transmission channel from the previous literature. In the textbook by Galí (2015), Chapter 1 shows monetary policy is neutral in the classical frictionless monetary models, and Chapter 3, discusses how the presence of sticky prices makes monetary policy non-neutral.[6,7] In contrast to those approaches, in this model, monetary policy could influence to the real variable without presence of sticky prices.

To inspect the mechanism, recall the Equation (22), $A_{t+1} F_K(K_{t+1}, N_{t+1}) = \rho_{t+1} + \delta$. A decrease in the real lending rate unambiguously lowers marginal production of capital. To see anatomy of what constitutes marginal production of capital, rewrite the marginal product of capital as below:

$$\underbrace{F_K(K_{t+1}, N_{t+1})}_{\text{marginal product of capital}} = \underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})} - 1 + \delta}_{\text{standard neoclassical term}} + \underbrace{\frac{\eta'(\tilde{\ell}_{t+1})}{1 + \pi_{t+1}}}_{\text{bank's marginal cost of lending}}$$

[6] The term monetary neutrality is often used differently in the literature. In New Keynesian literature, the monetary neutrality implies that changes in nominal interest rates and money supply both do not have impact on real variables. In contrast, as discussed in Head, Liu, Menzio and Wright (2012), in the New Monetarist models, although money is not superneutral, since real effects result from changes in nominal interest rates, inflation, or money growth, money is neutral because changes in aggregate money supply does not have real effects on real variable.

[7] It is worthwhile to mention that presence of sticky prices does not necessarily make monetary policy non-neutral. Head et al. (2012) provides a monetary search model where the price stickiness emerges endogenously in contrast to the models imposing price stickiness exogenously. While Head et al. (2012) explains price stickiness and micro-level price level changes which can match with microdata, money is neutral in the model.

What distinguishes this model from the neoclassical growth model is the last term of the above equation. The banks' enforcement cost provides a wedge to finance the investments. For example, as shown in Proposition 1, an increase in interest on reserves lowers $\tilde{\ell}_{t+1}$ when the banks hold excess reserves. This reduces the marginal cost of financing investment and influences the real lending rate through general equilibrium impact.

The channel is different from other micro-founded monetary models with capital. Aruoba and Wright (2003) is the one of first papers that introduced the neoclassical growth model to the Lagos and Wright (2005) environment. As pointed out by Waller (2003), it features a strong neoclassical dichotomy, meaning the outcomes in the DM and the CM can be solved independently. Later Aruoba, Waller and Wright (2011) and Waller (2011) break this dichotomy by introducing the role of capital in the DM where capital accumulation lowers the cost of producing DM goods. The other way to break down the dichotomy is to introduce pledgeable capital, allowing more credit trade across agents by holding more capital. (e.g.,Venkateswaran and Wright, 2013 and Gu, Jiang and Wang, 2019) Here, what breaks down the neoclassical dichotomy is the limited commitment problem between a firm and households.

**DM Trades** Now we look into to payment system. Given DM trade, we have

$$\frac{i_t}{\nu} = \sigma_1 \lambda(q_{1t}) + \sigma_3 \lambda(q_{3t}) \tag{27}$$

$$\left\{ \frac{1+i}{1+i_d} - 1 \right\} \frac{1}{\nu} = \sigma_2 \lambda(q_{2t}) + \sigma_3 \lambda(q_{3t}) \tag{28}$$

It is straightforward to see that DM consumption is efficient, $q_1 = q_3 = q^*$, when $i_t = 0$, i.e. the Friedman rule applies. However, if credit limit $\bar{b}_t$ is sufficiently high or the central pays sufficient interest on reserves, DM2 and DM3 consumption can be efficient. Therefore, the DM2 meeting consumption can be efficient even though the economy is not under the Friedman rule. This result can be formally summarized in the following proposition.

**Proposition 4.** *In the monetary equilibrium with $i_t > 0$, DM2 and DM3 consumptions are efficient $q_{2t} = q_{3t} = q^*$ when one of the following conditions holds: (i) $i_{rt} > \hat{\imath}_r$; (ii) $\bar{b} \geq \hat{b}_t$ where*

$$\hat{b}_t \equiv p^* - \Lambda^{-1}\left(\frac{i_t}{\nu\sigma_1}\right)$$

*and $\Lambda(z) \equiv \lambda \circ v^{-1}(z)$.*

The intuition behind the efficient DM2 consumption under the first condition is straightforward. In many monetary models, a high inflation or interest rate increases the opportunity cost of holding money. In the environment where money is valued as a medium of exchange, having less liquidity in the economy because of an opportunity cost of holding money is inefficient. However, the interest on reserves provides a proportional return. If this return is properly distributed across households, it eliminates the inefficiency that arises from the opportunity cost of holding money, which results in efficient DM2 consumption. The second condition simply says that the credit limit is high enough to give efficient consumption in the DM2 trade. As $\bar{b}_t \to \hat{b}_t^-$, the household's transaction deposit balance $d_t$ converges to 0. This is reminiscent of a result by Gu et al. (2016): if credit is easy, money is irrelevant; if credit is tight, money is essential, but credit becomes irrelevant. One difference is that, even though credit is easy, the household always holds cash $m > 0$ as long as $i_t < \bar{\iota}$ because the household only can trade using cash in DM1 meeting. In the following sections, I assume $\bar{b}_t < \hat{b}_t$ to allow transaction deposit to be used in the DM2 trade.

In the remaining of this section, I focus on a symmetric stationary monetary equilibrium in which the agents make the same decisions and the real balances, the credit limit, and aggregate productivity are constant over time; $K_{t+1} = K_t = K$, and $\bar{b}_{t+1} = \bar{b}_t = \bar{b}$. Given that $\phi_t/\phi_{t+1} = B_{t+1}/B_t = M_{t+1}/M_t = 1 + \mu$, the net inflation rate, $\pi$, is equal to the currency growth rate, $\mu$, in the stationary monetary equilibrium. By the Fisher equation, $1 + i = (1 + \mu)/\beta$.[8] This leads to the following definitions:

**Definition 2** (**Stationary Monetary Equilibrium**). *Given monetary policy, $i$, $i_r$, and $\chi$ and credit limit $\bar{b}$, a stationary monetary equilibrium consists of real balances, $(m, d, s)$, allocation $(q_1, q_2, q_3, C, K)$, the measure of banks $n$, and prices $(i_d, i_\ell)$, satisfying Definition 1 except for initial conditions.*

**Definition 3.** *The stationary monetary equilibrium is a scarce-reserves equilibrium when $\tilde{r} = \chi\tilde{d}$ and an ample-reserves equilibrium, when $\tilde{r} > \chi\tilde{d}$, respectively.*

From above definitions and Proposition 2, we have the following result.

**Proposition 5.** *Assume $i < \bar{\iota}$. Given $(i, \chi, \bar{b})$: (i) $\exists!$ ample-reserves equilibrium if and only if $i_r > \hat{\iota}_r$; (ii) $\exists!$ scarce-reserves equilibrium if and only if $\hat{\iota}_r \leq i_r$.*

For each type of equilibrium, the following results are proved in Appendix B.

[8]Note that $i \geq 0$ is necessary for the existence of equilibrium. Whereas the lower bound of the nominal interest rate is zero in this setting, one can relax this by introducing liquid assets or threats of theft. See Rocheteau, Wright and Xiao (2018b), Lee (2016) and Williamson (2019) for details.

**Proposition 6.** *In the ample-reserve equilibrium, $\partial i_\ell/\partial i > 0$, $\partial i_\ell/\partial i_r < 0$, $\partial i_d/\partial i = 0$ and $\partial \rho/\partial i_r < 0$ but $\partial \rho/\partial i$ is ambiguous. In the scarce-reserve equilibrium, when $\sigma_1$ is small, $\partial i_\ell/\partial i > 0$, $\partial i_\ell/\partial i_r < 0$, $\partial i_d/\partial i > 0$ and $\partial \rho/\partial i_r < 0$ but $\partial \rho/\partial i$ is ambiguous.*

Proposition 6 tells us that the monetary policy rates pass through the lending rate and deposit rate. In both types of equilibrium, the lending rate is strictly increasing in the nominal interest rate but is strictly decreasing in interest on reserves. In the scarce-reserves equilibrium, the deposit rate is strictly increasing in the nominal interest rate and the interests on reserves. However, in the ample-reserves equilibrium there is no pass-through to deposit rate. The pass-through from the nominal interest rate to the real lending rate is ambiguous, however, real lending rate is strictly decreasing in interest on reserves.

One also can check the effect of changes in the credit condition, $\bar{b}$. Its effect depends on the type of equilibrium.

**Proposition 7.** *In the scarce-reserve equilibrium, $\partial i_d/\partial \bar{b} > 0$, $\partial i_\ell/\partial \bar{b} > 0$, and $\partial \rho/\partial \bar{b} > 0$. In the ample reserve equilibrium, $\partial i_d/\partial \bar{b} = \partial i_\ell/\partial \bar{b} = \partial \rho/\partial \bar{b} = 0$.*

In the scarce-reserve equilibrium, better credit condition increases deposit rate, lending rate, and real lending rate. One implication for pass-through to deposit rate is that neutrality between money and credit does not hold.[9] This is because transaction deposit money incurs costs for operating banks. An increase in credit limit lowers the social cost of operating deposits which increases DM2 and DM3 trades through the increase in the deposit rate. In contrast to the scarce-reserve equilibrium, the changes in credit conditions do not have any impact on other market interest rates in the ample reserve equilibrium.

The above results came from the real balance channel. One can check the effect of credit conditions on real balances. As long as there are positive amounts of transaction deposit balances, $d > 0$, an increase in $\bar{b}$ lowers $d$. This channel also has an impact on the money multiplier. Define money multiplier $\xi \equiv (m + d)/(\phi B)$, then we have the following results.

**Proposition 8.** *In both types of stationary monetary equilibria, better credit conditions decrease the real balance of deposit i.e., $\partial d/\partial \bar{b} < 0$. In the ample-reserves equilibrium, the better credit condition lowers the money multiplier i.e., $\partial \xi/\partial \bar{b} < 0$. In the scarce-reserves equilibrium, as long as $\chi < 1$, the better credit condition lowers the money multiplier i.e., $\partial \xi/\partial \bar{b} < 0$.*

[9] For more discussion on the neutrality of money and credit, see Gu et al. (2016) and Wang et al. (2020).

This result is consistent with the findings in Section 2. As more unsecured credit becomes available, the real balance of deposit money decreases, which leads to a lower money multiplier. The model can provide more results on the money multiplier. Consistent with the observation illustrated in Figure 1, the following proposition shows that an increase in interest rate increases the money multiplier while an increase in interest on reserves lowers the money multiplier. Appendix B verifies the following:

**Proposition 9.** *In the ample-reserves equilibrium, we have*

$$\frac{\partial \xi}{\partial i} > 0, \quad \frac{\partial \xi}{\partial i_r} < 0.$$

Thus, the model can successfully address the mechanism illustrated in Section 1 and 2. We can interpret the decline in the money multiplier in the pre-2008 economy as a result of improved availability of consumer credit under the scarce-reserve equilibrium. For the post-2008 period, after the Fed started paying interest on reserves the economy moved to the ample-reserves equilibrium. The model suggests that the changes in the money multiplier and the excess reserves are the results of the Fed's management of two interest rates: the nominal policy rate and the interest on reserves.

