Money Creation and Banking: Theory and Evidence*

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Abstract

This paper develops a monetary-search model where the money multiplier is endogenously determined. I show that, when the central bank pays interest on reserves, the money multiplier and the quantity of reserve can depend on the nominal interest rate and the interest on reserves. The calibrated model can explain the evolution of the money multiplier and the excess reserve-deposit ratio in the pre-2008 and post-2008 period. The quantitative analysis suggests that the dramatic changes in the money multiplier after 2008 are driven by the introduction of the interest on reserves with the low nominal interest rate.

JEL Classification Codes: E42, E51
Keywords: Money, Credit, Interest on Reserves, Banking, Monetary Policy

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> a model of the banking system in which currency, reserves, and deposits play distinct roles ... seems essential if one wants to consider policies like reserves requirements, interest on deposits, and other measures that affect different components of the money stock differently. **Lucas (2000)**

1 Introduction

After the Great Recession, the U.S. economy experienced substantial changes in the conduct of monetary policy. The target interest rates were stuck near zero during 2009-2016, and the Federal Reserve has been paying interest on reserves since October 2008. For the post-2008 era, one of the important features of the monetary policy is a dramatic decline in M1 to monetary base ratio (M1 money multiplier, hereafter) which is accompanied by a huge increase in excess reserves. The M1 money multiplier dropped from 1.61 to 0.77, and the excess reserves to checkable deposit ratio increased from 0 to 1.43 during 2007-2015.

Most of the monetary models have been silent on how the money multiplier is determined when banks are holding excess reserves. The textbook models of money creation only focus on the case where banks lend until they reach zero excess reserves. Since the banks have been holding excess reserves since 2008, natural questions that arise are: Why are banks holding excess reserves but they did not before 2008? What drives the huge drop in the money multiplier? Most importantly, what determines the money multiplier?

The first goal of this paper is to build a model that endogenously determines bank's demand for reserves and its creation of inside money given the availability of different means of payments. To that end, I extend the search-theoretic monetary model of Lagos and Wright (2005) by introducing fractional reserve banking and unsecured credit. The model includes the explicit structure of monetary exchange and the role of financial intermediation. Agents can trade by using cash, claims on deposits (e.g., check or debit card), banknotes, and unsecured credit (e.g., credit card). The bank creates banknotes by making loans and its lending is constrained by the reserve requirement. The equilibrium is in one of the three cases: a scarce-reserves equilibrium, an ample-reserves equilibrium and a no-banking equilibrium.

In the *scarce-reserves equilibrium*, the nominal interest rate is sufficiently high. The bank's lending limit binds. If the central bank lowers the nominal interest rate, the reserve balance increases. The bank creates banknotes proportional to reserves. Since the reserve requirement determines this proportion, the reserve requirement affects the money multiplier.

If the central bank pays interest on reserves and sets the nominal interest rate at some moderate level, the bank holds excess reserves. We call this an *ample-reserves*

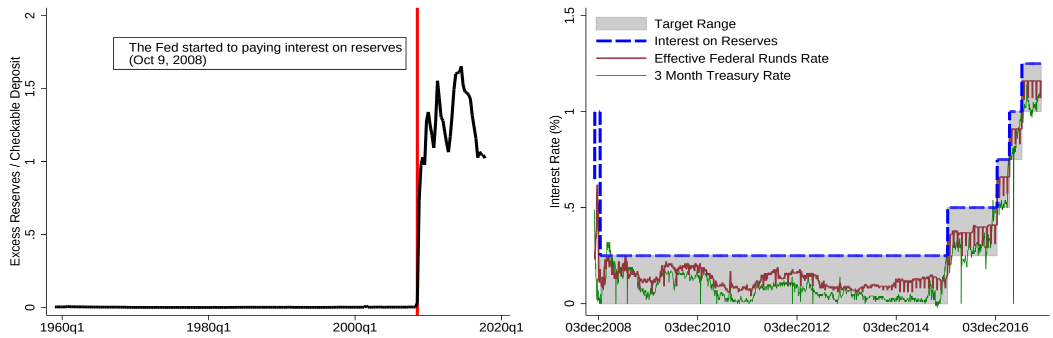

Figure 1: Excess reserves to checkable deposit ratio and interest rates

equilibrium. In the ample-reserves equilibrium, the bank's lending limit does not bind. The reserve requirement does not change the money multiplier. Instead the money multiplier is determined by the nominal interest rate and the interest on reserves. Lowering the nominal interest rate increases reserves but the banks do not create banknotes proportionally, which lowers the money multiplier. A higher interest on reserves decreases the money multiplier since the bank has more incentive to hold reserve and less incentive to create banknotes. These are new findings compared to the literature. Most of the literature focuses on the independence of the quantity of reserve from interest rate management in an abundant reserve regime. (e.g., Kashyap and Stein, 2012; Cochrane, 2014; Keister, Martin and McAndrews, 2008; Curdia and Woodford, 2011; Bech and Klee, 2011; Ennis, 2018) The rationale is that the interest rate paid on reserves provides a floor for the short-term interest rate that the central bank is seeking to control. Then, as the central bank's target reaches the same rate, paying interest on reserves "divorces" money from the interest rate management and the central bank can determine the amount of reserves independently of the interest rate. This implies market rates are equal to or higher than the rate paid by the central bank.[1] In my model, however, the interest on reserves and the nominal interest rate play distinct roles and they jointly determine the quantity of reserves.

When the nominal interest rate is low enough, there is sufficient outside money to trade so that the buyers do not need to deposit their balances to the bank. The bank can not create the inside money because it does not hold any reserves. We call this a *no-banking equilibrium*.

The interaction between unsecured credit and other means of payment is in line with Gu, Mattesini and Wright (2016) and Lester, Postlewaite and Wright (2012). Some fraction of agents can trade using unsecured credit, and the real balances respond endogenously as the credit condition changes. Better credit conditions reduce the real

[1]However, as Figure 1 shows, the interest on reserves had been higher than federal funds and had been equal to upper-limit of the target range.

balance of inside money and reserves, but not cash, which results in a lower money multiplier.

The second goal of this paper is to quantify the model to determine the impact of the monetary policy and the introduction of consumer credit on reserves and money multiplier. The model is parameterized to match the pre-2008 U.S data. Quantitatively, the calibrated model can account for the behavior of money creation before and after 2008. The model generated series can mimic the historical behavior of the M1 money multiplier, the excess reserves to deposit ratio, and the currency deposit ratio. The welfare analysis shows that lowering reserve requirement or paying interest on reserves can reduce the welfare cost of inflation. Also, the quantitative analysis identifies the source of changes in the money multiplier and means of payment. The counter-factual analysis shows that a pre-2008 trend of decreasing money multiplier is driven by an increase in unsecured credit while a post-2008 trend of decreasing money multiplier is not attributed to the increase in unsecured credit. From the model and data, I provide evidence that suggests the dramatic changes in the money multiplier after 2008 are mainly driven by the Federal Reserve's monetary policy: the introduction of the interest on reserves with low nominal interest rate.

Related Literature This work builds on two branches of literature. First and foremost, this work relates to the literature that explicitly studies money, credit, and financial intermediation to understand monetary transmission. As competing means of payment, it is important to understand the interaction between money and credit. There are many ways to introduce credit to the monetary economy. For example, Sanches and Williamson (2010) study the environment where money and credit are competing means of payment due to imperfect memory, limited commitment, and theft. Lotz and Zhang (2016) develop a model of money and credit where sellers must invest in record-keeping technology to accept credit. Williamson (2016) introduces a model of banking that allows agents to use money and asset-backed credit, and Berentsen, Camera and Waller (2007) introduce banks as financial intermediaries so that they accept deposits and issue IOUs with an enforcement technology. Gu et al. (2016) show that credit is a substitute for money and it crowds out the real balance of money. This paper follows Gu et al. (2016) and Berentsen et al. (2007) in the sense that the model allows agents to use unsecured debt with an exogenous credit limit and banks to issue private IOUs using its enforcement technology.

Second, this paper contributes to a large literature on inside money and money creation. The difference between inside money and outside money has been highlighted since Sargent and Wallace (1982) and Freeman and Huffman (1991). Previous works capturing the explicit role of reserve requirements include Freeman (1987), Haslag and Young (1998), and Freeman and Kydland (2000). Freeman (1987) and Haslag and Young (1998) study the impact of money creation and the reserve requirements on seigniorage revenue. Freeman and Kydland (2000) develop a tractable model of endogenous money multiplier. They show that the money-output correlation can be explained by the endogenous money supply resulting from households' choices in response to the business cycle. Recent advances in monetary economics based on search-theoretic framework provide a deeper understanding on banking and inside money. For example, in the Lagos and Wright (2005) framework, Gu, Mattesini, Monnet and Wright (2013) study the environment where banking arises endogenously, and they show that banking can improve the economy by facilitating trade using inside money. Andolfatto, Berentsen and Martin (2018) integrate the Diamond (1997) model of bank and financial markets into the Lagos and Wright (2005) framework and deliver a model where the fractional reserve banking arises in the equilibrium. This paper contributes to the literature by constructing a monetary-search model of banking to explain the money creation process.

This paper is organized as follows. Section 2 provides motivating evidence. Section 3 constructs the search-theoretic monetary model of money creation. Section 4 calibrates the model to quantify the theory. Section 5 concludes.

2 Motivating Evidence

The decrease in the money multiplier itself is not is a recent phenomenon. The top-left panel of Figure 2 shows that the M1 money multiplier has been decreasing since 1992. However, the declining trends before and after 2008 are different. As top-right panel of Figure 2 shows, a decline during 1992-2007 is accompanied by a huge increase in the ratio of currency to deposit while a decline after 2008 is accompanied by a huge drop in the ratio of currency to deposit. The bottom-left panel of Figure 2 reports two structural breaks in the relationship between M1 multiplier and required reserve ratio: 1992Q3 and 2008Q4.[2,3] It also shows that there is no negative relationship between the M1 money multiplier and required reserve ratio during 1992Q2 to 2008Q3 while the excess reserve ratio had been zero until 2008, which suggests that the required reserve

[2]The required reserve ratio presented in Figure 2 is computed by (Required Reserves)/(Total Checkable Deposits). The legal reserve requirement for net transaction accounts has been 10% from April 2, 1992 to March 25, 2020, but some banks are imposed upon by lower requirements or exempt depending on the size of their liabilities. These criteria have been changed 27 times from the 1st quarter 1992 to the last quarter of 2019. From March 2020, all the required reserve ratios became zero. See Feinman (1993) and https://www.federalreserve.gov/monetarypolicy/reservereq.htm for more details on the historical evolution of reserve requirement policy of the United States.

[3]Appendix B.2 contains more detail on the Chow tests reported in Figure 2.

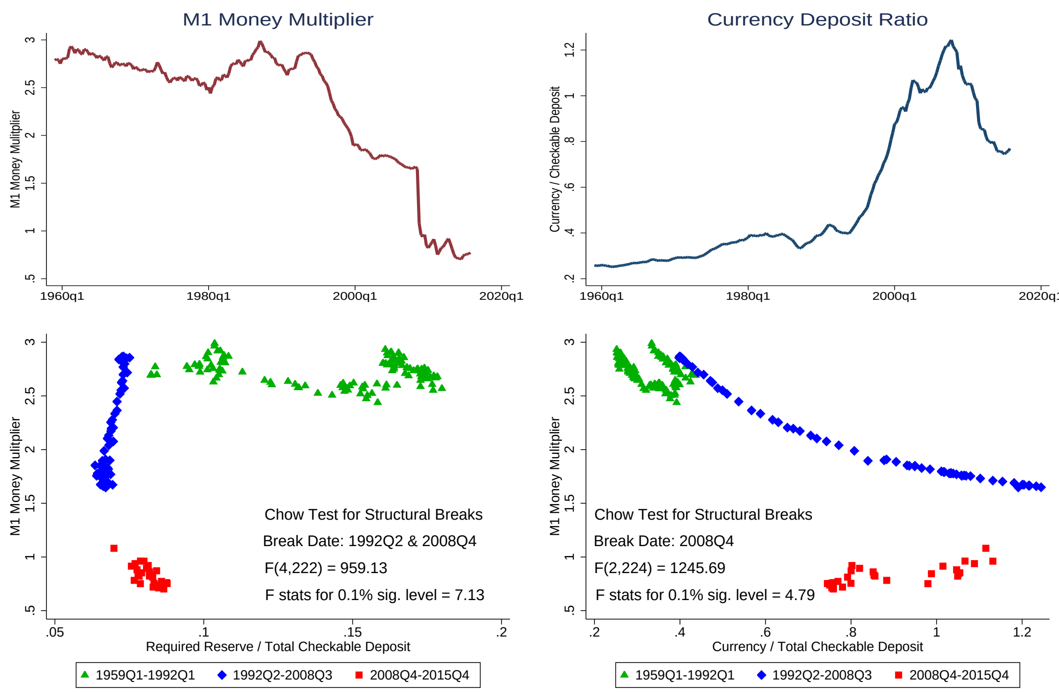

Figure 2: Money multiplier, currency/deposit ratio and required reserve ratio

Chow tests for structural breaks are implemented. The bottom-left panel reports test statistic with the null hypothesis of no structural breaks in 1992Q2 & 2008Q4 and the bottom-right panel reports test statistic with the null hypothesis of no structural break in 2008Q4. Sample periods are 1959Q1-2015Q4. Appendix B.1 contains details of the Chow tests.

ratio does not drive the money multiplier.

To get a better picture of what had happened at the structural break of 1992, Figure 3 plots the ratio of M1 to GDP for the US and the ratio of M1's components to GDP against the 3 month Treasury Bill rate. There is a breakdown in M1 in 1992 that coincides with the structural break in Figure 2. As pointed out by Lucas and Nicolini (2015), this is due to the breakdown of deposit component, not currency component. My hypothesis is that the increased availability of consumer credit crowded out deposit but not cash, which implies that once we account for the substituting impact of newly available consumer credit, there should still be a negative relationship between the real money balance and interest rate.[4]

Following Ireland (2009) and Cagan (1956), I relate the natural logarithm of m, the ratio of nominal money balances to nominal income, to, the short-term nominal interest rate, denoted by r. I also regress r on the natural logarithm of d, the ratio of

[4]One may think that this could be due to the relaxation of bank deposit regulation in the 1980s and 1990s that stimulated financial innovations such as money market deposit accounts (MMDAs) in the 1980s or retail sweep accounts in the 1990s (e.g., VanHoose and Humphrey, 2001, Teles and Zhou, 2005, Lucas and Nicolini, 2015, Berentsen, Huber and Marchesiani, 2015). However, Appendix B.3 shows that there is a breakdown in M2 in 1992 as well and MMDAs and retail sweeps are part of M2.

