Money Creation and Banking: Theory and Evidence[*]

HEON LEE[†]
Wells Fargo

February 12, 2024

Abstract

This paper studies the role of banks' money creation in monetary transmission. I develop a monetary-search model in which the demand for the monetary base and the money multiplier are endogenously determined through the banks' money creation channel. Both the model and the data show that the quantity of reserves is not independent of interest rate management policy, and short-term policy rates and interest on reserves play distinct roles in monetary transmission. I evaluate the theory by matching it with data, and the calibrated model can account for the evolution of the quantity of reserves, excess reserves, and the money multiplier from 1968 to 2015.

JEL Classification Codes: E42, E51
Keywords: Money, Credit, Interest on Reserves, Banking, Monetary Policy

[A] model of the banking system in which currency, reserves, and deposits play distinct roles ... seems essential if one wants to consider policies like reserves requirements, interest on deposits, and other measures that affect different components of the money stock differently.   **Lucas** (**2000**)

1.   Introduction

This paper develops a theory of money and banking that articulates banks' role in money creation and its interaction with credit. The key focal points are the endogenous determination of demand for base money through banking, and the central bank's control of base money to peg the short-term interest rate, which in turn influences the bank's money creation activity and other macroeconomic variables.

As pointed out by Romer (2000), the central bank conducts monetary policy through interventions in the market for base money. However, most leading models of monetary policy analysis do not consider the transmission mechanism via the market for the monetary base. For instance, New Keynesian models abstract from the money market mechanism. Moreover, many models assert the independence of the quantity of reserves from interest management policy, especially since the Federal Reserve started paying interest on reserves. In the New Monetarist framework, where monetary frictions are explicit, most models focus on monetary aggregates rather than the transmission mechanism from the market for the monetary base to the monetary aggregate through the banking sector. Instead of abstracting from the key mechanism, this paper revisits the issue of money creation to understand the role of banking in monetary transmission and to account for a number of observations within a unified framework.

Specifically, Section 2 provides four empirical findings from US data, which guide the modeling.

1. The reserves are not independent of interest rates during 2009-2015. The reserves and the banks' money creation activities are closely related to the interest rates.
2. The excess reserves to deposit ratio had been close to zero until 2007. The excess reserve ratio skyrocketed as the Fed introduced the interest on reserves.
3. The required reserve ratio does not have a negative relationship with the M1 money multiplier, and there were two structural breaks in the evolution of the M1 money multiplier: 1992 and 2008.
4. Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping and stable M1 money demand.

2

The first finding challenges the conventional approach that assumes independence between the quantity of reserves and the interest rate during the 2009-2015 period, known as the zero lower bound period. Contrary to this widely used assumption which has been a cornerstone in many unconventional monetary policy analyses, the finding implies that the quantity of reserves still plays a significant role in interest rate management. The second finding, which was already noted by Nakamura (2018), is worth emphasizing again, especially in light of the first finding. Taken together, the first and second findings suggest that the short-term policy rate and the interest on reserves may have distinct roles to play in interest rate management.

The third finding contradicts the undergraduate textbook theory of the money multiplier which predicts a negative relationship between the money multiplier and the required reserves ratio. Since the textbook theory assumes zero excess reserves, the absence of a negative relationship may not be a surprising finding during the post-2008 period with excess reserves. However, there is no negative relationship during the pre-2008 period as well. Finally, the last finding suggests that considering the role of unsecured credit could be essential to understand money demand. This paper advances a theory of money and banking, which accounts for all four observations mentioned above as well as the money creation process in the US economy.

This paper builds a monetary banking model based on Lagos and Wright (2005) to understand the monetary transmission. The model features the explicit structure of monetary exchange and the role of financial intermediations in money creation. Agents can trade by using cash, transaction deposit, and unsecured credit. The banks create deposits by making loans, which can be either transaction deposits or non-transaction deposits. However, the creation of transaction deposits is constrained by reserve requirements. The banks' demand for reserves, whether banks hold excess reserves, and the money multiplier are endogenously determined. They depend on the banks' deposit creation, which result from their profit maximization problem, given the monetary policy and credit conditions of the economy

When banks hold excess reserves, the bank's reserve requirement constraint does not bind and a change in reserve requirement does not change the money multiplier. Instead, the money multiplier is determined by the short-term policy rate and the interest on reserves. Lowering the short-term policy rate increases reserves, but the banks do not create deposit money proportionally, which lowers the money multiplier. Higher interest on reserves decreases the money multiplier because the bank has more incentive to hold reserves and less incentive to deposit money. The interest on reserves and the short-term rate play distinct roles, and they jointly determine the quantity of reserves. These rates also have different impacts on the lending rate. An increase in the short-term rate raises the lending rate, while

an increase in the interest on reserves lowers the lending rate.

Another ingredient of the model is unsecured credit which can substitute for other means of payment as in Gu, Mattesini and Wright (2016).[1] Better credit conditions lower transaction deposit balances as credit can substitute for money. This decrease in deposit balances leads to a lower money multiplier, regardless of whether banks hold excess reserves or not.

Next, I quantify the model by calibrating the model and asking how it accounts for observations in Section 2 and the money creation process. Given monetary policy behaved as it did, how well can the model account for the behavior of reserves, excess reserves, and money multiplier? The analysis shows the model can explain the historical evolution of the money creation process including all the observations from Section 2. Consistent with data, the model generates zero excess reserves between 1968 and 2007 as well as massive increases in excess reserves after 2008. The model gives the counterfactual reserves to output ratio which closely tracks its actual behavior from 1968 to 2015. It also generates drops in money multiplier during the 1990s and 2000s without excess reserves and its more drastic drops after 2008 were accompanied by the massive increase in excess reserves.

The quantitative exercise shows that dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves whereas the decrease in the money multiplier before 2008 is driven by better credit conditions. Contrary to previous approaches that assumed no changes in the short-term policy rate during 2009-2015 (so-called zero lower bound period) and merely focused on the quantity of reserves, this study confirms that the short-term interest rate did change during that period, and its movements directly reflected in the quantity of reserves.

This paper contributes to three strands of literature. First, it contributes to the growing literature on unconventional monetary policy and bank reserves. Many models of unconventional monetary policy take the zero lower bound constraint as a given and focus on the effect of asset purchases by issuing reserves, which is assumed to be independent of short-term interest rates. (Curdia and Woodford, 2011, Gertler and Karadi, 2011 and Lee, 2021). By focusing on which assets the central bank purchases, some recent works (e.g., Williamson, 2016 and Bhattarai, Eggertsson and Gafarov, 2015) study quantitative easing as reducing maturity of government debt. They take ZLB bound constraint seriously and treat bank reserves and short-term bonds as perfect substitutes under ZLB. Given this ZLB constraint, they focus on the effect of maturity transformation. Whereas these models assume independence of reserves to short-term interest rates, this paper shows that this independence assumption does

---

[1]By modeling unsecured credit with an exogenous credit limit, this paper follows Gu et al. (2016). For other approaches to introducing credit to the monetary economy, see Sanches and Williamson (2010), Lotz and Zhang (2016), and Williamson (2016).

not hold in the data. In contrast to the conventional approach to unconventional monetary policy, the quantitative analysis shows that the changes in reserves correspond to the changes in short-term interest rates. Rather than relying on the ad hoc assumption of independence, this paper articulates an explicit mechanism that determines the demand for reserves and the money multiplier.

Second, this paper contributes to a large literature on inside money and money creation. Previous works capturing the explicit role of reserve requirements and money creation include Freeman (1987), Haslag and Young (1998), and Freeman and Kydland (2000). Freeman (1987) and Haslag and Young (1998) study the impact of money creation and the reserve requirements on seigniorage revenue. Freeman and Kydland (2000) develop a tractable model of the endogenous money multiplier. They show that the money-output correlation can be explained by the endogenous money supply resulting from households' choices in response to the business cycle. Recent advances in monetary economics based on a search-theoretic framework provide a deeper understanding of banking and inside money. For example, Gu, Mattesini, Monnet and Wright (2013) study the environment where banking arises endogenously, and show that banking can improve the economy by facilitating trade using inside money. Andolfatto, Berentsen and Martin (2020) integrate a model of bank and financial markets by Diamond (1997) with Lagos and Wright (2005) framework and deliver a model where the fractional reserve banking arises in the equilibrium. Altermatt (2022) studies an economy in which a bank creates inside money by extending loans to entrepreneurs, which can be used for investment. This paper contributes to the literature by constructing a model of money and banking, which establishes the conditions under which banks hold excess reserves, provides an explicit mechanism for determining the demand for reserves and the money multiplier, and explains the money creation process as observed in the data.

Third, this work relates to the literature that studies money and credit explicitly. Gu et al. (2016) show that if money is essential, the nonmonetary credit is irrelevant. Changes in credit conditions only crowd out real balances. This neutrality result can be overturned if one introduces the costly credit (Bethune, Choi and Wright, 2020; Wang, Wright and Liu, 2020). Han and He (2019) show that in an economy in which both monetary credit and nonmonetary credit are used, credit is relevant even when money is essential. In this paper, there are fiat money, costly monetary credit (deposit money), and nonmonetary credit (unsecured credit), and the neutrality result does not hold. This is because there exists monetary credit and the bank's intermediation of monetary credit is costly.

This paper is organized as follows. Section 2 provides motivating evidence. Section 3 constructs the search-theoretic monetary model of money creation. Section 4 calibrates the model to quantify the theory. Section 5 concludes.

2. Motivating Evidence

This section presents a list of observations about money creation and money demand, which motivates the theoretical framework developed in the next sections.

Observation 1. The reserves are not independent of interest rates during 2009-2015. The interest rates are closely related to the quantity of reserves and the bank's money creation activity.

The left panel of Figure 1 shows the reserves to deposit ratio and the spread between the federal funds rate and the interest on reserves during 2009-2015, which is often called the zero lower bound (ZLB) period. Their opposite movements are evident. It suggests that the quantity of reserves still plays a role in interest rate management. This contradicts many monetary models of unconventional monetary policies in the post-2008 era, which assume the independence of the quantity of reserves from interest rate management in an ample-reserves regime. (e.g., Bech and Klee, 2011; Curdia and Woodford, 2011; Kashyap and Stein, 2012; Cochrane, 2014; Ennis, 2018; Piazzesi, Rogers and Schneider, 2019).

The assumption of independence is based on the idea that the interest rate paid on reserves sets a floor for the short-term policy rate. When the target policy rate reaches the interest rate on reserves, money is "divorced" from interest rate management, allowing the central bank to determine the quantity of reserves independently of the interest rate. However, in contrast to this notion, interest on reserves has been the upper bound of target policy rates. Appendix A provides a further discussion on interest on reserves and the overnight reverse repurchase facility.
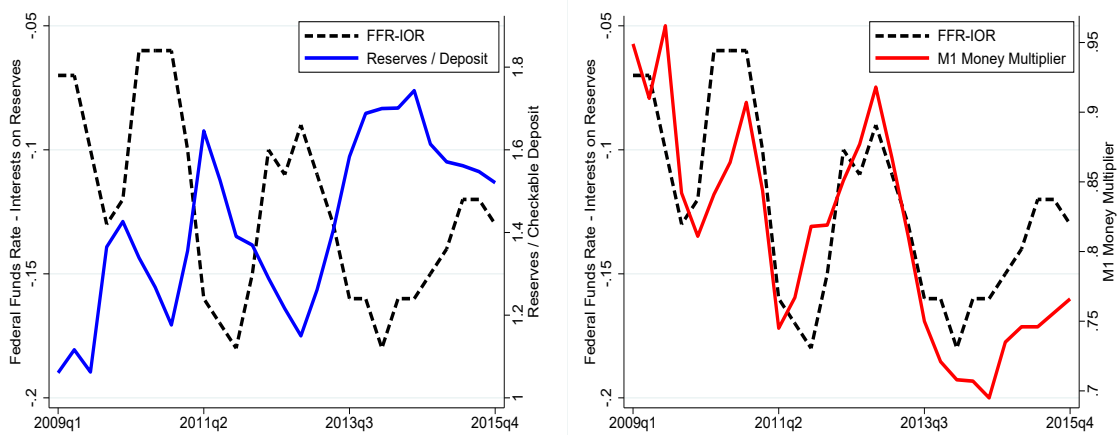
Figure 1: US excess reserves and M1 multiplier in the post-2008 period
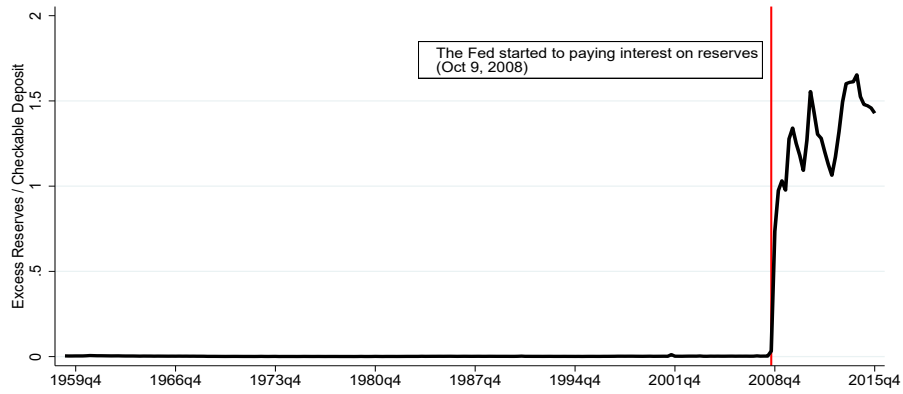
Figure 2: Excess reserves ratio

In addition, the right panel of Figure 1 shows that the M1 money multiplier moves together with the interest rate spread. This implies that the monetary policy during the post-2008 period is closely related to the bank's money creation activity, which was not given much attention in many monetary models of the last few decades.

Observation 2. The excess reserves to deposit ratio had been zero until 2007. The excess reserve ratio skyrocketed as the Fed introduced the interest on reserve.

Figure 2 plots the excess reserves to deposit ratio from 1959Q1 to 2015Q4. Before the 2008 Great Recession, the ratio remained at zero. However, it rose drastically after the recession and exceeded the value of 1, implying that banks have held more excess reserves than the amount of checking accounts they issued. Figure 2 also shows that the dramatic increase in excess reserves coincided with the Fed's introduction of the interest on reserves, as pointed out by Nakamura (2018). Taken together, the first and second findings suggest that short-term policy rates and interest on reserves may have distinct roles to play in interest rate management.

Observation 3. The required reserve ratio does not have a negative relationship with the money multiplier, and there were two structural breaks in the evolution of the money multiplier: 1992 and 2008.

The top-left panel of Figure 3 plots the US M1 multiplier over time. Whereas the money multiplier decreased drastically since 2008, the US M1 money multiplier already had been declining since the early 1990s. An increase in required reserves did not accompany this decrease in the money multiplier. The bottom-left panel of Figure 3 plots the M1 multiplier
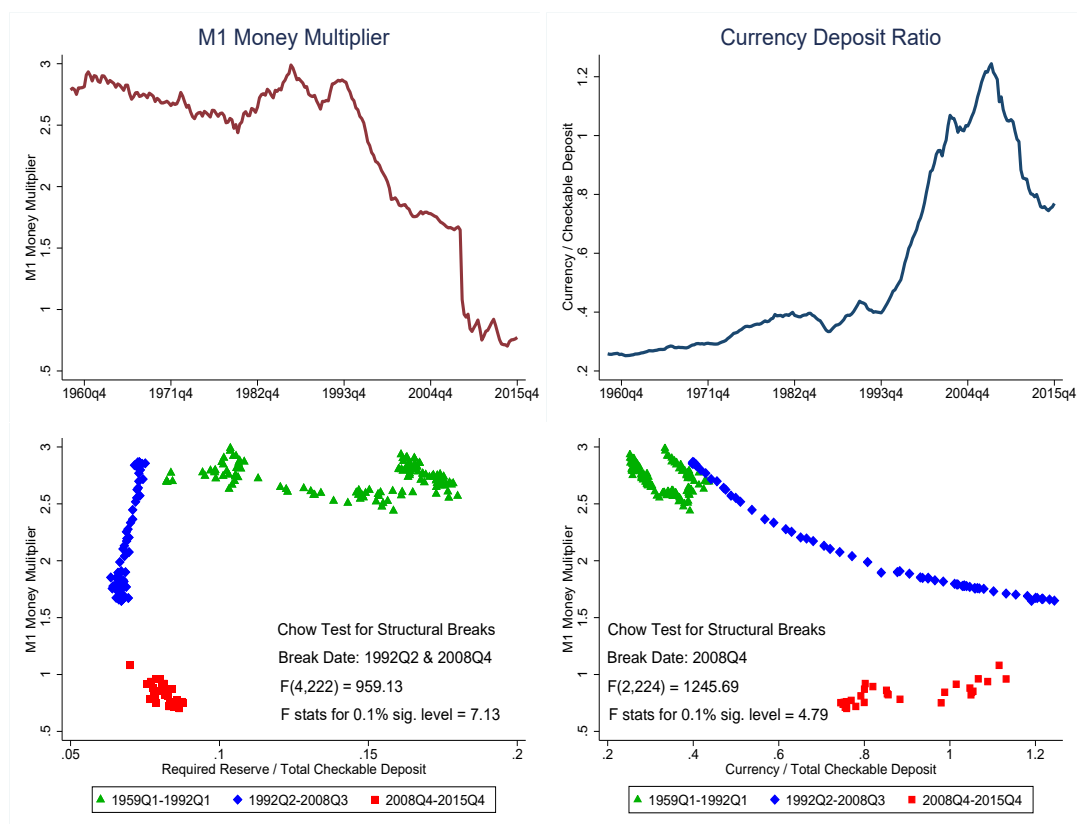
Figure 3: Money multiplier, currency/deposit ratio and required reserve ratio

Chow tests for structural breaks are implemented. The bottom-left panel reports a test statistic with the null hypothesis of no structural breaks in 1992Q2 and 2008Q4 and the bottom-right panel reports a test statistic with the null hypothesis of no structural break in 2008Q4. Sample periods are 1959Q1-2015Q4. Appendix D.1 contains details of the Chow tests.

against the required reserves.[2] There is no negative relationship, which differs from the textbook explanation of the money multiplier.

The left panels of Figure 3 show that the US M1 money multiplier has been decreasing since 1992. However, the declining trends before and after 2008 are different. As the top-right panel of Figure 3 shows, the decline during 1992-2007 is accompanied by a huge increase in the ratio of currency to deposit, whereas the decline after 2008 is accompanied by a huge drop in the ratio of currency to deposit. It is also worth noting that the M1 multiplier has been lower than 1 since 2009 which contradicts to the textbook theory of money creation.

---

[2]The required reserve ratio presented in Figure 3 is computed by (Required Reserves)/(Total Checkable Deposits). The legal reserve requirement for net transaction accounts was 10% from April 2, 1992, to March 25, 2020, but some banks are imposed upon by lower requirements or exempt depending on the size of their liabilities. These criteria changed 27 times from the 1st quarter of 1992 to the last quarter of 2019. From March 2020, all the required reserve ratios have become zero. See Feinman (1993) and https://www.federalreserve.gov/monetarypolicy/reservereq.htm for more details on the historical evolution of the reserve requirement policy of the United States.
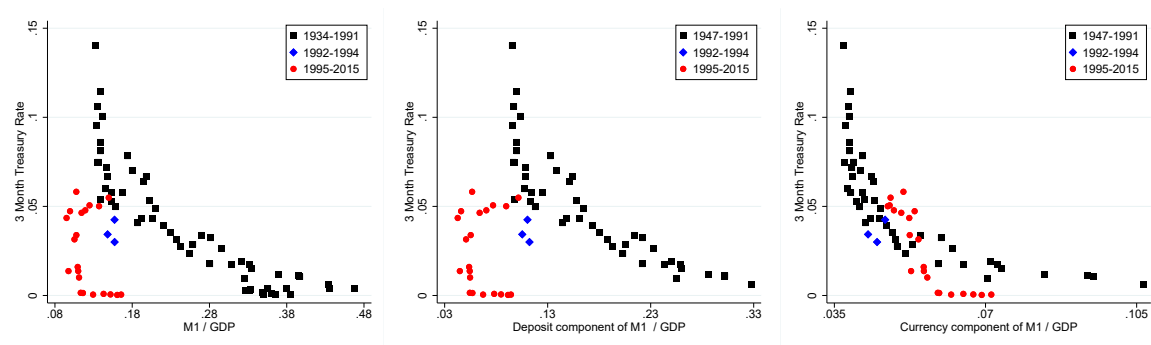
Figure 4: US Money demand for M1 and its components

The bottom-left panel of Figure 3 displays two structural breaks in the relationship between the M1 multiplier and the required reserve ratio: one in 1992Q3 and another in 2008Q4. In addition, the top-left panel of Figure 3 shows a structural break in the relationship between the M1 multiplier and the currency deposit ratio, which occurred in 2008Q4. The Chow tests for these breaks are described in more detail in Appendix D.2. The structural break of 2008Q4 coincided with the dramatic increase in excess reserves, which is illustrated in Fact 2.

Observation 4. Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping, stable M1 money demand.

Observation 3, discussed earlier, identified two structural breaks in 1992 and 2008. To gain a better understanding of the 1992 break, M1 can be decomposed into its deposit and currency components, as illustrated in Figure 4. The figure plots the ratio of M1 and its components to GDP against the 3-month Treasury Bill rate, revealing a breakdown in M1 around 1992 that coincided with the structural break observed in Figure 3.[3] As noted by Lucas and Nicolini (2015), this breakdown was caused by the deposit component rather than the currency component, which displays stable downward-sloping demand.

If an increased availability of consumer credit crowds out deposits but not cash, which implies that once one account for the substitution effect of the newly available consumer credit, there should still be a negative relationship between the real money balance and the interest rate. Following Cagan (1956), and Ireland (2009), I relate the natural logarithm of m, the ratio of money balances to income, to the short-term nominal interest rate, denoted

---

[3]One may think this is due to the introduction of retail sweep accounts and Automatic Transfer Service (ATS) in the 1990s. However, ATS was introduced in 1994 and the break of M1 occurred in 1992 which was before 1994. In addition to that, using sweep adjusted M1, Kejriwal, Perron and Yu (2022) also found a similar structural break. See Appendix B for more discussion.