### 3.6 Some Limits of *onetary* Economics

The previous literature on monetary policy has favored an approach that excludes explicit consideration of "money." The widely accepted interpretation of such model economies is that they represent a cashless limit of an economy in which households value and hold money. The justification for this approach is based on continuity results at a certain cashless limit obtained using reduced-form models (e.g., money-in-utility or cash-in-advance), which are provided by Woodford (1998), Woodford (2003) and Galí (2015). However, this is not necessary true. Lagos and Zhang (2022) show that, in the micro-founded monetary model, monetary policy can have a real effect through the real balance channel even in highly developed credit economies where the share of monetary transactions is negligible, arguing the continuity cashless limit result is not complete.

In light of Lagos and Zhang (2022), one can consider the cashless limit in this model as well. Let $\sigma_1 \to 0$, then there is no cash only meeting and the economy converges to the cashless economy as long as $i_d > 0$. However, households still hold deposit money when $i_d > 0$ and $\bar{b} < p^*$. In this case, the banks still hold reserves, implying $\phi R = \phi B > 0$. Therefore, even in cashless limit where there are no currency in circulation, money plays transaction role as means of payment in addition to banks' portfolio in their balance sheet. While there is no cash, monetary policy is effective through the real balance channel.

Next, let's consider the economy where neither deposit money nor physical currency are used in the DM transactions, i.e., $\sigma_1 \to 0$ and $\bar{b} \geq p^*$. In this case, as long as policy rates satisfies $i_r > \hat{\imath}_r$, the central bank still have ability to set its policy interest rate, $i$, by controlling base money supply which influences consumption, investment, future inflation and output through banking channel.

Therefore, the quantity of monetary base is relevant even though there is no monetary transactions in the decentralized trade. The mechanism is different from Lagos and Zhang (2022), but results are similar. Monetary policy remains effective even in highly developed credit economies where the share of monetary transactions is negligible.

# 4 Quantitative Analysis

The above sections have developed a model of money creation analyzing monetary transmission. The model is tractable, and analytical results can be established. In this section, I calibrate the model and evaluate the model quantitatively.

## 4.1 Calibration

The model period length is set to one year. The utility functions for the DM and the CM are $u(q) = B[(q + \varsigma)^{1-\varphi} - \varsigma^{1-\varphi}]/(1 - \varphi)$ and $U(C) = \log(C)$. The cost function for the DM is $c(q) = q$. The production function takes the form of a standard Cobb-Douglas function, $F(K, N) = K^\alpha H^{1-\alpha}$. The enforcement cost for lending is assumed to be quadratic, $\eta(\tilde{\ell}) = \Psi\tilde{\ell}^2$, and the balance sheet cost for managing reserves balances takes the form, $\gamma(\tilde{r}) = G\tilde{r}^g$.

The calibration period is 1968–2007. The benchmark nominal interest rate is $i = 0.0593$, the average 3-month treasury rate, and the benchmark required reserve ratio is $\chi = 0.1111$, the average of the ratio between required reserves and total checkable deposits. Since the Federal Reserve did not pay interest on reserve before October 2008, I set $i_r = 0$ as the benchmark.

Some parameters are directly pinned down. The discount factor $\beta$ is set to match a 3% real interest rate. The capital share in CM output is set to $\alpha = 1/3$ as the standard, and the capital depreciation rate is matched with $I/K = \delta = 0.0825$. To ensure that changes in credit conditions do not affect currency holdings, $\sigma_3$ has been

**Table 2:** Model parametrization

| Parameter | Parameter Description | Value | Target Description | Target | Model |
|---|---|---|---|---|---|
| | | **External Parameters** | | | |
| $\delta$ | depreciation rate | 0.0825 | investment/capital, $I/K$ | | |
| $\alpha$ | capital share in production | 0.3333 | labor's share of income, 2/3 | | |
| $\beta$ | discount factor | 0.9709 | real interest rate, 3% | | |
| $\sigma_2$ | DM2 matching prob. | 0.6890 | share of credit meeting | | |
| $\varsigma$ | parameter of $u(\cdot)$ | 0.0001 | Aruoba et al. (2011) | | |
| $\theta$ | bargaining power | 1 | take-it-or-leave-it offer | | |
| $\nu$ | prob. of being a buyer | 0.5 | normalization | | |
| $g$ | parameter of $\gamma(\cdot)$ | 1.5 | - | | |
| | | **Internal Parameters** | | | |
| $\zeta$ | coeff. on labor supply | 2.4949 | labor supply, $H$ | 0.3333 | 0.3329 |
| $\sigma_1$ | DM1 matching prob. | 0.0005 | currency/output, $M/PY$ | 0.0443 | 0.0441 |
| $G$ | parameter of $\gamma(\cdot)$ | 0.0020 | reserves/output, $R/PY$ | 0.0121 | 0.0118 |
| $\bar{b}$ | credit condition | 0.0559 | unsecured credit/output | 0.0347 | 0.0347 |
| $\kappa$ | bank entry cost | 0.0090 | lending rate, $i_\ell$ | 0.0862 | 0.0870 |
| $B$ | parameter of $u(\cdot)$ | 0.0130 | capital/output, $K/Y$ | 2.1896 | 2.2160 |
| $\varphi$ | parameter of $u(\cdot)$ | 3.4756 | elast. of $M/PY$ to $i$ | $-0.1948$ | $-0.2784$ |
| $\Psi$ | parameter of $\eta(\cdot)$ | 0.0213 | semi-elast. of $i_\ell$ to $i$ | 11.3673 | 11.5130 |

set to 0, based on the stable downward sloping currency demand illustrated in Figure 5. The fraction of DM2 meetings, $\sigma_2$, has been set to 0.6890 so that the equilibrium percentage of unsecured credit users matches 68.9%, which is the average percentage of US households holding at least one credit card from 1970 to 2007, based on the Survey of Consumer Finances. For simplicity, the bargaining power has been set to $\theta = 1$, which implies the buyer makes a take-it-or-leave-it offer to the seller in the DM. The probability of being a buyer has been normalized to $\nu = 0.5$. The parameter $\varsigma$ has been introduced in $u(q)$ merely to ensure that $u(0) = 0$, and has been set to $\varsigma = 0.0001$, as in Aruoba et al. (2011). The curvature parameter of $\gamma(\cdot)$ has been set to $g = 1.5$ to ensure that $\gamma(\cdot)$ is less convex than $\eta(\cdot)$.

The remaining 8 parameters $(\zeta, \kappa, \sigma_1, \bar{b}, G, B, \varphi, \Psi)$ are set to match the following 8 targets: (i) the standard measure of work as a fraction of discretionary time, $H = 1/3$; (ii) average nominal lending rate, $i_\ell = 0.0862$; (iii) currency output ratio, $M/PY = 0.0443$; (iv) reserves output ratio, $R/PY = 0.0121$; (v) unsecured credit output ratio, 0.0347; (vi) capital output ratio, $K/Y = 2.1896$; (vii) elasticity of currency demand to nominal interest rate, $-0.1948$; (viii) semi elasticity of nominal lending rate to nominal interest rate, 12.9284. The targets are computed on the basis of 1968-2007 data.

All of the targets in the model, except the elasticity of currency demand and the semi-elasticity of lending rate, are directly computed using straightforward formulas given the benchmark nominal interest rate and the required reserve ratio. Similar to Aruoba et al. (2011), the elasticity of currency demand is computed using changes in

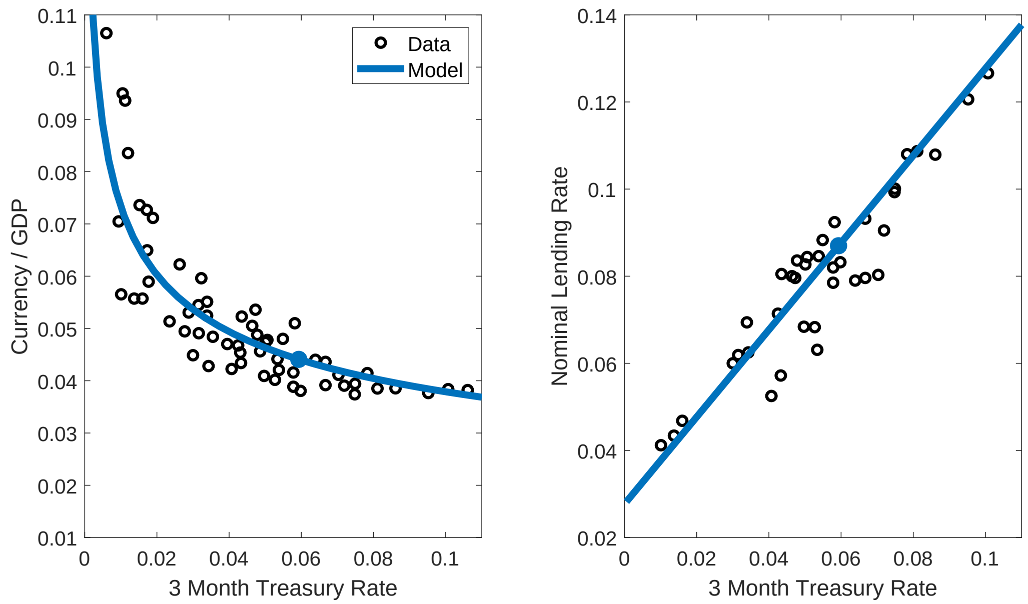


**Figure 5:** Money Demand for Currency and Interest rate Pass-through

money demand when the interest rate changes from $i - 0.05$ to $i + 0.05$. The semi-elasticity of lending rate is also computed using changes in the nominal lending rate when the interest rate changes from $i - 0.05$ to $i + 0.05$. More details of targets from the models can be found in Appendix D. The calibrated parameters and the targets are summarized in Table 2, and the calibrated money demand of currency and lending rate pass-through are shown in Figure 5.

## 4.2 Results

This section explores how well the model can account for the low-frequency behavior of reserves and the money creation process, assuming the only driving forces are monetary policy, $(i, i_r, \chi)$ and credit conditions, $\bar{b}$. Using the calibrated parameters, I compute the model equilibrium for given $(i, i_r, \chi, \bar{b})$. For monetary policy variables, I use 3 month treasury rate and interest on excess reserves,[10] and required reserves to total checkable deposit ratio. The unsecured credit limit $\bar{b}$ is computed using the unsecured credit to output ratio.

[10] Whereas the Federal Reserve had announced interest on excess reserves and interest on required reserves separately, the Fed had paid the same interest to both. As of March 2020, the Fed unified two interests on reserves as "Interest Rate on Reserve Balances" since the Board reduced reserve requirement to zero.