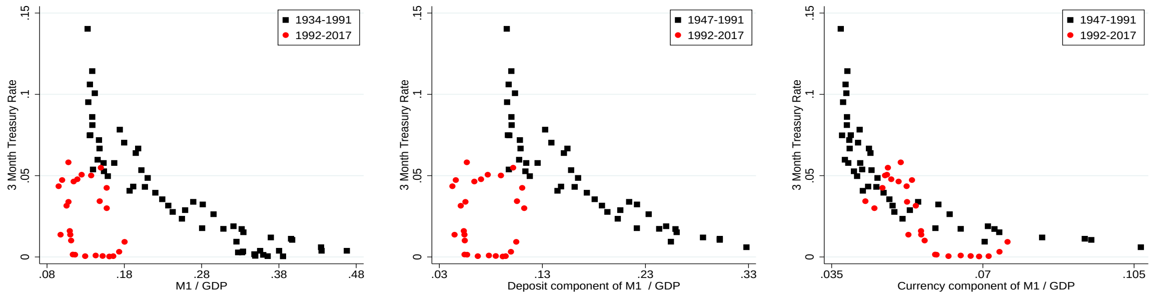

Figure 3: Money demand for M1 and its components

nominal deposit balances to nominal income.

$$ln(m_t) = \beta_0 + \beta_1 r_t + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \epsilon_t$$

In addition to the above specifications, to capture the impact of the improved availability of consumer credit that can substitute the deposit, I add logarithm of uc, the ratio of unsecured credit to nominal income as another regressor as follows.[5]

$$ln(m_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + \epsilon_t$$

I focus on the post-1980 period, until the arrival of the Great Recession. Column (1) and (3) report the estimates without unsecured credit and column (2) and (4) report the estimates with unsecured credit. The Johansen tests in column (1) and (3) fail to reject the null hypothesis of no cointegration which confirm the apparent breakdowns from Figure 3 and OLS estimates from column (1) and (3) both report positive coefficients on r_t which contradict to the conventional notion of money demand: the stable downward-sloping relationship between real balances and interest rates. In column (2) and (4), however, the Johansen tests reject their null of no cointegration at 99 percent confidence level, suggesting there exists a stable relationship between real money balances, interest rates and real balances of unsecured credit. To estimate the cointegration relationship, I implemented the canonical cointegrating regression, proposed by Park (1992), in column (2) and (4). The estimated coefficients on r_t and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once we account for the substitution effect of consumer credit, there still exists a stable negative relationship between real money balances and the interest rates. This substitution effect is a potential explanation for the decline of the money multiplier during 1992Q2 to 2008Q3.

The second structural break at 2008Q3, which can be detected from the bottom-

[5]Following to Krueger and Perri (2006), I use revolving consumer credit.

Table 1: Cointegration regressions and tests

Dependent Variable:	$ln(m_t)$		$ln(d_t)$	
	OLS	CCR	OLS	CCR
	(1)	(2)	(3)	(4)
r_t	0.016***	−0.027***	0.049***	−0.053***
	(0.004)	(0.004)	(0.009)	(0.009)
$ln(uc_t)$		−0.279***		−0.574***
		(0.033)		(0.040)
$adjR^2$	0.109	0.970	0.229	0.962
N	112	112	112	112
Johansen $r = 0$	**15.004**	41.744	**14.934**	49.174
5% Critial Value for $r = 0$	15.41	29.68	15.41	29.68
1% Critial Value for $r = 0$	20.04	35.65	20.04	35.65
Johansen $r = 1$	0.027	**12.163**	0.26	**14.319**
5% Critial Value for $r = 1$	3.76	15.41	3.76	15.41
1% Critial Value for $r = 1$	6.65	20.04	6.65	20.04

Notes: Column (1),(3) report OLS estimates and column (2),(4) report the canonical cointegrating regression (CCR) estimates. First stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in column (1)-(4). Appendix B.2 contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

right panel of Figure 2 as well as the bottom-right panel, coincides with the Fed's introduction of the interest on reserves.[6] Figure 4 plots the money multiplier and the excess reserves ratio with the spread between the federal funds rate and the interest on reserves. It shows that the money multiplier moves together with the spread while the excess reserves ratio moves in the opposite way. It suggests that, contrary to the previous work, the quantity of reserves may not be independent of the central bank's interest rate management even in the post-2008 period with ample reserves. To interpret this relationship, in the following section, I develop a model of money creation that can incorporate the evolution of the money creation process illustrated in this section.

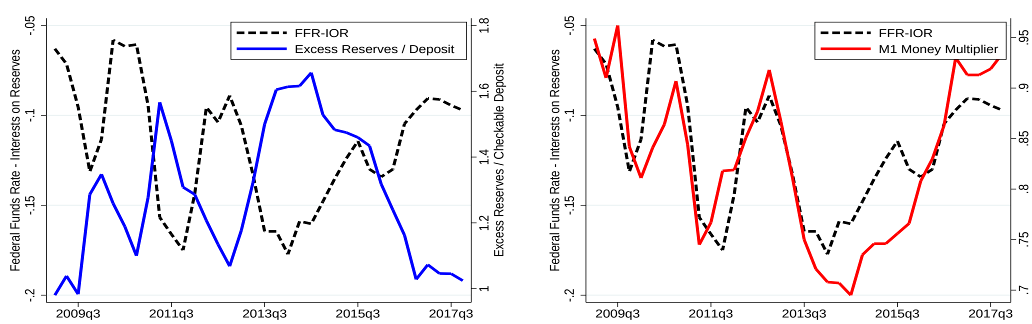

Figure 4: M1 multiplier and excess reserves in post-2008 period

[6]As Nakamura (2018) pointed out, the amount of reserves plumbed before interest rates hit zero and its dramatic increase was simultaneous with the Fed's introduction of the interest on reserves.

3 Model

The model constructed here extends the standard monetary search model (Lagos and Wright, 2005) by introducing fractional reserve banking and unsecured credit. Time is discrete and two markets convene sequentially in each time period: (1) a frictionless centralized market (CM, hereafter), where agents work, consume and adjust their balances, following after; (2) a decentralized market (DM, hereafter), where buyers and sellers meet and trade bilaterally. The DM trade features imperfect record-keeping and limited commitment. Due to these two frictions, some means of payment are needed in DM trades. Below I describe the economic agents in this economy and the different types of DM meetings.

Buyers and Sellers The economy consists of a unit mass of buyers and a unit mass of sellers who discount their utility each period by β. The preferences of buyers and sellers for each period are:

$$\mathcal{U}^b = U(X) - H + u(q) \quad \text{and} \quad \mathcal{U}^s = U(X) - H - c(q)$$

where X is the CM consumption, H is the CM disutility from production and q is the DM consumption. As standard, assume U', u', $c' > 0$, U'', $u'' < 0$, $c'' \geq 0$ and $u(0) = c(0) = 0$. Consumption goods are perishable. One unit of H produces one unit of X in the CM. The efficient consumption in the CM and DM are denoted by X^* and q^*, which solve $U'(X^*) = 1$ and $u'(q^*) = c'(q^*)$, respectively.

The Bank There are measure n of active banks that is endogenously determined by the free entry condition in the equilibrium. In the CM, the banks accept deposit and decide how much to deposit at the central bank as reserves and how much banknote to issue. Managing deposit payment facility incurs a cost. The cost is represented by a cost function, $\gamma(\tilde{d})$, where $\tilde{d}$ is the amount of deposit in real terms, $\gamma', \gamma'' > 0$ and $\gamma(0) = \gamma'(0) = 0$. The reserve earns a nominal interest rate of i_r. The bank extends loans by issuing banknotes. The loans are paid back with interest rate i_ℓ. Enforcing repayment is costly. The cost function is described by $\eta(\tilde{\ell})$, where $\tilde{\ell}$ is the loan in real terms, $\eta', \eta'' > 0$ and $\eta(0) = \eta'(0) = 0$. The bank's lending is constrained by the reserve requirement, i.e., the bank cannot lend more than $(1-\chi)\tilde{r}/\chi$, where χ is the reserve requirement and r is the real balance of reserves.

Types of DM meetings There are three types of DM meetings. In DM1, there is no record-keeping device, and the seller can only recognize cash. In DM2, the seller can recognize cash, the claims on bank accounts and private banknotes. So she accepts cash, deposit receipt and banknotes. In DM3, in addition to the means of payment accepted in DM2, the buyer can trade using unsecured credit with credit limit $\bar{\delta}$ as the trading is monitored imperfectly. The probability of joining a type j meeting is σ_j. The agents get to know which type of meeting they will be going in the preceding CM.[7]

The Central Bank The central bank controls the base money supply M in the CM. Let μ denote the base money growth rate. Then the changes in the real balance of base money can be written as

$$\mu M = M^+ - M$$

where x^+ is the value of (any variable) x in the next period. The base money is held in two ways: (1) C as currency in circulation i.e., outside money held by agents; (2) R as reserves held by a representative bank. Thus,

$$M = C + R$$

The central bank can control the base money supply in two ways. First, a central bank can conduct a lump-sum transfer or collect a lump-sum tax in the CM. Second, the central bank can increase the money supply by paying interest on reserves, i_r. Let T represents a lump-sum transfer (or tax if it is negative). The central bank's constraint is

$$\mu\phi M = \phi(M^+ - M) = T + i_r\phi R$$

where ϕ is the price of money in terms of the CM consumption good.

3.1 The CM Problem

Buyers' Decisions At the beginning of the CM, each buyer's subsequent DM meeting type is realized. Therefore, the buyers' CM problem depends on their DM meeting type. Let $W_j^B(m, d, b, \ell, \delta)$ denote the CM value function where j is the type of the following DM meeting, m is the cash holding, d is the deposit balance, b is the private banknote holding, ℓ is the debt borrowed from the bank during the last CM period, and δ is the unsecured debt owed to the seller from the previous DM. All the state variables are in unit of the current CM consumption good. Let τ denote the lump-sum transfer (or tax if it is negative) to the buyer in the CM. Now, consider the value of the CM. For an agent whose subsequent DM meeting type is realized as a j type DM meeting, the

[7]This can be modeled as endogenous similar to Lester et al. (2012) or Lotz and Zhang (2016) but here we assume the types of meetings are exogenously given. Lester et al. (2012) endogenize the meeting types by allowing sellers' costly *ex ante* choice to acquire the technology for recognizing certain type of assets. Similarly, Lotz and Zhang (2016) studies the environment with costly record-keeping technology where sellers must invest in a record-keeping technology to accept credit.

CM problem is

$$
\begin{aligned}
W_j^B(m,d,b,\ell,\delta) &= \max_{X,H,\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j} U(X) - H + \beta V_j^B(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j) \\
\text{s.t. } (1+\pi)\hat{m}_j + (1+\pi)\hat{d}_j + X &= m + (1+i_d)d + b - (1+i_\ell)\ell - \delta + H + \tau \\
\hat{b}_j &= \hat{\ell}_j,
\end{aligned} \tag{1}
$$

where $\hat{m}_j$, $\hat{d}_j$, $\hat{b}_j$ and $\hat{\ell}_j$ are the real cash holding, real deposit balance, and real private banknote balance, and real debt balance, respectively, carried to the next DM. The FOCs are $U'(X) = 1$ and

$$
-(1+\pi) + \beta\partial V_j^B(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j)/\partial\hat{m}_j \le 0, = \text{ if } \hat{m}_j > 0 \tag{2}
$$

$$
-(1+\pi) + \beta\partial V_j^B(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j)/\partial\hat{d}_j \le 0, = \text{ if } \hat{d}_j > 0 \tag{3}
$$

$$
\beta\partial V_j^B(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j)/\partial\hat{b}_j + \beta\partial V_j^B(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j)/\partial\hat{\ell}_j \le 0, = \text{ if } \hat{\ell}_j > 0. \tag{4}
$$

The first term on the LHS of equation (2) is the marginal cost of acquiring cash. The second term is the discounted marginal value of carrying cash to the following DM. Therefore, the choice of $\hat{m}_j > 0$ equates the marginal cost and the marginal return on cash. Similar interpretation applies to equation (3) for the decision on deposit. For equation (4), the first term on the LHS captures the discounted marginal value of carrying privately issued banknotes from the CM to the following DM, and the second term on the LHS captures the discounted marginal cost of getting a bank loan. The envelope conditions for $W_j^B(m,d,b,\ell,\delta)$ are

$$
\frac{\partial W_j^B}{\partial m} = 1, \quad \frac{\partial W_j^B}{\partial d} = 1 + i_d, \quad \frac{\partial W_j^B}{\partial b} = 1, \quad \frac{\partial W_j^B}{\partial \ell} = -(1+i_\ell), \quad \frac{\partial W_j^B}{\partial \delta} = 1,
$$

for all $j = 1, 2, 3$, which implies $W_j^B(m,d,b,\ell,\delta)$ is linear. This linearity allows us to write

$$
W_j^B(m,d,b,\ell,\delta) = m + (1+i_d)d + b - \delta - (1+i_\ell)\ell + W_j^B(0,0,0,0,0).
$$

Let $W^B(m,d,b,\ell,\delta)$ be the buyers' expected value function before the CM at period t opens, i.e., before their subsequent DM meeting type is realized. Then we can write the buyer's expected value function in the CM as $W^B(m,d,b,\ell,\delta) = \sum \sigma_j W_j^B(m,d,b,\ell,\delta)$.