Table 1: Cointegration regressions and tests

Dependent Variable:	$ln(m_t)$		$ln(d_t)$	
	OLS	CCR	OLS	CCR
	(1)	(2)	(3)	(4)
r_t	1.600***	−2.752**	4.928***	−4.990*
	(0.419)	(1.171)	(0.944)	(2.647)
$ln(uc_t)$		−0.282***		−0.583***
		(0.098)		(0.124)
$adjR^2$	0.109	0.981	0.230	0.959
Observation	112	112	112	112
Johansen $r = 0$	15.031	50.151	10.348	53.881
Johansen $r = 1$	0.034	9.842	0.563	10.538

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in columns (1)-(4). Appendix D.2 contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

by r. I also regress r on the natural logarithm of d, the ratio of deposit balances to income.

$$ln(m_t) = \beta_0 + \beta_1 r_t + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \epsilon_t$$

In addition to the above specifications, to capture the impact of the improved availability of consumer credit that can substitute the deposit, I add a logarithm of uc, the ratio of unsecured credit to income as another regressor as follows.[4]

$$ln(m_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t$$

I focus on the post-1980 period, until the arrival of the Great Recession. In Table 1, columns (1) and (3) report the estimates without unsecured credit, and columns (2) and (4) report the estimates with unsecured credit. The Johansen tests in columns (1) and (3) fail to reject the null hypothesis of no cointegration, which confirms the apparent breakdowns from Figure 4, and ordinary least squares (OLS) estimates from columns (1) and (3) both report positive coefficients on r_t that contradict the conventional notion of money demand: the stable downward-sloping relationship between real balances and interest rates. In columns (2) and (4), however, the Johansen tests reject their null of no cointegration at a 99 percent confidence level, suggesting there exists a stable relationship between real money balances, interest rates, and real balances of unsecured credit. To estimate the cointegration relationship, I implement the canonical cointegrating regression, proposed by Park (1992), in columns (2) and (4). The estimated coefficients on r_t and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document

---

[4]Following to Krueger and Perri (2006), I use revolving consumer credit.

the evidence that once one accounts for the substitution effect of consumer credit, there still exists a stable negative relationship between real money balances and the interest rates. This substitution effect is a potential explanation for the decline of the money multiplier during 1992Q2-2008Q3.

These findings suggest that a desirable monetary model for studying monetary transmission should have the following properties. First, the model should be capable of explaining how the amount of reserves is linked to interest rate management, regardless of whether banks hold excess reserves or not. Second, the model should feature the distinct roles of interest on reserves and nominal interest rates. Third, the model should be able to answer why banks are holding excess reserves, whereas they did not before 2008. Lastly, the model needs to capture the interaction between money and credit. In the following sections, by incorporating these four properties, I develop a theoretical model of the money creation process which is consistent with the above four observations, both qualitatively and quantitatively.

3. Model

The model constructed here extends the standard monetary search model (Lagos and Wright, 2005) by introducing fractional reserve banking, unsecured credit, and the standard growth theory with capital accumulation. Time is discrete and two markets convene sequentially in each time period: (1) a frictionless centralized market (CM, hereafter), where agents work, consume, and adjust their balances, following after (2) a decentralized market (DM, hereafter), where buyers and sellers meet and trade bilaterally. The DM trade features imperfect record-keeping and limited commitment. Due to these two frictions, some means of payment are needed in DM trades. Below I describe the economic agents in this economy and the different types of DM meetings.

There is a unit mass of households who discount their utility each period by β. The preferences of the households for each period are

$$\mathcal{U} = U(C) - \zeta H + u(q) - c(q)$$

where C is the CM consumption, $\zeta > 0$ is a parameter, H is the CM disutility from production, and q is the DM consumption. As standard, assume U', u', $c' > 0$, U'', $u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. Furthermore, I assume that the CM utility has constant relative risk aversion equal to 1, i.e., $-C\frac{U''(C)}{U'(C)} = 1$. Consumption goods are perishable. The efficient consumption in the DM is denoted by q^* which solves $u'(q^*) = c'(q^*)$. At the beginning of the DM, households get a preference shock such that they can be either buyers or sellers. A

11

household will be a buyer with probability ν while with probability $1 - \nu$ a household is a seller.

There is a representative firm. During the CM a young firm is born and becomes old and dies in the next CM. The firm maximizes its profit by producing CM consumption goods by hiring labor and using capital as inputs. The technology is given by $F(K, N)$ where K is the capital input and N is the labor input. The production function $F(K, N)$ satisfies $F_K, F_N > 0$, $F_{KK}, F_{NN} < 0$ and constant returns to scale. Given a constant returns to scale technology, we can define $f(k) \equiv F(k, 1) = F(K, N)/N$ where $k = K/N$. The capital is depreciated by δ every period. Assume that a firm and households are anonymous and a firm cannot commit to honour intertemporal promises. Therefore, a firm needs a medium of exchange to purchase capital goods. In order to acquire funding to pay for investment, firms can borrow loan from banks. There exists a capital producer whose technology can transform CM consumption goods into capital goods with cost $\Phi(\cdot)$.

There are measure n of active banks that will be endogenously determined by the free entry condition in the equilibrium. In the CM, the banks make portfolio choices for reserves, loans, transaction deposits, and non-transaction deposits. The banks extend loans by creating deposits that households can use as a means of payment to trade goods in the DM. The loans are paid back with interest rate i_ℓ. Enforcing repayment is costly. The cost function is described by $\eta(\tilde{\ell})$, where $\tilde{\ell}$ is the loan in real terms, $\eta', \eta'' > 0$, and $\eta(0) = 0$. Managing reserve balances incurs a cost. The cost is represented by a cost function $\gamma(\tilde{r})$, where $\tilde{r}$ is the amount of reserves in real terms, $\gamma', \gamma'' > 0$, and $\gamma(0) = 0$. The reserve earns a nominal interest rate of i_r. The bank's transaction deposit creation is constrained by the reserve requirement, i.e., the bank should hold reserves at least $\chi \tilde{d}$, where χ is the reserve requirement and $\tilde{d}$ is the real balance of transaction deposits.

There are three types of DM meetings that are distinguished by which the payment methods sellers accept. In DM1, there is no record-keeping device, and the seller can only recognize cash. In DM2, the seller accepts transaction deposits and unsecured credit. In DM3, she accepts cash, transaction deposits and unsecured credit. The buyer can trade using unsecured credit with credit limit $\bar{b}$ as the trading is monitored imperfectly.[5]

The central bank controls the base money supply B_t in the CM. Let μ denote the base money growth rate. Then the changes in the real balance of base money can be written as

---

[5]The acceptance of different means of payment can be endogenized as in Lester, Postlewaite and Wright (2012) or Lotz and Zhang (2016) but here we assume the types of meetings are exogenously given. Lester et al. (2012) endogenize the meeting types by allowing sellers' costly *ex ante* choice to acquire the technology for recognizing the certain type of assets. Similarly, Lotz and Zhang (2016) study the environment with costly record-keeping technology where sellers must invest in a record-keeping technology to accept credit.

$$\mu_{t+1} B_t = B_{t+1} - B_t,$$

The base money is held in two ways: (1) C as currency in circulation, i.e., outside money held by households; (2) R as reserves held by banks. Thus,

$$B_t = M_t + R_t.$$

The central bank can control the base money supply in two ways. First, it can conduct a lump-sum transfer or collect a lump-sum tax in the CM. Second, it can increase the money supply by paying interest on reserves, i_r. Let T represents a lump-sum transfer (or tax if it is negative). The central bank's constraint is

$$\mu_{t+1}\phi_{t+1} B_t = \phi_{t+1}(B_{t+1} - B_t) = T_{t+1} + i_{r,t+1}\phi_{t+1} R_{t+1},$$

where ϕ is the price of money in terms of the CM consumption good.

3.1. The CM Problem

Let $W(m, d, s, b)$ denote the CM value function where m is the cash holding, d is the transaction deposit balance, s is the non-transaction deposit balance, and $b > 0$ is the unsecured debt owed to the seller from the previous DM (or unsecured loans to buyer from the previous DM, if $b < 0$). All the state variables are in unit of the current CM consumption good. The CM problem is

$$W(m_t, d_t, s_t, b_t) = \max_{C_t, H_t, \hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}} U(C_t) - \zeta H_t + \beta V(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1})$$

$$\text{s.t.} \ \frac{\phi_t}{\phi_{t+1}}(\hat{m}_{t+1} + \hat{d}_{t+1} + \hat{s}_{t+1}) + C_t = w_t H_t + m_t + (1 + i_{dt})d_t + (1 + i_{st})s_t - b_t + T_t$$

where $\hat{m}_{t+1}$, $\hat{d}_{t+1}$, and $\hat{s}_{t+1}$ is the cash holding, transaction deposit balance, and non-transaction deposit balance, respectively, carried to the next DM. The first-order conditions (FOCs) are

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_m(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, = \ \text{if } \hat{m}_{t+1} > 0 \tag{1}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_d(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, = \ \text{if } \hat{d}_{t+1} > 0 \tag{2}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_s(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, = \ \text{if } \hat{s}_{t+1} > 0 \tag{3}$$

13

$$-\frac{\zeta}{w_t} + U'(C_t) = 0. \tag{4}$$

The first term on the left-hand side (LHS) of equation (1) is the marginal cost of acquiring cash. The second term is the discounted marginal value of carrying cash to the following DM. Therefore, the choice of $\hat{m}_{t+1} > 0$ equates the marginal cost and the marginal return on cash. A similar interpretation applies to equations (2) and (3) for the decision on deposits. The envelope conditions for $W(m, d, s, b)$ are

$$W_d(m_t, d_t, s_t, b_t) = (1 + i_{dt})\frac{\zeta}{w_t}, \quad W_m(m_t, d_t, s_t, b_t) = \frac{\zeta}{w_t}$$

$$W_s(m_t, d_t, s_t, b_t) = (1 + i_{st})\frac{\zeta}{w_t}, \quad W_b(m_t, d_t, s_t, b_t) = -\frac{\zeta}{w_t}$$

which implies $W(m_t, d_t, s_t, b_t)$ is linear. This linearity allows us to write

$$W(m_t, d_t, s_t, b_t) = \frac{\zeta}{w_t}\{m_t + (1 + i_{dt})d_t + (1 + i_{st})s_t\} - \frac{\zeta}{w_t}b_t + W(0, 0, 0, 0, 0).$$

3.2. The DM Problem

In the DM, the buyer and seller trade bilaterally. Let q_j and p_j be the DM consumption and payment in a type-j DM meeting. The bilateral trade is characterized by (p_j, q_j). This trade is subject to $p_j \leq z_j$ where z_j is the total liquidity of the buyer in a type-j meeting. The liquidity position for each type of buyer is

$$z_1 = m \tag{5}$$

$$z_2 = d(1 + i_d) + \bar{b} \tag{6}$$

$$z_3 = m + d(1 + i_d) + \bar{b} \tag{7}$$

The DM terms of trade are determined by Kalai (1977)'s proportional bargaining. Kalai bargaining solves the following problem:

$$\max_{p,q} u(q) - \frac{\zeta}{w}p \quad s.t \quad u(q) - \frac{\zeta}{w}p = \theta\left[u(q) - c(q)\right] \text{ and } p_j \leq z_j$$

where $\theta \in [0, 1]$ denotes the buyers' bargaining power. The payment, p, can be expressed as $p = v(q)w/\zeta = \{(1 - \theta)u(q) + \theta c(q)\}w/\zeta$. Define *liquidity premium*, $\lambda(q)$, as follows:

$$\lambda(q) = \frac{u'(q)}{v'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{(1 - \theta)u'(q) + \theta c'(q)}$$

14

where $\lambda(q) > 0$ and $\lambda'(q) < 0$ for $q < q^*$ and $\lambda(q^*) = 0$. When $z_j \geq p^*$, the buyer has sufficient liquidity to purchase efficient DM output q^*. In this case, the payment to the seller is $p^* = v(q^*)w/\zeta$.

The value function of an household at the beginning of DM is

$$V(m, d, s) = \nu V^B(m, d, s) + (1 - \nu)V^S(m, d, s)$$

where $V^B(m, d, s)$ and $V^S(m, d, s)$ denotes the value function for a buyer and a seller, respectively. By using the linearity of W, we can write a DM value function for a seller as follows:

$$V^S(m, d, s) = \sum_{j=1}^{3} \left\{ \frac{\sigma_j \nu}{1 - \nu} \left[\frac{\zeta}{w} p_j - c(q_j) \right] \right\} + W(m, d, s, 0)$$

and the value function of a buyer in the DM is

$$V^B(m, d, s) = \sum_{j=1}^{3} \left\{ \sigma_j \left[u(q_j) - \frac{\zeta}{w} p_j \right] \right\} + W(m, d, s, 0)$$

where $p_j \leq z_j$. The third term on the right-hand side (RHS) is the continuation value when there is no trade. The rest of the RHS is the surplus from the DM trade. The DM payments are constrained by $p_j \leq z_j$. For compact notation, define inflation rate as $\pi_{t+1} \equiv \phi_t/\phi_{t+1} - 1$. Assuming interior, differentiating V and substituting its derivatives into the FOCs from the CM problem yields

$$(1 + \pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_1\lambda(q_{1,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})] \tag{8}$$

$$(1 + \pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_2\lambda(q_{2,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})](1 + i_{d,t+1}) \tag{9}$$

$$(1 + \pi_{t+1})U'(C_t) = \beta U'(C_{t+1})(1 + i_{s,t+1}) \tag{10}$$

where $q_{j,t+1} = \min\{q^*, v^{-1}(\zeta z_{j,t+1}/w_{t+1})\}$ and $\lambda(q^*) = 0$.

3.3. The Bank's Problem

A bank maximizes its profit by choosing $\{\tilde{\ell}_t, \tilde{r}_t, \tilde{d}_t, \tilde{s}_t\}$ subject to its balance sheet identity constraint and reserve requirement constraint, where $\tilde{\ell}_t$ is lending, $\tilde{r}_t$ is reserve balance, $\tilde{d}_t$ the transaction deposit issuance, and $\tilde{s}_t$ the non-transaction deposit issuance, respectively, denoted in real terms. Since banking is competitive, a bank takes as given the interest rates: lending rate $i_{\ell,t}$, transaction deposit rate $i_{d,t}$, non-transaction deposit rate $i_{s,t}$. Interest on

reserves $i_{r,t}$ is determined by the central bank.

$$\max_{\tilde{r}_t, \tilde{d}_t, \tilde{\ell}_t, \tilde{s}_t} \quad (1 + i_{\ell,t})\tilde{\ell}_t + (1 + i_{r,t})\tilde{r}_t - (1 + i_{d,t})\tilde{d}_t - (1 + i_{s,t})\tilde{s}_t - \gamma(\tilde{r}_t) - \eta(\tilde{\ell}_t) \tag{11}$$

$$\text{subject to} \quad \tilde{\ell}_t + \tilde{r}_t = \tilde{d}_t + \tilde{s}_t \quad \text{and} \quad \tilde{r}_t \geq \chi \tilde{d}_t$$

In the first constraint, balance sheet identity, the LHS is assets, which include reserves and loans, and the RHS is liabilities, which include transaction deposits and non-transaction deposits. Let $\lambda_{\chi,t}$ denote the Lagrange multiplier for the reserve requirement constraint. Assuming interior, the FOCs for the bank's problem can be written as

$$0 = i_{\ell,t} - i_{s,t} - \eta'(\ell_t) \tag{12}$$

$$0 = i_{r,t} - i_{s,t} - \gamma'(\tilde{r}_t) + \lambda_{\chi,t} \tag{13}$$

$$0 = i_{s,t} - i_{d,t} - \chi\lambda_{\chi,t}. \tag{14}$$

The bank's *ex post* profit equals to the entry cost, κ

$$(1 + i_{\ell,t})\tilde{\ell}_t + (1 + i_{r,t})\tilde{r}_t - (1 + i_{d,t})\tilde{d}_t - (1 + i_{s,t})\tilde{s}_t - \gamma(\tilde{r}_t) - \eta(\tilde{\ell}_t) = \kappa. \tag{15}$$

Suppose there are active banks i.e., $n > 0$. Consider two cases. In the first case, the reserve requirement constraint is binding, i.e., $\lambda_{\chi,t} > 0$. In the second case, the reserve requirement constraint is loose, i.e., $\lambda_{\chi,t} = 0$. We call the first case a "scarce-reserves case," and the second an "ample-reserves case."

The Scarce-Reserves Case The bank does not have enough reserves. It needs to acquire reserves to issue more transaction deposit, which implies a binding constraint. With $\lambda_{\chi,t} > 0$, the bank's FOCs (12)-(14) give

$$i_{d,t} = (1 - \chi)i_{s,t} + \chi i_{r,t} - \chi\gamma'(\tilde{r}_t) \tag{16}$$

$$i_{\ell,t} = i_{s,t} + \eta'(\tilde{\ell}_t). \tag{17}$$

The Ample-Reserves Case The bank has sufficient reserves. Its reserve requirement constraint does not bind, $\lambda_{\chi,t} = 0$. Then the three FOCs for the bank's problem become

$$i_{d,t} = i_{s,t} \tag{18}$$

$$i_{r,t} = i_{s,t} + \gamma'(\tilde{r}_t) \tag{19}$$

$$i_{\ell,t} = i_{s,t} + \eta'(\tilde{\ell}_t). \tag{20}$$

3.4. The Firm and Capital Producer

A representative firm maximizes its profit by producing CM consumption goods and using its capital K_t and hiring labor N_t as inputs. In the CM of $t - 1$, the firm borrows funds L_t from banks, and purchases capital goods K_t using the funds. In the CM of t, the firm produces by hiring. The firm's problem can be written as follows:

$$\max_{N_t, K_t, L_t} L_t - Q_{t-1} K_t + \beta \left[F(K_t, N_t) - w_t N_t + Q_t (1 - \delta) K_t - (1 + i_{\ell,t}) \frac{L_t}{1 + \pi_t} \right]$$

subject to $L_t = Q_{t-1} K_t$, where Q_t is price of capital in terms of CM consumption good at period t. The firm's problem gives

$$F_N(K_t, N_t) = w_t, \quad F_K(K_t, N_t) = Q_{t-1} \frac{1 + i_{\ell,t}}{1 + \pi_t} - Q_t (1 - \delta) \tag{21}$$

A firm purchases capital from perfectly competitive capital producing firms at the end of period $t - 1$. This capital is used in production at t and its undepreciated $(1 - \delta) K_t$ part is resold to a capital producer once the production is over. Capital law of motion is given as:

$$K_{t+1} = (1 - \delta) K_t + I_t$$

A capital producer can transform CM consumption goods into capital goods with cost $\Phi(I_t)$. Formally, a capital producer solves the following profit-maximization problem:

$$\max_{I_t} Q_t I_t - \Phi(I_t)$$

which gives $Q_t = \Phi'(I_t)$. Assuming a linear production technology, $\Phi(I_t) = I_t$, gives $Q_t = 1$ for all t. Given above results, define the real lending rate:

$$\rho_t \equiv \frac{1 + i_{\ell,t}}{1 + \pi_t} - 1$$

Then we have the following equilibrium condition for the real lending rate and the marginal product of capital as follows:

$$F_K(K_t, N_t) = \rho_t + \delta \tag{22}$$

3.5. Equilibrium

In the equilibrium, the resource constraint for CM consumption goods and labor market clearing condition are satisfied.

$$C_t + K_{t+1} = F(K_t, N_t) + (1 - \delta)K_t, \text{ and } N_t = H_t \tag{23}$$

The money market clearing conditions are given as below

$$\phi_{t+1}M_{t+1} = m_{t+1}, \quad \phi_{t+1}R_{t+1} = n_{t+1}\tilde{r}_{t+1}, \quad \text{and} \quad B_{t+1} = M_{t+1} + R_{t+1} \tag{24}$$

and market clearing condition for lending and deposits are satisfied.

$$L_{t+1} = n_{t+1}\tilde{\ell}_{t+1}, \quad \text{and} \quad d_{t+1} = n_{t+1}\tilde{d}_{t+1} \tag{25}$$

Given agents' optimal choices and market clearing conditions, we define a monetary equilibrium as follows:

Definition 1. *A monetary equilibrium is a sequence of quantities* $\{K_t, N_t, m_t, d_t, r_t\}_{t=1}^{\infty}$, *prices* $\{\phi_t, i_{\ell t}, i_{dt}, w_t, \rho_t\}_{t=1}^{\infty}$, *and measures of active banks,* $\{n_t\}_{t=1}^{\infty}$ *that satisfies:*

1. *The Euler equations (8)-(10), and transversality conditions:*

$$\lim_{t\to\infty} \beta^t K_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t m_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t d_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t s_t = 0$$

2. *Optimality conditions of banks and the firm, (12)-(15) and (21);*

3. *Market clearing (23)-(25), and* $\phi_t B_t > 0$;

4. *Given a sequence for credit condition* $\{\bar{b}_t\}_{t=1}^{\infty}$;

5. *Given a sequence for monetary policy* $\{B_t, i_{rt}, \chi_t\}_{t=1}^{\infty}$;

6. *Given initial capital stock* K_0 *and monetary base stock* B_0.

As standard, the short-term policy rate i_t is given by the Fisher equation

$$1 + i_t = (1 + \pi_{t+1})\frac{U'(C_t)}{\beta U'(C_{t+1})} \tag{26}$$

implying $i_t = i_{s,t+1}$.