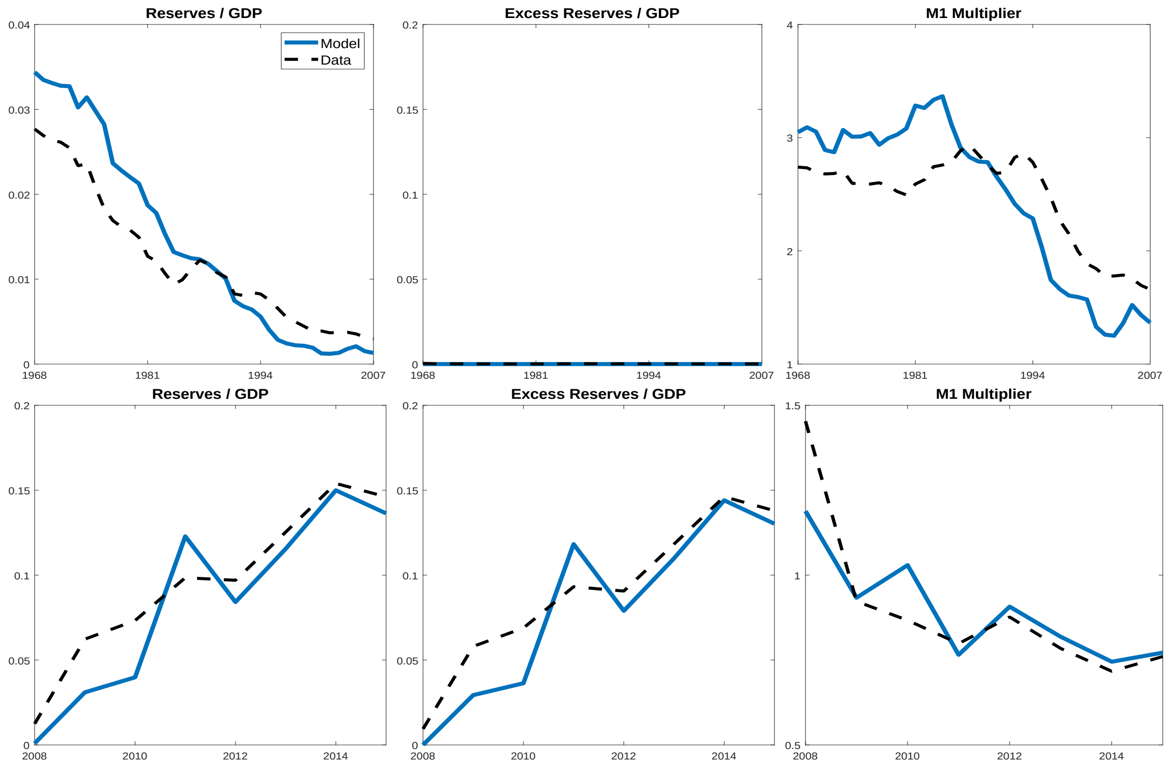


**Figure 6:** Model vs. Data

**Reserves and Money Multiplier** Figure 6 compares the model and data from 1968 to 2015. The top-left panel and the bottom-left panel shows reserves as a fraction of output from 1968 to 2007 and from 2008 to 2015, respectively. The model generates the quantity of reserves which can match the movement in the data. The top-middle panel and the bottom-middle panel of Figure shows excess reserves as fraction of output from 1968 to 2007 and from 2008 to 2015, respectively. The model also successfully generate the zero excess reserves during 1968-2007 and huge increase during 2008-2015. The top-right panel and the bottom-right panel of Figure 6 shows M1 money multiplier. While the model does not match all of the movement in the data, there is a very similar basic pattern. During 1968-2015 money multiplier consistently decreased, whereas there is no excess reserves. During 2008-2015, money multiplier decreased lower than one and reflected the behavior of excess reserves.

**Matching Observations** Now we will examine how the counterfactual equilibrium generated by the model aligns with the observed patterns discussed in Section 2.

Observation 1 shows that the quantity of reserves is not independent of the short-term policy rate. Figure 7 displays the counterfactual data from the model. As in Figure 1, Figure 7 show the evident opposite movements of the reserves to deposit

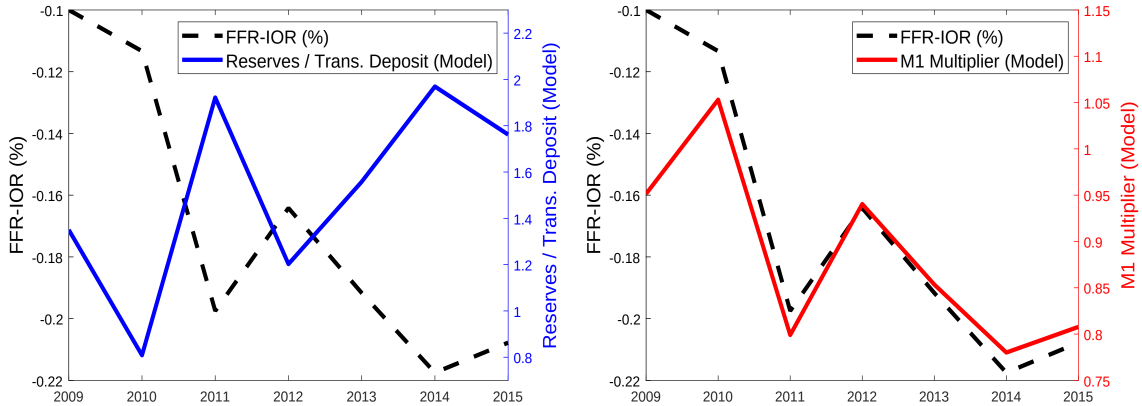


**Figure 7:** Reserves Deposit ratio and M1 multiplier: Model

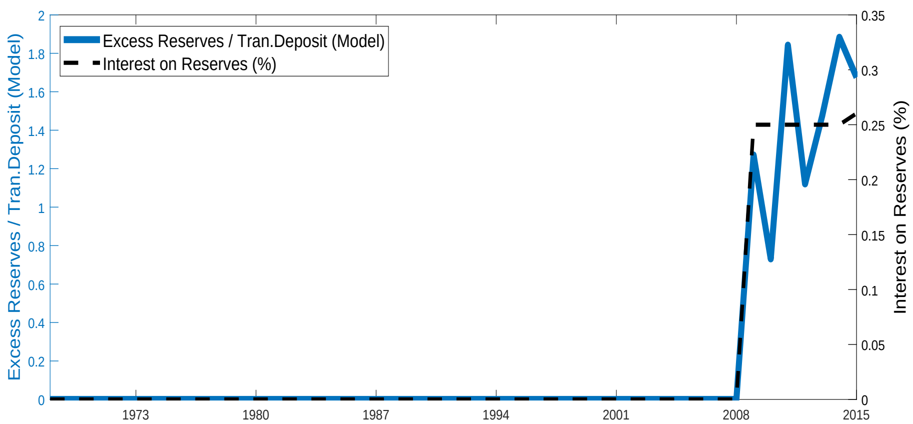


**Figure 8:** Excess reserves ratio (Model)

ratio with respect to the spread between short-term policy and interest on reserves. The counterfactual M1 money multiplier moves together with the spread as in Figure 1.

Observation 2 shows that excess reserves remained at zero until the introduction of interest on reserves, after which they skyrocketed. Similar to Figure 2, Figure 8 shows that. ... What happened here is that counterfactual.... Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus.

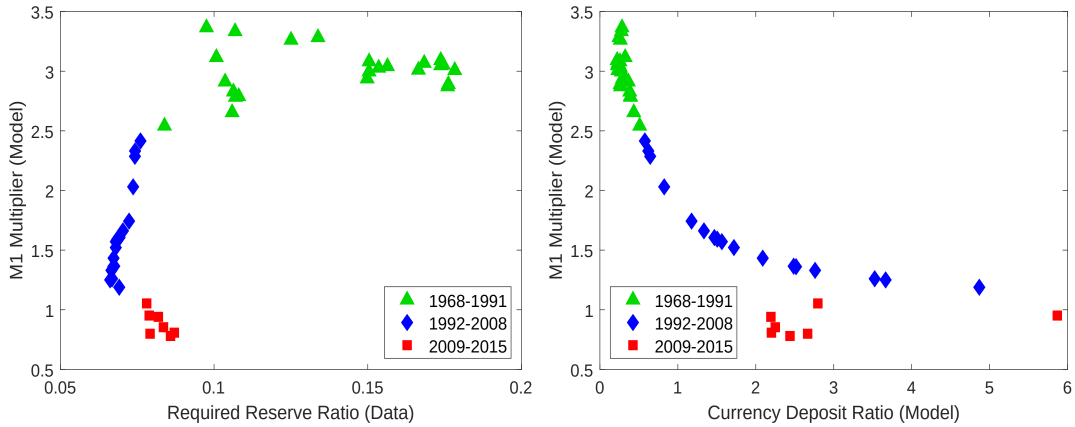


**Figure 9:** Money multiplier, currency/deposit ratio and required reserve ratio: Model

**Table 3:** Money demand and Credit: Model vs. Data

| Dependent Variable: $ln(m_t)$ | OLS | | CCR | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| | (1) | (2) | (3) | (4) |
| $r_t$ | 1.600*** | 7.097 | −2.755** | −6.206 |
| | (0.419) | | (1.171) | |
| $ln(uc_t)$ | | | −0.282*** | −1.039 |
| | | | (0.098) | |
| $adjR^2$ | 0.109 | 0.390 | 0.981 | 0.845 |

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Observation 3 shows that there is no negative relationship between the required reserve ratio and the money multiplier. Figure 3 shows that there is a consistent decrease in the money multiplier since 1992 which was not accompanied by decreases in the required reserve ratio but accompanied by increases in the currency deposit ratio. In Figure 9, the model successfully regenerates what we have observed. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris.

Observation 4 shows that consideration of unsecured credit recovers downward-sloping money demand. The model can reproduce this result. Using the counterfactual data generated from the model, Table 3 shows that unsecured credit consideration recovers the downward-sloping money demand model, as in the data. Column (2) show that only regression on interest rate gives positive estimates. Adding unsecured credit to output to the money demand equation recovers downward sloping money demand. Column (4) report ne .

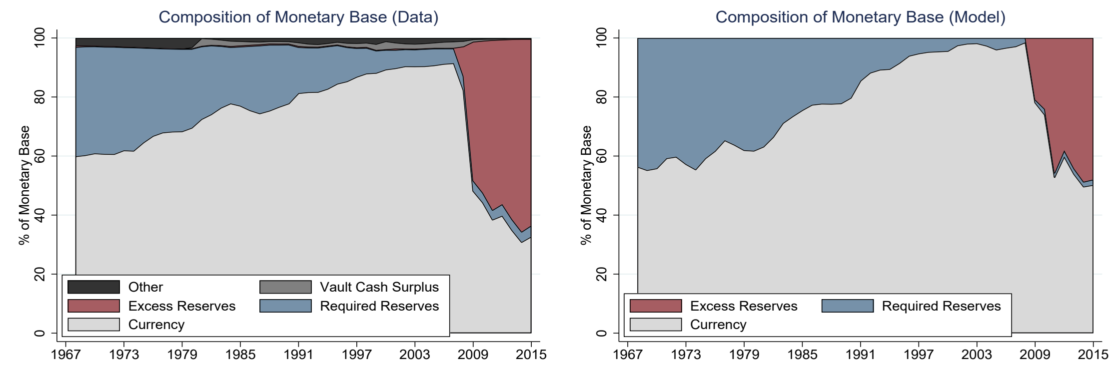


**Figure 10:** Composition of monetary base: data vs. model

**Composition of Monetary Base** The model also generates the composition of the monetary base over time. Figure 10 compares the composition of the monetary base from the data and the model. The model successfully generates the changes for each component of the monetary base - currency, required reserves, and excess reserves - both before and after 2008. The portion of currency consistently increased during 1968-2007 and it drastically decreased as the portion of excess reserves increased a lot. The portion of required reserves has been consistently decreased. It is worth emphasizing that a share of currency in the monetary base has been large. Until 2008, its share had increased from 60% to 90%. After 2008, its share has been around 40%. Even though a share of currency has been reduced, it still accounts for a large portion of the central bank's balance sheet. This implies that if one wants to consider the central bank's balance sheet as an important channel of monetary policy, one should incorporate the currency in the analysis.

## 4.3 Dynamics

In this section, I characterize the dynamics of the monetary equilibrium using the calibrated parameters. To solve the dynamic equilibrium defined in Definition 1, I apply a global projection method using Chebyshev polynomials, rather than relying on some approximations around the steady state of the economy (e.g., First-order, second-order, or higher-order Taylor expansion, or log-linearization).