Sellers' Decisions A seller enters the CM with cash, m, deposits, d, private banknotes, b, and unsecured credit δ that a buyer owes to the seller from previous DM. Let $W_j^S(m,d,b,0,\delta)$ be the sellers' value function in the CM at period t. It can be written as follows:

$$
\begin{aligned}
W_j^S(m,d,b,0,\delta) = & \max_{X,H,\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j} U(X) - H + \beta V_j^S(\hat{m}_j,\hat{d}_j,\hat{b}_j,\hat{\ell}_j) \\
\text{s.t. } (1+\pi)\hat{m}_j + (1+\pi)\hat{d}_j + X = & \, m + (1+i_d)d + b + \delta + H + \tau \\
\hat{b}_j = & \, \hat{\ell}_j
\end{aligned}
\tag{5}
$$

As we will see below, the DM terms of trade does not depend on the seller's portfolio, there is no incentive for the sellers to carry any liquidity to the next DM as the cost of holding liquidity is positive. The envelope conditions are

$$
\frac{\partial W_j^S}{\partial m} = 1, \quad \frac{\partial W_j^S}{\partial d} = 1 + i_d, \quad \frac{\partial W_j^S}{\partial b} = 1, \quad \frac{\partial W_j^B}{\partial \delta} = 1,
$$

for all $j \in \{1,2,3\}$, which implies $W_j^S(m,d,b,0,\delta)$ is linear. By linearity, we can write the CM value function as

$$
W_j^S(m,d,b,0,\delta) = m + (1+i_d) + b + \delta + W_j^S(0,0,0,0,0).
$$

3.2 The DM Problem

In the DM, the buyer and seller trade bilaterally. Let q_j and p_j be the DM consumption and payment in type-j DM meeting. The bilateral trade is characterized by (p_j, q_j). This trade is subject to $p_j \leq z_j$ where z_j is the total liquidity of the buyer in a type-j meeting. The liquidity position for each type of buyer is:

$$
z_1 = m_1 \tag{6}
$$

$$
z_2 = m_2 + d_2(1+i_d) + b_2 \tag{7}
$$

$$
z_3 = m_3 + d_3(1+i_d) + b_3 + \bar{\delta}. \tag{8}
$$

The DM terms of trade is determined by Kalai (1977) proportional bargaining. Kalai bargaining solves the following problem

$$
\max u(q) - p \quad s.t \quad u(q) - p = \theta\left[u(q) - c(q)\right]
$$

where $\theta \in [0,1]$ denotes the buyers' bargaining power. The payment, p, is a function of DM consumption, q. This can be expressed as $p = v(q) = (1-\theta)u(q) + \theta c(q)$. Now, define *liquidity premium*, $\lambda(q)$, as follows:

$$
\lambda(q) = \frac{u'(q)}{v'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{(1-\theta)u'(q) + \theta c'(q)}
$$

where $\lambda(q) > 0$ for $q < q^*$ and $\lambda(q^*) = 0$ with $\lambda'(q) < 0$ for $q \in [0, q^*)$. When $z_j \geq p^*$, the buyer has sufficient liquidity to purchase efficient DM output q^*. In this case, the payment to the seller will be $p^* = v(q^*)$.

By the linearity of W_j^B, we can write a DM value function for a buyer in a type-j meeting as follows:

$$V_j^B(m_j, d_j, b_j, \ell_j) = u(q_j) - p_j + W^B(m_j, d_j, b_j, \ell_j, 0) \tag{9}$$

where $p_j \leq z_j$. The third term on the RHS is the continuation value when there is no trade. The rest of the RHS is the surplus from the DM trade. DM payments are constrained by $p_j \leq z_j$. With $v(q_j) = p_j$ and $z_j \leq p^*$, differentiating V_j^B and substituting its derivatives into the FOCs from the CM problem yields

$$-(1+\pi) + \beta\big[1 + \lambda(q_1)\big] = 0 \tag{10}$$
$$-(1+\pi) + (1+i_d)\beta \leq 0, = \text{ if } d_1 > 0 \tag{11}$$
$$-i_\ell \leq 0, = \text{ if } \ell_1 > 0 \tag{12}$$
$$-(1+\pi) + \beta\big[1 + \lambda(q_i)\big] \leq 0, = \text{ if } m_j > 0 \text{ for } j = 2, 3 \tag{13}$$
$$-(1+\pi) + (1+i_d)\beta\big[1 + \lambda(q_i)\big] \leq 0, = \text{ if } d_j > 0 \text{ for } j = 2, 3 \tag{14}$$
$$-i_\ell + \lambda(q_i) \leq 0, = \text{ if } l_j > 0 \text{ for } j = 2, 3, \tag{15}$$

where $q_j = \min\{q^*, v^{-1}(z_j)\}$ and $\lambda(q^*) = 0$.

Given the linearity of W_j^S, the sellers' DM value function is

$$V_j^S(m_j, d_j, b_j, \ell_j) = p_j - c(q_j) + W_j^S(m_j, d_j, b_j, \ell_j, 0).$$

3.3 The Bank's Problem

A bank maximizes its profit subject to the lending constraint.

$$\max_{\tilde{r}, \tilde{d}, \tilde{\ell}} \quad i_r \tilde{r} - i_d \tilde{d} - \gamma(\tilde{d}) + i_\ell \tilde{\ell} - \eta(\tilde{\ell}) \tag{16}$$
$$s.t. \;\; \tilde{\ell} \leq \bar{\ell} = \frac{1-\chi}{\chi}\tilde{r} \tag{17}$$
$$\tilde{r} \leq \tilde{d} \tag{18}$$

Let λ_L denote the Lagrange multiplier for the lending constraint. It is straightforward to show $\tilde{r} = \tilde{d}$. The FOCs for the bank's problem can be written as

$$0 = i_r - i_d - \gamma'(\tilde{r}) + \lambda_L \left(\frac{1-\chi}{\chi} \right) \tag{19}$$

$$0 = i_\ell - \eta'(\tilde{\ell}) - \lambda_L \tag{20}$$

The bank's ex post profit equals to the entry cost, k

$$(i_r - i_d)\frac{r}{n} + i_\ell \frac{\ell}{n} - \gamma\left(\frac{r}{n}\right) - \eta\left(\frac{\ell}{n}\right) = k \tag{21}$$

where $\ell = n\tilde{\ell}$ and $r = n\tilde{r}$. Given $n > 0$, there are two cases. In the first case, the bank's lending constraint is binding, i.e., $\lambda_L > 0$. In the second case, the bank's lending constraint is not binding, i.e., $\lambda_L = 0$. We call the first case "scarce reserves case," and the second "ample reserves case."

The Scarce-Reserves Case When constraint (17) is tight, $\lambda_L > 0$. Since the bank does not have enough reserves, it needs to acquire reserves to make more loans, which implies a binding constraint. With $\lambda_L > 0$, the bank's FOCs (19) and (20) give

$$0 = i_r - i_d - \gamma'(\tilde{r}) + \left[i_\ell - \eta'(\tilde{\ell}) \right] \frac{1-\chi}{\chi}. \tag{22}$$

The Ample-Reserves Case When constraint (17) is loose, $\lambda_L = 0$. Since the bank already has sufficient reserves, its lending constraint does not bind. Then the two FOCs for the bank's problem are:

$$0 = i_r - i_d - \gamma'(\tilde{r}) \tag{23}$$

$$0 = i_\ell - \eta'(\tilde{\ell}) \tag{24}$$

The bank's unconstrained optimal lending, ℓ^*, satisfies:

$$i_\ell = \eta'(\ell^*).$$

and increases as i_ℓ rises.

3.4 Stationary Equilibrium

In this section, a monetary equilibrium is characterized. I focus on a symmetric stationary monetary equilibrium in which the same type of agents make the same decisions and the real balances are constant over time. Since $\phi/\phi^+ = M^+/M = C^+/C = 1 + \mu$, the net inflation rate, π, is equal to the currency growth rate, μ, in the stationary monetary equilibrium. By the Fisher equation, we have $1+i=(1+\mu)/\beta$. The market clearing conditions are

$$\sigma_2\ell_2+\sigma_3\ell_3=n\tilde{\ell}=\ell \tag{25}$$

$$\sigma_2 d_2+\sigma_3 d_3=n\tilde{r}=r=\phi R \tag{26}$$

$$\sigma_1 m_1+\sigma_2 m_2+\sigma_3 m_3=m=\phi C, \tag{27}$$

where $M=C+R$. Note that $i\geq 0$, i.e., $\mu\geq\beta-1$ is necessary for the existence of equilibrium.[8] Then we have the following definitions:

Definition 1 (**Stationary Equilibrium**). *Given monetary policy, i,i_r, and χ and credit limit $\bar{\delta}$, a stationary monetary equilibrium consists of real balances $(m_j,d_j,\ell_j)_{j=1}^3$, allocation (q_1,q_2,q_3,X), the measure of banks n and prices (i_ℓ,i_d), such that:*

(i) (i_d,i_ℓ,q_1,q_2,q_3) *solves* (10)-(15) *and* (19)-(21) *with* $q_1=v^{-1}(z_1)$, $q_2=\max\{q_1,v^{-1}(z_2)\}$ *and* $q_3=\min\{q^*,\tilde{q}_3\}$ *where* $\tilde{q}_3=\max\{q_2,v^{-1}(z_3)\}$, $z_1=m_1$, $z_2=m_2+(1+i_d)d_2+\ell_2$, *and* $z_3=m_3+(1+i_d)d_3+\ell_3+\bar{\delta}$.

(ii) The bank lending's constraint satisfies $\tilde{\ell}=\min(\bar{\ell},\ell^*)$, *where* $\bar{\ell}=(1-\chi)\tilde{r}/\chi$ *and* ℓ^* *solves* $i_\ell=\eta'(\ell^*)$

(iii) Asset markets clear (25)-(27).

Given Definition 1, there are three types of equilibria, which are defined as follows:

Definition 2. *In an ample-reserve equilibrium, $\bar{\ell}>\ell^*\geq 0$. In a scarce-reserves equilibrium, $\ell^*\geq\bar{\ell}>0$. In a no-banking equilibrium, $\bar{\ell}=r=n=0$.*

For given definitions, the following results are proved in Appendix A.

Proposition 1. *(i) In the ample-reserves and scarce-reserves equilibrium, $\partial i_d/\partial i>0$, $\partial i_\ell/\partial i>0$, $\partial i_d/\partial i_r>0$ and $\partial i_\ell/\partial i_r<0$. (ii) In the no-banking equilibrium, $\partial i_d/\partial i=\partial i_d/\partial i_r=\partial i_\ell/\partial i=\partial i_\ell/\partial i_r=0$.*

The result tells us that, as long as the measure of banking is positive the monetary policy rates pass through the deposit rate and lending rate. The deposit rate is strictly increasing in the nominal interest rate and the interests on reserves. The lending rate is strictly increasing in the nominal interest rate but is strictly decreasing in interest on reserves. I also establish conditions for the determination of equilibrium types.

[8]While the lower bound of the nominal interest rate is zero in this setting, one can relax this by introducing liquid assets or threats of theft. See Rocheteau, Wright and Xiao (2018b), Lee (2016) and Williamson et al. (2019) for detail.

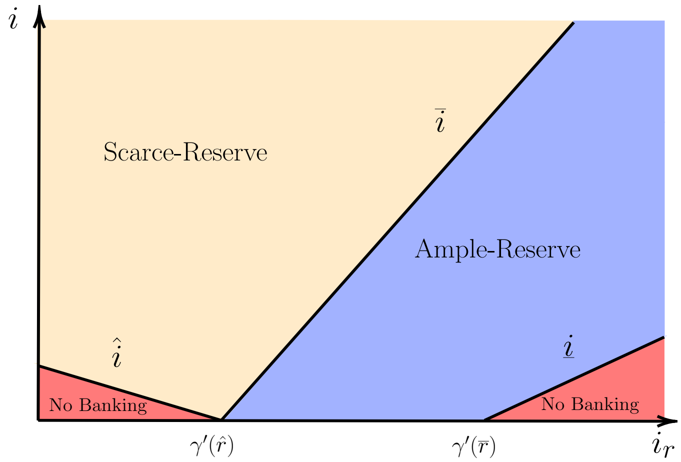

Figure 5: Monetary equilibrium regions in (i, i_r) space

Proposition 2. *Given (i_r, χ): (i) $\exists!$ ample-reserves equilibrium if and only if $i_r > \gamma'(\hat{r})$ and $i \in (\underline{i}, \bar{i})$; (ii) $\exists!$ scarce-reserves equilibrium if and only if either $i > \hat{i}$ and $i_r < \gamma'(\hat{r})$ or $i > \bar{i}$ and $i_r > \gamma'(\hat{r})$; (iii) $\exists!$ no banking equilibrium if and only if either $i \in [0, \hat{i})$ and $i_r < \gamma'(\hat{r})$, or $i \in [0, \underline{i})$ and $i_r > \gamma'(\underline{r})$; and the thresholds satisfy*

$$\hat{i} = \frac{\chi}{1-\chi}[\gamma'(\hat{r}) - i_r] + \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right), \quad \bar{i} = [1 + i_r - \gamma'(\hat{r})]\left[1 + \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right] - 1$$

and $\underline{i} = i_r - \gamma'(\underline{r})$, where $\hat{r}$ solves $\gamma'(\hat{r})\hat{r} - \gamma(\hat{r}) + \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\frac{1-\chi}{\chi}\hat{r} - \eta\left(\frac{1-\chi}{\chi}\hat{r}\right) = k$ and $\underline{r}$ solves $\gamma'(\underline{r})\underline{r} - \gamma(\underline{r}) = k$.

The banks hold excess reserves when the central bank pays sufficiently high interest on reserves with the nominal interest rate at some moderate level. To see the intuition consider the case where the bank holds reserves. The reserve requirement and the reserve balances determine the lending limit. Due to monotone pass-through from the nominal interest rate to the bank's lending rate, the bank's unconstrained optimal lending ℓ^* is increasing in the nominal interest rate. There exists a threshold of the nominal interest rate below which the lending limit is lower than the bank's unconstrained lending (scarce-reserves), and above which the lending limit is higher than the bank's unconstrained lending (ample-reserves). In other words, there is a critical value $\bar{i}$ that satisfies $\ell^* = \bar{\ell} = (1-\chi)\tilde{r}/\chi$.

However, when the equilibrium deposit rate is zero, agents have no incentive to deposit their balance to the bank, implying the no-banking equilibrium. As shown in the Proposition 1, the deposit rate is monotone in the nominal interest rate and the deposit rate could be zero given some nominal interest rate. There exists a threshold $\hat{i}$, below which the deposit rate is zero and above which the deposit rate is positive. The bank's constraint plays a crucial role in determining the equilibrium type. Lowering nominal interest rate or increasing interest on reserves make the bank's lending constraint loose. These lead to the following results:

Corollary 1. *The thresholds $\bar{i}$ and $\underline{i}$ are increasing in i_r and $\hat{i}$ is decreasing in i_r.*

As Figure 5 illustrates, the equilibrium are determined by (i, i_r). While Proposition 1 shows that how deposit rate can change as the central bank set the nominal interest rate, the pass-through of nominal interest rate to deposit rate depends on other policy variables and the equilibrium type.