Similar to Sargent and Wallace (1975) and Gu, Han and Wright (2020), a central bank can peg i_t by letting B_{t+1} evolve endogenously as long as the base money is valued, $\phi_{t+1} B_{t+1} > 0$. Here, monetary policy implementation is different from previous literature. For example, New Keynesian models simply assume that the central bank can determine interest rates. In the other monetary models with fiat money, the central bank implements the monetary policy by controlling the aggregate money supply, or by controlling the growth rate of aggregate money supply. In this model, the central bank can set interest rates by controlling the supply of base money which eventually influences the supply of monetary aggregate, and other macroeconomic variables. Here, for the central bank's monetary policy implementation, it is crucial to have monopoly power over the supply of base money which is the sum of reserves and currency in circulation.

Given this environment, we have the following results:

Proposition 1. *When the central bank does not pay interest on reserves i.e., $i_{r,t+1} = 0$, the banks do not hold excess reserves i.e., $\tilde{r}_{t+1} = \chi \tilde{d}_{t+1}$. When the banks hold excess reserves, $\partial \tilde{\ell}_{t+1}/\partial i_t < 0$, $\partial \tilde{\ell}_{t+1}/\partial i_{r,t+1} > 0$, $\partial \tilde{r}_{t+1}/\partial i_t < 0$, $\partial \tilde{r}_{t+1}/\partial i_{r,t+1} > 0$, $\partial i_{\ell,t+1}/\partial i_t > 0$, and $\partial i_{\ell,t+1}/\partial i_{r,t+1} < 0$.*

Proposition 1 says that paying interest on reserves is necessary to have banks hold excess reserves. It also states that changes in the short-term policy rate and the changes in interest on reserves have different effects on the banking sector when banks are holding excess reserves. For example, an increase in the short-term policy rate reduces each bank's reserve balances, while an increase in the interest on reserves raises the bank's reserve balances. This is because an increase in the short-term policy rate reduces returns on reserves while an increase in the interest on reserves raises returns on reserves. Also, an increase in the policy rate reduces each bank's lending and raises the lending rate, whereas an increase in the interest on reserves increases the bank's lending and lowers the lending rate.

Monetary Transmission and Breaking of Neoclassical Dichotomy In the classical frictionless monetary models, output and the real interest rate are determined independently of monetary policy. In other words, monetary policy is neutral with respect to those real variables. Here, monetary policy could influence the real variable such as investment. It is worth discussing the difference in monetary transmission channels from the previous literature. In the textbook by Galí (2015), Chapter 1 shows that monetary policy is neutral in the classical frictionless monetary models, and Chapter 3, discusses how the presence of sticky

prices makes monetary policy non-neutral.[6,7] In contrast to those approaches, in this model, monetary policy could influence the real variable without the presence of sticky prices.

To inspect the mechanism, recall the Equation (22), $F_K(K_{t+1}, N_{t+1}) = \rho_{t+1} + \delta$. A decrease in the real lending rate unambiguously lowers the marginal production of capital. To see the anatomy of what constitutes marginal production of capital, rewrite the marginal product of capital as below:

$$\underbrace{F_K(K_{t+1}, N_{t+1})}_{\text{marginal product of capital}} = \underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})} - 1 + \delta}_{\text{standard neoclassical term}} + \underbrace{\frac{\eta'(\tilde{\ell}_{t+1})}{1 + \pi_{t+1}}}_{\text{bank's marginal cost of lending}} \tag{27}$$

What distinguishes this model from the neoclassical growth model is the last term of the above equation. The banks' enforcement cost provides a wedge to finance the investments. For example, as shown in Proposition 1, an increase in interest on reserves lowers $\tilde{\ell}_{t+1}$ when the banks hold excess reserves. This reduces the marginal cost of financing investment and influences the real lending rate through general equilibrium impact.

The channel is different from other micro-founded monetary models with capital. Aruoba and Wright (2003) is the one of first papers that introduced the neoclassical growth model to the Lagos and Wright (2005) environment. As pointed out by Waller (2003), it features a strong neoclassical dichotomy, meaning the outcomes in the DM and the CM can be solved independently. Later Aruoba, Waller and Wright (2011) and Waller (2011) break this dichotomy by introducing the role of capital in the DM where capital accumulation lowers the cost of producing DM goods. The other way to break down the dichotomy is to introduce pledgeable capital, allowing more credit trade across agents by holding more capital. (e.g.,Venkateswaran and Wright, 2013 and Gu, Jiang and Wang, 2019) Here, what breaks down the neoclassical dichotomy is the limited commitment problem between a firm and households.

Stationary Monetary Equilibrium In the remaining of this section, I focus on a symmetric

---

[6]The term monetary neutrality is often used differently in the literature. In New Keynesian literature, monetary neutrality implies that changes in short-term policy rates and money supply both do not have impacts on real variables. In contrast, as discussed in Head, Liu, Menzio and Wright (2012), in the New Monetarist models, although money is not superneutral, since real effects result from changes in nominal interest rates, inflation, or money growth rate, money is neutral because changes in aggregate money supply do not have real effects on the real variable.

[7]It is worthwhile to mention that presence of sticky prices does not necessarily make monetary policy non-neutral. Head et al. (2012) provides a monetary search model where the price stickiness emerges endogenously in contrast to the models imposing price stickiness exogenously. While Head et al. (2012) explains price stickiness and micro-level price level changes which can match with microdata, money is neutral in their model.

stationary monetary equilibrium in which the agents make the same decisions and all real variables are constant over. Given that $\phi_t/\phi_{t+1} = B_{t+1}/B_t = M_{t+1}/M_t = 1 + \mu$, the net inflation rate, π, is equal to the currency growth rate, μ, in the stationary monetary equilibrium. By the Fisher equation, $1 + i = (1 + \mu)/\beta$.[8] This leads to the following definitions:

Definition 2 (**Stationary Monetary Equilibrium**). *Given monetary policy, i, i_r, and χ and credit limit $\bar{b}$, a stationary monetary equilibrium consists of real balances, (m, r, d, s, ℓ), allocation (q_1, q_2, q_3, C, K, N), the measure of banks n, and prices (i_d, i_ℓ), satisfying Definition 1 except for initial conditions.*

Definition 3. *The stationary monetary equilibrium is a scarce-reserves equilibrium when $\tilde{r} = \chi\tilde{d}$ and an ample-reserves equilibrium, when $\tilde{r} > \chi\tilde{d}$, respectively.*

Given the above definitions, we have the following result.

Proposition 2. *Given $(i, \chi, \bar{b})$: (i) $\exists!$ ample-reserves equilibrium if and only if $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$; (ii) $\exists$ scarce-reserves equilibrium if and only if $i_r \leq \bar{\iota}_r$; and the thresholds satisfy $\bar{\Delta} = \gamma'(\underline{r})$ and $\bar{\iota}_r = \gamma'(\bar{r}) + i$ where $\underline{r}$ solves $\kappa = \gamma'(\underline{r})\underline{r} - \gamma(\underline{r})$ and $(\bar{r}, \bar{\ell}, \bar{K}, \bar{N}, \bar{C}, \bar{n})$ solves*

$$F_K(\bar{K}, \bar{N}) = \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\bar{\ell})}{\beta(1+i)}, \qquad \max\left\{0, \frac{\chi\left\{v(q^*)F_N(\bar{K}, \bar{N})/\zeta - \bar{b}\right\}}{\bar{n}}\right\} = (1+i)\bar{r},$$

$\eta'(\bar{\ell})\bar{\ell} + \gamma'(\bar{r})\bar{r} - \gamma(\bar{r}) - \eta(\bar{\ell}) = \kappa$, $\bar{C} + \delta\bar{K} = F(\bar{K}, \bar{N})$, $U'(\bar{C}) = \zeta/F_N(\bar{K}, \bar{N})$, and $\bar{K} = \bar{n}\bar{\ell}$.

When the central bank does not pays interest on reserves or pays low interest such that $i_r \leq \bar{\iota}_r$, banks only hold required reserves because the opportunity cost of holding reserves is higher than its benefit. When the central bank pays interest on reserves such that $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$, banks always hold positive amount of reserve balances that satisfies $\gamma'(\tilde{r}) = i_r - i$ and $\tilde{r} > \chi\tilde{d}$. In this case, banks hold a large amount of reserves not due to regulatory requirements, but because of the benefit of holding reserves which outweighs the opportunity cost of holding reserves.

Having solved the stationary monetary equilibrium, I proceed to establish the results on the money multiplier. Define money multiplier $\xi \equiv (m + d)/(\phi B)$, then we have the following results.

Proposition 3. *In the ample-reserves equilibrium, the money multiplier is increasing in i and decreasing in i_r, i.e, $\partial\xi/\partial i > 0$ and $\partial\xi/\partial i_r < 0$.*

---

[8]Note that $i \geq 0$ is necessary for the existence of equilibrium. Whereas the lower bound of the nominal interest rate is zero in this setting, one can relax this by introducing liquid assets or threats of theft. See Rocheteau, Wright and Xiao (2018b), Lee (2016) and Williamson (2019) for details.

Proposition 3 shows that the money multiplier is increasing in short-term policy rate while it is increasing in interest on reserves, which is consistent with the observation illustrated in Figure 1. This result is very intuitive. In the ample-reserves equilibrium, banks hold reserves because holding reserves itself is profitable regardless of the reserve requirement. The higher interest on reserves decreases the money multiplier because the banks have more incentive to hold reserves and less incentive to create transaction deposits. Increasing the short-term policy rate decreases reserves, but the banks do not create transaction deposits proportionally, which lowers the money multiplier. These are new findings compared to the literature.

To get more analytical results, I assume that the following restriction is satisfied in any equilibrium[9]:

Assumption 1. $1 > \gamma'(\tilde{r}) + \gamma''(\tilde{r})\tilde{r}$.

For each type of equilibrium, the following results are proved in Appendix C.

Proposition 4. *Suppose $p^* > \bar{b}$. In the ample-reserves equilibrium, $\partial i_d / \partial \bar{b} = 0$ and $\partial d / \partial \bar{b} < 0$. In the scarce-reserves equilibrium, when the equilibrium is unique, $\partial i_d / \partial \bar{b} > 0$ and $\partial d / \partial \bar{b} < 0$ if $\bar{b}$ or σ_3 is small.*

Proposition 4 shows that a better credit condition can increase the deposit rate in the scarce-reserve equilibrium but not in the ample-reserve equilibrium. One implication for the deposit rate changes is that neutrality between money and credit does not hold in the scarce-reserve equilibrium.[10] This is because the use of transaction deposit money incurs costs for operating banks. An increase in credit limit lowers the cost of operating deposits by reducing real balances which increases DM2 and DM3 trades through the increase in the deposit rate. In contrast to the scarce-reserve equilibrium, changes in credit conditions do not impact the deposit rate in the ample-reserve equilibrium; it merely crowds out real balances. Therefore, an increase in $\bar{b}$ can reduce d in both types of equilibrium.

As can be seen from the results above, the model can successfully address the mechanism illustrated in Section 1 and 2. We can interpret the breakdown of the money demand relationship in the pre-2008 economy as a result of improved availability of consumer credit under the scarce-reserve equilibrium. For the post-2008 period, after the Fed started paying interest on reserves, the economy shifted to the ample-reserves equilibrium. The model suggests that the changes in the money multiplier and the excess reserves during the post-2008 period are the results of the Fed's management of two interest rates: the nominal policy rate and the interest on reserves.

---

[9]This assumption implies the convex cost function $\gamma(\cdot)$ is not too convex.
[10]For more discussion on the neutrality of money and credit, see Gu et al. (2016) and Wang et al. (2020).

One can also check the interest rate pass-through of the monetary policy. Its pass-through depends on the type of equilibrium.

Proposition 5. *(i) In the ample-reserve equilibrium, $\partial i_\ell / \partial i > 0$, $\partial i_\ell / \partial i_r < 0$, $\partial i_d / \partial i = 1 > 0$, $\partial i_d / \partial i_r = 0$, and $\partial \rho / \partial i_r < 0$ but $\partial \rho / \partial i$ is ambiguous. (ii) In the scarce-reserve equilibrium, when the stationary monetary equilibrium is unique, $\partial i_\ell / \partial i_r < 0$, and $\partial \rho / \partial i_r < 0$ if $\bar{b}$ or σ_3 is small. When σ_3 is small, $\partial i_\ell / \partial i > 0$, and $\partial i_d / \partial i > 0$ but $\partial \rho / \partial i$ is ambiguous.*

Proposition 5 tells us that the monetary policy rates pass through the lending rate and deposit rate. Similar to Proposition 1, changes in the short-term policy rate and the changes in interest on reserves have different effects. In both types of equilibrium, the lending rate is strictly increasing in the short-term policy rate but is strictly decreasing in interest on reserves. In the scarce-reserves equilibrium, the deposit rate is strictly increasing in the short-term policy rate and the interest on reserves. The pass-through from the short-term policy to the real lending rate is ambiguous, however, the real lending rate is strictly decreasing in interest on reserves.

We now look into DM trades more closely. From (8) and (9), we have

$$\frac{i_t}{\nu} = \sigma_1 \lambda(q_1) + \sigma_3 \lambda(q_3) \tag{28}$$

$$\left\{ \frac{1+i}{1+i_d} - 1 \right\} \frac{1}{\nu} = \sigma_2 \lambda(q_2) + \sigma_3 \lambda(q_3). \tag{29}$$

where $i \geq i_d$ and $i \geq 0$. It is straightforward to see that DM1 and DM3 consumptions are efficient, $q_1 = q_3 = q^*$, when $i_t = 0$, i.e. the Friedman rule applies. However, if the central bank pays sufficient interest on reserves, we have $i_d = i$ which gives efficient consumptions in DM3 as well as in DM2 even when $i > 0$. This result can be formally summarized in the following proposition.

Proposition 6. *Let the short-term policy rate is positive $i > 0$. Then the DM consumptions in DM2 and DM3 are efficient $q_2 = q_3 = q^*$ when $i_r \geq \bar{\iota}_r$.*

The intuition behind the efficient DM consumptions when $i_r > \bar{\iota}_r$ is straightforward. In many monetary models, a higher inflation or interest rate increases the opportunity cost of holding money. In the environment where money is valued as a medium of exchange, having less liquidity in the economy because of the opportunity cost of holding money is inefficient. However, the interest on reserves provides a proportional return. If this return is properly distributed across households, it eliminates the inefficiency arising from the opportunity cost of holding money, leading to efficient consumption in DM2 and DM3. Therefore, when the central bank pays sufficient interest on reserves, the DM2 and DM3 meeting consumptions can be efficient even though the economy is not under the Friedman rule.

In addition to that, if credit limit $\bar{b}$ is sufficiently high, DM2 and DM3 consumptions also can be efficient even though $i > 0$. Appendix C verifies the following:

Proposition 7. *The threshold $\bar{\iota}_r$ is decreasing in $\bar{b}$. When $\bar{b} \geq p^*$, $\bar{\iota}_r = i$, $d = 0$ and $q_2 = q_3 = q^*$.*

Proposition 7 simply states that if the credit limit is high enough, it results in efficient consumption in both DM2 and DM3 trades. As $\bar{b} \to p^{*-}$, the household's transaction deposit balance d converges to 0. This is reminiscent of a result by Gu et al. (2016): if credit is easy, money is irrelevant; if credit is tight, money is essential, but credit becomes irrelevant. One difference is that even though credit is easy (i.e., $\bar{b} \geq p^*$) the household always holds cash $m > 0$ as long as $i < \nu\sigma_1\lambda(0)$ because the household only can trade using cash in the DM1 meeting. Since the better credit condition lowers transaction deposit balance, required reserves also shrink accordingly, leading to decreases in $\bar{\iota}_r$.

3.6. Some Limits of onetary Economics

The previous literature on monetary policy has favored an approach that excludes explicit consideration of "money." The widely accepted interpretation of such model economies is that they represent a cashless limit of an economy in which households value and hold money. The justification for this approach is based on continuity results at a certain cashless limit obtained using reduced-form models (e.g., money-in-utility or cash-in-advance), which are provided by Woodford (1998), Woodford (2003) and Galí (2015). However, this is not necessarily true. Lagos and Zhang (2022) show that, in a micro-founded monetary model, where financial intermediaries possess some degree of market power, monetary policy can exert a real effect through the real balance channel. This holds true even in highly developed credit economies where the share of monetary transactions is negligible. They argue that the continuity cashless limit result is incomplete in this context. This is because when financial intermediaries have some degree of market power, money can be an effective and credible outside option that makes monetary policy effective through medium-of-exchange transmission channels even when a negligible amount of money is used.

In light of Lagos and Zhang (2022), one can consider the cashless limit in this model as well. Let $\sigma_1 \to 0$, then there is no cash only meeting and the economy converges to the cashless economy as long as $i_d > 0$. However, households still hold deposit money when $i_d > 0$ and $\bar{b} < p^*$. In this case, the banks still hold reserves, implying $\phi R = \phi B > 0$. Therefore, even in the cashless limit where there is no currency in circulation, money plays transaction role as means of payment in addition to banks' portfolio in their balance sheet. While there is no cash, monetary policy is effective through the real balance channel.

Next, let's consider the economy where neither deposit money nor physical currency is used in the DM transactions, i.e., $\sigma_1 \to 0$ and $\bar{b} \geq p^*$. In this case, as long as policy rates satisfy $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$, the central bank still has the ability to set its policy interest rate, i, by controlling base money supply which influences consumption, investment, future inflation and output through banking channel.

Therefore, the quantity of monetary base is relevant even though there are no monetary transactions in the decentralized trade. The mechanism is different from Lagos and Zhang (2022), but the results are similar. Monetary policy remains effective even in highly developed credit economies where the share of monetary transactions is negligible.

4. Quantitative Analysis

The above sections have developed a model of money creation analyzing monetary transmission. The model is tractable, and analytical results can be established. In this section, I calibrate the model and evaluate the model quantitatively.

4.1. Calibration

The model period length is set to one year. The utility functions for the DM and the CM are $u(q) = B[(q + \varsigma)^{1-\varphi} - \varsigma^{1-\varphi}]/(1 - \varphi)$ and $U(C) = \log(C)$. The cost function for the DM is $c(q) = q$. The production function takes the form of a standard Cobb-Douglas function, $F(K, N) = K^\alpha H^{1-\alpha}$. The enforcement cost for lending is assumed to be quadratic, $\eta(\tilde{\ell}) = \Psi\tilde{\ell}^2$, and the balance sheet cost for managing reserves balances takes the form, $\gamma(\tilde{r}) = G\tilde{r}^g$.

The calibration period is 1968–2007. The benchmark nominal interest rate is $i = 0.0593$, the average 3-month treasury rate, and the benchmark required reserve ratio is $\chi = 0.1111$, the average of the ratio between required reserves and total checkable deposits. Since the Federal Reserve did not pay interest on reserve before October 2008, I set $i_r = 0$ as the benchmark.

Some parameters are directly pinned down. The discount factor β is set to match a 3% real interest rate. The capital share in CM output is set to $\alpha = 1/3$ as the standard, and the capital depreciation rate is matched with $I/K = \delta = 0.0825$. To ensure that changes in credit conditions do not affect currency holdings, σ_3 has been set to 0, based on the stable downward sloping currency demand illustrated in Figure 5. The fraction of DM2 meetings, σ_2, has been set to 0.6890 so that the equilibrium percentage of unsecured credit users matches 68.9%, which is the average percentage of US households holding at least one credit card from 1970 to 2007, based on the Survey of Consumer Finances. For simplicity, the bargain-

Table 2: Model parametrization

Parameter	Description	Value	Target Description	Target	Model
		External Parameters			
δ	depreciation rate	0.0825	investment/capital, I/K		
α	capital share in F	0.3333	labor's share of income, $2/3$		
β	discount factor	0.9709	real interest rate, 3%		
σ_2	DM2 matching prob.	0.6890	share of credit meeting		
ς	parameter of $u(\cdot)$	0.0001	Aruoba et al. (2011)		
θ	bargaining power	1	take-it-or-leave-it offer		
ν	prob. of being a buyer	0.5	normalization		
g	parameter of $\gamma(\cdot)$	1.5	-		
		Internal Parameters			
ζ	coeff. on labor supply	2.4934	labor supply, H	0.3333	0.3333
σ_1	DM1 matching prob.	0.0001	currency/output, M/PY	0.0443	0.0464
G	parameter of $\gamma(\cdot)$	0.0019 (fixed)	reserves/output, R/PY	0.0121	0.0110
$\bar{b}$	credit condition	0.0559	unsecured credit/output	0.0347	0.0347
κ	bank entry cost	0.0103	lending rate, i_ℓ	0.0862	0.0866
B	parameter of $u(\cdot)$	0.0022	capital/output, K/Y	2.1896	2.2275
φ	parameter of $u(\cdot)$	4.7562	elast. of M/PY to i	-0.1948	-0.2099
Ψ	parameter of $\eta(\cdot)$	0.0181	semi-elast. of i_ℓ to i	11.3673	11.5729
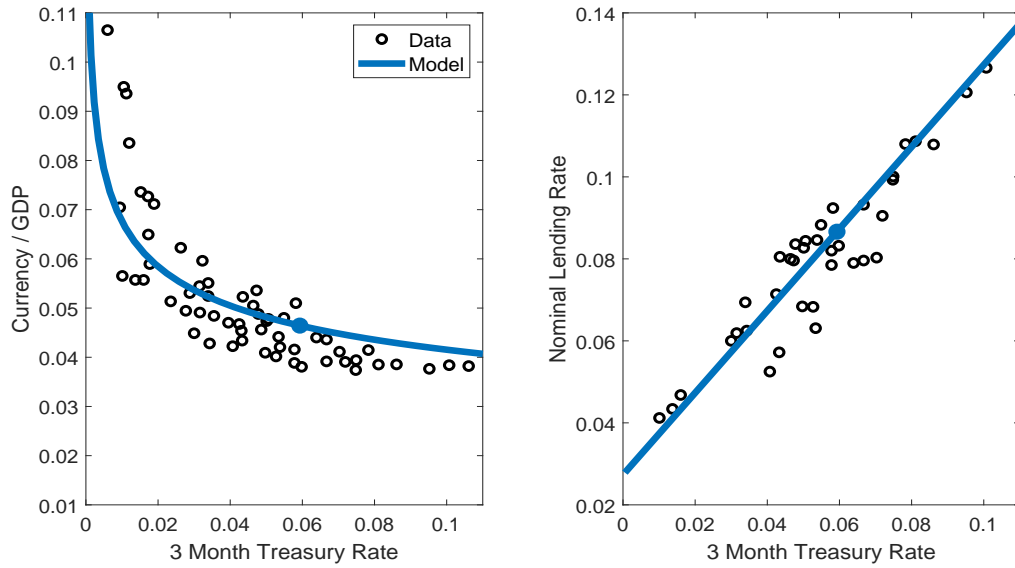
Figure 5: Money Demand for Currency and Interest rate Pass-through

ing power has been set to $\theta = 1$, which implies the buyer makes a take-it-or-leave-it offer to the seller in the DM. The probability of being a buyer has been normalized to $\nu = 0.5$. The parameter ς has been introduced in $u(q)$ merely to ensure that $u(0) = 0$, and has been set to $\varsigma = 0.0001$, as in Aruoba et al. (2011). The curvature parameter of $\gamma(\cdot)$ has been set to $g = 1.5$ to ensure that $\gamma(\cdot)$ is less convex than $\eta(\cdot)$.