Figure 11 displays the policy functions under different target interest rates, $i$. In the top-left panel, we observe that the real lending rate $\rho_t$ decreases as $K_t$ increases, which aligns with the standard neoclassical growth model. However, as the target interest rate changes, the policy function shifts accordingly, with lower interest rates shifting

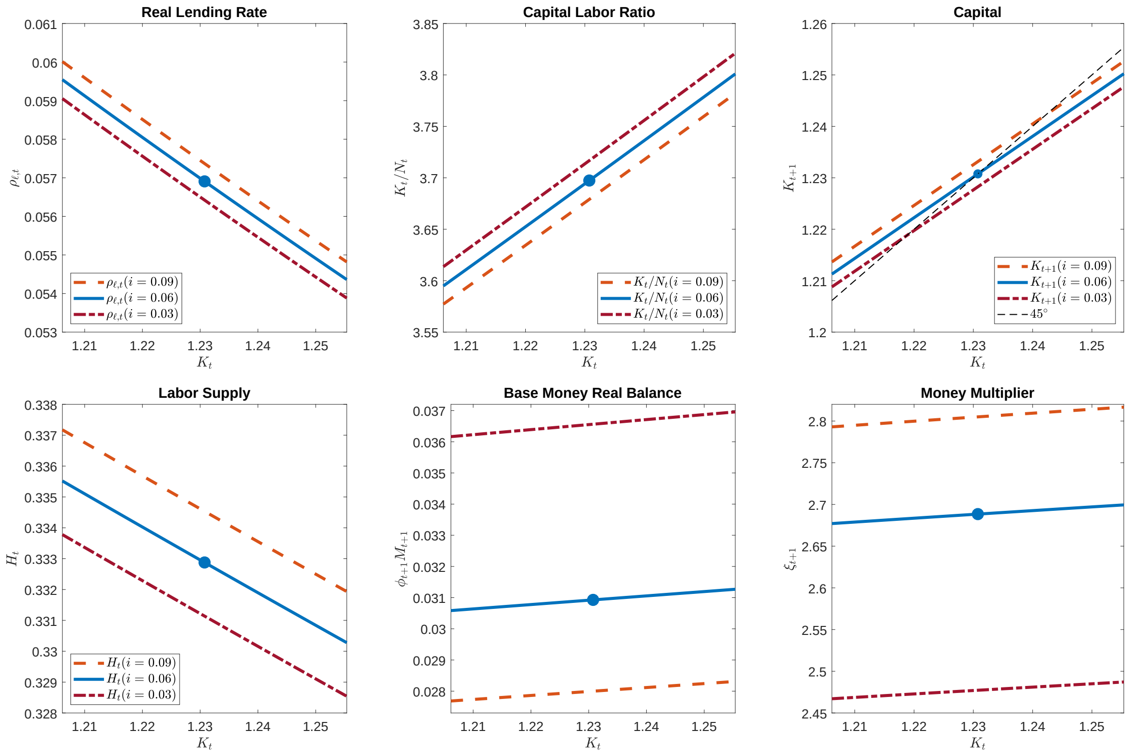


**Figure 11:** Policy Functions

left and higher interest rates shifting right. Since $\rho_t = F_K(K_t, N_t) - \delta$, changes in target interest rate can influence the marginal product of the capital and capital-labor ratio which eventually influences investments as well. But the changes in the real rate are not big. High $k$, .. lower labor supply.

As observed, the money multiplier is positively correlated with output, which aligns with the findings of Freeman and Kydland (2000). So does monetary base balances.

While the central bank can control the interest rate, it cannot control the money supply once it set the interest rate target, because the central bank needs to supply a monetary base to meet market demand. As $K_t$ increases market needs more investment which in turn increases the firm's demand for lending. Then The banks lend loans to fulfill the firm's demand by creating deposit money. Consequently, the central bank supplies reserves to banks to accommodate the banks' lending. Therefore, by setting short-term policy rates, the money supply is endogenously determined by the demand of the firm and households. Through the bank's balance sheet, money and the real sector are linked to each other.

## 4.4 Implication for Monetary Transmission

In the model and above quantitative exercise, central bank sets interest rates by controlling the monetary base. This is consistent with the conventional notion of monetary policy. Recall Romer (2000):

> [The] appropriate concept of money is unambiguously high-powered money. Here $M$ is not a variable the central bank is targeting, but rather one it is manipulating to make interest rates behave in the way it desires. This is an excellent description of high-powered money. Moreover, for high-powered money, the assumption that the opportunity cost of holding money is the nominal rate is appropriate. In addition, the assumption that the central bank can control the money stock is a much better approximation for high-powered money than for broader measures of the money stock.

The exercise shows that, given credit conditions, there exists a one-to-one correspondence from a set of monetary policy variables to the quantity of currency, the quantity of reserves, the quantity of excess reserves, and the money multiplier. In this approach, one does not need to rely on the assumption that the central bank directly controls monetary aggregates or the even more abstract assumption that the central bank controls interest rates out of nothing. Instead, the model gives an explicit process of monetary policy from setting short-term policy rates by controlling the monetary base to its influence on the monetary aggregate. It not only provides an explicit monetary transmission mechanism but also gives different views on monetary policy. For example, unconventional monetary policies, such as quantitative easing, are not asset purchases independent of the target level of the short-term policy rate. Rather, they are active uses of two different interest rates: interest on reserves and short-term policy rates. Changes in reserves reflect the changes in short-term policy rates and vice versa, implying we also can interpret them as the uses of interest on reserves and quantity of monetary base (or reserves) as well. The model does not require the assumption of independence between equilibrium prices (short-term policy rates) and quantities (monetary base or reserves).

# 5 Concluding Remarks

This paper develops a monetary-search model with fractional reserve banking and unsecured credit, and studies the money creation process. In the fractional reserve banking system, money is created when banks make loans. The bank's inside money creation, however, can be constrained by the reserve requirement and the reserves.

Banks hold excess reserves when the central bank pays sufficiently high interest on reserves. In this case, the money multiplier and the quantity of the reserve depend on the nominal interest rate and the interest on reserves rather than the reserve requirement. In contrast to the previous works, these two interest rates play distinct roles and the quantity of reserves are not independent from the interest rate management. Paying interest on reserves can move the economy from the scarce reserves regime to the ample reserves regime, which is consistent with what we have seen in the US economy. The quantitative analysis can generate simulated data that resemble the actual data. This paper provides evidence from the model and the data that suggests that the dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves with the low nominal interest rate.

This work can be extended in various ways. Although I focus on the centralized market for the reserves with homogeneous banks, in reality, the market for reserves is a decentralized interbank market and banks have different portfolios. Therefore, one can further investigate how much the market structure and heterogeneity matter for the transmission of monetary policy (e.g., Afonso and Lagos, 2015; Armenter and Lester, 2017; Afonso, Armenter and Lester, 2019). Second, I assume that bank assets are composed of loans and reserves. But commercial banks' assets are mainly composed of securities, loans, and reserves. Extending the model to incorporate banks' portfolio choices and analyzing the role of investment, financial regulation, and monetary policy can open up other research avenues. (e.g., Rocheteau, Wright and Zhang, 2018a).

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## Appendix A Interest on reserves and floor system?

This section discusses details about the interest on reserves and the floor system. The Federal Reserve started paying interest on reserves (IOR) in Oct 2008. In contrast to the idea that paying interest on reserves provides a floor, Figure 12 shows that interest on reserves has been equal to the upper bound of the Fed's target range. Instead, the interest rate of the Overnight Reverse Repurchase (ON RRP) has been equal to the lower bound of the target rate. To put it simply, the interest on reserves serves as the upper bound of the target range, while the ON RRP rate acts as the lower bound of the target range for most of the period. Therefore, the short-term policy rate is tightly controlled within this target range.

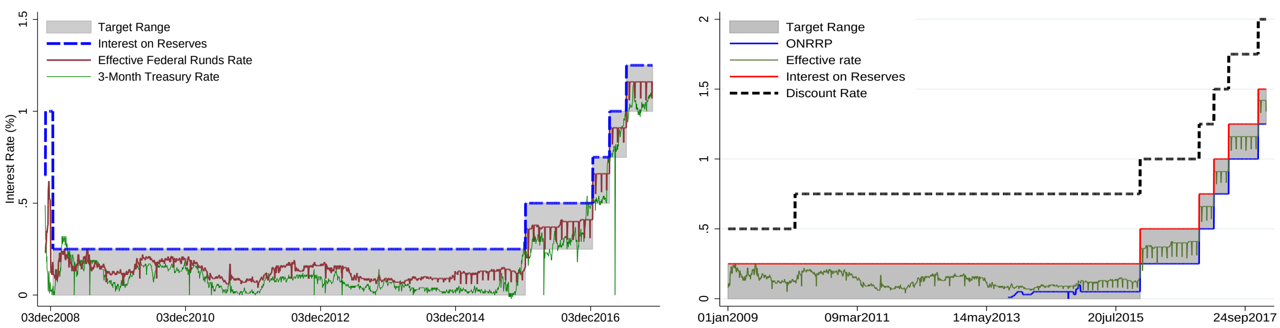


**Figure 12:** Interest on reserves, ON RRP and target range

Given that the federal funds market is a market for unsecured bilateral interbank lending of reserves, a lower federal funds rate than the IOR may seem counterintuitive. This discrepancy is due to the institutional framework. While both depository institutions (DIs) and government-sponsored enterprises (GSEs) can trade in the federal funds market, only DIs are eligible to earn IOR. The GSEs, such as the Federal Home Loan Banks, can earn arbitrage profits in the federal funds market by borrowing from the repo market and lending to DIs who can earn interest on reserves. Armenter and Lester (2017) present a model of interbank trade in which GSEs' lending to DIs that earn IOR determines the federal funds rates. Since GSEs also participate in the repo market, the ON RRP facility's interest rate serves as an alternative for financial institutions not eligible for the IOR, establishing a lower bound on the federal funds rate. See Afonso, Entz, LeSueur et al. (2013b) and Afonso, Entz, LeSueur et al. (2013a) for the related discussion on the trade in the federal funds market.

As Figure 12 shows, the federal funds rate has never touched the lower bound of the target range or reached zero. This contradicts some recent models that emphasize the occasionally-binding ZLB constraint (e.g., Billi and Galí, 2020; Fernández-Villaverde, Marbet, Nuño and Rachedi, 2021; Holden, 2021). Instead, the rate has fluctuated within the target ranges over time.

# Appendix B Proofs

**Proof of Proposition 2.** Recall (8)-(9) without the assumption of interior, and substitute (26):

$$\sigma_1\lambda(q_{1t}) + \sigma_3\lambda(q_{3t}) \leq \frac{i_{t-1}}{\nu} \tag{29}$$

$$\sigma_2\lambda(q_{2t}) + \sigma_3\lambda(q_{3t}) \leq \frac{1}{\nu}\left(\frac{1+i_{t-1}}{1+i_{d,t}} - 1\right) \tag{30}$$

If $i_{t-1} \geq \nu(\sigma_1+\sigma_3)\lambda(0)$, then DM consumptions are zero, $q_{1t} = m = 0$. In this case, the real balances of currency are zero, $\phi M = m = 0$. As long as $i_{t-1} < \nu(\sigma_1+\sigma_3)\lambda(0)$, the household holds positive currency balances $\phi M = m > 0$ and the real balances of the monetary base are positive $\phi B = \phi(M+R) > 0$.