Proposition 3. *Higher reserve requirement weakens the pass-through from the nominal interest rate to the deposit rate in the scarce-reserve equilibrium i.e.,*

$$\frac{\partial^2 i_d}{\partial \chi \partial i} < 0 \quad \text{if } \bar{\ell} < \ell^*$$

while higher interest on reserve raises the pass-through from the nominal interest rate to the deposit rate in the ample-reserve equilibrium i.e.,

$$\frac{\partial^2 i_d}{\partial i_r \partial i} > 0 \quad \text{if } \bar{\ell} \geq \ell^*$$

The Scarce-Reserves Equilibrium In the scarce-reserves case, the lending limit is less than the bank's unconstrained optimal lending. Therefore, the lending constraint (17) binds, i.e., $\ell^* > \tilde{\ell} = \bar{\ell} = (1-\chi)\tilde{r}/\chi$, where $\tilde{r}$ represents the equilibrium reserve balance. The equilibrium lending is pinned down by total reserve balance r. In the scarce-reserves equilibrium, given i_ℓ and i_d, the equilibrium real balance of reserves satisfies

$$r = \begin{cases} \dfrac{(\sigma_2+\sigma_3)\chi}{1+i_d\chi} L^{-1}(i_\ell) - \dfrac{\sigma_3 \bar{\delta}\chi}{1+i_d\chi} & \text{if } \hat{\delta} \leq \bar{\delta} \\ \dfrac{\sigma_2\chi}{1+i_d\chi} L^{-1}(i_\ell) & \text{if } \hat{\delta} > \bar{\delta} \end{cases} \tag{28}$$

where $\hat{\delta} = L^{-1}(i_\ell)$ and the equilibrium reserve balance is decreasing in i,

$$\frac{\partial r}{\partial i} = \begin{cases} \dfrac{(\sigma_2+\sigma_3)\chi L'^{-1}(i_\ell)}{1+i_d\chi}\dfrac{\partial i_\ell}{\partial i} - \dfrac{\chi[(\sigma_2+\sigma_3)L'^{-1}(i_\ell) - \sigma_3\bar{\delta}]}{(1+i_d\chi)}\dfrac{\partial i_d}{\partial i} < 0 & \text{if } \hat{\delta} \leq \bar{\delta} \\ \dfrac{\sigma_2\chi L'^{-1}(i_\ell)}{1+i_d\chi}\dfrac{\partial i_\ell}{\partial i} - \dfrac{\sigma_2\chi L^{-1}(i_\ell)}{(1+i_d\chi)^2}\dfrac{\partial i_d}{\partial i} < 0 & \text{if } \hat{\delta} > \bar{\delta} \end{cases}$$

since $\partial i_\ell/\partial i < 0$ and $\partial i_d/\partial i > 0$. Thus, there is a downward sloping demand curve for reserves. Since total inside money balance is proportional to reserve balances, we have downward-sloping relationship between the real balance and the interest rates as well.

The Ample-Reserves Equilibrium When $i \in (\underline{i}, \bar{i})$ and $i_r > \gamma'(\hat{r})$, each bank holds sufficient reserves to lend ℓ^*. Thus, the unconstrained optimal lending is less than the lending limit. In the ample-reserves equilibrium, $q_2 = q^*$ if $i_r - i = \gamma'(\underline{r})$. Therefore, the DM2 meeting consumption can be efficient even though the economy is not under the Friedman rule. This result can be formally summarized in the following proposition.

Proposition 4. *In the ample-reserves equilibrium with $i > 0$, DM2 consumption is efficient if $i_r = i + \gamma'(\underline{r})$.*

The intuition behind the efficient DM2 consumption is straightforward. In many monetary models, a high inflation or a high interest rate increases the opportunity cost of holding money. In the environment where money is valued as a medium of exchange, having less liquidity in the economy because of an opportunity cost of holding money is inefficient. However, the interest on reserves provides a proportional return on reserves. If this return is properly distributed across agents, it eliminates the inefficiency which arises from the opportunity cost of holding money, which results in efficient DM2 consumption.

The No-Banking Equilibrium In the no-banking case, the deposit interest rate is zero, $i_d = 0$. Since the return to holding deposits is dominated or equal to the return to holding currency, agents do not have any incentive to deposit their balances. With zero reserve, the lending limit is zero. Therefore, in this equilibrium, agents only use cash for DM trading. All agents hold the same balance of cash and consume the same amount of consumption goods.

$$i = L(m_j) \text{ for } j = 1, 2 \tag{29}$$

and $m_3 = \max\{0, L^{-1}(i) - \bar{\delta}\}$. In this equilibrium, it is straightforward to see that DM consumptions are efficient when $i = 0$, i.e. the Friedman rule applies.

Regardless of the equilibrium type, there exists a downward-sloping demand curve for total liquidity. To illustrate this, I define the monetary aggregate as a sum of cash holdings, reserves, and banknotes in the economy. The right (left) panel of Figure 6 shows that there exist stable downward-sloping demand curves for monetary aggregates (reserves) with different interest on reserves. An increase in i_r shifts the money demand to the right, both in the scarce-reserves equilibrium and in the ample-reserves equilibrium. In the scarce-reserves case, higher i_r raises $\tilde{r}$ and $\tilde{\ell}$ with a more loose lending constraint, and shifts the money demand to the right. However, in the ample-reserves equilibrium, higher i_r raises reserves $\tilde{r}$ but it decreases $\tilde{\ell}$. Lastly, a rise in i_r increases $\bar{i}$, which allows the monetary authority to induce the ample-reserves equilib-

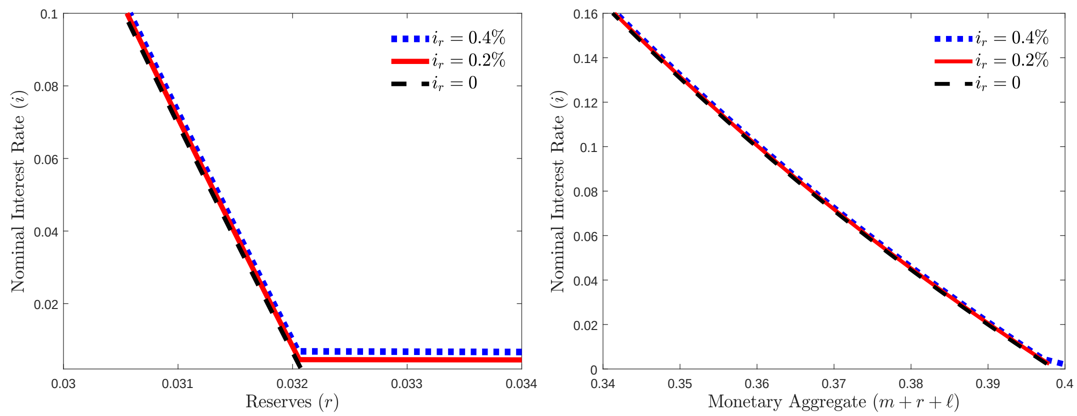

Figure 6: Demand for reserves and the monetary aggregate

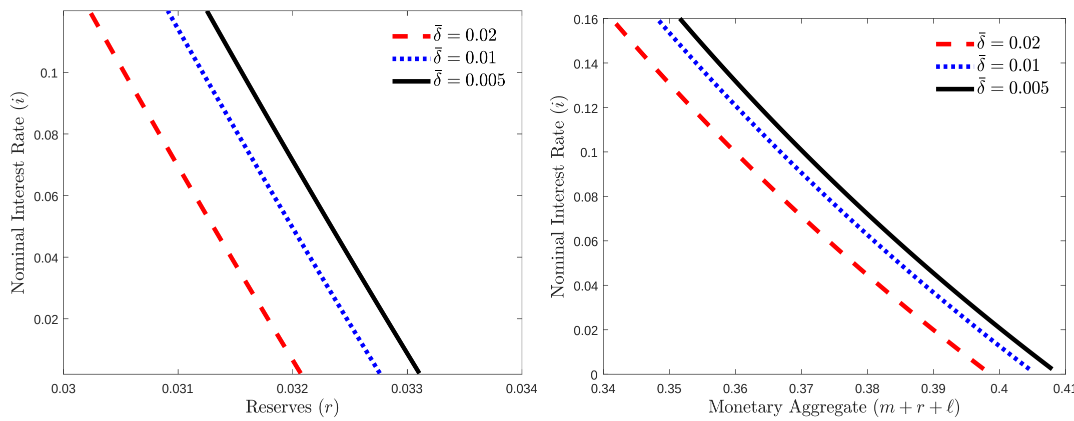

Figure 7: Demand for reserves and the monetary aggregate with different credit limits

rium with higher nominal interest rates. As the central bank pays interest on reserves, the equilibrium can be shifted to ample-reserves equilibrium which has flatter demand for reserves. This is consistent with the observation from Nakamura (2018), the quantity of reserves plumbed before interest rates hit zero but its dramatic increase was simultaneous with the Fed's introduction of the interest on reserves.

The Role of Access to Unsecured Credit We also can check the effect of changes in the access to unsecured credit, σ_3 or changes in the credit limit, $\bar{\delta}$. An increase in σ_3 implies that more buyers can use unsecured credit in the DM. Some fraction of DM2 buyers are changed into DM3 buyers and they hold less amount of deposit balance compared to the amount they had. With higher $\bar{\delta}$, while the measure of DM3 buyers stays same, DM3 buyers can use more unsecured credit for the DM trade. As long as there are positve amount of reserve balances, $r > 0$, an increase in σ_3 or $\bar{\delta}$ lowers r. The Appendix A verifies the following:

Proposition 5. *Let $\hat{\delta} \leq \bar{\delta}$. Then in scarce and ample reserve equilibrium, better credit condition and more credit access both decrease the real balance of reserves i.e.,*

$$\frac{\partial r}{\partial \bar{\bar{\delta}}} < 0, \quad \frac{\partial r}{\partial \sigma_3} < 0.$$

This result is consistent with the finding in Section 2. As more unsecured credit is available, real balances of inside money decreases. By summing up the results, we now establish the results on money multiplier. Define money multiplier $\zeta \equiv (m + r + \ell)/(m + r)$ then we have following results:

Proposition 6. *In the ample-reserve and scarce-reserve equilibrium, better credit condition lowers money multiplier as long as $m > 0$ and $\chi < 1$ and i.e.,*

$$\frac{\partial \zeta}{\partial \bar{\bar{\delta}}} < 0 \quad \text{if } m > 0 \ \&\ \chi < 1$$

In ample reserve equilibrium, for small m, we have

$$\frac{\partial \zeta}{\partial i} > 0, \quad \frac{\partial \zeta}{\partial i_r} < 0$$

Thus, the model can successfully address the mechanism illustrated in Section 2. We can interpret the decline in the money multiplier in the pre-2008 economy as a result of improved availability of consumer credit under the scarce-reserve equilibrium. For the post-2008, after the Fed started paying interest on reserves economy moved to the ample-reserve equilibrium. The model and the evidence suggest that the changes in the money multiplier and excess reserve ratio are the results from the Fed's management of two interest rates, nominal interest rate and interest on reserves.

4 Quantitative Analysis

To evaluate the theory quantitatively, I calibrate the model to match several targets using pre-2008 data. Using calibrated parameters, I compare the model predictions with the data pre-2008 and post-2008 periods. Given the parameters, the stationary equilibrium is characterized by $(i, i_r, \chi, \bar{\delta})$. The required reserves ratio is computed by dividing the required reserves by total checkable deposits. While the first three series are easy to obtain, it is hard to get the unsecured credit limit, $\bar{\delta}$, either from macro and micro data. The unsecured credit limit is computed using the unsecured credit to output ratio. In the model, the unsecured credit to output ratio is given by $\sigma_3\bar{\delta}/(B + \sum_j^3 \sigma_j z_j)$, so we can compute $\bar{\delta}$ using the model with the given policies (i, i_r, χ) and parameters. Following Krueger and Perri (2006), the revolving consumer credit is used as the unsecured credit. For this exercise, I generate simulated data by using 4 series: (i) nominal interest rates[9]; (ii) the interest on reserves; (iii) the required reserve ratio; and (iv) the unsecured credit to GDP ratio.

4.1 Calibration

The utility functions for the DM and the CM are $u(q) = Bq^{1-b}/(1-b)$ and $U(X) = \log(X)$ implying $X^* = 1$ (a normalization). The cost function for the DM is $c(q) = q$. The enforcement cost for lending is assumed to be quadratic, $\eta(\tilde{\ell}) = E\tilde{\ell}^2$ and the management cost for deposit facility takes the form, $\gamma(\tilde{d}) = A\tilde{d}^{1.2}$.[10] The fraction of buyers who can use unsecured credit is set as $\sigma_3 = 0.69$.[11] When σ_1 is set, σ_2 will be set directly since $\sigma_2 = 1 - \sigma_1 - \sigma_3$. The remaining 8 parameters $(\theta, A, B, b, k, E, \sigma_1)$ are set to match the following eight targets: (i) the average retail market markup; (ii) the average credit share of the DM transactions, $\sigma_3\delta/DM$; (iii) the average currency to deposit ratio, C/D; (iv) the average reserves to output ratio, R/Y; (v) the average currency to output ratio, C/Y; (vi) the semi-elasticity of C/Y to i where i denotes the nominal interest rate; (vii) the average net income of banks to output ratio;[12] (viii) the average net income of banks to deposit ratio. The targets are computed based on 1968-2007 data, except for the markup which uses the average from 1993 to 2007 and the net income of banks which uses the average from 1984 to 2007.

[9] I use 3-month Treasury-bill rates as standard. Section 4.5 and Appendix C.1 check robustness by using different measures of monetary policy, which suggests that the main results are not overly sensitive to the choice for the measure of monetary policy.

[10] Since there is not sufficient justifications for the curvature parameter 1.2 of deposit operating cost, sensitivity analyses are included in Appendix C.2.

[11] The Survey of Consumer Finances provides triennial series for the percentage of US households holding at least one credit card from 1970 to 2007. Average percentage during 1970-2007 is 69%.

[12] I use the net attributable income of FDIC-insured commercial banks and savings institutions as the net income of banks.

Table 2: Model parametrization

Parameter	Value	Target/source	Data	Model
External Parameters				
DM3 matching prob, σ_3	0.69	SCF 1970-2007		
Internal Parameters				
bargaining power, θ	0.454	avg. retail markup	1.384	1.384
enforcement cost level, E	0.001	avg. UC/DM	0.387	0.370
deposit operating cost level, A	0.0017	avg. R/Y	0.014	0.017
entry cost, k	0.0011	avg. Π/Y	0.0016	0.0011
DM1 matching prob, σ_1	0.187	avg. C/D	0.529	0.520
DM utility level, B	0.618	avg. C/Y	0.044	0.044
DM utility curvature, b	0.398	semi-elasticity of C/Y to i	-3.713	-3.712

Note: C, R, DM, D, UC, Y denote currency in circulation, reserves, DM transactions, deposit, unsecured credit and nominal output, respectively. Π denotes the net income of banks.