The remaining 8 parameters $(\zeta, \kappa, \sigma_1, \bar{b}, G, B, \varphi, \Psi)$ are set to match the following 8 targets:[11] (i) the standard measure of work as a fraction of discretionary time, $H = 1/3$; (ii) the average nominal lending rate, $i_\ell = 0.0862$; (iii) the currency output ratio, $M/PY = 0.0443$; (iv) the reserves output ratio, $R/PY = 0.0121$; (v) the unsecured credit output ratio, 0.0347; (vi) the capital output ratio, $K/Y = 2.1896$; (vii) the elasticity of currency demand to the nominal interest rate, -0.1948; (viii) the semi-elasticity of the nominal lending rate to the nominal interest rate, 12.9284. The targets are computed based on 1968-2007 data.

All of the targets in the model, except the elasticity of currency demand and the semi-elasticity of lending rate, are directly computed using straightforward formulas given the benchmark nominal interest rate and the required reserve ratio. Similar to Aruoba et al. (2011), the elasticity of currency demand is computed using changes in money demand when the interest rate changes from $i - 0.05$ to $i + 0.05$. The semi-elasticity of lending rate is also computed using changes in the nominal lending rate when the interest rate changes from $i - 0.05$ to $i + 0.05$. The calibrated parameters and the targets are summarized in Table 2, and the calibrated money demand of currency and lending rate pass-through are shown in Figure 5.

4.2.   Results

This section explores how well the model can account for the low-frequency behavior of reserves and the money creation process, assuming the only driving forces are monetary policy, (i, i_r, χ) and credit conditions, $\bar{b}$. The unsecured credit limit $\bar{b}$ is computed using the unsecured credit to output ratio. Using the calibrated parameters, I compute the model equilibrium for given $(i, i_r, \chi, \bar{b})$. For monetary policy variables, I use 3 month treasury rate and interest on excess reserves,[12] and required reserves to total checkable deposit ratio.

Reserves and Money Multiplier Figure 6 compares the model and data from 1968 to 2015.

---

[11]I exogenously fix the parameter G to 0.00185. The identification of this model could be weak; there are different combinations of these parameters that can fit the data similarly. Table 6 in the Appendix shows an alternative parametrization that still yields the same predictions.

[12]Whereas the Federal Reserve had announced interest on excess reserves and interest on required reserves separately, the Fed had paid the same interest to both. As of March 2020, the Fed unified two interests on reserves as "Interest Rate on Reserve Balances" since the Board reduced reserve requirement to zero.
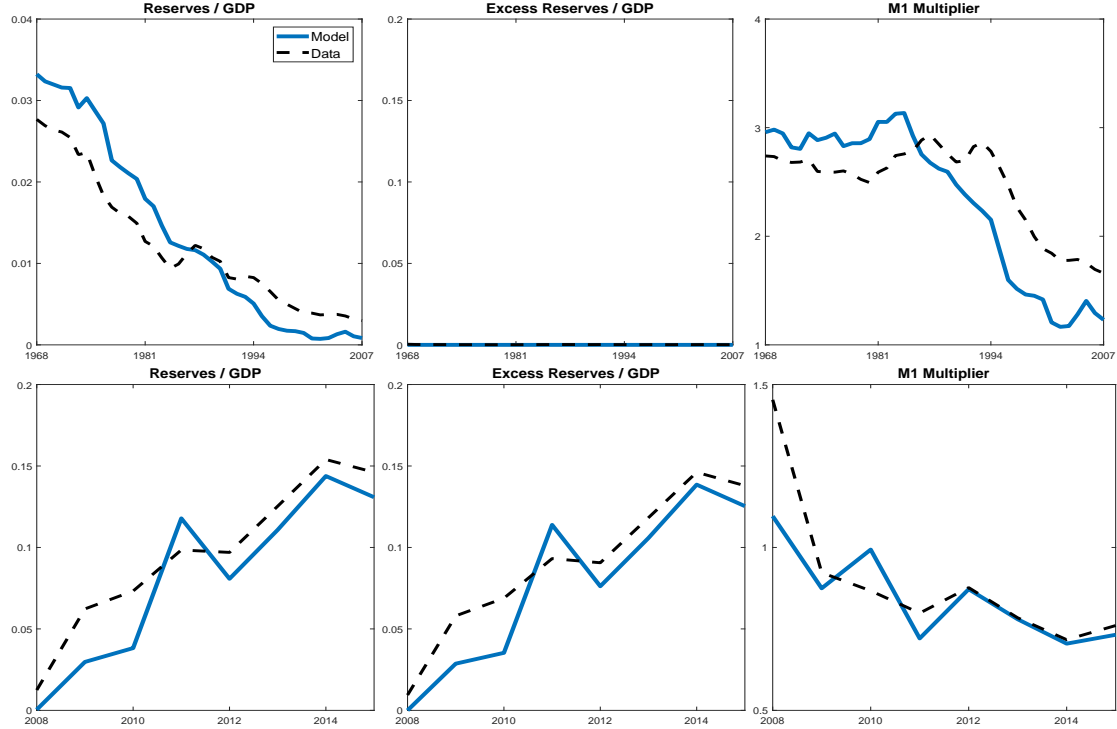
Figure 6: Model vs. Data

The top-left panel and the bottom-left panel shows reserves as a fraction of output from 1968 to 2007 and from 2008 to 2015, respectively. The model generates the quantity of reserves which can match the movement in the data. The top-middle panel and the bottom-middle panel of Figure shows excess reserves as fraction of output from 1968 to 2007 and from 2008 to 2015, respectively. The model also successfully generate the zero excess reserves during 1968-2007 and huge increase during 2008-2015. The top-right panel and the bottom-right panel of Figure 6 shows M1 money multiplier. While the model does not match all of the movement in the data, there is a very similar basic pattern. During 1968-2015 money multiplier consistently decreased, whereas there is no excess reserves. During 2008-2015, money multiplier decreased lower than one and reflected the behavior of excess reserves.

Matching Observations We now examine the model's ability to reproduce the observed patterns discussed in Section 2.

Observation 1 shows that the quantity of reserves is not independent of the short-term policy rate. Figure 7 displays the counterfactual data from the model. As in Figure 1, Figure 7 show the evident opposite movements of the reserves to deposit ratio with respect to the spread between short-term policy and interest on reserves. Also, the counterfactual M1 money multiplier moves together with the spread as in Figure 1. As already shown in Propo-

28
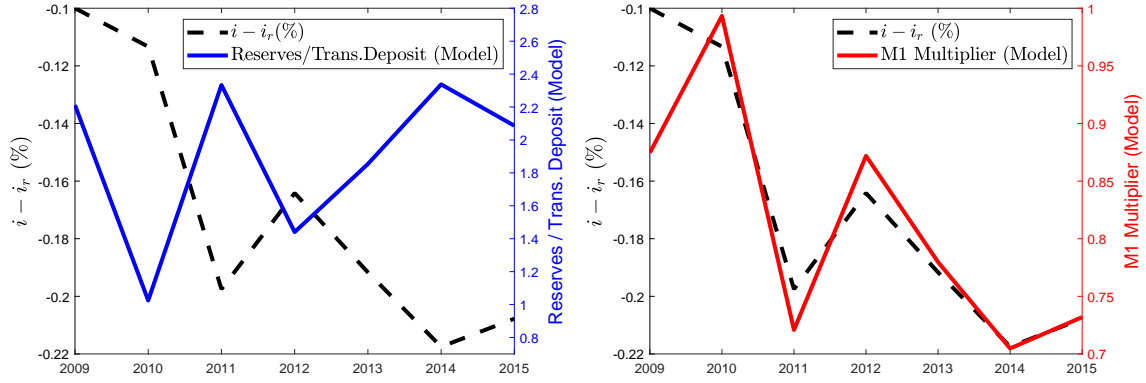
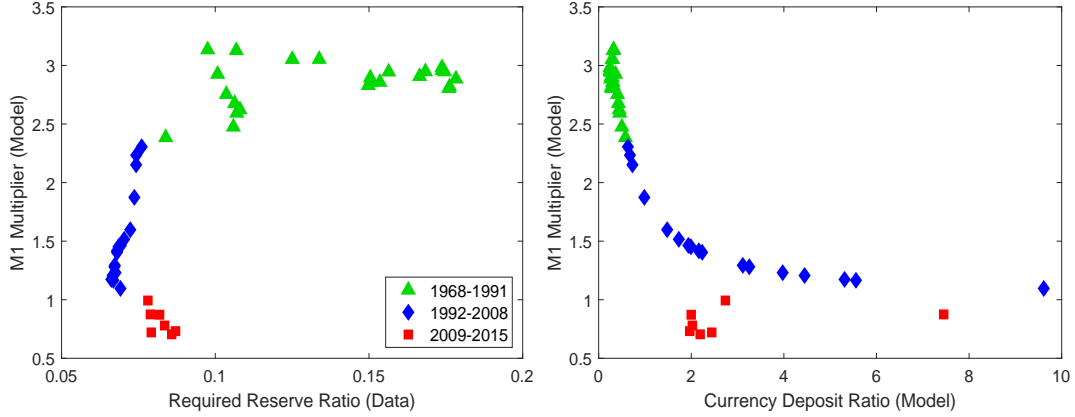**Figure 7:** Reserves Deposit ratio and M1 multiplier: Model
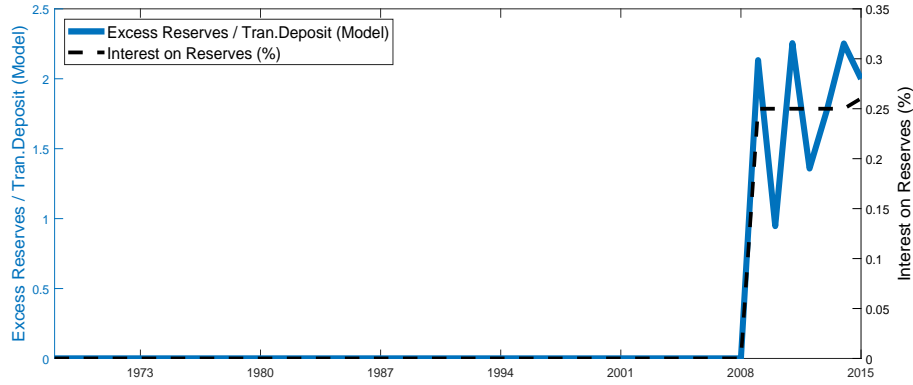
Figure 8: Excess reserves ratio (Model)

sition 3, a higher interest on reserves increases the incentive hold to reserves because the banks earn interest just by holding reserves. However, banks do not create transaction deposits proportional to the increases in reserves which eventually lowers the money multiplier. In contrast, a higher short-term policy rate lowers reserve balances by disincentivizing the bank to hold reserves which eventually increases money multipliers.

Observation 2 shows that the excess reserves deposit ratio remained at zero until the introduction of interest on reserves, after which they skyrocketed. Similar to Figure 2, Figure 8 shows the excess reserves deposit ratio remains zero before the introduction of interest on reserves as well as a huge increase in excess reserves. Paying interest reserves gives more incentive to hold reserves, resulting in banks holding more reserves than the required level.

Observation 3 shows that there is no negative relationship between the required reserve ratio and the money multiplier, and it also shows two structural breaks, one in 1992 and another in 2008. As shown in Figure 3, there has been a consistent decrease in the money multiplier since 1992, which was not accompanied by a decrease in the required reserve ratio but rather by an increase in the currency deposit ratio. Notably, the decrease in the money

29

Figure 9: Money multiplier, currency/deposit ratio and required reserve ratio: Model

Table 3: Money demand and Credit: Model vs. Data

| Dependent Variable: $ln(m_t)$	OLS		CCR	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
r_t	1.600***	8.101	−2.755**	−5.057
	(0.419)		(1.171)	
$ln(uc_t)$			−0.282***	−1.046
			(0.098)	
$adjR^2$	0.109	0.433	0.981	0.840

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported.

multiplier after 2008 was not accompanied by an increase in the currency deposit ratio. In Figure 9, the model successfully reproduces this observation, showing a persistent decrease in the money multiplier since 1992 that was not accompanied by a decrease in the required reserve ratio but rather by an increase in the currency deposit ratio. Additionally, the model demonstrates that a decrease in the money multiplier did not accompany an increase in the currency deposit ratio after 2008.

Observation 4 shows that consideration of unsecured credit recovers downward-sloping M1 money demand. The model can reproduce this result as well. Using the counterfactual data generated from the model, Table 7 shows that in the model unsecured credit consideration recovers the downward-sloping money demand, as in the data.[13] Column (2) shows that the regressing only on interest rate gives positive estimates. Column (4) shows that adding unsecured credit to output ratio to the money demand equation recovers downward-sloping money demand, as in the data.

---

[13]To compare the result from Table 1, the model parts of Table 7 use annualized quarterly series instead of annual data.

[14]The monetary base and currency are sourced from 'H.6 Money Stock Measures', published by the Federal
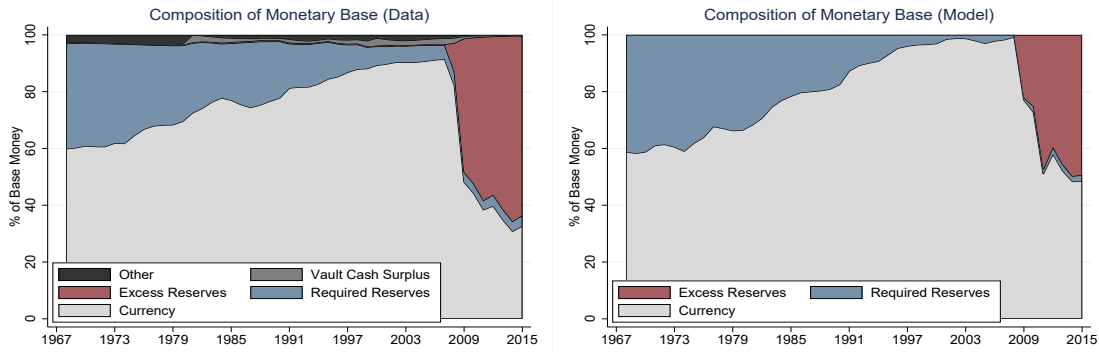
Figure 10: Composition of monetary base: data vs. model[14]

Composition of Monetary Base The model also generates the composition of the monetary base over time. Figure 10 compares the composition of the monetary base between the data and the model. The model successfully captures the changes in each component of the monetary base - currency, required reserves, and excess reserves - both before and after 2008. The currency portion consistently increased from 1968 to 2007 and then drastically decreased as the portion of excess reserves significantly increased. The portion of required reserves has consistently decreased. It is worth emphasizing that the share of currency in the monetary base has been substantial. Until 2008, its share had increased from 60% to 90%. After 2008, its share has hovered around 40%. Even though the share of currency has been reduced, it still accounts for a large portion of the central bank's balance sheet. This implies that if one wants to consider the central bank's balance sheet as an important channel of monetary policy, incorporating the currency in the analysis is necessary

4.3. Implication for Monetary Transmission

In the model economy, the central bank sets interest rates by controlling the monetary base. This is consistent with the conventional notion of monetary policy. Recall Romer (2000):

> [The] appropriate concept of money is unambiguously high-powered money.
> Here M is not a variable the central bank is targeting, but rather one it is manip-
> ulating to make interest rates behave in the way it desires. This is an excellent
> description of high-powered money. Moreover, for high-powered money, the

---

Reserve. These data can be found in the Federal Reserve Economic Data (FRED) database under the series BOGMBASE and CURRSL. Additionally, data on required reserves, excess reserves, and vault cash surplus are obtained from 'H.3 Aggregate Reserves of Depository Institutions and the Monetary Base', also published by the Federal Reserve. These figures are compiled in the FRED under the series EXCRESNS, EXCSRESNS REQRESNS, and VAULTSUR. The category 'Other' represents the monetary base that does not fall into any of these specified categories.

assumption that the opportunity cost of holding money is the nominal rate is appropriate. In addition, the assumption that the central bank can control the money stock is a much better approximation for high-powered money than for broader measures of the money stock.

However, the recent literature does not seem to align in this understanding of monetary policy. Many macroeconomic models got rid of this monetary transmission, especially after the Fed introduced the interest on reserves. For example, Goodfriend (2011) put it as "Interest on reserves frees monetary policy to fund credit policy independently of interest rate policy." and Cochrane (2014) put as "[The] Fed can separate interest rate changes ... from balance sheet policy" Cochrane (2014) goes even further by putting "Federal Reserve policy in the future goes so far past "monetary" that the label will no longer be appropriate." The result in this paper is the opposite. This paper shows that paying interest on reserves does not make the quantity of reserves independent of interest rate policy.

The quantitative analysis shows that, given credit conditions, there exists a one-to-one mapping from a set of monetary policy variables to the quantity of currency, the quantity of reserves, the quantity of excess reserves, and the money multiplier. In this approach, one does not need to rely on the assumption that the central bank directly controls monetary aggregates or the even more abstract assumption that the central bank controls interest rates out of nothing. Instead, the model gives an explicit process of monetary policy from setting short-term policy rates by controlling the monetary base to its influence on the monetary aggregate.

It not only provides an explicit monetary transmission mechanism but also gives different views on monetary policy. For example, unconventional monetary policies, such as quantitative easing, are not asset purchases independent of the target level of the short-term policy rate. Rather, they are active uses of two different interest rates: interest on reserves and short-term policy rates. Changes in reserves reflect the changes in short-term policy rates and vice versa, implying we also can interpret them as the uses of interest on reserves and quantity of monetary base (or reserves) as well. The model does not require the assumption of independence between equilibrium prices (short-term policy rates) and quantities (monetary base or reserves).

4.4. Dynamics

In this section, I characterize the dynamics of the monetary equilibrium using the calibrated parameters. To solve the dynamic equilibrium defined in Definition 1, I apply a global projection method using Chebyshev polynomials, rather than relying on some approximations
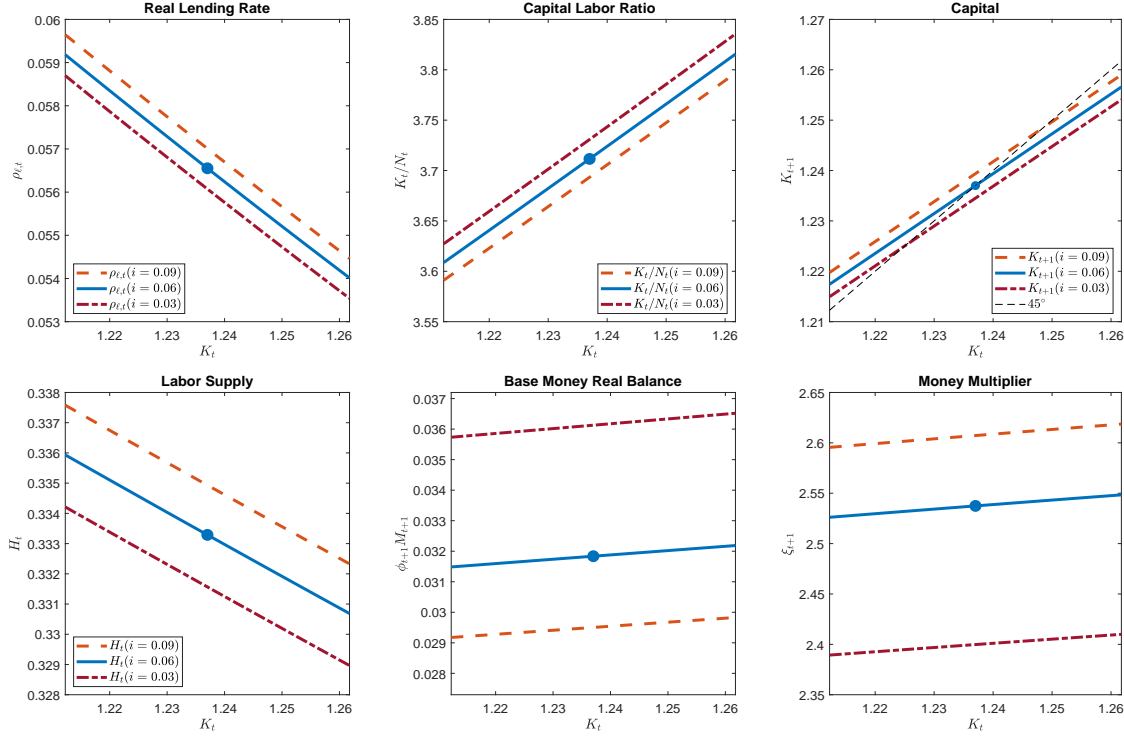
Figure 11: Policy Functions

around the steady state of the economy (e.g., First-order, second-order, or higher-order Taylor expansion, or log-linearization).