When the central bank pays interest on reserves such that $i_r > i$, the bank holds positive reserve balances that satisfy $i_r - i = \gamma'(\tilde{r})$. Since the real reserve balances are positive, $\phi R = n\tilde{r} > 0$, the real balances of the monetary base are positive $\phi B = \phi(M+R) > 0$. When $i_t > \nu(\sigma_1+\sigma_2)\lambda(0)$, but $i_t > i_{rt}$. The bank's are not holding any reserves because positive reserve balance should satisfies $i_r - i = \gamma'(\tilde{r})$. In this case, $\phi B = 0$. Consider a case with $i_t < \nu(\sigma_1+\sigma_2)\lambda(0)$. If $i_{dt} > 0$, $\frac{i_{dt}}{1+i_{dt}}(1+i_t) = \nu\sigma_1\lambda(q_{1t})$ and $\frac{1+i_t}{1+i_{dt}} - 1 = \nu\lambda(q_{2t})$ implying $q_{1t}$, $q_{2t} > 0$. If $i_{dt} = 0$, then $i_t/\nu = \sigma_1\lambda(q_{1t}) + \sigma_2\lambda(q_{2t})$ with $q_{1t} = q_{2t} > 0$. In both cases, there exists positive real balances of currency, $m > 0$, which implies $\phi B > 0$.

Consider a case with $i_{rt} < i_t$. From the bank's optimality condition with non-binding reserve requirement, Equation (19) cannot be satisfied with $\tilde{r} \geq 0$ but the bank's optimality condition with binding reserve requirement, Equation (16) can be satisfied. Therefore, the bank does not hold excess reserves when $i_{rt} < i_t$. When $i_{rt} = i_t$, with non-binding reserve requirement, $\tilde{r} = 0$. With binding reserve requirement, $i_t - i_{dt} = \chi\gamma'(\tilde{r})$. Since the scarce-reserves case should satisfies $i_t \geq i_{dt} \geq 0$, the bank holds positive reserves balances with binding reserve requirement constraint as long as $i_t > i_{dt}$. Therefore, when $i_{rt} = i_t$ the bank does not hold excess reserves. If $i_{rt} > i_t$, there always exists $(\tilde{r}, \tilde{\ell})$ satisfies (12)-(14) and (15) with $\lambda_\chi$=0, non-binding reserve requirement constraint. Given $i_{rt} - i_t = \gamma'(\tilde{r})$, the bank holds positive amount of reserve $\tilde{r} > 0$. ■

**Proof of Proposition 4.** Consider the first case. When $i_{rt} > \bar{\iota}$ and $i_t > 0$, $i_{dt} = i_t > 0$ implying $\lambda(q_{2t}) = 0$. Then $q_{2t} = q^*$. Consider the second case. When $i_d = 0$, $i_t/\nu = \sigma_1\Lambda(m_t) + \sigma_2\Lambda(\max\{p^*, m_t + \bar{b}\})$. Then, if $\bar{b} > p^* - m$, we have $q_{2t} = q^*$ and

$m = \Lambda^{-1}\left(\frac{i_t}{\nu\sigma_1}\right)$. Therefore, when $i_d = 0$, $q_{2t} = q^*$ under the following condition:

$$\bar{b} > \hat{b}_t \equiv p^* - \Lambda^{-1}\left(\frac{i_t}{\nu\sigma_1}\right)$$

When $i_d > 0$, we have

$$\begin{aligned} i_t - (i_t - i_{dt})/(1+i_{dt}) &= \nu\sigma_1\Lambda(m_t) \\ (i_t - i_{dt})/(1+i_{dt}) &= \nu\sigma_2\Lambda\Big(\max\{p^*, m_t + d_t + \bar{b}\}\Big). \end{aligned}$$

Since $m_t > \Lambda^{-1}\left(\frac{i_t}{\nu\sigma_1}\right)$ when $i_d > 0$, it is straightforward to show that $\bar{b} > \hat{b}_t$ suffices to have $q_{2t} = q^*$. ■

**Proof of Proposition 5.** ■

**Comparative Statics under the ample-reserves equilibrium** Recall (15) and (18)-(20)

$$0 = G^1 \equiv (1+i_\ell)\tilde{\ell} + (1+i_r)\tilde{r} - (1+i)(\tilde{\ell}+\tilde{r}) - \gamma(\tilde{r}) - \eta(\tilde{\ell}) - \kappa \tag{31}$$

$$0 = G^2 \equiv i_r - i - \gamma'(\tilde{r}) \tag{32}$$

$$0 = G^3 \equiv i_\ell - i - \eta'(\tilde{\ell}) \tag{33}$$

Applying the implicit function theorem yields

$$\underbrace{\begin{bmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{bmatrix}}_{\equiv \boldsymbol{G}} \begin{bmatrix} d\tilde{\ell} \\ d\tilde{r} \\ di_\ell \end{bmatrix} = - \begin{bmatrix} G^1_{i_r} di_r + G^1_i di \\ G^2_{i_r} di_r + G^2_i di \\ G^3_{i_r} di_r + G^3_i di \end{bmatrix}$$

where

$$\begin{aligned} G^1_{i_r} &= \tilde{r}, & G^1_i &= -(\tilde{\ell}+\tilde{r}) \\ G^2_{i_r} &= 1, & G^2_i &= -1 \\ G^3_{i_r} &= 0, & G^3_i &= -1. \end{aligned} \qquad |\boldsymbol{G}| = \begin{vmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{vmatrix} = \begin{vmatrix} 0 & 0 & \tilde{\ell} \\ 0 & -\gamma''(\cdot) & 0 \\ -\eta''(\cdot) & 0 & 1 \end{vmatrix} = -\eta''(\cdot)\gamma''(\cdot)\tilde{\ell} < 0$$

By Cramer's rule, we have

$$\frac{\partial\tilde{\ell}}{\partial i}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}-G_i^1 & G_{\tilde{r}}^1 & G_{i_\ell}^1\\ -G_i^2 & G_{\tilde{r}}^2 & G_{i_\ell}^2\\ -G_i^3 & G_{\tilde{r}}^3 & G_{i_\ell}^3\end{vmatrix}>0,\quad \frac{\partial\tilde{\ell}}{\partial i_r}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}-G_{i_r}^1 & G_{\tilde{r}}^1 & G_{i_\ell}^1\\ -G_{i_r}^2 & G_{\tilde{r}}^2 & G_{i_\ell}^2\\ -G_{i_r}^3 & G_{\tilde{r}}^3 & G_{i_\ell}^3\end{vmatrix}<0$$

$$\frac{\partial\tilde{r}}{\partial i}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}G_{\tilde{\ell}}^1 & -G_i^1 & G_{i_\ell}^1\\ G_{\tilde{\ell}}^2 & -G_i^2 & G_{i_\ell}^2\\ G_{\tilde{\ell}}^3 & -G_i^3 & G_{i_\ell}^3\end{vmatrix}<0,\quad \frac{\partial\tilde{r}}{\partial i_r}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}G_{\tilde{\ell}}^1 & -G_{i_r}^1 & G_{i_\ell}^1\\ G_{\tilde{\ell}}^2 & -G_{i_r}^2 & G_{i_\ell}^2\\ G_{\tilde{\ell}}^3 & -G_{i_r}^3 & G_{i_\ell}^3\end{vmatrix}>0$$

$$\frac{\partial i_\ell}{\partial i}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}G_{\tilde{\ell}}^1 & G_{\tilde{r}}^1 & -G_i^1\\ G_{\tilde{\ell}}^2 & G_{\tilde{r}}^2 & -G_i^2\\ G_{\tilde{\ell}}^3 & G_{\tilde{r}}^3 & -G_i^3\end{vmatrix}>0,\quad \frac{\partial i_\ell}{\partial i_r}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}G_{\tilde{\ell}}^1 & G_{i_r}^1 & -G_{i_r}^1\\ G_{\tilde{\ell}}^2 & G_{i_r}^2 & -G_{i_r}^2\\ G_{\tilde{\ell}}^3 & G_{i_r}^3 & -G_{i_r}^3\end{vmatrix}<0.$$

**Comparative Statics under the scarce-reserves equilibrium**

$$\begin{aligned}
0=J^1 &\equiv -i+\nu\sigma_1\Lambda(m)+\nu\sigma_2\Lambda\left(m+\bar{b}+\frac{n\tilde{r}}{\chi}\left\{1+(1-\chi)i+\chi i_r-\chi\gamma'(\tilde{r})\right\}\right)\\
0=J^2 &\equiv -\frac{1+i}{1+i_d}+1+\nu\sigma_2\Lambda\left(m+\bar{b}+\frac{n\tilde{r}}{\chi}\left\{1+(1-\chi)i+\chi i_r-\chi\gamma'(\tilde{r})\right\}\right)\\
0=J^3 &\equiv -\kappa+\left\{1+i+\eta'(\tilde{\ell})\right\}\tilde{\ell}+(1+i_r)\tilde{r}-\left\{1+(1-\chi)i+\chi i-\chi\gamma'(\tilde{r})\right\}\tilde{r}/\chi\\
&\quad -(1+i)\left(\tilde{\ell}+\tilde{r}-\tilde{r}/\chi\right)-\eta(\tilde{\ell})-\gamma(\tilde{r})\\
0=J^4 &\equiv -U'\left[AF\left(n\tilde{\ell},N\right)-\delta n\tilde{\ell}\right]+\zeta/AF_N\left(n\tilde{\ell},N\right)\\
0=J^5 &\equiv -AF_K\left(n\tilde{\ell},N\right)+\frac{1}{\beta}-1+\delta+\frac{\eta'(\tilde{\ell})}{\beta(1+i)}
\end{aligned}$$

5 equations 5 unknowns $(q_1,q_2,k,h,\tilde{b}^c)$ given $(\rho_1,\psi,\phi,\tilde{B})$

$$\underbrace{\begin{bmatrix}J_m^1 & J_{\tilde{r}}^1 & J_{\tilde{\ell}}^1 & J_n^1 & J_N^1\\ J_m^2 & J_{\tilde{r}}^2 & J_{\tilde{\ell}}^2 & J_n^2 & J_N^2\\ J_m^3 & J_{\tilde{r}}^3 & J_{\tilde{\ell}}^3 & J_n^3 & J_N^3\\ J_m^4 & J_{\tilde{r}}^4 & J_{\tilde{\ell}}^4 & J_n^4 & J_N^4\\ J_m^5 & J_{\tilde{r}}^5 & J_{\tilde{\ell}}^5 & J_n^5 & J_N^5\end{bmatrix}}_{\equiv\boldsymbol{J}}\begin{bmatrix}dm\\ d\tilde{r}\\ d\tilde{\ell}\\ dn\\ dN\end{bmatrix}=-\begin{bmatrix}J_i^1di+J_{i_r}^1di_r+J_{\bar{b}}^1d\bar{b}\\ J_i^2di+J_{i_r}^2di_r+J_{\bar{b}}^2d\bar{b}\\ J_i^3di+J_{i_r}^3di_r+J_{\bar{b}}^3d\bar{b}\\ J_i^4di+J_{i_r}^4di_r+J_{\bar{b}}^4d\bar{b}\\ J_i^5di+J_{i_r}^5di_r+J_{\bar{b}}^5d\bar{b}\end{bmatrix}$$

$$\boldsymbol{J} = \begin{bmatrix} J^1_m & J^1_{\tilde{r}} & J^1_{\tilde{\ell}} & J^1_n & J^1_N \\ J^2_m & J^2_{\tilde{r}} & J^2_{\tilde{\ell}} & J^2_n & J^2_N \\ J^3_m & J^3_{\tilde{r}} & J^3_{\tilde{\ell}} & J^3_n & J^3_N \\ J^4_m & J^4_{\tilde{r}} & J^4_{\tilde{\ell}} & J^4_n & J^4_N \\ J^5_m & J^5_{\tilde{r}} & J^5_{\tilde{\ell}} & J^5_n & J^5_N \end{bmatrix}$$