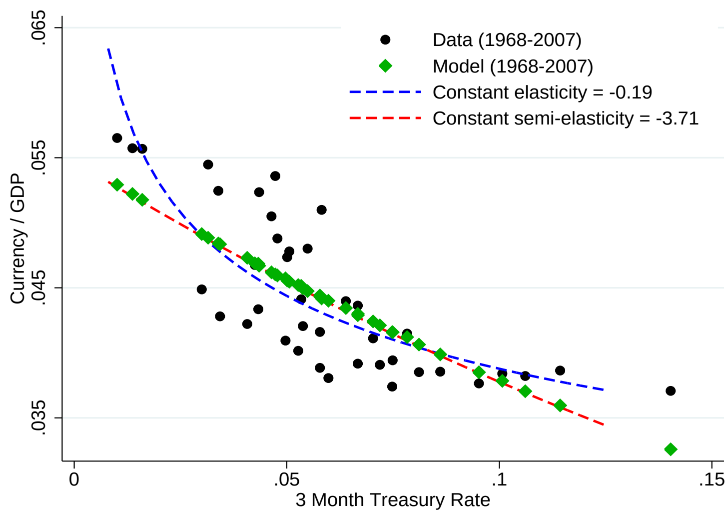

Figure 8: Money demand for currency

The bargaining power θ is set to match the DM markup to the retail markup.[13] Setting (B, b) matches the currency to output ratio and the semi-elasticity of C/Y with respect to i. The costs of operating deposit services and issuing loans from the bank, captured by (A, a) and E, are set to match the reserves to output ratio, the unsecured credit to DM transaction ratio and the banking industry profit to deposit ratio. The entry cost k is set to get banking industry profit to output ratio. Lastly, I set σ_1 to match the currency-deposit ratio. The calibrated parameters and the targets are summarized in Table 2, and the calibrated money demand of currency is shown in Figure 8.

[13]In the Annual Retail Trade Survey, the average ratio of gross margins to sales from 1993-2007 is 0.2776, implying the average markup is 1.3844.

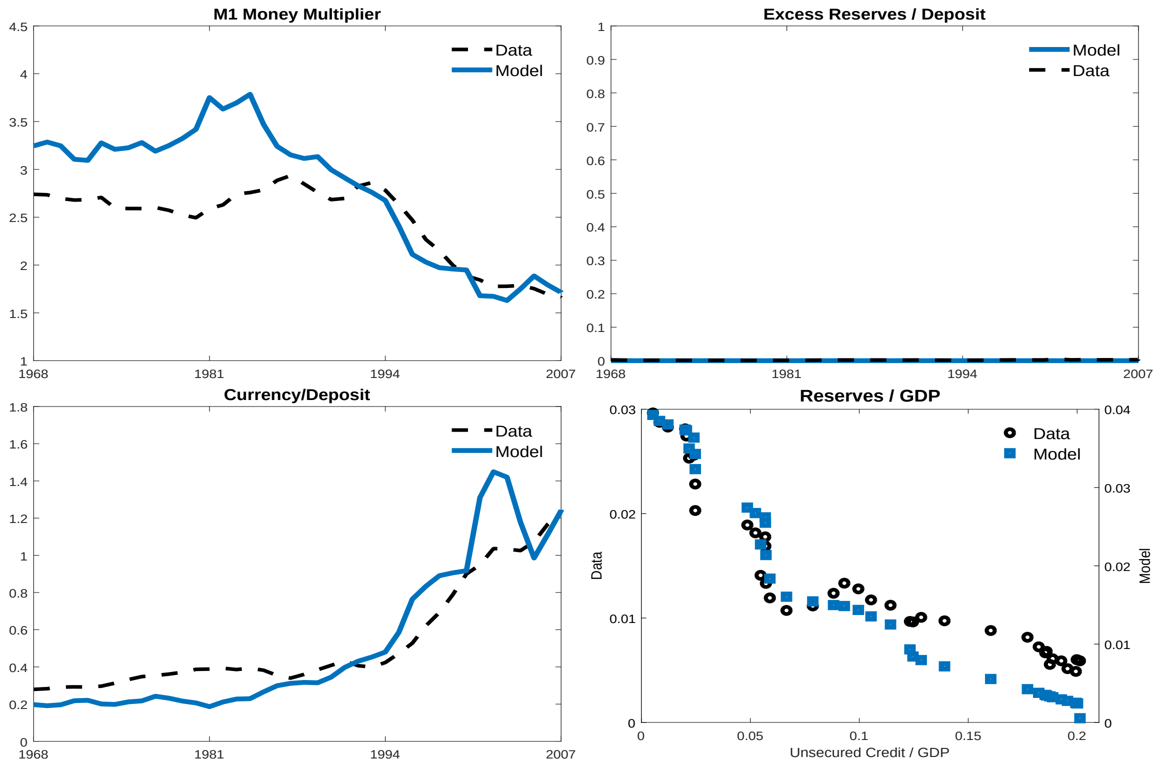

Figure 9: In-sample fit: 1968-2007

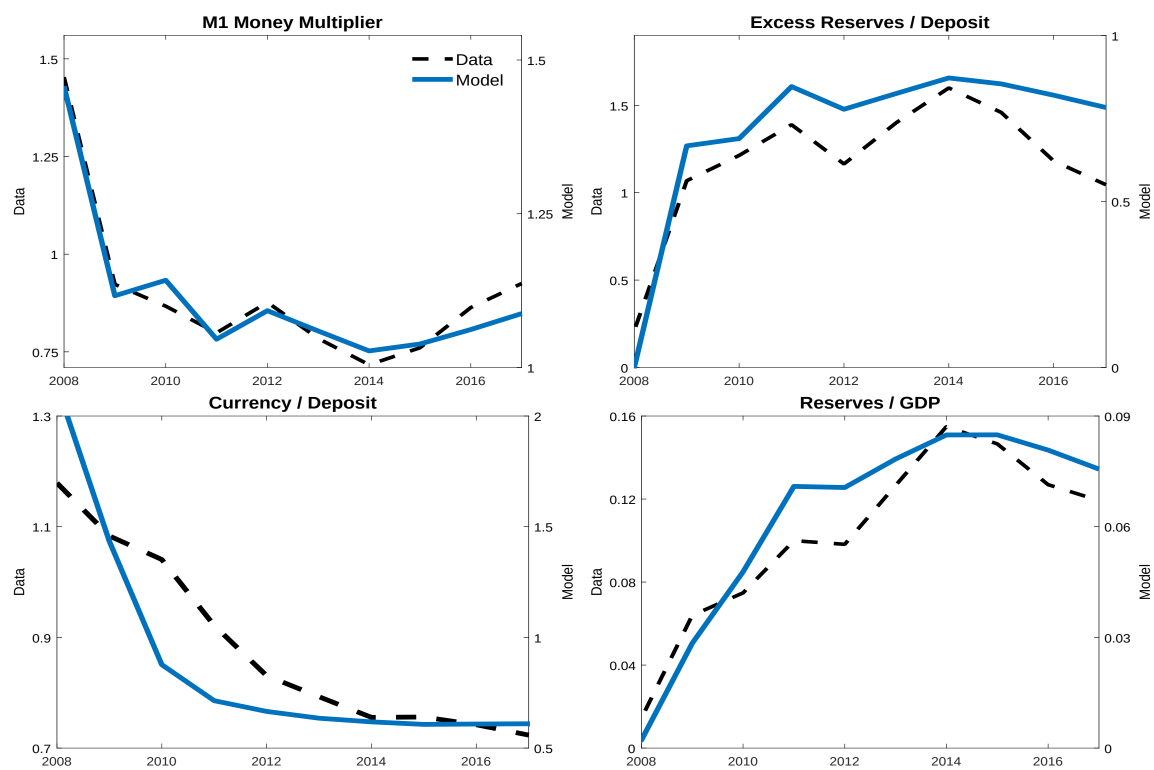

Figure 10: Out-of-sample fit: 2008-2017

4.2 Results and the Model Fit

Figure 9 compares the model and data for the sample period, 1968 -2007. The top-left panel of Figure 9 shows the M1 money multiplier from 1968 to 2007. The model generates decreases in the M1 multiplier during 1987-2007, while the peak was in 1987 in the data and 1984 in the model. During this period, the excess reserves to deposit ratio had been almost zero both in the data and in the model, suggesting the US economy had been in the scarce-reserves equilibrium. In the model, the declining trend of the M1 multiplier in the pre-2008 period is driven by the increase in unsecured credit, which crowds out the inside money (private banknotes, and reserves) but not currency. This induces increases in the currency to deposit ratio, as shown in the bottom-left panel of Figure 9. The bottom-right panel of Figure 9 compares how unsecured credit crowds out reserves in the model and the data.

The next step is to evaluate model projections by comparing them with the data after 2007. Overall, the model can match the patterns in the data. Timeplots of Figure 10 compares the model projections for the M1 money multiplier, the excess reserves to deposit ratio, the currency to deposit ratio, and the reserves to output ratio with data from 2008 to 2017. The model implied series show similar patterns to the actual data series. The model can generate the change in the equilibrium type, from scarce-reserves to ample-reserves, and a similar pattern of excess reserves to deposit ratio. This change in the equilibrium type is represented by a huge drop in the money multiplier in the top-left panel and a huge increase in the excess reserves to deposit ratio in the top-right panel.

Regression estimates shown in Table 3 illustrate the main mechanism of the model. Columns (1) and (2) show the regression coefficient estimates using the following equation for 1968-2007.

$$\text{Reserves}_t/\text{GDP}_t = \beta_0 + \beta_1 \text{UnsecuredCredit}_t/\text{GDP}_t + \beta_2 \texttt{Tbill3}_t + \epsilon_t$$

Since all three series have a unit root and are cointegrated, both in the data and in the model-generated series, the coefficients are estimated using the canonical cointegrating regression.[14] The estimated negative coefficient on the 3 month T-bill rate suggests a downward sloping demand for reserves with respect to the interest rate; but other coefficients on unsecured credit suggest that this demand for reserves can shift as the credit condition changes, as shown in Figure 7. This is consistent with Proposition 5, and the model-implied regression gives similar results. Columns (3)-(4) regress the M1 multiplier on the 3 month T-bill rate and the interest on reserves and Columns (5)-(6) regress the excess reserves ratio on the same variables. Because

[14]Unit root and cointegration test results are reported in Appendix B.2.

Table 3: Model-implied regression coefficients, model vs. data

Dependent Variable:	Reserves/GDP (1968-2007)		M1 Money Multiplier (2009-2017)		Excess Reserve/Deposit (2009-2017)	
	Data (1)	Model (2)	Data (3)	Model (4)	Data (5)	Model (6)
Unsecured Credit/GDP	-0.125^{***}	-0.200				
	(0.010)					
3 Month T-bill Rate	-0.001^{***}	-0.001	0.686^{***}	0.868	-1.561^{***}	-1.697
	(0.000)		(0.184)		(0.498)	
Interest on Reserves			-0.648^{***}	-0.853	1.461^{**}	1.688
			(0.210)		(0.567)	
$adjR^2$	0.830	0.814	0.656	0.920	0.577	0.997

Notes: Columns (1)-(2) report the canonical cointegrating regression (CCR) estimates. First stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. Columns (3)-(6) report OLS estimates. For (3) and (5) Newey-West standard errors with lag 1 are reported in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Intercepts are included but not reported.

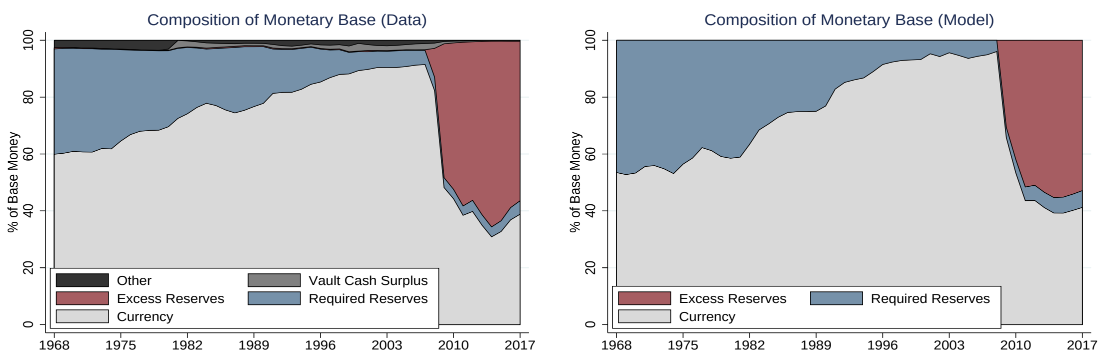

Figure 11: Composition of monetary base: data vs. model

the number of observations in the data are too small, Columns (3) and (5) use data from the 1st quarter of 2009 to the 4th quarter of 2017. Regressions using the data and the model-implied series provide similar results. Based on the regression results in (3)-(6), for a given interest on reserves, raising the 3 month T-bill rates increases the M1 multiplier while it decreases excess reserves. At the same time, for a given 3-month treasury rate, lowering the interest on reserves decreases the M1 multiplier while it increases excess reserves. In the model, when the bank faces higher interests on reserves the bank holds more reserves and does not lend as much as before. This is because interest on reserves yields profits to the bank with low cost but lending is associated with the enforcement cost. This increases the excess reserve ratio and lowers the money multiplier. The model also provides the composition of the monetary base over time. Figure 11 compares the composition of the monetary base from the data and the model. The model successfully generates the changes for each component of the monetary base - currency, required reserves, and excess reserves - both before and after 2008.

A Digression on Model Fit For the post-2007 period, while the model projections can match the patterns in the data well, they do not fit very well in levels. This discrepancy is from the fact that the theoretical lower bound for the money multiplier is one in the model. In reality, however, the U.S. economy has experienced M1 multipliers lower than 1.[15] There are two potential explanations for this.