Figure 11 displays the policy functions under different target policy rates, i. The top panels show that the real lending rate ρ_t decreases as the capital stock K_t increases. Additionally, the capital-labor ratio K_t/N_t and the future capital stock K_{t+1} increase as the capital stock K_t increases. These results align with the standard neoclassical growth model. However, as the target interest rate changes, the policy function shifts accordingly. Lowering the policy rate shifts the policy function of real lending downward while shifting the policy functions of capital-labor ratio and future capital upward. However, when the target policy rate changes, the policy functions shift accordingly. Lowering the policy rate causes the policy function of real lending to shift downward while causing the policy functions of capital-labor ratio and future capital to shift upward.

The bottom panels of Figure 11 display the policy functions of labor supply, base money balance, and the money multiplier. The labor supply is decreasing in K_t and the money multiplier, money supply, and capital stock, K_{t+1} are increasing in K_t. This finding aligns with the result from Freeman and Kydland (2000) which shows the money multiplier and money supply are positively correlated with investment.

While the central bank can control the supply of base money, the supply of money in the

economy is endogenous. This is because, once it sets the interest rate target, the central bank needs to supply base money to meet the market demand. As K_t increases, the economy needs more investment, which leads to an increase in the firm's demand for lending. The banks then lend loans to fulfill the firm's demand by creating deposit money. Consequently, the central bank supplies reserves to banks to accommodate their lending. Therefore, by setting short-term policy rates, the money supply is endogenously determined by the demands of firms and households.

5. Concluding Remarks

This paper develops a monetary-search model with fractional reserve banking and unsecured credit, and studies the role of money creation in monetary transmission. In the fractional reserve banking system, money is created when banks make loans. The bank's inside money creation, however, can be constrained by the reserve requirement and the reserves. Whether the reserve requirement constraint binds or not is endogenously determined by the banks' profit maximization.

Banks hold excess reserves when the central bank pays sufficiently high interest on reserves. In this case, the money multiplier and the quantity of the reserve depend on the short-term policy rate and the interest on reserves rather than the reserve requirement. In contrast to previous works, these two interest rates play distinct roles, and the quantity of reserves is not independent of interest rate management. Furthermore, their impact on the lending rate are different: the lending rate increases with the short-term interest rate, while it decreases with interest on reserves. Quantitative analysis can generate simulated data that resemble the actual data. This paper provides evidence from the model and the data that suggests that the dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves.

This work can be extended in various ways. Although I focus on the centralized market for the reserves with homogeneous banks, in reality, the market for reserves is a decentralized interbank market and banks have different portfolios. Therefore, one can further investigate how much the market structure and heterogeneity matter for the transmission of monetary policy (e.g., Afonso and Lagos, 2015; Armenter and Lester, 2017; Afonso, Armenter and Lester, 2019). Second, I assume that bank assets are composed of loans and reserves. But commercial banks' assets are composed of securities, loans, and reserves. Extending the model to incorporate banks' portfolio choices and analyzing the role of investment, financial regulation, and monetary policy can open up other research avenues. (e.g., Rocheteau, Wright and Zhang, 2018a).

References

Afonso, Gara and Ricardo Lagos, "Trade Dynamics in the Market for Federal Funds," *Econometrica*, 2015, *83* (1), 263–313.

Afonso, Gara M, Alex Entz, Eric LeSueur et al., "Who's Borrowing in the Fed Funds Market?," Technical Report, Federal Reserve Bank of New York 2013.

— , — , — , **and** — , "Who's Lending in the Federal Funds Market?," Technical Report, Federal Reserve Bank of New York 2013.

Afonso, Gara, Roc Armenter, and Benjamin Lester, "A Model of the Federal Funds Market: Yesterday, Today, and Tomorrow," *Review of Economic Dynamics*, 2019, *33*, 177–204.

Altermatt, Lukas, "Inside money, investment, and unconventional monetary policy," *International Economic Review*, 2022, *63* (4), 1527–1560.

Andolfatto, David, Aleksander Berentsen, and Fernando M Martin, "Money, banking, and financial markets," *The Review of Economic Studies*, 2020, *87* (5), 2049–2086.

Armenter, Roc and Benjamin Lester, "Excess Reserves and Monetary Policy Implementation," *Review of Economic Dynamics*, 2017, *23*, 212–235.

Aruoba, S Borağan and Randall Wright, "Search, money, and capital: a neoclassical dichotomy," *Journal of Money, Credit and Banking*, 2003, pp. 1085–1105.

— , **Christopher J Waller, and Randall Wright**, "Money and capital," *Journal of Monetary Economics*, 2011, *58* (2), 98–116.

Bech, Morten L and Elizabeth Klee, "The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market," *Journal of Monetary Economics*, 2011, *58* (5), 415–431.

Berentsen, Aleksander, Samuel Huber, and Alessandro Marchesiani, "Financial Innovations, Money Demand, and the Welfare Cost of Inflation," *Journal of Money, Credit and Banking*, 2015, *47* (S2), 223–261.

Bethune, Zachary, Michael Choi, and Randall Wright, "Frictional Goods Markets: Theory and Applications," *The Review of Economic Studies*, 2020, *87* (2), 691–720.

Bhattarai, Saroj, Gauti B Eggertsson, and Bulat Gafarov, "Time consistency and the duration of government debt: A signalling theory of quantitative easing," Technical Report, National Bureau of Economic Research 2015.

Cagan, Phillip, "The Monetary Dynamics of Hyperinflation," *Studies of the Quantity Theory if Money*, 1956.

Cochrane, John H, "Monetary Policy with Interest on Reserves," *Journal of Economic Dynamics and Control*, 2014, *49*, 74–108.

Curdia, Vasco and Michael Woodford, "The Central-Bank Balance Sheet as an Instrument of Monetary Policy," *Journal of Monetary Economics*, 2011, *58* (1), 54–79.

Cynamon, Barry Z, Donald H Dutkowsky, and Barry E Jones, "Redefining the monetary aggregates: a clean sweep," *Eastern Economic Journal*, 2006, *32* (4), 661–672.

Diamond, Douglas W, "Liquidity, Banks, and Markets," *Journal of Political Economy*, 1997, *105* (5), 928–956.

Elliott, Graham and Ulrich K Müller, "Minimizing the Impact of the Initial Condition on Testing for Unit Roots," *Journal of Econometrics*, 2006, *135* (1-2), 285–310.

Ennis, Huberto M, "A Simple General Equilibrium Model of Large Excess Reserves," *Journal of Monetary Economics*, 2018, *98*, 50–65.

Feinman, Joshua N, "Reserve Requirements: History, Current Practice, and Potential Reform," *Federal Reserve Bulletin*, 1993, *79*, 569.

Freeman, Scott, "Reserve Requirements and Optimal Seigniorage," *Journal of Monetary Economics*, 1987, *19* (2), 307–314.

_ **and Finn E Kydland**, "Monetary Aggregates and Output," *American Economic Review*, 2000, *90* (5), 1125–1135.

Galí, Jordi, *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press, 2015.

Gertler, Mark and Peter Karadi, "A model of unconventional monetary policy," *Journal of monetary Economics*, 2011, *58* (1), 17–34.

Goodfriend, Marvin, "Central banking in the credit turmoil: An assessment of Federal Reserve practice," *Journal of Monetary Economics*, 2011, *58* (1), 1–12.

Gu, Chao, Fabrizio Mattesini, and Randall Wright, "Money and Credit Redux," *Econometrica*, 2016, *84* (1), 1–32.

_ , _ , **Cyril Monnet, and Randall Wright**, "Banking: A New Monetarist Approach," *Review of Economic Studies*, 2013, *80* (2), 636–662.

_ , **Han Han, and Randall Wright**, "The effects of news when liquidity matters," *International Economic Review*, 2020, *61* (4), 1411–1435.

_ , **J Jiang, and Liang Wang**, "Credit Condition, Inflation and Unemployment," Technical Report, Working Paper 2019.

Han, Han and Chao He, "Money and Credit Revisited," *Available at SSRN 3497788*, 2019.

Harvey, David I, Stephen J Leybourne, and AM Robert Taylor, "Unit Root Testing in Practice: Dealing with Uncertainty over the Trend and Initial Condition," *Econometric Theory*, 2009, *25* (3), 587–636.

Haslag, Joseph H and Eric R Young, "Money Creation, Reserve Requirements, and Seigniorage," *Review of Economic Dynamics*, 1998, *1* (3), 677–698.

Head, Allen, Lucy Qian Liu, Guido Menzio, and Randall Wright, "Sticky prices: A new monetarist approach," *Journal of the European Economic Association*, 2012, *10* (5), 939–973.

Ireland, Peter N, "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand," *American Economic Review*, 2009, *99* (3), 1040–52.

Kalai, Ehud, "Proportional solutions to bargaining situations: interpersonal utility comparisons," *Econometrica: Journal of the Econometric Society*, 1977, pp. 1623–1630.

Kashyap, Anil K and Jeremy C Stein, "The Optimal Conduct of Monetary Policy with Interest on Reserves," *American Economic Journal: Macroeconomics*, 2012, *4* (1), 266–82.

Kejriwal, Mohitosh, Pierre Perron, and Xuewen Yu, "A two-step procedure for testing partial parameter stability in cointegrated regression models," *Journal of Time Series Analysis*, 2022, *43* (2), 219–237.

Krueger, Dirk and Fabrizio Perri, "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *The Review of Economic Studies*, 2006, *73* (1), 163–193.

Lagos, Ricardo and Randall Wright, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 2005, *113* (3), 463–484.

_ **and Shengxing Zhang**, "The Limits of onetary Economics: On Money as a Constraint on Market Power," *Econometrica*, 2022, *90* (3), 1177–1204.

Lee, Donggyu, "Quantitative Easing and Inequality," 2021.

Lee, Seungduck, "Money, Asset Prices, and the Liquidity Premium," *Journal of Money, Credit and Banking*, 2016.

Lester, Benjamin, Andrew Postlewaite, and Randall Wright, "Information, Liquidity, Asset Prices, and Monetary Policy," *The Review of Economic Studies*, 2012, *79* (3), 1209–1238.

Lotz, Sebastien and Cathy Zhang, "Money and Credit as Means of Payment: A New Monetarist Approach," *Journal of Economic Theory*, 2016, *164*, 68–100.

Lucas, Robert E, "Inflation and Welfare," *Econometrica*, 2000, pp. 247–274.

_ **and Juan Pablo Nicolini**, "On the Stability of Money Demand," *Journal of Monetary Economics*, 2015, *73*, 48–65.

Nakamura, Emi, "Discussion of Monetary Policy: Conventional and Unconventional," May 2018. Nobel Symposium, Swedish House of Finance.

Park, Joon Y, "Canonical Cointegrating Regressions," *Econometrica: Journal of the Econometric Society*, 1992, pp. 119–143.

Piazzesi, Monika, Ciaran Rogers, and Martin Schneider, "Money and banking in a New Keynesian model," *Standford WP*, 2019.

Rocheteau, Guillaume, Randall Wright, and Cathy Zhang, "Corporate Finance and Monetary Policy," *American Economic Review*, 2018, *108* (4-5), 1147–1186.

_ **, _ , and Sylvia Xiaolin Xiao**, "Open Market Operations," *Journal of Monetary Economics*, 2018, *98*, 114–128.

Romer, David H, "Keynesian macroeconomics without the LM curve," *Journal of economic perspectives*, 2000, *14* (2), 149–169.

Sanches, Daniel and Stephen Williamson, "Money and Credit with Limited Commitment and Theft," *Journal of Economic Theory*, 2010, *145* (4), 1525–1549.

Sargent, Thomas J and Neil Wallace, """ Rational" expectations, the optimal monetary instrument, and the optimal money supply rule," *Journal of political economy*, 1975, *83* (2), 241–254.

Teles, Pedro and Ruilin Zhou, "A Stable Money Demand: Looking for the Right Monetary Aggregate," *Economic Perspectives*, 2005, *29* (1), 50.

VanHoose, David D and David B Humphrey, "Sweep accounts, Reserve Management, and Interest Rate Volatility," *Journal of Economics and Business*, 2001, *53* (4), 387–404.

Venkateswaran, Venky and Randall Wright, "Pledgability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity," *NBER Working Paper*, 2013, (w19009).

Waller, Christopher, "Comment on" Search, Money, and Capital: A Neoclassical Dichotomy"," *Journal of Money, Credit, and Banking*, 2003, *35* (6), 1111–1117.

Waller, Christopher J, "Random matching and money in the neoclassical growth model: Some analytical results," *Macroeconomic Dynamics*, 2011, *15* (S2), 293–312.

Wang, Liang, Randall Wright, and Lucy Qian Liu, "Sticky prices and costly credit," *International Economic Review*, 2020, *61* (1), 37–70.

Williamson, Stephen, "Central Bank Digital Currency: Welfare and Policy Implications," 2019 Meeting Papers No. 386, 2019. Society for Economic Dynamics.

Williamson, Stephen D, "Scarce Collateral, the Term Premium, and Quantitative Easing," *Journal of Economic Theory*, 2016, *164*, 136–165.

Woodford, Michael, "Doing without money: controlling inflation in a post-monetary world," *Review of Economic Dynamics*, 1998, *1* (1), 173–219.

_ , "Interest and prices," 2003.

APPENDIX FOR ONLINE PUBLICATION

A.   Interest on reserves and floor system?

This section discusses details about the interest on reserves and the floor system. The Federal Reserve started paying interest on reserves (IOR) in Oct 2008. In contrast to the idea that paying interest on reserves provides a floor, Figure 12 shows that interest on reserves has been equal to the upper bound of the Fed's target range. Instead, the interest rate of the Overnight Reverse Repurchase (ON RRP) has been equal to the lower bound of the target rate. To put it simply, the interest on reserves serves as the upper bound of the target range, while the ON RRP rate acts as the lower bound of the target range for most of the period. The short-term policy rate is tightly controlled within this target range.
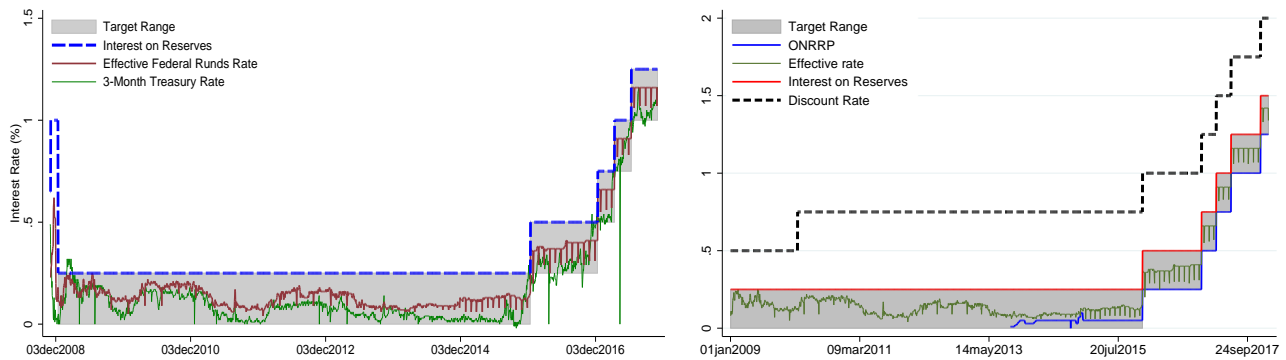
**Figure 12:** Interest on reserves, ON RRP and target range

Given that the federal funds market is a market for unsecured bilateral interbank lending of reserves, a lower federal funds rate than the IOR may seem counterintuitive. This discrepancy is due to the institutional framework. While both depository institutions (DIs) and government-sponsored enterprises (GSEs) can trade in the federal funds market, only DIs are eligible to earn IOR. The GSEs, such as the Federal Home Loan Banks, can earn arbitrage profits in the federal funds market by borrowing from the repo market and lending to DIs who can earn interest on reserves. Armenter and Lester (2017) present a model of interbank trade in which GSEs' lending to DIs that earn IOR determines the federal funds rates. Since GSEs also participate in the repo market, the ON RRP facility's interest rate serves as an alternative for financial institutions not eligible for the IOR, establishing a lower bound on the federal funds rate. See Afonso, Entz, LeSueur et al. (2013a) and Afonso, Entz, LeSueur et al. (2013b) for more discussion on the trade in the federal funds market.

B. Retail Sweep

One may think that the structural break in 1992 found in Section 2 can be attributed to the relaxation of bank deposit regulation in the 1990s that stimulated financial innovations such as retail sweep accounts in the 1990s (e.g., VanHoose and Humphrey, 2001, Teles and Zhou, 2005, Lucas and Nicolini, 2015, Berentsen, Huber and Marchesiani, 2015). Base on this idea, previous works have used an alternative measure of M1 as monetary aggregates, "M1 Adjusted for Retail Sweeps" (M1S, hereafter).

The rationale for using M1S is that the introduction of the automatic transfer system (ATS) in the early 1990s made highly liquid transaction balances outside M1. The ATS enables convenient transfer from money market deposit account (MMDA) to sweep account. The sweep account is a transaction deposit, whereas the MMDA is a saving deposit that is classified as Non-M1 M2. While the MMDA pays higher deposit rates than other saving deposits, same as other saving deposits, the MMDA had been subject to the restriction on the number of convenient transactions due to Regulation D.[15] Thus, the introduction of ATS may enable people to use MMDA as a liquid deposit. The claim is that this may result in the appearance of highly liquid transaction balances within instruments outside M1. Cynamon, Dutkowsky and Jones (2006) discuss the benefits of using M1S instead of M1 as a measure of money supply.
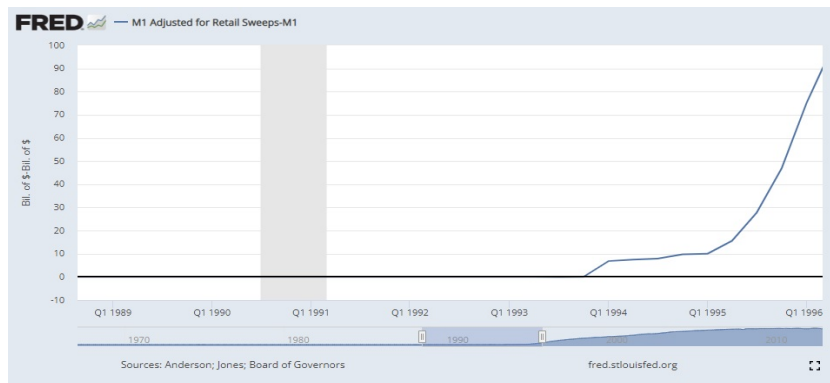
Figure 13: Difference between M1 adjusted for retail sweep and M1

However, it is worth noting that the ATS was introduced to commercial banks in 1994 while the structural break of M1 from Section 2 occurred in 1992. Consistent with this, Figure 13 shows that the difference between M1S and M1 was zero before 1994. Using M1S, Berentsen et al. (2015) and Kejriwal et al. (2022) also found the structural break of M1 in the early 1990s, suggesting retail sweep consideration cannot explain the break in the early 1990s.

---

[15]The MMDA was introduced in the early 1980s after the US Congress permitted its creation as of December 1982. Due to Regulation D, no more than six convenient transactions using the MMDA could be made per statement period. As of March 2020, the Federal Reserve removed this restriction.

C. Proofs

Proof of Proposition 1. Consider the case where the reserve requirement constraint is not binding. Recall equation (19) and $i_t = i_{s,t+1}$:

$$i_{r,t+1} - i_t = \gamma'(\tilde{r}_{t+1})$$

Given $i_{r,t+1} = 0$ and $i_t \geq 0$, solving (19) yields $\tilde{r}_{t+1} \leq 0$ which could not be a equilibrium. Therefore, when the central bank does not pay interest on reserves $i_{r,t+1} = 0$ there is no equilibrium with excess reserves $\tilde{r} > \chi\tilde{d} \geq 0$.

Next, we move on to the comparative static analysis. Recall (15) and (18)-(20)

$$
\begin{aligned}
0 = G^1 &\equiv (1 + i_{\ell,t+1})\tilde{\ell}_{t+1} + (1 + i_{r,t+1})\tilde{r}_{t+1} \\
&\quad - (1 + i_t)(\tilde{\ell}_{t+1} + \tilde{r}_{t+1}) - \gamma(\tilde{r}_{t+1}) - \eta(\tilde{\ell}_{t+1}) - \kappa
\end{aligned}
\tag{30}
$$

$$0 = G^2 \equiv i_{r,t+1} - i_t - \gamma'(\tilde{r}_{t+1}) \tag{31}$$

$$0 = G^3 \equiv i_{\ell,t+1} - i_t - \eta'(\tilde{\ell}_{t+1}) \tag{32}$$

To simplify the notation, drop the time subscripts. Applying the implicit function theorem yields

$$
\underbrace{\begin{bmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{bmatrix}}_{\equiv G}
\begin{bmatrix} d\tilde{\ell} \\ d\tilde{r} \\ di_\ell \end{bmatrix}
= -\begin{bmatrix} G^1_{i_r} di_r + G^1_i di \\ G^2_{i_r} di_r + G^2_i di \\ G^3_{i_r} di_r + G^3_i di \end{bmatrix}
$$

where

$$
\begin{array}{ll}
G^1_{i_r} = \tilde{r}, & G^1_i = -(\tilde{\ell} + \tilde{r}), \\
G^2_{i_r} = 1, & G^2_i = -1, \\
G^3_{i_r} = 0, & G^3_i = -1.
\end{array}
\qquad
|G| = \begin{vmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{vmatrix} = \begin{vmatrix} 0 & 0 & \tilde{\ell} \\ 0 & -\gamma'' & 0 \\ -\eta'' & 0 & 1 \end{vmatrix} = -\eta''\gamma''\tilde{\ell} < 0
$$

By Cramer's rule, we have

$$
\frac{\partial\tilde{\ell}}{\partial i} = \frac{1}{|G|}\begin{vmatrix} \tilde{\ell} + \tilde{r} & 0 & \tilde{\ell} \\ 1 & -\gamma'' & 0 \\ 1 & 0 & 1 \end{vmatrix} > 0, \qquad
\frac{\partial\tilde{\ell}}{\partial i_r} = \frac{1}{|G|}\begin{vmatrix} -\tilde{r} & 0 & \tilde{\ell} \\ -1 & -\gamma'' & 0 \\ 0 & 0 & 1 \end{vmatrix} < 0,
$$

$$\frac{\partial \tilde{r}}{\partial i} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} 0 & \tilde{\ell} + \tilde{r} & \tilde{\ell} \\ 0 & 1 & 0 \\ -\eta'' & 1 & 1 \end{vmatrix} < 0, \quad \frac{\partial \tilde{r}}{\partial i_r} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} 0 & -\tilde{r} & \tilde{\ell} \\ 0 & -1 & 0 \\ -\eta'' & 0 & 1 \end{vmatrix} > 0,$$

$$\frac{\partial i_\ell}{\partial i} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} 0 & 0 & \tilde{\ell} + \tilde{r} \\ 0 & -\gamma'' & 1 \\ -\eta'' & 0 & 1 \end{vmatrix} > 0, \quad \frac{\partial i_\ell}{\partial i_r} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} 0 & 0 & -\tilde{r} \\ 0 & -\gamma'' & -1 \\ -\eta'' & 0 & 0 \end{vmatrix} < 0.$$

∎

Proof of Proposition 2. This proof is divided into 2 parts. (i) Existence and uniqueness of the ample-reserve equilibrium; (ii) Existence of the scarce-reserve equilibrium.