$$= \begin{bmatrix} \nu[\sigma_1\Lambda'(z_1) + \sigma_2\Lambda'(z_2)] & J^1_{\tilde{r}} & 0 & J^1_n & 0 \\ \nu\sigma_2\Lambda'(z_2) & J^2_{\tilde{\ell}} & 0 & a & 0 \\ 0 & \gamma''(\cdot)\tilde{r} & \eta''(\cdot)\tilde{\ell} & 0 & 0 \\ 0 & 0 & -[]U'' - \frac{\zeta F_{KN} n}{AF_N^2} & -[]U'' - \frac{\zeta F_{KN} \ell}{AF_N^2} & -AF_{KN}U''(\cdot) - \frac{\zeta F_{NN}}{AF_N^2} \\ 0 & 0 & -AF_{KK}n + \frac{\eta''}{\beta(1+i)} & -AF_{KK}\tilde{\ell} & -AF_{KN} \end{bmatrix}$$


$$\boldsymbol{J} = \begin{bmatrix} J^1_m & J^1_{\tilde{r}} & J^1_{\tilde{\ell}} & J^1_n & J^1_N \\ J^2_m & J^2_{\tilde{r}} & J^2_{\tilde{\ell}} & J^2_n & J^2_N \\ J^3_m & J^3_{\tilde{r}} & J^3_{\tilde{\ell}} & J^3_n & J^3_N \\ J^4_m & J^4_{\tilde{r}} & J^4_{\tilde{\ell}} & J^4_n & J^4_N \\ J^5_m & J^5_{\tilde{r}} & J^5_{\tilde{\ell}} & J^5_n & J^5_N \end{bmatrix}$$


$$= \begin{bmatrix} \nu\sum_{j=1}^{2}\sigma_j\Lambda'(z_j) & J^1_{\tilde{r}} & 0 & J^1_n & 0 \\ \nu\sigma_2\Lambda'(z_2) & J^2_{\tilde{\ell}} & 0 & a & 0 \\ 0 & \gamma''(\cdot)\tilde{r} & \eta''(\cdot)\tilde{\ell} & 0 & 0 \\ 0 & 0 & -[]U'' - \frac{\zeta F_{KN} n}{AF_N^2} & -[]U'' - \frac{\zeta F_{KN} \ell}{AF_N^2} & -AF_{KN}U''(\cdot) - \frac{\zeta F_{NN}}{AF_N^2} \\ 0 & 0 & -AF_{KK}n + \frac{\eta''}{\beta(1+i)} & -AF_{KK}\tilde{\ell} & -AF_{KN} \end{bmatrix}$$

$$|\boldsymbol{J}| = \nu\sigma_1\Lambda'(z_1)\left\{a_{22}\eta''\tilde{\ell}|\bar{\boldsymbol{D}}| + \gamma''\tilde{r}\Lambda'(z_2)\frac{\tilde{r}\nu\sigma_2}{\chi}(1+i_d)|\bar{\boldsymbol{B}}|\right\}$$
$$+ \nu\sigma_2\Lambda'(z_2)\left\{(a_{22} - a_{11})\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} < 0$$

where

$$
\begin{aligned}
a_{22} &= -\left\{\frac{1+i}{(1+i_d)^2} + \frac{\nu\sigma_2 n\tilde{r}}{\chi}L'(z_2)\right\}\gamma'' + \frac{\nu\sigma_2 n}{\chi}(1+i_d)L'(z_2) < 0 \\
a_{22} - a_{11} &= -\frac{1+i}{(1+i_d)^2}\gamma'' < 0 \\
|\bar{\boldsymbol{D}}| &= -U'' A^2 F_N F_{KK} < 0 \\
|\bar{\boldsymbol{B}}| &= -U'' n A^2 F_N F_{KK} + \frac{\eta''(\cdot)}{\beta(1+i)}\left(U'' A F_N + \frac{\zeta}{A F_N^2} F_{NN}\right) < 0
\end{aligned}
$$

**Proposition 10.** *In the ample-reserve equilibrium, $\partial i_\ell/\partial i > 0$, $\partial i_\ell/\partial i_r < 0$, $\partial i_d/\partial i = 0$ and $\partial \rho/\partial i_r < 0$ but $\partial \rho/\partial i$ is ambiguous. In the scarce-reserve equilibrium, when $\sigma_1$ is small, $\partial i_\ell/\partial i > 0$, $\partial i_\ell/\partial i_r < 0$, $\partial i_d/\partial i > 0$ and $\partial \rho/\partial i_r < 0$ but $\partial \rho/\partial i$ is ambiguous.*

**Proof of Proposition 6.** Consider the ample-reserves equilibrium. It can be summarized as 4 equations 4 unknowns as follows.

$$
\begin{aligned}
0 = G^1 &\equiv -\gamma'(\tilde{r}) + i_r - i \\
0 = G^2 &\equiv -\kappa + \{1 + i + \eta'(\tilde{\ell})\}\tilde{\ell} + (1 + i_r)\tilde{r} - (1 + i)(\tilde{\ell} + \tilde{r}) - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \\
0 = G^3 &\equiv -1/\beta + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + f'(k) \\
0 = G^4 &\equiv -f(k) + f'(k)k + \zeta/U'[(f(k) - \delta k)N]
\end{aligned}
$$

where $k = K/H$. Applying the implicit function theorem yields

$$
\underbrace{\begin{bmatrix} G^1_k & G^1_N & G^1_\ell & G^1_r \\ G^2_k & G^2_N & G^2_\ell & G^2_r \\ G^3_k & G^3_N & G^3_\ell & G^3_r \\ G^4_k & G^4_N & G^4_\ell & G^4_r \end{bmatrix}}_{\equiv \boldsymbol{G}} \begin{bmatrix} dk \\ dN \\ d\tilde{\ell} \\ d\tilde{r} \end{bmatrix} = - \underbrace{\begin{bmatrix} G^1_{i_r} & G^1_i & G^1_b \\ G^2_{i_r} & G^2_i & G^2_b \\ G^3_{i_r} & G^3_i & G^3_b \\ G^4_{i_r} & G^4_i & G^4_b \end{bmatrix}}_{\equiv \bar{\boldsymbol{G}}} \begin{bmatrix} di \\ di_r \\ d\bar{b} \end{bmatrix} \tag{34}
$$

where

$$
\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 0 & -\gamma'' \\ 0 & 0 & \eta''\tilde{\ell} & 0 \\ f'' & 0 & -\frac{\eta''}{\beta(1+i)} & 0 \\ f''k - N(f' - \delta)U''\zeta/(U')^2 & -(f - \delta k)U''\zeta/(U')^2 & 0 & 0 \end{bmatrix}
$$

and

$$
\bar{\boldsymbol{G}} = \begin{bmatrix} -1 & 1 & 0 \\ -\tilde{r} & \tilde{r} & 0 \\ \frac{\eta'}{\beta(1+i)^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
$$

By Cramer's rule, we have

$$
\frac{\partial \tilde{\ell}}{\partial i} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} G^1_k & G^1_N & -G^1_i & G^1_r \\ G^2_k & G^2_N & -G^2_i & G^2_r \\ G^3_k & G^3_N & -G^3_i & G^3_r \\ G^4_k & G^4_N & -G^4_i & G^4_r \end{vmatrix} = \frac{-U''(f - \delta k)\zeta/(U')^2 f''\gamma''\tilde{r}}{|\boldsymbol{G}|} > 0,
$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} G_k^1 & G_N^1 & -G_{i_r}^1 & G_r^1 \\ G_k^2 & G_N^2 & -G_{i_r}^2 & G_r^2 \\ G_k^3 & G_N^3 & -G_{i_r}^3 & G_r^3 \\ G_k^4 & G_N^4 & -G_{i_r}^4 & G_r^4 \end{vmatrix} = \frac{U''(f-\delta k)\zeta/(U')^2 f''\gamma''\tilde{r}}{|\boldsymbol{G}|} < 0$$

since

$$|\boldsymbol{G}| = -\gamma''(f-\delta k)U''\frac{\zeta}{(U')^2}f''\eta''\tilde{\ell} < 0.$$

Because $\partial\tilde{\ell}/\partial i > 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, $\partial i_\ell/\partial i = 1 + \eta''(\tilde{\ell})\frac{\partial\tilde{\ell}}{\partial i} > 0$. Similarly, because $\partial\tilde{\ell}/\partial i_r < 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, $\partial i_\ell/\partial i_r = \eta''(\tilde{\ell})\frac{\partial\tilde{\ell}}{\partial i_r} < 0$. The real lending rate can be written as $\rho = 1/\beta - 1 + \frac{\eta'(\tilde{\ell})}{\beta(1+i)}$. Then we have $\partial\rho/\partial i_r = \frac{\partial\tilde{\ell}}{\partial i_r}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} < 0$ and $\partial\rho/\partial i = \frac{\partial\tilde{\ell}}{\partial i}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} - \frac{\eta'(\tilde{\ell})}{\beta(1+i)^2} \lesseqgtr 0$.

**Comparative Statics under the scarce-reserves equilibrium** Consider the scarce-reserves equilibrium. It can be expressed as 5 equations 5 unknowns as follows.

$$\begin{aligned}
0 = J^1 \equiv & -i + \nu\sigma_1\Lambda(m) + \nu\sigma_2\Lambda\left(m + \bar{b} + \frac{n\tilde{r}}{\chi}\left\{1+(1-\chi)i + \chi i_r - \chi\gamma'(\tilde{r})\right\}\right) \\
0 = J^2 \equiv & -\frac{1+i}{1+i_d} + 1 + \nu\sigma_2\Lambda\left(m + \bar{b} + \frac{n\tilde{r}}{\chi}\left\{1+(1-\chi)i + \chi i_r - \chi\gamma'(\tilde{r})\right\}\right) \\
0 = J^3 \equiv & -\kappa + \left\{1+i+\eta'(\tilde{\ell})\right\}\tilde{\ell} + (1+i_r)\tilde{r} - \left\{1+(1-\chi)i + \chi i - \chi\gamma'(\tilde{r})\right\}\tilde{r}/\chi \\
& - (1+i)\left(\tilde{\ell} + \tilde{r} - \tilde{r}/\chi\right) - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \\
0 = J^4 \equiv & -U'\left[AF\left(n\tilde{\ell}, N\right) - \delta n\tilde{\ell}\right] + \zeta/AF_N\left(n\tilde{\ell}, N\right) \\
0 = J^5 \equiv & -AF_K\left(n\tilde{\ell}, N\right) + \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\tilde{\ell})}{\beta(1+i)}
\end{aligned}$$