One possible reason is that monetary policy can be conducted in different ways compared to the lump-sum transfer in the model. In the model, all the base money is distributed to agents through the lump-sum transfer, and they keep some fraction in their bank accounts. Reserves are in the bank deposits in this setup, and this implies the money multiplier can not be lower than one. In contrast to most of the monetary models that assume money is injected as a lump-sum transfer across the agents (buyers and sellers, in this model), much money injection is made to the banking system directly in the real economy. For example, in the quantitative easing program, the Fed purchased large amounts of financial assets from financial intermediaries and gave them the same amount in reserves. These reserves are directly injected into the banking system and this is different from lump-sum transfers. In this case, reserves can only be held by banks, not by the public, and banks do not lend out reserves. One may need to consider a more explicit mechanism for monetary policy implementation.[16]

The other possible reason is that reserves could be kept in saving account or time deposits, which is in M2 but not in M1. Even though one assumes that monetary base is distributed through a lump-sum transfer, it does not have to be kept in checkable account. In this case, there is no discrepancy between the data and the theoretical lower bound for the money multiplier since the M2 money multiplier has never been lower than 1.[17] From a balance sheet point of view, reserves are recorded as cash asset on the commercial bank's balance sheet because reserves are held as an account for the commercial banks at the Federal Reserve Bank, but deposits are liabilities. In this case, reserves cannot exceed total liabilities (total deposits) but it can exceed checkable deposits.

[15] The M1 multiplier of the US was lower than one from December 2008 (0.975) until June 2018 (0.991).

[16] Previous works on the explicit model of the interbank market with monetary policy implementation include Armenter and Lester (2017), Afonso, Armenter and Lester (2019), Bianchi and Bigio (2014) and Chiu, Eisenschmidt and Monnet (2020). Those models explicitly describe search frictions and the market structure of the interbank market for reserves while this paper assumes a centralized market for reserves. Noting that the Fed controls the effective federal funds rates which are interbank rates, introducing the interbank market can allow more realistic monetary transmission.

[17] The lowest M2 money multiplier during 1959-2019 was 2.812 at Aug of 2014.

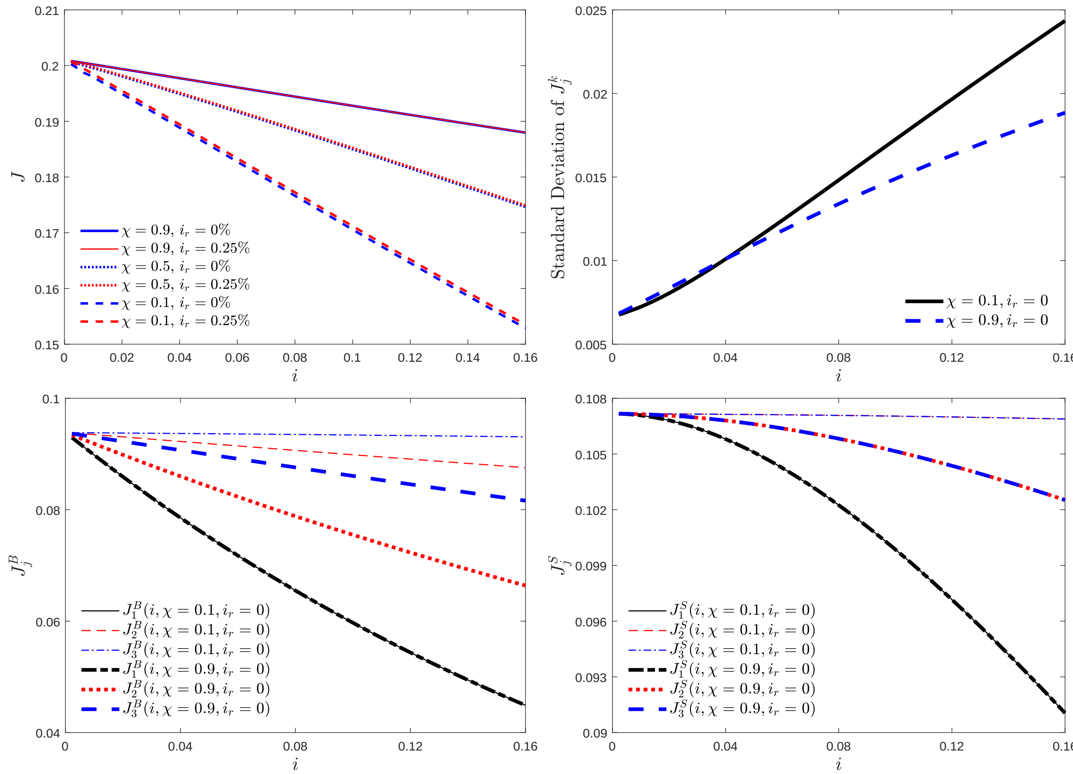

Figure 12: Welfare

4.3 Welfare

So far, I have shown that different tools of monetary policy have distinct roles in monetary transmission. This section focus on the impacts of these different tools of monetary policy in terms of welfare. I measure the welfare of the seller in j type DM meeting using her DM trade surplus.

$$J_j^S(i, \chi, i_r) = (1-\theta)[u(q_j) - c(q_j)]$$

and the welfare of the buyer who trades in the j type DM meeting is DM trade surplus with the cost for acquiring the cash and reserves.

$$J_j^B(i, \chi, i_r) = -im_j(i, \chi, i_r) - (i - i_d)r_j(i, \chi, i_r) + (1-\theta)[u(q_j) - c(q_j)]$$

Then, I can define the total welfare as a weighted sum of each agents' welfare.

$$J(i, \chi, i_r) = \sum_{j=1}^{3} \sigma_j [J_j^B(i, \chi, i_r) + J_j^S(i, \chi, i_r)]$$

The top-left panel of Figure 12 illustrates the effects of monetary policy, i, on total welfare J ranging from 0 to 16% and how its impact can change depending on different reserve requirements and different interest rates on reserves. Each curve denotes the welfare under the different reserve requirements and interest rates on reserves. The welfare is monotonically decreasing in i and each curve can be shifted up by paying interest on reserves or lowering the reserve requirement. The difference becomes smaller as the economy faces lower i, and the Friedman rule gives the optimal level of welfare.

In addition to total welfare, one can examine the distributional effect of monetary policy. The top-right panel of Figure 12, plots the standard deviation of each agent's welfare, J_j^k under different policies. Clearly, the effects of the monetary policy are different across the agents. The bottom-left (bottom-right) panels of Figure 12 plots buyer's (seller's) welfare in different DM meetings depending on different policies. Among buyers, DM1 buyer's welfare is lower than others. While DM2 and DM3 buyers consume the same amount in the DM, since the access to unsecured credit allows the DM3 buyer not to need money as much as DM2 buyer, DM3's welfare is higher than DM2 buyer. For sellers, since the consumption in the DM2 and DM3 are equal, they enjoy the same welfare. Same to the buyers' case, DM1 seller's welfare lowest among sellers. Lowering reserves requirement can make both buyers and sellers in DM2 better off as well as DM3. However, DM1 agents' welfare is independent of reserves requirement, access to credit, and the interest on reserves but only depends on the nominal interest rate.

4.4 Counterfactual Analysis

In this section, I use calibrated parameters to assess how the money multiplier and currency deposit ratio would be changed by setting different reserve requirements. I also demonstrate that it is important to distinguish the effect of reserve requirement from the effect of credit.

The top panel of Figure 13 shows the counterfactual under different reserve requirement while keeping $(i, i_r, \bar{\delta})$ the same as in the benchmark case. With lower reserve requirement, the money multiplier increases. However, we see a trend of decreasing multiplier regardless of reserve requirement. As illustrated in Table 3, this gradual decrease in the money multiplier since the late 1980s is driven by an increase in unsecured credit in the model. The currency deposit ratio increases with higher reserve requirement and all the cases show the similar trend.

The bottom panel of Figure 13 shows the counterfactual under different reserve requirements and $\bar{\delta} = 0$ with (i, i_r) the same as in the data. In the bottom-right panel, we see almost no changes in the currency deposit ratio. Since the money demand for currency and inside money is stable, if unsecured credit did not crowd out inside money,

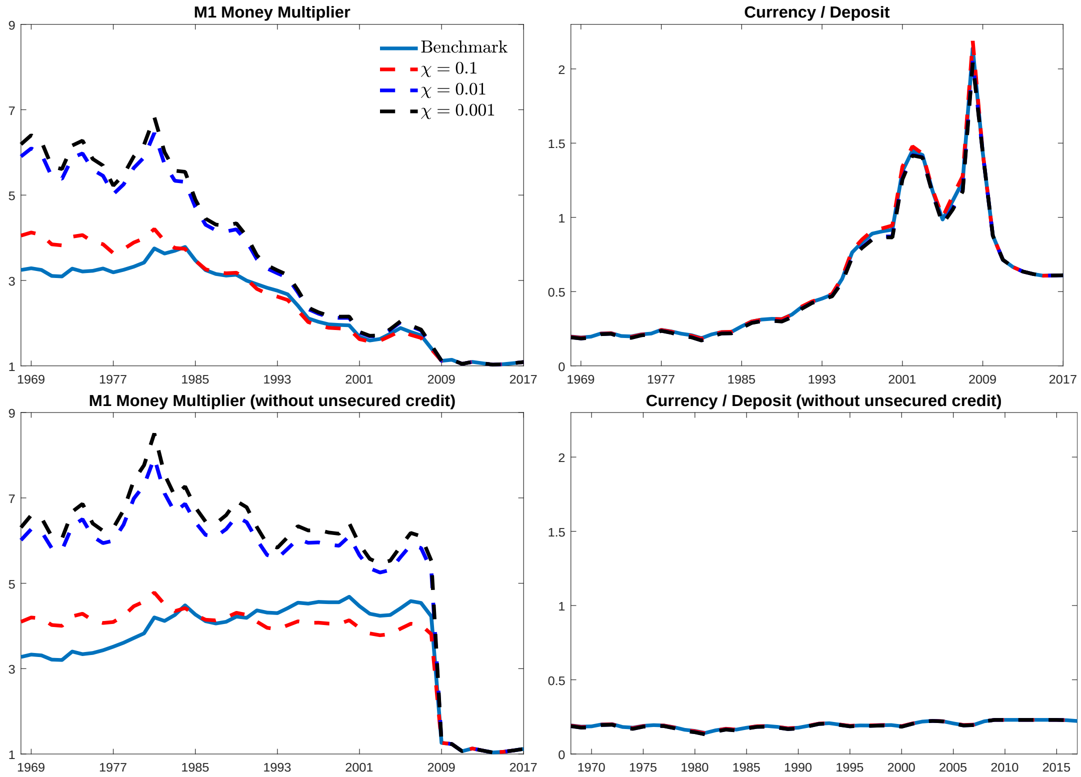

Figure 13: Counterfactual analysis

there would not be substantial increase in the currency deposit ratio as the U.S. economy witnessed. In the case of the money multiplier, while it shows a stationary pattern before 2009, it drops drastically after the Federal Reserve started paying interest on reserves and lowered the nominal interest rate. This suggests that the gradual decline of the money multiplier from the late 1980s to 2007 can be attributed to an increase in unsecured credit, while the dramatic decline in the money multiplier since 2008 can be explained by the monetary policies of the Federal Reserve.

4.5 Robustness

This section briefly summaries a few results from Appendix C. Appendix C.1 examines the sensitivity of the results using different measures of monetary policy target: the federal funds rate, the commercial paper rate. Using different measures does not change the main results. In Appendix C.2, to check whether the curvature parameter 1.2 for the deposit operating cost is sensitive, I recalibrate the parameter as 1.15 or 1.25 instead of 1.2 which is the benchmark parameter. Changing these parameters does not have a significant impact on the results.

5 Concluding Remarks

This paper develops a monetary-search model with fractional banking and unsecured credit, and studies the money creation process. In the fractional reserve system, the money is created when the bank makes loans. The bank's lending, however, can be constrained by the reserve requirement and the reserves. In the model, there are three types of equilibrium: (i) the scarce-reserves equilibrium; (ii) the ample-reserves equilibrium; and (iii) the no-banking equilibrium.

In the scarce-reserves equilibrium, the bank's lending is constrained by the reserve requirement and reserves. So, the money creation process depends on the reserves and the reserve requirement. In the ample-reserve equilibrium, the bank's lending is not constrained by the reserve requirement or reserves because of sufficient reserves. In this case, the bank's lending depends on the nominal interest rates and the interest on reserves. In contrast to previous works, even in the ample-reserves equilibrium, the amount of reserves depends on the nominal interest rates and the interest on reserves. In the no-banking equilibrium, all the base money is held by the buyers and there are no deposits in the economy. Regardless of the equilibrium types, there exists a money demand relationship, and the Friedman rule is optimal. In the ample-reserves equilibrium, there exists an optimal interest on reserves which makes the non-cash DM meeting consumption efficient without the Friedman rule.

There exists a threshold of the nominal interest rate between the scarce-reserves equilibrium and the ample-reserves equilibrium. If the nominal interest rate is lower than the threshold, the equilibrium is the ample-reserves equilibrium, while if the nominal interest rate is higher than the threshold, the equilibrium is the scarce-reserves equilibrium. In addition, this threshold is increasing in interest on reserves. Therefore, paying interest on reserves with low nominal interest rates can move the economy from the scarce reserves equilibrium to the ample reserves equilibrium, which is consistent with what we have seen in the US economy. The quantitative analysis can generate simulated data that resemble the actual data. This paper provides evidence from the model and the data that suggests that the dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves with the low nominal interest rate.

The paper can be extended in various ways. While I focus on the centralized market for the reserves with homogeneous banks, in reality, the market for reserves is a decentralized interbank market and each bank has a heterogeneous balance sheet with a different portfolio. Therefore, one can further investigate how much the market structure and heterogeneity matter for the transmission of monetary policy (e.g., Afonso and Lagos, 2015; Armenter and Lester, 2017; Afonso, Armenter and Lester, 2019). Second, it would be worthwhile to study how inside creation via making loans is be related to investment and firms' dynamics (e.g., Ennis, 2018; Bianchi and Bigio, 2014; Altermatt, 2019). This will allow us to understand the investment channels of monetary policy more explicitly. Moreover, I assume that bank assets are composed of loans and reserves. But commercial banks' assets are mainly composed of securities, loans, and reserves. Extending the model to incorporate banks portfolio choice and analyzing the role of investment, financial regulation, and monetary policy can open up other research avenues (e.g., Rocheteau, Wright and Zhang, 2018a).