Part 1 (Existence and uniqueness of the ample-reserve equilibrium) Consider the ample-reserve equilibrium. The ample-reserve equilibrium satisfies $i_d = i$, $\lambda_\chi = 0$, and $\tilde{r} > \chi \tilde{d}$. Given $i_d = i$ and $\lambda_\chi = 0$, the equilibrium solves the following system of equations.

$$0 = -\gamma'(\tilde{r}) + i_r - i \tag{33}$$

$$0 = -\kappa + \eta'(\tilde{\ell})\tilde{\ell} + \gamma'(\tilde{r})\tilde{r} - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \tag{34}$$

$$0 = -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + f'(k) \tag{35}$$

$$0 = -f(k) + f'(k)k + \zeta/U'\big(\{f(k) - \delta k\}N\big) \tag{36}$$

$$0 = -(1+i)n\tilde{d} + \max\big\{0, p^* - \bar{b}\big\} \tag{37}$$

$$0 = -kN + n\tilde{\ell} \tag{38}$$

$$0 = -i + \nu\sigma_1\lambda(q) \tag{39}$$

First, let's check equations (33)-(38) since i pins down q_1. Given (i, i_r) where $i_r \geq i$, equation (33) uniquely pins down $\tilde{r} \geq 0$ and $\partial\tilde{r}/\partial i_r > 0$ because $\gamma'' > 0$. When $i_r < i$, there is no solution $\tilde{r} \geq 0$ satisfying equation (33). Since $\eta'' > 0$ and $\gamma'' > 0$, equation (34) uniquely pins down $\tilde{\ell}$ as long as $\bar{\Delta} + i \geq i_r$ where $\bar{\Delta} = \gamma'(\underline{r})$ and $\underline{r}$ solves $\kappa = \gamma'(\underline{r})\underline{r} - \gamma(\underline{r})$. When $i_r > \bar{\Delta} + i$, there is no solution for (34) given $\tilde{r} \geq 0$ satisfying (33). Similarly, since $U'' < 0$ and $f'' < 0$, equation (35) uniquely pins down k. Given $(\tilde{r}, \tilde{\ell}, k)$ from (33)-(35), it is straightforward to show that (36)-(38) give (n, N, d) uniquely. Therefore, the system of equations (33)-(38) provides a unique solution for $(\tilde{r}, \tilde{\ell}, k, n, N, d)$ when $i_r \in (i, \bar{\Delta} + i)$.

Next, we will check under what condition $\tilde{r} > \chi\tilde{d}$ is satisfied. Consider a case satisfying $\tilde{r} = \chi\tilde{d}$ where $i_d = i$ which is a threshold between ample-reserve case and scarce reserve case. In this case, we have $(\tilde{r}, \tilde{\ell}, k, n, N) = (\bar{r}, \bar{\ell}, \bar{k}, \bar{n}, \bar{N})$ where $(\bar{r}, \bar{\ell}, \bar{K}, \bar{N}, \bar{C}, \bar{n})$ solves

$$\eta'(\bar{\ell})\bar{\ell} + (i_r - i)\bar{r} - \gamma(\bar{r}) - \eta(\bar{\ell}) = \kappa,$$

$$F_K(\bar{K}, \bar{N}) = \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\bar{\ell})}{\beta(1+i)}, \qquad \max\left\{0, \frac{\chi\left\{v(q^*)F_N(\bar{K}, \bar{N})/\zeta - \bar{b}\right\}}{\bar{n}}\right\} = (1+i)\bar{r},$$

and $\bar{C} + \delta\bar{K} = F(\bar{K}, \bar{N})$, $U'(\bar{C}) = \zeta/F_N(\bar{K}, \bar{N})$, and $\bar{K} = \bar{n}\bar{\ell}$.

Since $\partial\tilde{r}/\partial i_r > 0$, showing $\partial\tilde{d}/\partial i_r < 0$ will suffice to show (i) part of the Proposition 2. Consider the following system of equations.

$$0 = \tilde{G}^1 \equiv -\gamma'(\tilde{r}) + i_r - i$$

$$0 = \tilde{G}^2 \equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} + \gamma'(\tilde{r})\tilde{r} - \eta(\tilde{\ell}) - \gamma(\tilde{r})$$

$$0 = \tilde{G}^3 \equiv -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + F_K(n\tilde{\ell}, N)$$

$$0 = \tilde{G}^4 \equiv -F_N(n\tilde{\ell}, N) + \frac{\zeta}{U'(F(n\tilde{\ell}, N) - \delta n\tilde{\ell})}$$

$$0 = \tilde{G}^5 \equiv -n\tilde{d} + \max\left\{\frac{v(q^*)F_N(n\tilde{\ell}, N)/\zeta - \bar{b}}{1+i}, 0\right\}$$

Applying the implicit function theorem gives the following:

$$\underbrace{\begin{bmatrix} 0 & \tilde{G}^1_r & 0 & 0 & 0 \\ 0 & \tilde{G}^2_r & \tilde{G}^2_\ell & 0 & 0 \\ \tilde{G}^3_n & 0 & \tilde{G}^3_\ell & \tilde{G}^3_N & 0 \\ \tilde{G}^4_n & 0 & \tilde{G}^4_\ell & \tilde{G}^4_N & 0 \\ \tilde{G}^5_n & 0 & \tilde{G}^5_\ell & \tilde{G}^5_N & \tilde{G}^5_d \end{bmatrix}}_{=\tilde{G}} \begin{bmatrix} dn \\ d\tilde{r} \\ d\tilde{\ell} \\ dN \\ d\tilde{d} \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} di_r - \begin{bmatrix} -1 \\ 0 \\ \frac{\eta'}{\beta(1+i)^2} \\ 0 \\ -\frac{v(q^*)F_N}{\zeta(1+i)^2} \end{bmatrix} di - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{1+i} \end{bmatrix} d\bar{b}$$

$$(40)$$

where

$$\tilde{G} = \begin{bmatrix} 0 & -\gamma'' & 0 & 0 & 0 \\ 0 & \gamma''\tilde{r} & \eta''\tilde{\ell} & 0 & 0 \\ F_{KK}\tilde{\ell} & 0 & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)} + F_{KK}n & F_{KN} & 0 \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & 0 & -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} & 0 \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & 0 & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & -n \end{bmatrix}$$

Using Cramer's rule, we have the following comparative statics:

44

$$\frac{\partial \tilde{r}}{\partial i_r} = \frac{n\gamma''\eta''\tilde{\ell}^2}{|\tilde{G}|} \begin{vmatrix} F_{KK}\tilde{\ell} & F_{KN} \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -F_{NN} - \frac{\zeta F_N U''}{U'^2} \end{vmatrix} > 0$$

$$\frac{\partial \tilde{d}}{\partial i_r} = \frac{\gamma''\tilde{r}}{|\tilde{G}|} \begin{vmatrix} F_{KK}\tilde{\ell} & -\frac{\eta''}{\beta(1+i)} + F_{KK}n & F_{KN} \\ -\{F_{KN} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -\{F_{KN} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} \end{vmatrix} < 0$$

since $(F_{KN})^2 = F_{KK}F_{NN}$, $F_{KN} = F_{NK}$ and $F_N > F_{NK}K$. Given $\partial \tilde{r}/\partial i_r > 0 \; \partial \tilde{d}/\partial i_r < 0$, we can conclude that given $(i, \chi, \bar{b})$, $\exists!$ ample-reserves equilibrium if and only if $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$.

Part 2 (Existence of the scarce-reserve equilibrium): From Part 1, we already know that there is no scarce-reserve equilibrium when $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$. Hence, we only focus on that case where $i_r \in [0, \bar{\iota}_r]$. The scarce-reserve equilibrium solves the following system of equations.

$$0 = -i_d + (1 - \chi)i_s + \chi i_r - \chi\gamma'(\tilde{r}) \tag{41}$$

$$0 = -i_\ell + i + \eta'(\tilde{\ell}) \tag{42}$$

$$0 = -\kappa + \eta'(\tilde{\ell})\tilde{\ell} + \gamma'(\tilde{r})\tilde{r} - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \tag{43}$$

$$0 = -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + f'(k) \tag{44}$$

$$0 = -f(k) + f'(k)k + \zeta/U'\big(\{f(k) - \delta k\}N\big) \tag{45}$$

$$0 = -\tilde{r} + \chi\tilde{d} \tag{46}$$
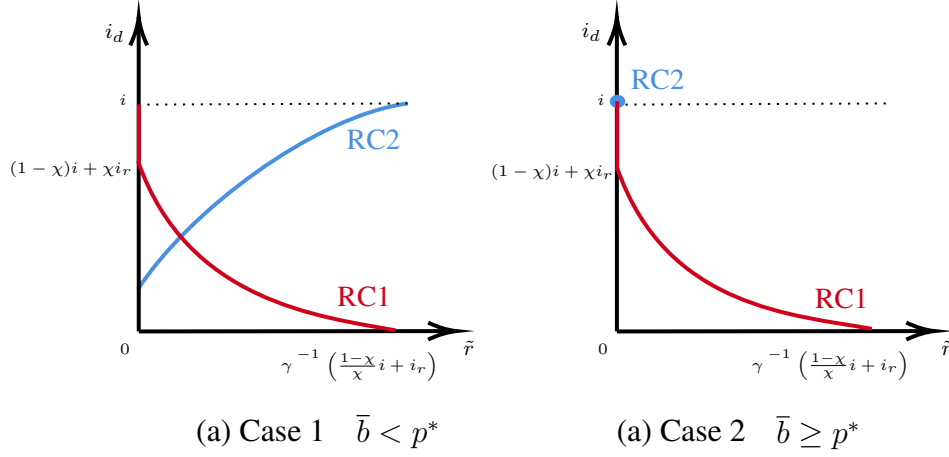
$$0 = -kN + n\tilde{\ell} \tag{47}$$

$$0 = -i + \nu\sigma_1\lambda(q_1) + \nu\sigma_3\lambda(q_3) \tag{48}$$

$$0 = -\frac{1+i}{1+i_d} + 1 + \nu\sigma_2\lambda(q_2) + \nu\sigma_3\lambda(q_3) \tag{49}$$

$$0 = -v(q_1) + \min\left\{v(q^*), \frac{\zeta m}{f(k) - f'(k)k}\right\} \tag{50}$$

$$0 = -v(q_2) + \min\left\{v(q^*), \frac{\zeta(n\tilde{d} + \bar{b})}{f(k) - f'(k)k}\right\} \tag{51}$$

$$0 = -v(q_3) + \min\left\{v(q^*), \frac{\zeta(m + n\tilde{d} + \bar{b})}{f(k) - f'(k)k}\right\} \tag{52}$$

(a) Case 1 $\quad \bar{b} < p^*$ $\qquad$ (a) Case 2 $\quad \bar{b} \geq p^*$

Figure 14: RC1 and RC2

The equations (42)-(52) can be simplified as follows:

$$0 = \tilde{H}^1 \equiv -i + \nu\sigma_1\Lambda\left(\frac{\zeta}{f(k) - f'(k)k}m\right) \tag{53}$$

$$+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k) - f'(k)k}\left[m + \bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right]\right\}\right)$$

$$0 = \tilde{H}^2 \equiv -\frac{1 + i}{1 + i_d} + 1 \tag{54}$$

$$+ \nu\sigma_2\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k) - f'(k)k}\left[\bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right]\right\}\right)$$

$$+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k) - f'(k)k}\left[m + \bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right]\right\}\right)$$

$$0 = \tilde{H}^3 \equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} - \eta(\tilde{\ell}) + \gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) \tag{55}$$

$$0 = \tilde{H}^4 \equiv -f'(k) + \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\tilde{\ell})}{\beta(1 + i)} \tag{56}$$

$$0 = \tilde{H}^5 \equiv \frac{\zeta}{f(k) - f'(k)k} - U'([f(k) - \delta k]N) \tag{57}$$

$$0 = \tilde{H}^6 \equiv -k + \frac{n\tilde{\ell}}{N} \tag{58}$$

Relaxing the equality of (41), we obtain $i_d \geq (1 - \chi)i_s + \chi i_r - \chi\gamma'(\tilde{r})$. From equation (41), it is given that $\tilde{r}$ decreases with i_d for $i_d \in [0, (1 - \chi)i_s + \chi i_r]$. The curve that satisfies equation (41) in the $(i_d, \tilde{r})$ space, labeled as RC1, represents the amount of real reserve balances a bank is willing to hold for a given i_d. The curve that satisfies equations (53) to (58) in the $(i_d, \tilde{r})$ space, labeled as RC2, represents the required reserves for transaction deposit demand. This takes into account the values of i_d and the corresponding activities of

46

households and firms. The RC1 curve is a downward-sloping single-valued function, while RC2 can be either a single-valued or a multi-valued curve.

The RC1 curve starts at $(\tilde{r}, i_d) = (0, (1 - \chi)i + \chi i_r)$, with a continuum of points where $\tilde{r} = 0$ and $i \geq i_d \geq (1 - \chi)i + \chi i_r$. Along the RC1 curve, i_d weakly decreases with $\tilde{r}$ until it reaches the point $(\tilde{r}, i_d) = \left(\gamma^{-1}\left(\frac{1-\chi}{\chi}i + i_r\right), 0\right)$. In equilibrium, since $i \geq i_d$, the right end of the RC2 curve solves (53)-(58) with $i_d = i$. For a given $(i_r, \bar{b})$ with $i_d = i$, this holds when $\tilde{r} > 0$ as long as $\bar{b} < p^*$. If $\bar{b} \geq p^*$, then $\tilde{r} = 0$. At the left end of the RC2 curve, it solves (53)-(58) where $\tilde{r} = 0$ and $i \geq i_d \geq 0$. It is evident that when $\bar{b} \geq p^*$, the RC2 curve becomes a degenerate case with a single point at $(\tilde{r}, i_d) = (0, i)$. When $\bar{b} < p^*$, the left end of the RC2 curve satisfies $\tilde{r} > 0$, and $i > i_d \geq 0$. Therefore, there is at least one point where the RC1 curve intersects the RC2 curve where the RC1 curve crosses the RC2 curve from below. Consequently, we can conclude that a scarce-reserves equilibrium exists if and only if $0 \leq i_r \leq \bar{\iota}_r$. ∎

Proof of Proposition 3. Recall money multiplier ξ:

$$\xi = \frac{m + d}{\phi B} = \frac{m + d}{m + r} = \frac{m + n\tilde{d}}{m + n\tilde{r}}$$

Showing $\partial\tilde{r}/\partial i_r > 0$, $\partial\tilde{r}/\partial i < 0$, $\partial\tilde{d}/\partial i_r < 0$, and $\partial\tilde{d}/\partial i > 0$ suffices to show $\partial\xi/\partial i > 0$ and $\partial\xi/\partial i_r < 0$. Recall $\gamma'(\tilde{r}) = i_r - i$. It is straightforward to show that $\partial\tilde{r}/\partial i_r > 0$ and $\partial\tilde{r}/\partial i < 0$. Part 1 of Proposition 2 already has shown $\partial\tilde{d}/\partial i_r < 0$. To show $\partial\tilde{d}/\partial i > 0$, recall (40). Applying implicit function theorem and comparative statics gives

$$\frac{\partial\tilde{d}}{\partial i} = \frac{|\tilde{\boldsymbol{G}}_{i,d}|}{|\tilde{\boldsymbol{G}}|}$$

where

$$\tilde{\boldsymbol{G}}_{i,d} = \begin{bmatrix} 0 & -\gamma'' & 0 & 0 & 1 \\ 0 & \gamma''\tilde{r} & \eta''\tilde{\ell} & 0 & 0 \\ F_{KK}\tilde{\ell} & 0 & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)} + F_{KK}n & F_{KN} & -\frac{\eta'}{\beta(1+i)^2} \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & 0 & -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} & 0 \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & 0 & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & \frac{v(q^*)F_N}{\zeta(1+i)^2} \end{bmatrix}.$$

Solving this yields the following expression:

$$
\begin{aligned}
\frac{\partial \tilde{d}}{\partial i} &= \frac{|\tilde{\boldsymbol{G}}_{i,d}|}{|\tilde{\boldsymbol{G}}|} = -\frac{\gamma'' \eta'' \tilde{\ell}}{|\tilde{\boldsymbol{G}}|}
\begin{vmatrix}
F_{KK}\tilde{\ell} & F_{KN} & -\frac{\eta'}{\beta(1+i)^2} \\
-\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -F_{NN} - \frac{\zeta F_N U''}{U'^2} & 0 \\
-\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & \frac{v(q^*)F_N}{\zeta(1+i)^2}
\end{vmatrix} \\
&- \frac{\gamma'' \tilde{r}}{|\tilde{\boldsymbol{G}}|}
\begin{vmatrix}
F_{KK}\tilde{\ell} & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)} + F_{KK}n & F_{KN} \\
-\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} \\
-\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)}
\end{vmatrix} > 0.
\end{aligned}
$$

∎

Proof of Proposition 4. This proof is divided into 2 parts: (i) Ample reserves equilibrium and (ii) scarce reserve equilibrium.

Ample reserves equilibrium In the ample reserve equilibrium, since $i_d = i$, it is easy to show that $\partial i_d / \partial \bar{b} = 0$. When $p^* > \bar{b}$ we have

$$
d = \frac{v(q^*)\{f(k) - f'(k)k\}/\zeta - \bar{b}}{1 + i}
$$

in the ample reserves equilibrium. From comparative statics, we have

$$
\frac{\partial \tilde{\ell}}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{b},\tilde{\ell}}|}{|\tilde{\boldsymbol{G}}|} = 0, \quad
\frac{\partial n}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{b},n}|}{|\tilde{\boldsymbol{G}}|} = 0, \quad
\frac{\partial N}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{b},N}|}{|\tilde{\boldsymbol{G}}|} = 0
$$

since

$$
|\tilde{\boldsymbol{G}}_{\bar{b},\tilde{\ell}}| = -\frac{1}{1+i}
\begin{vmatrix}
0 & \tilde{G}_r^1 & 0 & 0 \\
0 & \tilde{G}_r^2 & 0 & 0 \\
\tilde{G}_n^3 & 0 & \tilde{G}_N^4 & 0 \\
\tilde{G}_n^4 & 0 & \tilde{G}_N^4 & 0
\end{vmatrix} = 0, \quad
|\tilde{\boldsymbol{G}}_{\bar{b},n}| = -\frac{1}{1+i}
\begin{vmatrix}
\tilde{G}_r^1 & 0 & 0 & 0 \\
\tilde{G}_r^2 & \tilde{G}_\ell^2 & 0 & 0 \\
0 & \tilde{G}_\ell^3 & \tilde{G}_N^3 & 0 \\
0 & \tilde{G}_\ell^4 & \tilde{G}_N^4 & 0
\end{vmatrix} = 0
$$

$$
|\tilde{\boldsymbol{G}}_{\bar{b},N}| = \frac{1}{1+i}
\begin{vmatrix}
0 & \tilde{G}_r^1 & 0 & 0 \\
0 & \tilde{G}_r^2 & \tilde{G}_\ell^2 & 0 \\
\tilde{G}_n^3 & 0 & \tilde{G}_\ell^3 & 0 \\
\tilde{G}_n^4 & 0 & \tilde{G}_\ell^4 & 0
\end{vmatrix} = 0.
$$

The above comparative statics implies $\frac{\partial k}{\partial \bar{b}} = 0$ since $k = K/N = n\tilde{\ell}/N$. Then it is straightforward to show $\partial d / \partial \bar{b} < 0$ and $\partial \tilde{d} / \partial \bar{b} < 0$.

Scarce reserves equilibrium First, I define $\tilde{z}_j \equiv \frac{\zeta}{f(k) - f'(k)k} \max\{p^*, z_j\}$ to simplify

48

notations. The scarce reserve equilibrium solves the following system of equations:

$$0 = J^1 \equiv -i + \nu\sigma_1\Lambda\left(\frac{\zeta}{f(k) - f'(k)k}m\right) \tag{59}$$

$$+ \nu\sigma_3\Lambda\left(\frac{\zeta}{f(k) - f'(k)k}\left\{m + \bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right\}\right)$$

$$0 = J^2 \equiv -\frac{1 + i}{1 + i_d} + 1 + \nu\sigma_2\Lambda\left(\frac{\zeta}{f(k) - f'(k)k}\left\{\bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right\}\right) \tag{60}$$

$$+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k) - f'(k)k}\left[m + \bar{b} + \frac{[1 + i_d]n\tilde{r}}{\chi}\right]\right\}\right)$$

$$0 = J^3 \equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} - \eta(\tilde{\ell}) + \gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) \tag{61}$$

$$0 = J^4 \equiv \frac{\zeta}{f(k) - f'(k)k} - U'\left([f(k) - \delta k]N\right) \tag{62}$$

$$0 = J^5 \equiv -k + \frac{n\tilde{\ell}}{N} \tag{63}$$

$$0 = J^6 \equiv -f(k) + \frac{1}{\beta} - 1 + \delta + \frac{\eta(\tilde{\ell})}{\beta(1 + i)} \tag{64}$$

where $i_d = (1 - \chi)i + \chi i_r - \chi\gamma'(\tilde{r})$.