5 equations 5 unknowns $(q_1, q_2, k, h, \tilde{b}^c)$ given $(\rho_1, \psi, \phi, \tilde{B})$

$$\underbrace{\begin{bmatrix} J_m^1 & J_{\tilde{r}}^1 & J_{\tilde{\ell}}^1 & J_n^1 & J_N^1 \\ J_m^2 & J_{\tilde{r}}^2 & J_{\tilde{\ell}}^2 & J_n^2 & J_N^2 \\ J_m^3 & J_{\tilde{r}}^3 & J_{\tilde{\ell}}^3 & J_n^3 & J_N^3 \\ J_m^4 & J_{\tilde{r}}^4 & J_{\tilde{\ell}}^4 & J_n^4 & J_N^4 \\ J_m^5 & J_{\tilde{r}}^5 & J_{\tilde{\ell}}^5 & J_n^5 & J_N^5 \end{bmatrix}}_{\equiv \boldsymbol{J}} \begin{bmatrix} dm \\ d\tilde{r} \\ d\tilde{\ell} \\ dn \\ dN \end{bmatrix} = - \begin{bmatrix} J_i^1 di + J_{i_r}^1 di_r + J_{\bar{b}}^1 d\bar{b} \\ J_i^2 di + J_{i_r}^2 di_r + J_{\bar{b}}^2 d\bar{b} \\ J_i^3 di + J_{i_r}^3 di_r + J_{\bar{b}}^3 d\bar{b} \\ J_i^4 di + J_{i_r}^4 di_r + J_{\bar{b}}^4 d\bar{b} \\ J_i^5 di + J_{i_r}^5 di_r + J_{\bar{b}}^5 d\bar{b} \end{bmatrix} = - \underbrace{\begin{bmatrix} J_i^1 & J_{i_r}^1 & J_{\bar{b}}^1 \\ J_i^2 & J_{i_r}^2 & J_{\bar{b}}^2 \\ J_i^3 & J_{i_r}^3 & J_{\bar{b}}^3 \\ J_i^4 & J_{i_r}^4 & J_{\bar{b}}^4 \\ J_i^5 & J_{i_r}^5 & J_{\bar{b}}^5 \end{bmatrix}}_{\equiv \bar{\boldsymbol{J}}} \begin{bmatrix} di \\ di_r \\ d\bar{b} \end{bmatrix}$$

where

$$\boldsymbol{J} = \begin{bmatrix} \nu\sum_{j=1}^{2}\sigma_j\Lambda'(z_j) & J_{\tilde{r}}^1 & 0 & J_n^1 & 0 \\ \nu\sigma_2\Lambda'(z_2) & J_{\tilde{\ell}}^2 & 0 & a & 0 \\ 0 & \gamma''(\cdot)\tilde{r} & \eta''(\cdot)\tilde{\ell} & 0 & 0 \\ 0 & 0 & -[]U'' - \frac{\zeta F_{KN} n}{AF_N^2} & -[]U'' - \frac{\zeta F_{KN}\ell}{AF_N^2} & -AF_{KN}U''(\cdot) - \frac{\zeta F_{NN}}{AF_N^2} \\ 0 & 0 & -AF_{KK}n + \frac{\eta''}{\beta(1+i)} & -AF_{KK}\tilde{\ell} & -AF_{KN} \end{bmatrix}$$

and

$$\bar{\boldsymbol{J}} = \begin{bmatrix} -1 + \frac{1-\chi}{\chi}n\tilde{r}\nu\sigma_2\Lambda'(z_2) & \upsilon\sigma_2\Lambda'(z_2) & \upsilon\sigma_2\Lambda'(z_2) \\ -\frac{1}{1+i_d} + \frac{1-\chi}{\chi}n\tilde{r}\nu\sigma_2\Lambda'(z_2) + \frac{1+i}{(1+i_d)^2}(1-\chi) & \upsilon\sigma_2\Lambda'(z_2) + \frac{1+i}{(1+i_d)^2}\chi & \upsilon\sigma_2\Lambda'(z_2) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{\eta'(\cdot)}{\beta(1+i)^2} & 0 & 0 \end{bmatrix}.$$

The determinant of $\boldsymbol{J}$, $|\boldsymbol{J}|$, is

$$\begin{aligned} |\boldsymbol{J}| =& \nu\sigma_1\Lambda'(z_1)\left\{a_{22}\eta''\tilde{\ell}|\bar{\boldsymbol{D}}| + \gamma''\tilde{r}\Lambda'(z_2)\frac{\tilde{r}\nu\sigma_2}{\chi}(1+i_d)|\bar{\boldsymbol{B}}|\right\} \\ &+ \nu\sigma_2\Lambda'(z_2)\left\{(a_{22}-a_{11})\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} < 0 \end{aligned}$$

where

$$a_{22} = -\left\{\frac{1+i}{(1+i_d)^2} + \frac{\nu\sigma_2 n\tilde{r}}{\chi}L'(z_2)\right\}\gamma'' + \frac{\nu\sigma_2 n}{\chi}(1+i_d)L'(z_2) < 0$$

$$a_{22} - a_{11} = -\frac{1+i}{(1+i_d)^2}\gamma'' < 0$$

$$|\bar{\boldsymbol{D}}| = U''A^2\tilde{\ell}\left\{F_K F_{KN}\left(\frac{1}{\delta}-1\right) - F_N F_{KK}\frac{1}{\delta}\right\} < 0$$

$$|\bar{\boldsymbol{B}}| = U''A^2 n\left\{F_K F_{KN}\left(\frac{1}{\delta}-1\right) - F_N F_{KK}\frac{1}{\delta}\right\} + \frac{\eta''}{\beta(1+i)}\left(U''AF_N + \frac{\zeta}{AF_N^2}F_{NN}\right) < 0$$

$$|\boldsymbol{J}_{\boldsymbol{i_r},\tilde{\boldsymbol{r}}}| = \nu\sigma_1\Lambda'(z_2)\left\{b_{22}\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} + \nu\sigma_2\left\{(b_{22}-b_{12})\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} < 0$$

where

$$b_{22} = -\left\{\frac{1+i}{(1+i_d)^2}\chi + \nu\sigma_2 n\tilde{r}\Lambda'(z_2)\right\} < 0, \qquad b_{22} - b_{12} = -\frac{1+i}{(1+i_d)^2}\chi < 0$$

$$|\boldsymbol{J_{i_r,\tilde{\ell}}}| = \tilde{r}\gamma''|\bar{\boldsymbol{D}}|\left[\nu\sigma_2\Lambda'(z_2)\frac{1+i}{(1+i_d)^2}\chi + \nu\sigma_1\Lambda'(z_1)\left\{\nu\sigma_2\Lambda'(z_2) + \frac{1+i}{(1+i_d)^2}\chi\right\}\right] > 0$$

$$\begin{aligned}
|\boldsymbol{J_{i_r,m}}| =& \frac{1+i}{(1+i_d)^2}\chi\left\{a_{22}\eta''\tilde{\ell}|\bar{\boldsymbol{D}}| + \chi\gamma''\tilde{r}\Lambda'(z_2)\frac{\tilde{r}\nu\sigma_2}{\chi}(1+i_d)|\bar{\boldsymbol{B}}|\right\} \\
& - \left\{\frac{1+i}{(1+i_d)^2}\chi + \nu\sigma_2\Lambda'(z_2)n\tilde{r}\right\}\left\{(a_{22}-a_{12})\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} \\
|\boldsymbol{J_{i,\tilde{r}}}| =& \\
|\boldsymbol{J_{i,\tilde{\ell}}}| =& \\
|\boldsymbol{J_{i,m}}| =&
\end{aligned}$$

$$\begin{aligned}
|\boldsymbol{J_{i,\tilde{r}}}| =& -\nu\sigma_1\Lambda'(z_1)\left\{c_{22}\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} - \nu\sigma_2\Lambda'(z_2)\left\{(c_{22}-c_{12})\eta''\tilde{\ell}|\bar{\boldsymbol{D}}|\right\} - \nu\sigma_1\Lambda'(z_1)\frac{\eta'}{\beta(1+i)^2}|\bar{\boldsymbol{G}}| \\
|\boldsymbol{J_{i,\tilde{\ell}}}| =& \gamma''\tilde{r}\left\{\nu\sigma_1\Lambda'(z_1)\left(U''AF_N + \frac{\eta F_{NN}}{AF_N^2}\right)\frac{\nu\sigma_2\tilde{r}}{\chi}(1+i_d)\Lambda'(z_2)\right\} \\
& + \nu\sigma_2\Lambda'(z_2)\left[\frac{i_d}{1+i_d} + \frac{1+i}{(1+i_d)^2}(1-\chi)\right] \\
& - \nu\sigma_1\Lambda'(z_1)\left[\frac{1}{1+i_d} - \frac{1-\chi}{\chi}n\tilde{r}\nu\sigma_2\Lambda'(z_2) - \frac{1+i}{(1+i_d)^2}(1-\chi)\right] \\
|\boldsymbol{J_{i,m}}| =&
\end{aligned}$$

$$\begin{aligned}
\lim_{\sigma_1\to 0}|\boldsymbol{J_{i,\tilde{r}}}| &> 0 \\
\lim_{\sigma_1\to 0}|\boldsymbol{J_{i,\tilde{\ell}}}| &< 0 \\
|\boldsymbol{J_{i,m}}| &=
\end{aligned}$$

$$\begin{aligned}
|\boldsymbol{J_{\bar{b},\tilde{r}}}| &= \\
|\boldsymbol{J_{\bar{b},\tilde{\ell}}}| &= \\
|\boldsymbol{J_{\bar{b},m}}| &=
\end{aligned}$$

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{|\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{\ell}}}|}{|\boldsymbol{J}|} > 0, \qquad \frac{\partial \tilde{r}}{\partial i} = \frac{|\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} < 0, \qquad \frac{\partial \tilde{m}}{\partial i} = \frac{|\boldsymbol{J}_{\boldsymbol{i},\boldsymbol{m}}|}{|\boldsymbol{J}|} < 0$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{|\boldsymbol{J}_{\boldsymbol{i_r},\tilde{\boldsymbol{\ell}}}|}{|\boldsymbol{J}|} < 0, \qquad \frac{\partial \tilde{r}}{\partial i_r} = \frac{|\boldsymbol{J}_{\boldsymbol{i_r},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} > 0, \qquad \frac{\partial \tilde{m}}{\partial i_r} = \frac{|\boldsymbol{J}_{\boldsymbol{i_r},\boldsymbol{m}}|}{|\boldsymbol{J}|} =?$$

$$\frac{\partial \tilde{\ell}}{\partial \bar{b}} = \tag{35}$$

$$\frac{\partial \tilde{r}}{\partial \bar{b}} = \tag{36}$$

$$\frac{\partial \tilde{m}}{\partial \bar{b}} = \tag{37}$$

$$\tag{38}$$

■

**Proposition 11.** *In the scarce-reserve equilibrium, $\partial i_d/\partial \bar{b} > 0$, $\partial i_\ell/\partial \bar{b} > 0$, and $\partial \rho/\partial \bar{b} > 0$. In the ample reserve equilibrium, $\partial i_d/\partial \bar{b} = \partial i_\ell/\partial \bar{b} = \partial \rho/\partial \bar{b} = 0$.*

**Proof of Proposition 7.** ■

**Proof of Proposition 8.** In the ample-reserves equilibrium, we have $i_d = i$ which implies $d = p^* - m - \bar{b}$ where $m = \Lambda^{-1}[i/(\nu\sigma_1)]$. In this case, $m$ is invariant with respect to $\bar{b}$. Therefore, $\partial d/\partial \bar{b} = -1 < 0$. In the scarce-reserves equilibrium, from the comparative statics result in Proposition 7, we already have $\partial i_\ell/\bar{b} > 0$ and $\partial \rho/\bar{b} > 0$. This implies $\partial n/\bar{b} < 0$. Since $\partial \tilde{r}/\bar{b} < 0$, $\partial r/\bar{b} < 0$ where $r = n\tilde{r}$. Given $d = r/\chi$, $\partial d/\bar{b} < 0$ as well. For money multiplier, $\xi$, in the ample-reserves equilibrium, we have

$$\begin{aligned}
\frac{\partial \xi}{\partial \bar{b}} &= \frac{1}{m+r}\left(\partial m/\partial \bar{b} + \partial d/\partial \bar{b}\right) - \frac{m+d}{(m+r)^2}\left(\partial m/\partial \bar{b} + \partial r/\partial \bar{b}\right) \\
&= \frac{1}{(m+r)}\frac{\partial d}{\partial \bar{b}} < 0
\end{aligned}$$

since $\partial m/\partial \bar{b} = \partial r/\partial \bar{b} = 0$ and $\partial d/\partial \bar{b} < 0$. In the scarce-reserves equilibrium, we have

$$\begin{aligned}
\frac{\partial \xi}{\partial \bar{b}} &= \frac{1}{m+r}\left\{\partial m/\partial \bar{b} + (1/\chi)\partial r/\partial \bar{b}\right\} - \frac{m+r/\chi}{(m+r)^2}\left(\partial m/\partial \bar{b} + \partial r/\partial \bar{b}\right) \\
&= \frac{1}{(m+r)^2}\left\{r\left(1-\frac{1}{\chi}\right)\partial m/\partial \bar{b} + m\left(\frac{1}{\chi}-1\right)\partial r/\partial \bar{b}\right\}.
\end{aligned}$$