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Appendix

Appendix A Proofs

Proof of Proposition 1 and 2. First, consider the ample-reserve equilibrium. The real balance of reserves and lending for each bank, $\tilde{\ell}$, $\tilde{r}$, are determined by following two equations.

$$0 = \underbrace{\frac{1+i}{1+i_r-\gamma'(\tilde{r})} - 1 - \eta'(\tilde{\ell})}_{P} \tag{30}$$

$$0 = \underbrace{\gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) + \eta'(\tilde{\ell})\tilde{\ell} - \eta(\tilde{\ell}) - k}_{E} \tag{31}$$

where $\frac{1+i}{1+i_r-\gamma'(\tilde{r})} \geq 1$. By implicit function theorem we have

$$P_r d\tilde{r} + P_\ell d\tilde{\ell} + P_i di + P_{i_r} di_r = 0 \tag{32}$$

$$E_r d\tilde{r} + E_\ell d\tilde{\ell} + E_i di + E_{i_r} di_r = 0 \tag{33}$$

where

$$\begin{array}{ll} P_r = (1+i)\gamma''(\tilde{r})/\{1+i_r-\gamma'(\tilde{r})\}^2 & E_r = \gamma''(\tilde{r})\tilde{r} \\ P_\ell = -\eta''(\tilde{\ell}) & E_\ell = \eta''(\tilde{\ell})\tilde{\ell} \\ P_i = 1/\{1+i_r-\gamma'(\tilde{r})\} & E_i = 0 \\ P_{i_r} = -(1+i)/\{1+i_r-\gamma'(\tilde{r})\}^2 & E_{i_r} = 0. \end{array}$$

Using (32), we have

$$\frac{\partial \tilde{r}}{\partial \tilde{\ell}} = -\frac{\partial P_\ell}{\partial P_r} = \frac{\eta''(\tilde{\ell})[1+i_r-\gamma'(\tilde{r})]^2}{(1+i)\gamma''(\tilde{r})} > 0$$

and equation (33) gives $\frac{\partial \tilde{r}}{\partial \tilde{\ell}} = -\frac{\partial E_\ell}{\partial E_r} = -\frac{\gamma''(\tilde{r})\tilde{r}}{\eta''(\tilde{\ell})\tilde{\ell}} < 0$. In equation (30), $\tilde{r}_1 = \max\{0, \gamma'^{-1}(i_r - i)\}$ when $\tilde{\ell} = 0$ and in equation (31) when $\tilde{\ell} = 0$, $\tilde{r}_2 > 0$ where $\tilde{r}_2$ solves $\gamma'(\tilde{r}_2)\tilde{r}_2 - \gamma(\tilde{r}_2) = k$. Therefore equations (30)-(31) have a single crossing point in $\mathbb{R}^2_+$ space as long as $\tilde{r}_1 < \tilde{r}_2$. This condition satisfies under the ample reserve equilibrium, which will be shown at the end of this proof.

By applying Cramer's Rule to (32)-(33) , we have following comparative statics results.

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{P_i E_r}{P_r E_\ell - P_\ell E_r} > 0, \quad \frac{\partial \tilde{r}}{\partial i} = \frac{-P_i E_\ell}{P_r E_\ell - P_\ell E_r} < 0$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{P_{i_r} E_r}{P_r E_\ell - P_\ell E_r} < 0, \quad \frac{\partial \tilde{r}}{\partial i_r} = \frac{-P_{i_r} E_\ell}{P_r E_\ell - P_\ell E_r} > 0.$$

Given above result, we can get immediate results $\partial i_d/\partial i > 0$ and

$$\frac{\partial i_d}{\partial i_r} = 1 - \frac{1+i}{1+i+[1+i_r-\gamma'(\tilde{r})]} > 0 \tag{34}$$

since $i_d = i_r - \gamma'(\tilde{r}) > 0$. Given the above results, n solves

$$(\sigma_2+\sigma_3)L^{-1}(i_\ell) - \sigma_3\bar{\delta} = n(\tilde{r}+\tilde{\ell}) \tag{35}$$

when $\hat{\delta} > \bar{\delta}$ while n solves $\sigma_2 L^{-1}(i_\ell) = n(\tilde{r}+\tilde{\ell})$ when $\hat{\delta} \leq \bar{\delta}$ where $i_\ell = (1+i)/(1+i_d)-1$.

Given $\partial\tilde{\ell}/\partial i_r < 0$, $\partial\tilde{\ell}/\partial i > 0$ and $i_\ell = \eta(\tilde{\ell})$, we have $\partial i_\ell/\partial i_r < 0$ and $\partial i_\ell/\partial i > 0$ which implies DM2 trade, $L^{-1}(i_\ell)$, is increasing in i_r and decreasing in i.

Now, consider the no-banking equilibrium. The no-banking equilibrium satisfies

$$i = L(m_j) \text{ for } j = 1, 2$$

and $m_3 = \max\{0, L^{-1}(i) - \bar{\delta}\}$ with $r = 0$, $i_d = 0$ and $n = 0$. In the no-banking case, it is straight forward to show

$$\frac{\partial r}{\partial i} = 0, \quad \frac{\partial i_d}{\partial i} = 0, \quad \frac{\partial r}{\partial i_r} = 0, \quad \frac{\partial i_d}{\partial i_r} = 0.$$

In the scarce-reserve equilibrium $\tilde{r}$ determined by

$$(i_r - i_d)\tilde{r} + i_\ell\frac{1-\chi}{\chi}\tilde{r} - \gamma(\tilde{r}) - \eta\left(\frac{1-\chi}{\chi}\tilde{r}\right) = k$$

$$i_r - i_d - \gamma'(\tilde{r}) + \left[i_\ell - \eta'\left(\frac{1-\chi}{\chi}\tilde{r}\right)\right]\frac{1-\chi}{\chi} = 0,$$

where $i_\ell = \frac{1+i}{1+i_d} - 1$ and this implies $\tilde{r}$ solves

$$\gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) + \eta'\left(\frac{1-\chi}{\chi}\tilde{r}\right)\frac{1-\chi}{\chi}\tilde{r} - \eta\left(\frac{1-\chi}{\chi}\tilde{r}\right) = k \tag{36}$$

which is independent of i and i_r. Let's define this $\tilde{r}$ as $\hat{r}$. Since left-hand side of (36) is strictly increasing in $\hat{r}$ and equal to 0 when $\hat{r} = 0$ and right-hand side is given as constant parameter, (36) uniquely pins down $\hat{r}$. The equilibrium deposit rate can be expressed as

$$i_d = i_r - \gamma'(\hat{r}) + \left[\frac{1+i}{1+i_d} - 1 - \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right]\frac{1-\chi}{\chi} > 0. \tag{37}$$

Since $\hat{r}$ is independent of i and i_r, by implicit function theorem we have $\frac{\partial i_d}{\partial i_r} = 1 + \frac{(1-\chi)(1+i)}{\chi(1+i_d)^2} > 0$ and

$$\frac{\partial i_d}{\partial i} = \left[\frac{\chi(1+i_d)}{1-\chi} + \frac{1+i}{1+i_d}\right]^{-1} > 0 \tag{38}$$

Using above results, one can show that

$$\frac{\partial i_\ell}{\partial i} = \frac{(1+i_d)\chi}{(1+i)(1-\chi)+\chi(1+i_d)^2} > 0, \quad \frac{\partial i_\ell}{\partial i_r} = -\frac{1+i}{(1+i_d)^2}\frac{\partial i_d}{\partial i_r} < 0.$$

Next step is characterizing the equilibrium type for given policies. When bank's unconstrained optimal lending $\ell^* = \eta'^{-1}(i_\ell)$ is bigger than $\bar{\ell}$, the equilibrium is a scarce reserve equilibrium while when $\bar{\ell} > \ell^* = \eta'^{-1}(i_\ell)$, the equilibrium is an ample reserve equilibrium. There exists a threshold $\bar{i}$ that satisfies $\bar{\ell} = \ell^*$,

$$\bar{i} = [1+i_r-\gamma'(\hat{r})]\left[1+\eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right]-1.$$

Since $\partial\tilde{r}/\partial i < 0$ and $\bar{\ell} = \tilde{r}(1-\chi)/\chi$, $\partial\bar{\ell}/\partial i < 0$ while $\partial\ell^*/\partial i > 0$. Therefore, $\bar{i}$ is a critical value above which the equilibrium is scarce reserve equilibrium and below which the equilibrium is ample reserve equilibrium. The no-banking equilibrium arise when the equilibrium deposit rate i_d is zero and lowering i decreases the deposit rate. The threshold between no-banking case and scarce-reserve case $\hat{i}$ satisfies $i_d = i_r - \gamma'(\tilde{r}) + \left[\frac{1+\hat{i}}{1+i_d} - 1 - \eta'\left(\frac{1-\chi}{\chi}\tilde{r}\right)\right]\frac{1-\chi}{\chi} = 0$, implying

$$\hat{i} = \frac{\chi}{1-\chi}[\gamma'(\hat{r})-i_r]+\eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)$$

When $i < i_d$, there is no equilibrium in the market for reserves since demand for reserves are infinite. In this this case, there is no equilibrium with banks, implying the no-banking equilibrium. The threshold for $i = i_d$ is $\underline{i} = i_r - \gamma'(\underline{r})$ where $\underline{r}$ solves $\gamma'(\underline{r})\underline{r} - \gamma(\underline{r}) = k$. Since the ample-reserve equilibrium always satisfies $i > \underline{i}$, the condition of unique existence of the ample-reserve equilibrium, $\tilde{r}_1 < \tilde{r}_2$, is satisfied.

Therefore, we can conclude that given (i_r, χ): (i) $\exists!$ ample-reserves equilibrium if and only if $i_r > \gamma'(\hat{r})$ and $i \in (\underline{i}, \bar{i})$; (ii) $\exists!$ scarce-reserves equilibrium if and only if either $i > \hat{i}$ and $i_r < \gamma'(\hat{r})$ or $i > \bar{i}$ and $i_r > \gamma'(\hat{r})$; (iii) $\exists!$ no banking equilibrium if and only if either $i \in [0, \hat{i})$ and $i_r < \gamma'(\hat{r})$, or $i \in [0, \underline{i})$ and $i_r > \gamma'(\underline{r})$

■

Proof of Corollary 1. Since $\underline{r}$ solves $\gamma'(\underline{r})\underline{r} - \gamma(\underline{r}) = k$, $\underline{r}$ is independent of i_r. Therefore, given $\underline{i} = i_r - \gamma'(\underline{r})$, $\partial\underline{i}/\partial i_r = 1 > 0$. Similarly, since $\hat{r}$ solves $\gamma'(\hat{r})\hat{r} - \gamma(\hat{r}) + \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\frac{1-\chi}{\chi}\hat{r} - \eta\left(\frac{1-\chi}{\chi}\hat{r}\right) = k$, $\hat{r}$ is independent of i_r. Therefore, given

$$\hat{i} = \frac{\chi}{1-\chi}[\gamma'(\hat{r})-i_r]+\eta'\left(\frac{1-\chi}{\chi}\hat{r}\right), \quad \bar{i} = [1+i_r-\gamma'(\hat{r})]\left[1+\eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right]-1,$$

we have $\partial\hat{i}/\partial i_r = \frac{-\chi}{1-\chi} < 0$ and $\partial\bar{i}/\partial i_r = \left[1+\eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right] > 0$.

■

Proof of Proposition 3. First, consider the scarce-reserve equilibrium. Recall equation (36) and (37)

$$0 = \underbrace{\gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) + \eta'\left(\frac{1-\chi}{\chi}\tilde{r}\right)\frac{1-\chi}{\chi}\tilde{r} - \eta\left(\frac{1-\chi}{\chi}\tilde{r}\right) - k}_{\Omega}$$

$$0 = \underbrace{-i_d + i_r - \gamma'(\tilde{r}) + \left[\frac{1+i}{1+i_d} - 1 - \eta'\left(\frac{1-\chi}{\chi}\tilde{r}\right)\right]\frac{1-\chi}{\chi}}_{\Lambda}$$

Applying the implicit function theorem gives

$$\left[\begin{array}{cc} \Omega_{i_d} & \Omega_r \\ \Lambda_{i_d} & \Lambda_r \end{array}\right]\left[\begin{array}{c} di_d/d\chi \\ d\tilde{r}/d\chi \end{array}\right] = \left[\begin{array}{c} -\Omega_\chi \\ -\Lambda_\chi \end{array}\right]$$

where

$$\begin{array}{ll} \Omega_{i_d} = 0, & \Lambda_{i_d} = -1 - \frac{(1+i)(1-\chi)}{\chi(1+i_d)^2} \\ \Omega_r = \gamma''(\cdot)\tilde{r} + \eta''(\cdot)\left(\frac{1-\chi}{\chi}\right)^2\tilde{r}, & \Lambda_r = \gamma''(\cdot) - \left(\frac{1-\chi}{\chi}\right)^2\eta''(\cdot) \\ \Omega_\chi = -\frac{1-\chi}{\chi^3}\tilde{r}^2\eta''(\cdot), & \Lambda_\chi = \frac{1-\chi}{\chi^3}\tilde{r}\eta''(\cdot) - \frac{1}{\chi^2}\left[\frac{1+i}{1+i_d} - 1 - \eta'(\cdot)\right]. \end{array}$$

Using Cramer's rule gives following comparative statics results.

$$\frac{\partial i_d}{\partial \chi} = \frac{\left|\begin{array}{cc} -\Omega_\chi & \Omega_r \\ -\Lambda_\chi & \Lambda_r \end{array}\right|}{\left|\begin{array}{cc} \Omega_{i_d} & \Omega_r \\ \Lambda_{i_d} & \Lambda_r \end{array}\right|} = -\frac{\frac{1}{\chi^2}\left[\frac{1+i}{1+i_d} - 1 - \eta'\left(\frac{1-\chi}{\chi}\hat{r}\right)\right]}{1 + \frac{(1+i)(1-\chi)}{\chi(1+i_d)^2}} < 0$$

Recall (38) $\frac{\partial i_d}{\partial i} = \left[\frac{\chi(1+i_d)}{1-\chi} + \frac{1+i}{1+i_d}\right]^{-1}$, by taking derivative with respect to χ we have

$$\frac{\partial i_d}{\partial \chi \partial i} = \frac{-(1+i_d)}{(1-\chi)^2[1+i+\frac{\chi}{1-\chi}(1+i_d)]^2} + \frac{1+i}{[1+i+\frac{\chi}{1-\chi}(1+i_d)^2]^2}\frac{\partial i_d}{\partial \chi} < 0$$

since $\frac{\partial i_d}{\partial \chi} < 0$.