Assuming interior, applying the implicit function theorem yields

$$\underbrace{\begin{bmatrix} J_m^1 & J_{\tilde{r}}^1 & 0 & J_k^1 & J_n^1 & 0 \\ J_m^2 & J_{\tilde{r}}^2 & 0 & J_k^2 & J_n^2 & 0 \\ 0 & J_{\tilde{r}}^3 & J_{\tilde{\ell}}^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_{\tilde{\ell}}^5 & J_k^5 & J_n^5 & J_N^5 \\ 0 & 0 & J_{\tilde{\ell}}^6 & J_k^6 & 0 & 0 \end{bmatrix}}_{\equiv J} \begin{bmatrix} dm \\ d\tilde{r} \\ d\tilde{\ell} \\ dk \\ dn \\ dN \end{bmatrix} = -\underbrace{\begin{bmatrix} J_{\bar{b}}^1 & J_{i_r}^1 & J_i^1 \\ J_{\bar{b}}^2 & J_{i_r}^2 & J_i^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & J_i^6 \end{bmatrix}}_{\equiv \bar{J}} \begin{bmatrix} d\bar{b} \\ di_r \\ di \end{bmatrix} \tag{65}$$

where

$$J_m^1 = \frac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}, \qquad J_m^2 = \frac{\nu\zeta}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3),$$

$$J_{\tilde{r}}^1 = \frac{n\nu\zeta\{1+i_d-\tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sigma_3\Lambda'(\tilde{z}_3), \qquad J_r^2 = -\frac{1+i}{(1+i_d)^2}\chi\gamma''(\tilde{r}) + \frac{n\nu\zeta\{1+i_d-\tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sum_{j=2}^{3}\{\sigma_j\Lambda'(\tilde{z}_j)\},$$

$$J_{\tilde{r}}^3 = \gamma''(\tilde{r})\tilde{r}, \qquad J_{\tilde{\ell}}^3 = \eta''(\tilde{\ell})\tilde{\ell},$$

$$J_{\tilde{\ell}}^5 = \frac{n}{N}, \qquad J_{\tilde{\ell}}^6 = \frac{\eta''(\tilde{\ell})}{\beta(1+i)},$$

$$J_k^1 = \frac{\nu\zeta f''(k)k}{\{f(k)-f'(k)k\}^2}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}, \qquad J_k^2 = \frac{\nu\zeta f''(k)k}{\{f(k)-f'(k)k\}^2}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\},$$

$$J_k^4 = \frac{\zeta f''(k)k}{\{f(k)-f'(k)k\}^2} - \{f'(k)-\delta\}NU''(C), \qquad J_k^5 = -1,$$

$$J_k^6 = -f''(k), \qquad J_n^1 = \frac{\nu\zeta(1+i_d)\tilde{r}}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3),$$

$$J_n^2 = \frac{\nu\zeta(1+i_d)\tilde{r}}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}, \qquad J_n^5 = \frac{\ell}{N},$$

$$J_N^4 = -\{f-\delta k\}U''(C), \qquad J_N^5 = -\frac{n\ell}{N^2},$$

$$J_{\tilde{b}}^1 = \frac{\nu\zeta}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3), \qquad J_{\tilde{b}}^2 = \frac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\},$$

$$J_{i_r}^1 = \frac{\sigma_3 n\tilde{r}\nu\zeta}{f(k)-f'(k)k}\Lambda'(\tilde{z}_3), \qquad J_{i_r}^2 = \frac{n\tilde{r}\nu\zeta}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\},$$

$$J_i^1 = -1 + \frac{(1-\chi)d\sigma_3\nu\zeta}{f-f'k}\Lambda(\tilde{z}_3), \qquad J_i^2 = -\frac{1}{1+i_d} + \frac{(1-\chi)d\nu\zeta}{f-f'k}\{\sigma_2\Lambda(\tilde{z}_2) + \sigma_3\Lambda(\tilde{z}_3)\} + \frac{1+i}{(1+i_d)^2}\chi,$$

$$J_i^6 = -\frac{\eta'(\tilde{\ell})}{\beta(1+i)^2}.$$

The determinant of $\boldsymbol{J}$ is

$$\begin{aligned}
|\boldsymbol{J}| = & -U''\gamma''\tilde{r}f''\frac{n}{N}(f-\delta k)\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{(1+i_d)d}{n} \\
& \times \{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
& -U''\eta''\tilde{\ell}f''\frac{\ell}{N}(f-\delta k)\frac{\nu\zeta}{f-f'k}\left\{-\frac{1+i}{(1+i_d)^2}\chi\gamma''\right\}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\} \\
& +U''\gamma''\tilde{r}\frac{\eta''}{\beta(1+i)}\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{(1+i_d)d}{n}(f-f'k) \\
& \times \{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
& -U''\eta''\tilde{\ell}f''\frac{\ell}{N}(f-\delta k)[1+i_d-\tilde{r}\chi\gamma'']\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{d}{\tilde{r}} \\
& \times \{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
& +U'\frac{\tilde{\ell}}{N^2}\gamma''\tilde{r}\frac{\eta''}{\beta(1+i)}\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{f''k}{f-f'k} \\
& \times \Big[(1+i_d)d\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
& \qquad - \{z_2\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + (z_3-z_1)\sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + z_2\sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\}\Big]
\end{aligned}$$

Since $z_2 = z_3 - z_1 \to (1 + i_d)d$ when $\bar{b} \to 0$, we have

$$\lim_{\bar{b} \to 0} |\boldsymbol{J}| < 0$$

under Assumption 1. One can also show $\lim_{\sigma_3 \to 0} |\boldsymbol{J}| < 0$ as well. Therefore, for sufficiently small $\bar{b}$ or σ_3, we have $|\boldsymbol{J}| < 0$. From comparative statics we have

$$\frac{\partial \tilde{r}}{\partial \bar{b}} = \frac{|\boldsymbol{J}_{\bar{b},\tilde{r}}|}{|\boldsymbol{J}|}$$

$$= \frac{1}{|\boldsymbol{J}|} \underbrace{\left(\frac{\nu\zeta}{f - f'k}\right)^2}_{\oplus} \underbrace{\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_3\sigma_2\Lambda(\tilde{z}_3)\Lambda(\tilde{z}_2)\}}_{\oplus} \underbrace{\frac{f''U''\eta''\ell(f - \delta k)}{\beta(1 + i)N}}_{\oplus}$$

$$\frac{\partial n}{\partial \bar{b}} = \frac{|\boldsymbol{J}_{\bar{b},n}|}{|\boldsymbol{J}|}$$

$$= \frac{1}{|\boldsymbol{J}|} \underbrace{\left(\frac{\nu\zeta}{f - f'k}\right)^2}_{\oplus} \underbrace{\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_3\sigma_2\Lambda(\tilde{z}_3)\Lambda(\tilde{z}_2)\}}_{\oplus}$$

$$\times \underbrace{\left[\frac{(f'k - f)U''\gamma''\tilde{r}k\eta''}{\beta(1 + i)} + (f - \delta k)U''f''\frac{n}{N} + \frac{-\zeta f''k\eta''k}{\{f - f'k\}^2 N\beta(1 + i)}\right]}_{\oplus}$$

where

$$\boldsymbol{J}_{\bar{b},\tilde{r}} = \begin{bmatrix} J_m^1 & -J_{\bar{b}}^1 & 0 & J_k^1 & J_n^1 & 0 \\ J_m^2 & -J_{\bar{b}}^2 & 0 & J_k^2 & J_n^2 & 0 \\ 0 & 0 & J_\ell^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_\ell^5 & J_k^5 & J_n^5 & J_N^5 \\ 0 & 0 & J_\ell^6 & J_k^6 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{J}_{\bar{b},n} = \begin{bmatrix} J_m^1 & J_{\tilde{r}}^1 & 0 & J_k^1 & -J_{\bar{b}}^1 & 0 \\ J_m^2 & J_{\tilde{r}}^2 & 0 & J_k^2 & -J_{\bar{b}}^2 & 0 \\ 0 & J_{\tilde{r}}^3 & J_\ell^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_\ell^5 & J_k^5 & 0 & J_N^5 \\ 0 & 0 & J_\ell^6 & J_k^6 & 0 & 0 \end{bmatrix}.$$

Then we have

$$\lim_{\bar{b} \to 0} \frac{|\boldsymbol{J}_{\bar{b},\tilde{r}}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\bar{b} \to 0} \frac{|\boldsymbol{J}_{\bar{b},n}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\sigma_3 \to 0} \frac{|\boldsymbol{J}_{\bar{b},\tilde{r}}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\sigma_3 \to 0} \frac{|\boldsymbol{J}_{\bar{b},n}|}{|\boldsymbol{J}|} < 0$$

because $|\boldsymbol{J}_{\bar{b},\tilde{r}}| > 0$ and $|\boldsymbol{J}_{\bar{b},n}| > 0$. Since $d = n\tilde{d}/\chi$, it is easy to show that $\partial d/\partial \bar{b} < 0$ for small $\bar{b}$ or σ_3. Since $i_d = (1 - \chi)i + \chi i_r - \chi\gamma'(\tilde{r})$ and $\partial \tilde{r}/\partial \bar{b} < 0$ for small $\bar{b}$ or σ_3, we have

51

$\partial i_d / \partial \bar{b} > 0$ for small $\bar{b}$ or σ_3. ■

Proof of Proposition 5. This proof is divided into 4 parts:

Part 1 (Comparative Statics in the Ample-Reserve Equilibrium): Consider the ample-reserves equilibrium. It can be summarized as 4 equations 4 unknowns as follows.

$$0 = G^1 \equiv -\gamma'(\tilde{r}) + i_r - i$$

$$0 = G^2 \equiv -\kappa + \{1 + i + \eta'(\tilde{\ell})\}\tilde{\ell} + (1 + i_r)\tilde{r} - (1 + i)(\tilde{\ell} + \tilde{r}) - \eta(\tilde{\ell}) - \gamma(\tilde{r})$$

$$0 = G^3 \equiv -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1 + i)} + f'(k)$$

$$0 = G^4 \equiv -f(k) + f'(k)k + \zeta/U'(\{f(k) - \delta k\}N)$$

where $k = K/H$. Applying the implicit function theorem yields

$$\underbrace{\begin{bmatrix} G_k^1 & G_N^1 & G_\ell^1 & G_r^1 \\ G_k^2 & G_N^2 & G_\ell^2 & G_r^2 \\ G_k^3 & G_N^3 & G_\ell^3 & G_r^3 \\ G_k^4 & G_N^4 & G_\ell^4 & G_r^4 \end{bmatrix}}_{\equiv G} \begin{bmatrix} dk \\ dN \\ d\tilde{\ell} \\ d\tilde{r} \end{bmatrix} = -\underbrace{\begin{bmatrix} G_{i_r}^1 & G_i^1 & G_b^1 \\ G_{i_r}^2 & G_i^2 & G_b^2 \\ G_{i_r}^3 & G_i^3 & G_b^3 \\ G_{i_r}^4 & G_i^4 & G_b^4 \end{bmatrix}}_{\equiv \bar{G}} \begin{bmatrix} di \\ di_r \\ d\bar{b} \end{bmatrix} \tag{66}$$

where

$$G = \begin{bmatrix} 0 & 0 & 0 & -\gamma'' \\ 0 & 0 & 0 & \eta''\tilde{\ell} & 0 \\ f'' & 0 & -\frac{\eta''}{\beta(1+i)} & 0 \\ f''k - N(f'-\delta)U''\zeta/(U')^2 & -(f-\delta k)U''\zeta/(U')^2 & 0 & 0 \end{bmatrix}$$

and

$$\bar{G} = \begin{bmatrix} -1 & 1 & 0 \\ -\tilde{r} & \tilde{r} & 0 \\ \frac{\eta'}{\beta(1+i)^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

By Cramer's rule, we have

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} G_k^1 & G_N^1 & -G_i^1 & G_r^1 \\ G_k^2 & G_N^2 & -G_i^2 & G_r^2 \\ G_k^3 & G_N^3 & -G_i^3 & G_r^3 \\ G_k^4 & G_N^4 & -G_i^4 & G_r^4 \end{vmatrix} = \frac{-(f-\delta k)U''\frac{\zeta}{(U')^2}f''\gamma''\tilde{r}}{|\boldsymbol{G}|} > 0,$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} G_k^1 & G_N^1 & -G_{i_r}^1 & G_r^1 \\ G_k^2 & G_N^2 & -G_{i_r}^2 & G_r^2 \\ G_k^3 & G_N^3 & -G_{i_r}^3 & G_r^3 \\ G_k^4 & G_N^4 & -G_{i_r}^4 & G_r^4 \end{vmatrix} = \frac{(f-\delta k)U''\frac{\zeta}{(U')^2}f''\gamma''\tilde{r}}{|\boldsymbol{G}|} < 0$$

since

$$|\boldsymbol{G}| = -(f-\delta k)\frac{\gamma''U''}{(U')^2}\zeta f''\eta''\tilde{\ell} < 0.$$

Because $\partial \tilde{\ell}/\partial i > 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, we have $\partial i_\ell/\partial i = 1 + \eta''(\tilde{\ell})\frac{\partial \tilde{\ell}}{\partial i} > 0$. Similarly, because $\partial \tilde{\ell}/\partial i_r < 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, $\partial i_\ell/\partial i_r = \eta''(\tilde{\ell})\frac{\partial \tilde{\ell}}{\partial i_r} < 0$. The real lending rate can be written as $\rho = 1/\beta - 1 + \frac{\eta'(\tilde{\ell})}{\beta(1+i)}$. It is straightforward to show $\partial \rho/\partial i_r = \frac{\partial \tilde{\ell}}{\partial i_r}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} < 0$ and $\partial \rho/\partial i = \frac{\partial \tilde{\ell}}{\partial i}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} - \frac{\eta'(\tilde{\ell})}{\beta(1+i)^2} \lessgtr 0$. Since $i_d = i$, $\partial i_d/\partial i = 1 > 0$ and $\partial i_d/\partial i_r = 0$.

Part 2 (Comparative Statics in the Scarce-Reserve Equilibrium with respect to i_r):

$$\boldsymbol{J}_{i_r,\tilde{r}} = \begin{bmatrix} J_m^1 & -J_{i_r}^1 & 0 & J_k^1 & J_n^1 & 0 \\ J_m^2 & -J_{i_r}^2 & 0 & J_k^2 & J_n^2 & 0 \\ 0 & 0 & J_{\tilde{\ell}}^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_{\tilde{\ell}}^5 & J_k^5 & J_n^5 & J_N^5 \\ 0 & 0 & J_{\tilde{\ell}}^6 & J_k^6 & 0 & 0 \end{bmatrix}.$$

$$|\boldsymbol{J}_{i_r,\tilde{r}}| = (J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1) \underbrace{J_{\tilde{\ell}}^3 J_k^6 J_N^4 J_n^5}_{\oplus}$$

where

$$J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 =$$
$$\left(\frac{\nu\zeta}{f-f'k}\right)\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}\chi\left[\frac{1+i}{(1+i_d)^2} + \left\{\frac{(n\tilde{r}/\chi)\nu\zeta}{f-f'k}\right\}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}\right]$$
$$- n\tilde{r}\left(\frac{\nu\zeta}{f-f'k}\right)^2\{\sigma_3\Lambda'(\tilde{z}_3)\}^2.$$

Therefore, as long as

$$J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0 \tag{67}$$

holds (which we will check later), we have $\partial \tilde{r}/\partial i_r > 0$ for small $\bar{b}$ which also implies $\partial \tilde{\ell}/\partial i_r < 0$ for small $\bar{b}$. Immediate results of above results are $\partial i_\ell/\partial i_r < 0$ and $\partial \rho/\partial i_r < 0$. The next part (Part 3) verifies the condition (67).

Part 3 (Confirming $J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0$ holds when scarce-reserve equilibrium is unique): To confirm $J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0$, recall (53)-(58) and apply implicit function theorem:

$$\underbrace{\begin{bmatrix} \tilde{H}_m^1 & \tilde{H}_{\tilde{r}}^1 & 0 & \tilde{H}_k^1 & \tilde{H}_n^1 & 0 \\ \tilde{H}_m^2 & \tilde{H}_{\tilde{r}}^2 & 0 & \tilde{H}_k^2 & \tilde{H}_n^2 & 0 \\ 0 & \tilde{H}_{\tilde{r}}^3 & \tilde{H}_{\tilde{\ell}}^3 & 0 & 0 & 0 \\ 0 & 0 & \tilde{H}_{\tilde{\ell}}^4 & \tilde{H}_k^4 & 0 & 0 \\ 0 & 0 & 0 & \tilde{H}_k^5 & 0 & \tilde{H}_N^5 \\ 0 & 0 & \tilde{H}_{\tilde{\ell}}^6 & \tilde{H}_k^6 & \tilde{H}_n^6 & \tilde{H}_N^6 \end{bmatrix}}_{\equiv \tilde{H}} \begin{bmatrix} dm \\ d\tilde{r} \\ d\tilde{\ell} \\ dk \\ dn \\ dN \end{bmatrix} = - \begin{bmatrix} (n\tilde{r}/\chi)\nu\sigma_3 \Lambda'(\tilde{z}_3) \\ \frac{1+i}{(1+i_d)^2} + (n\tilde{r}/\chi)\nu\left[\sigma_2 \Lambda'(\tilde{z}_2) + \sigma_3 \Lambda'(\tilde{z}_3)\right] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} di_d$$

where

$$H_m^1 = \frac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_1 \Lambda'(\tilde{z}_1) + \sigma_3 \Lambda'(\tilde{z}_3)\}, \quad H_m^2 = \frac{\nu\zeta}{f(k)-f'(k)k}\sigma_3 \Lambda'(\tilde{z}_3),$$

$$H_{\tilde{r}}^1 = \frac{n\nu\zeta\{1+i_d - \tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sigma_3 \Lambda'(\tilde{z}_3), \qquad H_r^2 = -\frac{1+i}{(1+i_d)^2} + \frac{n\nu\zeta(1+i_d)}{\chi\{f(k)-f'(k)k\}}\sum_{j=2}^3\{\sigma_j \Lambda'(\tilde{z}_j)\},$$

$$H_{\tilde{r}}^3 = J_{\tilde{r}}^3, \qquad H_{\tilde{\ell}}^3 = J_{\tilde{\ell}}^3,$$

$$H_{\tilde{\ell}}^5 = J_{\tilde{\ell}}^5, \qquad H_{\tilde{\ell}}^6 = J_{\tilde{\ell}}^6,$$

$$H_k^1 = J_k^1, \qquad H_k^2 = J_k^2,$$

$$H_k^4 = J_k^4, \qquad H_k^5 = J_k^5,$$

$$H_k^6 = J_k^6, \qquad H_n^1 = J_n^1,$$

$$H_n^2 = J_n^2, \qquad H_n^5 = J_n^5,$$

$$H_N^4 = J_N^4, \qquad H_N^5 = J_N^5,$$

From Part 2 of Proposition 2, there exists at least one point where RC1 crosses RC2 curve from below. If the equilibrium is unique, the equilibrium exists where RC1 crosses RC2 curve from below which implies that the equilibrium satisfies $\partial \tilde{r}/\partial i_d > 0$. The uniqueness

of scarce reserve implies $\partial \tilde{r}/\partial i_d > 0$ which also means $\frac{|\tilde{H}_{i_d,\tilde{r}}|}{|\tilde{H}|} > 0$. This holds if and only if

$$\tilde{H}_m^2 \tilde{H}_{i_d}^1 - \tilde{H}_m^1 \tilde{H}_{i_d}^2$$

$$= (n\tilde{r}/\chi)\left(\frac{\nu\zeta}{f - f'k}\right)^2 \{\sigma_3 \Lambda'(\tilde{z}_3)\}^2$$

$$- \frac{\nu\zeta}{f - f'k}\{\sigma_1 \Lambda'(\tilde{z}_1) + \sigma_3 \Lambda'(\tilde{z}_3)\}\left[\frac{1+i}{(1+i_d)^2} + \frac{(n\tilde{r}/\chi)\nu\zeta}{f - f'k}\{\sigma_1 \Lambda'(\tilde{z}_1) + \sigma_3 \Lambda'(\tilde{z}_3)\}\right] > 0$$

because

$$|\tilde{\boldsymbol{H}}| = (\tilde{H}_m^1 \tilde{H}_r^2 - \tilde{H}_r^1 \tilde{H}_m^2)\tilde{H}_\ell^3 \tilde{H}_k^4(-\tilde{H}_N^5 \tilde{H}_n^6) + (\tilde{H}_m^1 \tilde{H}_r^3 \tilde{H}_k^2 - \tilde{H}_m^2 \tilde{H}_r^3 \tilde{H}_k^1)\tilde{H}_{\tilde{\ell}}^3(-\tilde{H}_n^6 \tilde{H}_N^5) > 0$$

and $\tilde{H}_m^2 \tilde{H}_{i_d}^1 - \tilde{H}_m^1 \tilde{H}_{i_d}^2 = -(J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1)\chi$, we have $J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0$ when the equilibrium is unique.