In this case, $\partial \xi/\partial \bar{b} < 0$ as long as $\chi < 1$ since $\partial m/\partial \bar{b} > 0$ and $\partial r/\partial \bar{b} < 0$. ■

**Proof of Proposition 9.** Recall $\gamma'(\tilde{r}) = i_r - i$. It is straightforward to show that $\partial\tilde{r}/\partial i_r > 0$ and $\partial\tilde{r}/\partial i < 0$. From comparative statics, we already have $\partial n/\partial i_r > 0$. Combining these two results, we have $\partial r/\partial i_r > 0$ where $r = n\tilde{r}$. In the ample-reserves equilibrium, $i_d = i$ which implies $d = p^* - m - \bar{b}$ where $m = \Lambda^{-1}[i/(\nu\sigma_1)]$. Recall money multiplier $\xi$:

$$\xi = \frac{m+d}{\phi B} = \frac{p^* - \bar{b}}{m+r}$$

Since $r$ is increasing in $i_r$ and $\partial m/\partial i_r = \partial p^*/\partial i_r = \partial\bar{b}/\partial i_r = 0$, we can conclude that $\partial\xi/\partial i_r < 0$. Recall $\partial m/\partial i < 0$ and $\partial r/\partial i < 0$. Then it is straightforward to show that $\partial\xi/\partial i > 0$.

■

# Appendix C Additional Results

## C.1 Chow Test

Figure 3 includes the Chow test for structural breaks. The test result reported in the bottom-left panel of Figure 3 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{RequiredReserves/Deposit})_t \\ &+ \mathbf{1}_{t\geq 1992Q2}[\gamma_0 + \gamma_1(\text{RequiredReserves/Deposit})_t] \\ &+ \mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{RequiredReserves/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 4a reports $F$-statistics which are obtained by testing $\gamma_0 = \gamma_1 = \delta_0 = \delta_1 = 0$. The Chow test in the bottom-right panel of Figure 3 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{Currency/Deposit})_t \\ &+ \mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{Currency/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 4b reports $F$-statistics is obtained by testing $\delta_0 = \delta_1 = 0$. The regression estimates and the Chow test results are summarized at Table 4.

## C.2 Unit Root Test

Columns (2) and (4) in Table 3 includes the canonical cointegrating regression estimates and the cointegration tests. This section reports unit root tests for the series used in

**Table 4:** Chow test for structural breaks

**(a)** Require reserve ratio

| Dependent Variable: Money Multiplier | |
|---|---|
| RR | $-0.601$ |
| | (0.365) |
| RR $\times$ $\mathbf{1}_{t \geq 1992Q2}$ | $132.279^{***}$ |
| | (0.031) |
| RR $\times$ $\mathbf{1}_{t \geq 2008Q4}$ | $-147.943^{***}$ |
| | (8.574) |
| $\mathbf{1}_{t \geq 1992Q2}$ | $9.091^{***}$ |
| | (0.557) |
| $\mathbf{1}_{t \geq 2008Q4}$ | $0.074^{***}$ |
| | (0.611) |
| Constant | $2.813^{***}$ |
| | (0.053) |
| Obs. | 228 |
| $R^2$ | 0.963 |
| DF for numerator | 4 |
| DF for denominator | 222 |
| $F$ Statistic for Chow test | 1711.32 |
| $F$ Statistic for 1% sig. level | 3.40 |
| $F$ Statistic for 0.1% sig. level | 4.79 |

**(b)** Currency deposit ratio

| Dependent Variable: Money Multiplier | |
|---|---|
| CD | $-1.301^{***}$ |
| | (0.027) |
| CD $\times$ $\mathbf{1}_{t \geq 2008Q4}$ | $-52.018^{***}$ |
| | (4.995) |
| $\mathbf{1}_{t \geq 2008Q4}$ | $3.061^{***}$ |
| | (0.409) |
| Constant | $3.159^{***}$ |
| | (0.015) |
| Obs. | 228 |
| $R^2$ | 0.974 |
| DF for numerator | 2 |
| DF for denominator | 224 |
| $F$ Statistic for Chow test | 1245.69 |
| $F$ Statistic for 1% sig. level | 4.70 |
| $F$ Statistic for 0.1% sig. level | 7.13 |

Notes: Newey-West standard errors are in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Degree of freedom is denoted by DF.

Columns (2) and (4).

In addition to this Table 5

For all the four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference rejects the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test following to Elliott and Müller (2006) and Harvey, Leybourne and Taylor (2009), because the magnitude of the initial value can be problematic. Let ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. The data are quarterly from 1980Q1 to 2007Q4.

**Table 5:** Unit root test

| | | Phillips-Perron test | |
|---|---|---|---|
| | | $Z(\rho)$ | $Z(t)$ |
| $ln(m)$ | (M2) | 0.567 | 0.297 |
| $ln(d)$ | (M2) | 1.275 | 1.054 |
| $ln(m)$ | (M1) | 0.567 | 0.297 |
| $ln(d)$ | (M1) | 1.275 | 1.054 |
| $ln(uc)$ | | $-1.114$ | $-1.710$ |
| $r$ | | $-7.721$ | $-2.471$ |
| $\Delta ln(m)$ | (M2) | $-46.623^{***}$ | $-5.335^{***}$ |
| $\Delta ln(d)$ | (M2) | $-42.267^{***}$ | $-5.060^{***}$ |
| $\Delta ln(m)$ | (M1) | $-46.623^{***}$ | $-5.335^{***}$ |
| $\Delta ln(d)$ | (M1) | $-42.267^{***}$ | $-5.060^{***}$ |
| $\Delta ln(uc)$ | | $-41.998^{***}$ | $-5.107^{***}$ |
| $\Delta r$ | | $-94.183^{***}$ | $-9.263^{***}$ |

## C.3 Unit Root Test

Table 6 provides the unit root test results for M2 to output ratio and deposit component of the M2 to output ratio. For all the two variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first differences reject the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test

# Appendix D Appendix Calibration

- Money demand: money-output ratio

$$Z \equiv \frac{m}{PY} = \frac{m/p}{zF(K,H) + [zF_H(K,H)/\zeta]\sum_{j=1}^{2}\nu\sigma_j v(q_j,k,z)}$$
$$= \frac{v(q_1)[zF_H(K,H)/\zeta]}{zF(K,H) + [zF_H(K,H)/\zeta]\sum_{j=1}^{2}\nu\sigma_j v(q_j,k,z)}$$

- Average markup:

$$\sigma_1\frac{v(q_1,k,z)}{q_1 g_q(q_1,k,z)} + \sigma_2\frac{v(q_2,k,z)}{q_2 g_q(q_2,k,z)}$$

- Outstanding treasury debt/output ratio:

$$\frac{\bar{B}}{PY} = \frac{\bar{B}/p}{zF(K,H) + [zF_H(K,H)/\zeta]\sum_{j=1}^{2}\nu\sigma_j v(q_j,k,z)}$$
$$= \frac{\tilde{B}}{zF(K,H) + [zF_H(K,H)/\zeta]\sum_{j=1}^{2}\nu\sigma_j v(q_j,k,z)}$$

- Capital/output ratio:

$$\frac{K}{Y} = \frac{k}{zF(k,h) + [zF_H(K,H)/\zeta]\sum_{j=1}^{2}\nu\sigma_j v(q_j,k,z)}$$

- Long-term interest rate

$$\rho_j = (1+\rho_1)[1+\nu\sigma_2\lambda(q_2,k,z)\chi(1)]\left[\prod_{i=1}^{j}\frac{1}{1+\nu\sigma_2\lambda(q_2,k,z)\chi(i)}\right]^{1/j} - 1$$

- Labor supply: $h$

# Appendix E Data Sources and Variable Definitions

$$\frac{m}{p} = \frac{g(q,k)w(1-t_h)}{A}$$

$$\frac{M}{P} = m = \frac{v(q)w}{\zeta} = \frac{v(q)AF_H}{\zeta}$$

$$Y = m = AF(K,N) + [AF_N(K,N)/\zeta]\sum_{j=1}^{2} \nu\sigma_j v(q_j)$$

$$\frac{M}{PY} = \frac{v(q_1)[AF_N(K,N)/\zeta]}{AF(K,N) + [AF_N(K,N)/\zeta]\sum_{j=1}^{2} \nu\sigma_j v(q_j)}$$

$$\frac{R}{PY} = \frac{n\tilde{r}}{AF(K,N) + [AF_N(K,N)/\zeta]\sum_{j=1}^{2} \nu\sigma_j v(q_j)}$$

- 
- Average markup:

$$\sigma_1 \frac{v(q_1)}{q_1 c'(q_1)} + \sigma_2 \frac{v(q_2)}{q_2 c'(q_2)}$$

# Appendix F Data Sources and Variable Definitions

The quantitative analysis uses the annual average of the below series.

- Federal funds rates: "Effective Federal Funds Rate" (FRED series FEDFUNDS).
- Interest on reserves: "Interest Rate on Excess Reserves" (FRED series IOER) and "Interest Rate on Required Reserves" (FRED series IORR).
- 3-month treasury rate: "3-Month Treasury Bill: Secondary Market Rate" (FRED series TB3MS).
- Deposit / (Total checkable deposits): "Total Checkable Deposits" (FRED series TCDSL).
- Excess reserves: "Excess Reserves of Depository Institutions" (FRED series EXCRESNS) and (FRED series EXCSRESNS).
- Excess reserve ratio: $\frac{\text{Excess reserves}}{\text{Total checkable deposits}}$.
- Required reserves: "Required Reserves of Depository Institutions" (FRED series REQRESNS).

- Required reserves ratio: $\frac{\text{Required reserves}}{\text{Total checkable deposits}}$.
- Reserves: "Total Reserves of Depository Institutions" (FRED series TOTRESNS).
- M1: "M1 Money Stock" (FRED series M1SL).
- M2: "M2 Money Stock" (FRED series M2SL).
- Monetary base: "Monetary Base; Total" (FRED series BOGMBASE).
- M1 money multiplier: $\frac{\text{M1}}{\text{Monetary Base}}$.
- Currency: "Currency Component of M1" (FRED series CURRSL).
- Deposit Component of M1: M1−Currency.
- Deposit Component of M2: M2−Currency.
- Unsecured credit: "Revolving Consumer Credit Owned and Securitized" (FRED series REVOLSL) transformed from monthly to quarterly by summing monthly data.
- GDP: "Gross Domestic Product" (FRED series GDP), quarterly and "Gross Domestic Product" (FRED series GDPA), annual.
- Commercial paper rate: "Nominal interest rate" from Ireland (2009), updated to 2015 using "3-Month AA Nonfinancial Commercial Paper Rate" (FRED series CPN3M).

**Table 6:** Unit root test (M2)

| | Phillips-Perron test | |
|---|---|---|
| | $Z(\rho)$ | $Z(t)$ |
| $ln(m)$ | 0.567 | 0.297 |
| $ln(d)$ | 1.275 | 1.054 |
| $\Delta ln(m)$ | $-46.623^{***}$ | $-5.335^{***}$ |
| $\Delta ln(d)$ | $-42.267^{***}$ | $-5.060^{***}$ |

Notes: All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey et al. (2009). ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.