Part 4 (Comparative Statics in the Scarce-Reserve Equilibrium with respect to i): Recall (65). Using Cramer's rule, we have $\frac{\partial \tilde{r}}{\partial i} = \frac{|\boldsymbol{J}_{i,\tilde{r}}|}{|\boldsymbol{J}|}$ where

$$\boldsymbol{J}_{i,\tilde{r}} = \begin{bmatrix} J_m^1 & -J_i^1 & 0 & J_k^1 & J_n^1 & 0 \\ J_m^2 & -J_i^2 & 0 & J_k^2 & J_n^2 & 0 \\ 0 & 0 & J_\ell^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_{\tilde{\ell}}^5 & J_k^5 & J_n^5 & J_N^5 \\ 0 & -J_i^6 & J_{\tilde{\ell}}^6 & J_k^6 & 0 & 0 \end{bmatrix}$$

and

$$|\boldsymbol{J}_{i,\tilde{r}}| = (J_m^1 J_i^2 - J_m^2 J_i^1)J_{\tilde{\ell}}^3 J_k^6 J_n^5 J_N^4 - J_i^6 J_\ell^3 J_k^4(J_m^1 J_n^2 J_N^5 - J_m^2 J_n^1 J_N^5)$$
$$- J_i^6 J_\ell^3 J_N^4 \{J_m^1(J_k^2 J_n^5 - J_n^2 J_k^5) - J_m^2(J_k^1 J_n^5 - J_n^1 J_k^5)\}.$$

Given that, one can show that

$$\lim_{\sigma_3 \to 0} \frac{|\boldsymbol{J}_{i,\tilde{r}}|}{|\boldsymbol{J}|} < 0$$

which implies $\frac{\partial \tilde{r}}{\partial i} < 0$ and $\frac{\partial \tilde{\ell}}{\partial i} > 0$. Given these results, it is straightforward to show that $\partial i_\ell/\partial i > 0$ and $\partial i_d/\partial i > 0$ for small σ_3. ∎

Proof of Proposition 6. Recall equation (28)-(29).

$$\frac{i_t}{\nu} = \sigma_1 \lambda(q_1) + \sigma_3 \lambda(q_3)$$

$$\left\{ \frac{1+i}{1+i_d} - 1 \right\} \frac{1}{\nu} = \sigma_2 \lambda(q_2) + \sigma_3 \lambda(q_3)$$

where $v(q_1) = \zeta m / \{f(k) - f'(k)k\}$, $v(q_2) = \zeta(d + \bar{b}) / \{f(k) - f'(k)k\}$, and $v(q_3) = \zeta(m + d + \bar{b}) / \{f(k) - f'(k)k\}$. When $i_r \geq \bar{\iota}_r$, we have $i_d = i$ which gives $\lambda(q_2) = \lambda(q_3)$. Therefore, it is straightforward to show $q_2 = q_3 = q^*$. ∎

Proof of Proposition 7. Since $\partial \tilde{r} / \partial \bar{b} < 0$ in the ample reserve equilibrium and $\bar{\iota}_r = \gamma'(\tilde{r}) + i$, $\bar{\iota}_r$ is decreasing in $\bar{b}$. When the credit limit is sufficiently large $\bar{b} > p^*$, $z_2, z_3 > p^*$ which results in $q_2 = q_3 = q^*$. Since $\bar{b} > p^*$ the households do not have any incentive to hold transaction deposits, $d = 0$ which implies $\chi d = 0$. Therefore, the reserve requirement constraint does not bind. In this case, each bank's reserve balance is determined by $\gamma'(\tilde{r}) = i_r - i$. The bank holds excess reserves as long as $i_r > i$. ∎

D. Additional Results

D.1. Chow Test

Figure 3 includes the Chow test for structural breaks. The test result reported in the bottom-left panel of Figure 3 is implemented by estimating following regression.

$$\text{Money multiplier}_t = \beta_0 + \beta_1(\text{RequiredReserves/Deposit})_t$$
$$+ \mathbf{1}_{t \geq 1992Q2}[\gamma_0 + \gamma_1(\text{RequiredReserves/Deposit})_t]$$
$$+ \mathbf{1}_{t \geq 2008Q4}[\delta_0 + \delta_1(\text{RequiredReserves/Deposit})_t] + \epsilon_t$$

Table 4a reports F-statistics which are obtained by testing $\gamma_0 = \gamma_1 = \delta_0 = \delta_1 = 0$. The Chow test in the bottom-right panel of Figure 3 is implemented by estimating following regression.

$$\text{Money multiplier}_t = \beta_0 + \beta_1(\text{Currency/Deposit})_t$$
$$+ \mathbf{1}_{t \geq 2008Q4}[\delta_0 + \delta_1(\text{Currency/Deposit})_t] + \epsilon_t$$

Table 4b reports F-statistics is obtained by testing $\delta_0 = \delta_1 = 0$. The regression estimates and the Chow test results are summarized at Table 4.

Table 4: Chow test for structural breaks

(a) Require reserve ratio

Dependent Variable: Money Multiplier	
RR	−0.601
	(0.365)
RR × $\mathbf{1}_{t \geq 1992Q2}$	132.279***
	(0.031)
RR × $\mathbf{1}_{t \geq 2008Q4}$	−147.943***
	(8.574)
$\mathbf{1}_{t \geq 1992Q2}$	9.091***
	(0.557)
$\mathbf{1}_{t \geq 2008Q4}$	0.074***
	(0.611)
Constant	2.813***
	(0.053)
Obs.	228
R^2	0.963
DF for numerator	4
DF for denominator	222
F Statistic for Chow test	1711.32
F Statistic for 1% sig. level	3.40
F Statistic for 0.1% sig. level	4.79

(b) Currency deposit ratio

Dependent Variable: Money Multiplier	
CD	−1.301***
	(0.027)
CD × $\mathbf{1}_{t \geq 2008Q4}$	−52.018***
	(4.995)
$\mathbf{1}_{t \geq 2008Q4}$	3.061***
	(0.409)
Constant	3.159***
	(0.015)
Obs.	228
R^2	0.974
DF for numerator	2
DF for denominator	224
F Statistic for Chow test	1245.69
F Statistic for 1% sig. level	4.70
F Statistic for 0.1% sig. level	7.13

Notes: Newy-West standard errors are in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Degree of freedom is denoted by DF.

D.2. Unit Root Test

Columns (2) and (4) in Table 1 includes the canonical cointegrating regression estimates and the cointegration tests. This section reports unit root tests for the series used in Columns (2) and (4). For all four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference rejects the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test following to Elliott and Müller (2006) and Harvey, Leybourne and Taylor (2009), because the magnitude of the initial value can be problematic. Let ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. The data are quarterly from 1980Q1 to 2007Q4.

Table 5: Unit root test

|	Phillips-Perron test	
	$Z(\rho)$	$Z(t)$
$ln(m)$	0.567	0.297
$ln(d)$	1.275	1.054
$ln(uc)$	-1.114	-1.710
r	-7.721	-2.471
$\Delta ln(m)$	-46.623***	-5.335***
$\Delta ln(d)$	-42.267***	-5.060***
$\Delta ln(uc)$	-41.998***	-5.107***
Δr	-94.183***	-9.263***

E. Alternative Parametrization

As discussed in Section 4, there is an alternative set of parameters which can that can fit the data similarly and generate the same predictions. This section presents the alternative parameterization and discusses the results. Table 6 shows the list of parameter values and the model fit for the target used in the calibration in Section 4. Overall, model values fit well with the targeted values. Figure 15 also shows that currency demand and interest rate pass-through to lending rate fits are good as well.

Table 6: Alternative parametrization

| Parameter	Parameter Description	Value	Target Description	Target	Model
		External Parameters			
δ	depreciation rate	0.0825	investment/capital, I/K		
α	capital share in production	0.3333	labor's share of income, $2/3$		
β	discount factor	0.9709	real interest rate, 3%		
σ_2	DM2 matching prob.	0.6890	share of credit meeting		
ς	parameter of $u(\cdot)$	0.0001	Aruoba et al. (2011)		
θ	bargaining power	1	take-it-or-leave-it offer		
ν	prob. of being a buyer	0.5	normalization		
g	parameter of $\gamma(\cdot)$	1.5	-		
		Internal Parameters			
ζ	coeff. on labor supply	2.4949	labor supply, H	0.3333	0.3329
σ_1	DM1 matching prob.	0.0005	currency/output, M/PY	0.0443	0.0441
G	parameter of $\gamma(\cdot)$	0.0020	reserves/output, R/PY	0.0121	0.0118
$\bar{b}$	credit condition	0.0559	unsecured credit/output	0.0347	0.0347
κ	bank entry cost	0.0090	lending rate, i_ℓ	0.0862	0.0870
B	parameter of $u(\cdot)$	0.0130	capital/output, K/Y	2.1896	2.2160
φ	parameter of $u(\cdot)$	3.4756	elast. of M/PY to i	-0.1948	-0.2784
Ψ	parameter of $\eta(\cdot)$	0.0213	semi-elast. of i_ℓ to i	11.3673	11.5130
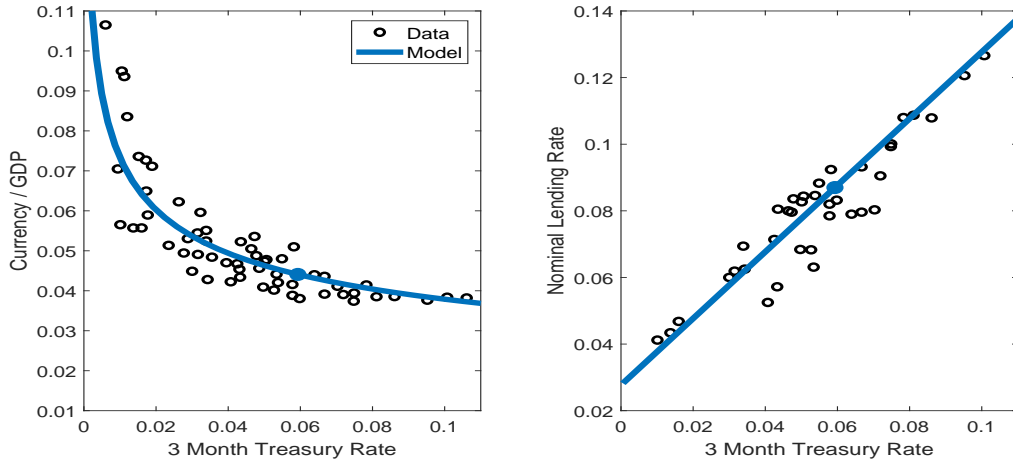
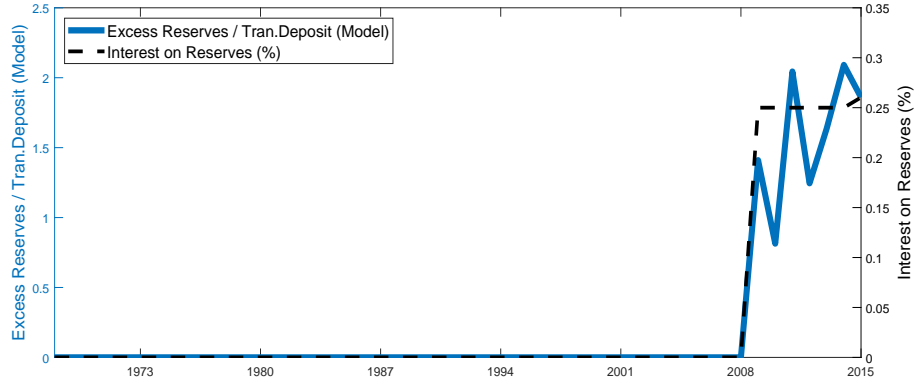**Figure 15:** Alternative parametrization: currency and lending Rate
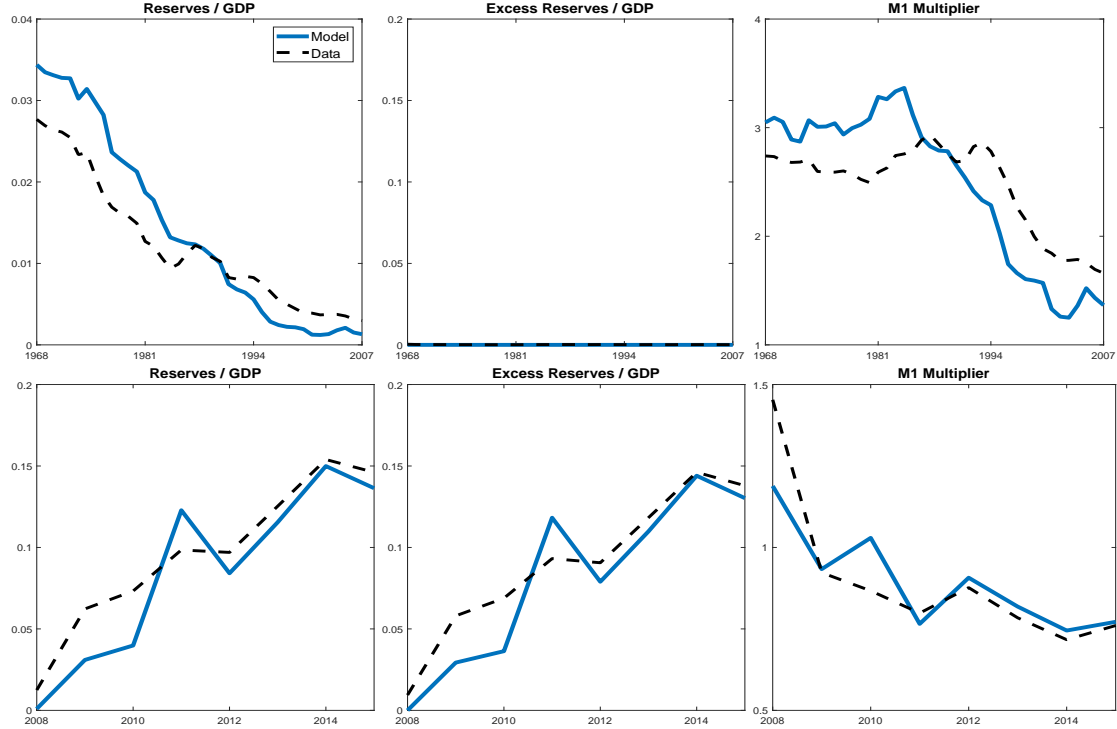
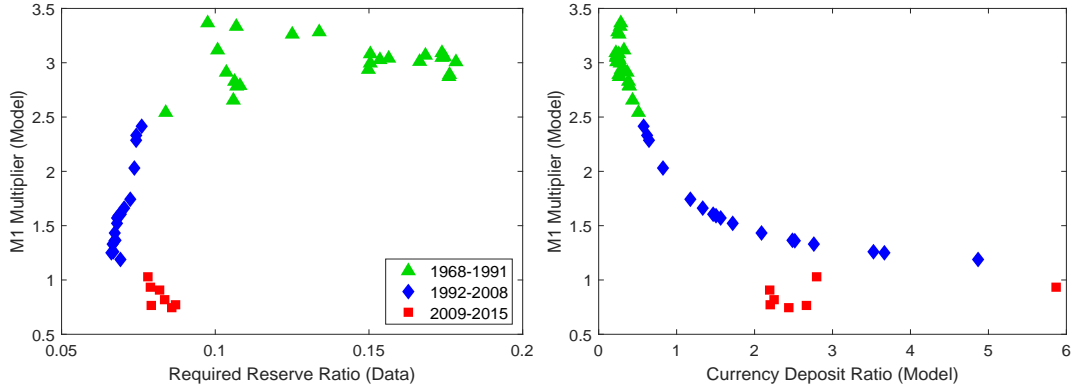**Figure 16:** Alternative parametrization: Model vs. Data

Figure 16 compares reserves, excess reserves, and M1 money multiplier. As in the benchmark calibration, the model generates similar patterns to the data. The exercise under alternative parametrization also can generate all the observations discussed in Section 2 and 4.
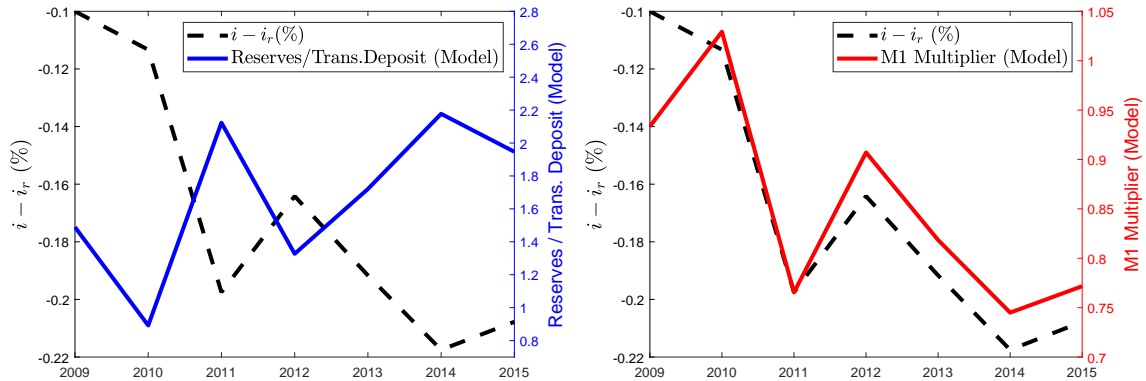
Figure 17: Alternative parametrization: Reserves Deposit ratio and M1 multiplier

Figure 18: Alternative parametrization: Excess reserves ratio

Figure 19: Alternative parametrization: Money multiplier, currency/deposit ratio and required reserve ratio
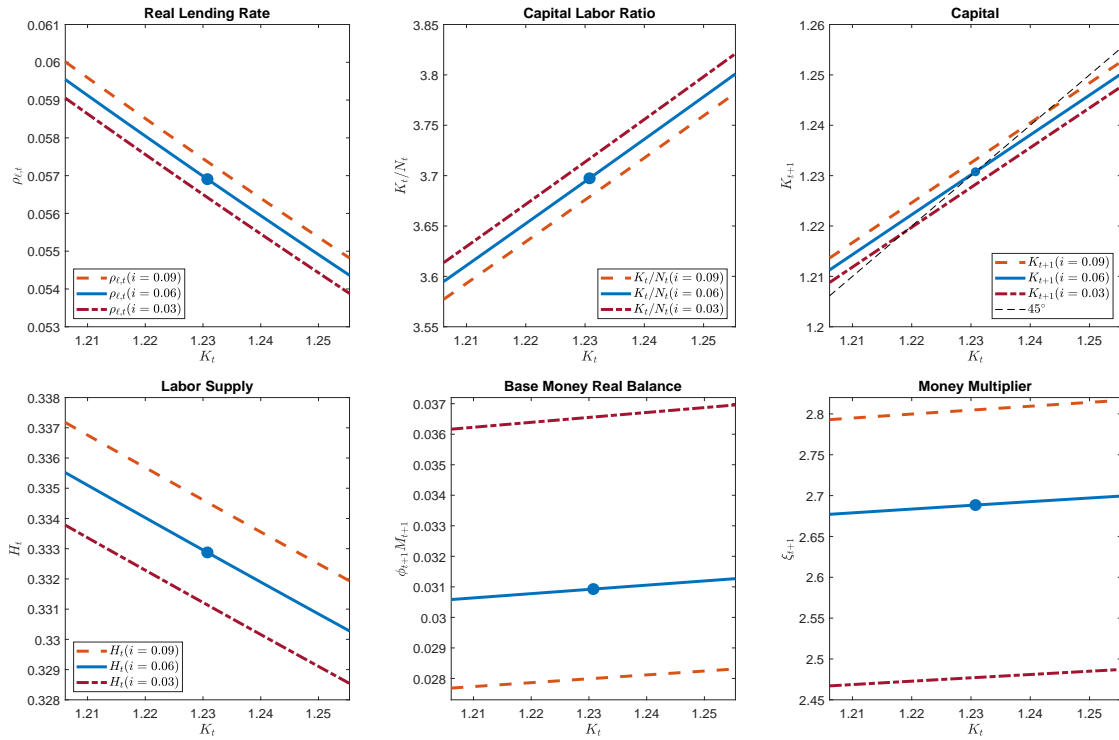
Table 7: Alternative parametrization: Money demand and Credit, Model vs. Data

Dependent Variable: $ln(m_t)$	OLS		CCR	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
r_t	1.600***	7.097	−2.755**	−6.206
	(0.419)		(1.171)	
$ln(uc_t)$			−0.282***	−1.039
			(0.098)	
$adjR^2$	0.109	0.390	0.981	0.845

Notes: Columns (1) and (3) report OLS estimates and columns (2) and (4) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Figure 20: Alternative parametrization: Composition of monetary base

Figure 21: Alternative parametrization: Policy Functions

F.   Data Sources and Variable Definitions

The quantitative analysis uses the annual average of the below series.

- Federal funds rates: "Effective Federal Funds Rate" (FRED series FEDFUNDS).

- Interest on reserves: "Interest Rate on Excess Reserves" (FRED series IOER) and "Interest Rate on Required Reserves" (FRED series IORR).

- 3-month treasury rate: "3-Month Treasury Bill: Secondary Market Rate" (FRED series TB3MS).

- Deposit / (Total checkable deposits): "Total Checkable Deposits" (FRED series TCDSL).

- Excess reserves: "Excess Reserves of Depository Institutions" (FRED series EXCRESNS) and (FRED series EXCSRESNS).

- Excess reserve ratio: $\frac{\text{Excess reserves}}{\text{Total checkable deposits}}$.

- Required reserves: "Required Reserves of Depository Institutions" (FRED series REQRESNS).

- Required reserves ratio: $\frac{\text{Required reserves}}{\text{Total checkable deposits}}$.

- Reserves: "Total Reserves of Depository Institutions" (FRED series TOTRESNS).

- M1: "M1 Money Stock" (FRED series M1SL).

- M2: "M2 Money Stock" (FRED series M2SL).

- Monetary base: "Monetary Base; Total" (FRED series BOGMBASE).

- M1 money multiplier: $\frac{\text{M1}}{\text{Monetary Base}}$.

- Currency: "Currency Component of M1" (FRED series CURRSL).

- Deposit Component of M1: M1$-$Currency.

- Deposit Component of M2: M2$-$Currency.

- Unsecured credit: "Revolving Consumer Credit Owned and Securitized" (FRED series REVOLSL)

- GDP: "Gross Domestic Product" (FRED series GDP), quarterly and "Gross Domestic Product" (FRED series GDPA), annual.