Now, consider the ample-reserve equilibrium. Recall (34) from the ample reserve case,

$$\frac{\partial i_d}{\partial i_r} = 1 - \frac{1+i}{1+i+[1+i_r-\gamma'(\tilde{r})]}$$

Taking derivative with respect to i gives following

$$\begin{aligned} \frac{\partial^2 i_d}{\partial i \partial i_r} &= \frac{-[1+i_r-\gamma'(\cdot)] - (1+i)\gamma''(\cdot)\frac{\partial \tilde{r}}{\partial i}}{\{2+i+i_r-\gamma'(\cdot)\}^2} \\ &= \frac{[1+i_r-\gamma'(\cdot)]^3}{\{2+i+i_r-\gamma'(\cdot)\}^2\{\ell+[1+i_r-\gamma'(\cdot)]^2\}} > 0 \end{aligned}$$

using the results from Proof of Proposition 1 and 2. ■

Proof of Proposition 5. Assume $\hat{\delta} \leq \bar{\delta}$. For the scarce reserve equilibrium, recall equation (28)

$$r = \frac{(\sigma_2 + \sigma_3)\chi}{1 + i_d\chi} L^{-1}(i_\ell) - \frac{\sigma_3 \bar{\delta}\chi}{1 + i_d\chi}.$$

Given $\hat{\delta} \leq \bar{\delta}$, it is straightforward to show that

$$\frac{\partial r}{\partial \bar{\delta}} = -\frac{\sigma_3 \chi}{1 + i_d\chi} < 0, \quad \frac{\partial r}{\partial \sigma_3} = -\frac{\bar{\delta}\chi}{1 + i_d\chi} < 0.$$

For the ample reserve equilibrium, recall equation (35)

$$(\sigma_2 + \sigma_3)L^{-1}(i_\ell) - \sigma_3\bar{\delta} = n(\tilde{r} + \tilde{\ell})$$

Since $\tilde{r}$, $\tilde{\ell}$, i_d, i_ℓ and n_1 are independent of $\bar{\delta}$, we have

$$\frac{\partial n}{\partial \bar{\delta}} = \frac{-\sigma_3}{\tilde{r} + \tilde{\ell}} < 0$$

implying $\frac{\partial r}{\partial \bar{\delta}} < 0$ where $r = n\tilde{r}$. ■

Proof of Proposition 6. Define money multiplier $\zeta \equiv (m + r + \ell)/(m + r)$. In the scarce reserves equilibrium, $\zeta = \frac{m + r/\chi}{m + r}$. In this case we have

$$\frac{\partial \zeta}{\partial \bar{\delta}} = \frac{m(1 - \chi)}{\chi(m + r)^2} \frac{\partial r}{\partial \bar{\delta}} < 0$$

as long as $m > 0$ and $\chi \in (0, 1)$. In the ample reserves equilibrium, we have

$$\frac{\partial \zeta}{\partial \bar{\delta}} = \frac{m\tilde{\ell}}{(m + r)^2} \frac{\partial n}{\partial \bar{\delta}} < 0$$

as long as $m > 0$. Now consider the effect of i and i_r under the ample reserve equilibrium. By taking derivative with respect to i, we have

$$\frac{\partial \zeta}{\partial i} = \frac{1}{(m + r)^2} \left[\underbrace{\frac{\partial \tilde{\ell}}{\partial i}}_{\oplus} n(m + r) - \underbrace{\frac{\partial m}{\partial i}}_{\ominus} \ell + \underbrace{\frac{\partial n}{\partial i}}_{?} \tilde{\ell} m \right],$$

which is positive when m is small enough. By taking derivative with respect to i_r,

$$\frac{\partial \zeta}{\partial i_r} = \frac{1}{(m + r)^2} \left[\underbrace{\frac{\partial \tilde{\ell}}{\partial i_r}}_{\ominus} n(m + r) + \underbrace{\frac{\partial n}{\partial i_r}}_{?} \tilde{\ell} m \right]$$

which is negative when m is small enough. Therefore, for small m, we have

$$\frac{\partial \zeta}{\partial i} > 0, \quad \frac{\partial \zeta}{\partial i_r} < 0$$

in the ample reserve equilibrium. ■

Appendix B Additional Results

B.1 Chow Test

Figure 2 includes the Chow test for structural breaks. The test result reported in the bottom-left panel of Figure 2 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{RequiredReserves/Deposit})_t \\ &+\mathbf{1}_{t\geq 1992Q2}[\gamma_0 + \gamma_1(\text{RequiredReserves/Deposit})_t] \\ &+\mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{RequiredReserves/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 4a reports F-statistics which are obtained by testing $\gamma_0 = \gamma_1 = \delta_0 = \delta_1 = 0$.

The Chow test in the bottom-right panel of Figure 2 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{Currency/Deposit})_t \\ &+\mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{Currency/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 4b reports F-statistics is obtained by testing $\delta_0 = \delta_1 = 0$. The regression estimates and the Chow test results are summarized at the below table.

Table 4: Chow test for structural breaks

(a) Require reserve ratio

Dependent Variable: Money Multiplier	
RR	-0.601
	(0.365)
RR × $\mathbf{1}_{t\geq 1992Q2}$	132.279^{***}
	(0.031)
RR × $\mathbf{1}_{t\geq 2008Q4}$	-147.943^{***}
	(8.574)
$\mathbf{1}_{t\geq 1992Q2}$	9.091^{***}
	(0.557)
$\mathbf{1}_{t\geq 2008Q4}$	0.074^{***}
	(0.611)
Constant	2.813^{***}
	(0.053)
Obs.	228
R^2	0.963
DF for numerator	4
DF for denominator	222
F Statistic for Chow test	1711.32
F Statistic for 1% sig. level	3.40
F Statistic for 0.1% sig. level	4.79

(b) Currency deposit ratio

Dependent Variable: Money Multiplier	
CD	-1.301^{***}
	(0.027)
CD × $\mathbf{1}_{t\geq 2008Q4}$	-52.018^{***}
	(4.995)
$\mathbf{1}_{t\geq 2008Q4}$	3.061^{***}
	(0.409)
Constant	3.159^{***}
	(0.015)
Obs.	228
R^2	0.974
DF for numerator	2
DF for denominator	224
F Statistic for Chow test	1245.69
F Statistic for 1% sig. level	4.70
F Statistic for 0.1% sig. level	7.13

Notes: Newy-West standard errors are in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Degree of freedom is denoted by DF.

B.2 Unit Root and Cointegration Test

Column (2) and (4) in Table 1 includes the canonical cointegrating regression estimates and the cointegration tests. This section reports unit root tests for those series used in Column (2) and (4). For all the four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference reject the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey, Leybourne and Taylor (2009). ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. The data are quarterly from 1980Q1 to 2007Q4.

Table 5: Unit root test

	Phillips-Perron test	
	$Z(\rho)$	$Z(t)$
$ln(m)$	0.567	0.297
$ln(d)$	1.275	1.054
$ln(uc)$	-1.114	-1.710
r	-7.721	-2.471
$\Delta ln(m)$	-46.623^{***}	-5.335^{***}
$\Delta ln(d)$	-42.267^{***}	-5.060^{***}
$\Delta ln(uc)$	-41.998^{***}	-5.107^{***}
Δr	-94.183^{***}	-9.263^{***}

Column (1) and (2) in Table 3 includes the canonical cointegrating regression estimates since all three series - R/Y, UC/Y, and 3 month treasury rate - have unit roots and cointegrated both for the data and the model-implied series. This section also reports unit root tests and cointegration tests and sensitivity check using federal funds rates. For all the four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference reject the null hypothesis of non-stationarity at 1% significance level. For all the five variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference reject the null hypothesis of non-stationarity at 1% significance level. The Johansen tests reject their null of no cointegration at 99 percent confidence level, suggesting there exists a stable relationship between real reserves balances, interest rates and real balances of unsecured credit both in the data and the model. This result is robust with repect to different measure of interest rate, the federal funds rate. The data are yearly from 1968 to 2007.

Table 6: Unit root test and additional CCR estimates

(a) Unit root test

	Phillips-Perron test	
	$Z(\rho)$	$Z(t)$
`Tbill3`	−7.683	−1.967
`ffr`	−8.683	−2.121
UC/Y	−0.315	−0.450
R/Y(Data)	−1.735	−2.240
R/Y(Model)	−1.100	−1.877
Δ`Tbill3`	−24.363***	−4.514***
Δ`ffr`	−25.127***	−4.747***
$\Delta UC/Y$	−24.204***	−4.202***
$\Delta R/Y$(Data)	−26.473***	−4.329***
$\Delta R/Y$(Model)	−33.663***	−5.189***

(b) Canonical cointegrating regression

Dependent Variable:	Reserves/GDP (1968-2007)
UC/Y	−0.122***
	(0.004)
`ffr`	−0.064***
	(0.009)
Constant	3.058***
	(0.095)
Obs.	40
R^2	0.854
adj R^2	0.846
Long run S.E.	0.141

Notes: All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey et al. (2009). ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 7: Johansen test for cointegration

(a) UC/Y, Tbill3 and R/Y (Data)

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	39.5289	29.68	35.65
1	**6.3521**	15.41	20.04
2	1.7359	3.76	6.65
Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	33.1768	20.97	25.52
1	**4.6162**	14.07	18.63
2	1.7359	3.76	6.65

(b) UC/Y, Tbill3 and R/Y (Model)

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	48.3176	29.68	35.65
1	**9.8317**	15.41	20.04
2	3.3576	3.76	6.65
Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	38.4859	20.97	25.52
1	**6.4741**	14.07	18.63
2	3.3576	3.76	6.65

(c) UC/Y, ffr and R/Y (Data)

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	42.2554	29.68	35.65
1	**6.1539**	15.41	20.04
2	1.7615	3.76	6.65
Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	36.1015	20.97	25.52
1	**4.3924**	14.07	18.63
2	1.7615	3.76	6.65

(d) UC/Y, ffr and R/Y (Model)

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	47.4812	29.68	35.65
1	**9.7295**	15.41	20.04
2	3.1840	3.76	6.65
Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	37.7517	20.97	25.52
1	**6.5455**	14.07	18.63
2	3.1840	3.76	6.65

B.3 Unit Root and Cointegration Test for M2

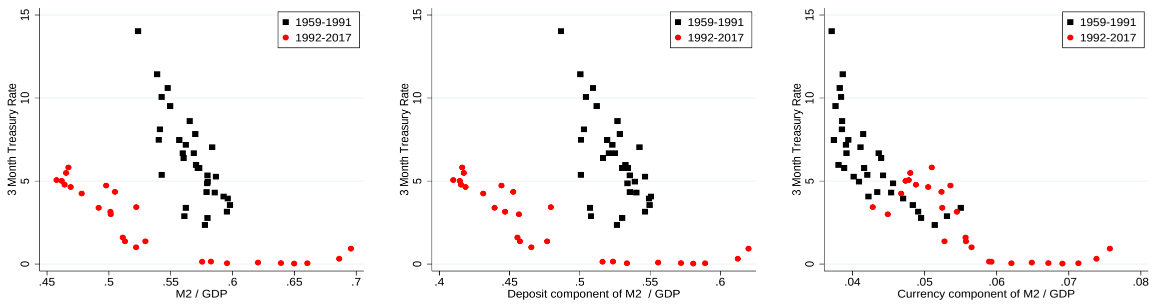

Figure 14: Money demand for M2 and its components

To check the empirical breakdown of the stable relation between M2 and interest rate, Figure 14 plots the ratio of M2 to GDP for the US and the ratio of M2's components to GDP against the 3 month Treasury Bill rate. There is a breakdown in M2 in 1992 as well that coincides with the structural break in Figure 2 and Figure 3.

Table 8: Cointegration regressions and tests (M2)

Dependent Variable:	$ln(m_t)$		$ln(d_t)$	
	OLS	CCR	OLS	CCR
	(1)	(2)	(3)	(4)
r_t	0.009***	−0.019***	0.013***	−0.020***
	(0.002)	(0.002)	(0.002)	(0.003)
$ln(uc_t)$		−0.182***		−0.225***
		(0.024)		(0.027)
$adjR^2$	0.133	0.306	0.201	0.288
N	112	112	112	112
Johansen $r = 0$	**18.582**	40.396	**19.210**	39.421
5% Critial Value for $r = 0$	15.41	29.68	15.41	29.68
1% Critial Value for $r = 0$	20.04	35.65	20.04	35.65
Johansen $r = 1$	2.762	**13.177**	2.713	**13.364**
5% Critial Value for $r = 1$	3.76	15.41	3.76	15.41
1% Critial Value for $r = 1$	6.65	20.04	6.65	20.04

Notes: Column (1),(3) report OLS estimates and column (2),(4) report the canonical cointegrating regression (CCR) estimates. First stage long-run variance estimation for CCR is based on Bartlett kernel and lag 1. For (1) and (2) Newey-West standard errors with lag 1 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported in column (1)-(4). The data are quarterly from 1980Q1 to 2007Q4.

To see whether the unsecured credit can account for this breakdown in M2, I repeated the analysis of Table 1 using M2 instead of M1. Table 8 reports the results. Again, I focus on the post-1980 period, until the arrival of the Great Recession. In column (2) and (4) the Johansen tests reject their null of no cointegration at 99 percent confidence level, suggesting there exists a stable relationship between M2 real money balances, interest rates and real balances of unsecured credit. The canonical cointegrating regression estimates in column (2) and (4) show that the estimated coefficients on r_t and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once we account for the substitution effect of consumer credit, there still exists a stable negative relationship between M2 real balances and the interest rates.

Table 9 provides the unit root test results for M2 to output ratio and deposit component of M2 to output ratio. For all the two variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference reject the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test

Table 9: Unit root test (M2)

	Phillips-Perron test	
	$Z(\rho)$	$Z(t)$
$ln(m)$	0.567	0.297
$ln(d)$	1.275	1.054
$\Delta ln(m)$	-46.623^{***}	-5.335^{***}
$\Delta ln(d)$	-42.267^{***}	-5.060^{***}

Notes: All series are demeaned before implementing the unit root test because the magnitude of the initial value can be problematic, as pointed out by Elliott and Müller (2006) and Harvey et al. (2009). ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Appendix C Quantitative Robustness

C.1 Robustness I: Different Measure of Monetary Policy

Different papers have use different series as monetary instruments for fitting the monetary model to the money demand. For example, Lucas (2000) and Lagos and Wright (2005) use commercial paper rate. Bethune, Choi and Wright (2020) uses 3 month treasury bill rate. New Keynesian literature usually use federal funds rate (e.g, Christiano, Eichenbaum and Evans, 2005 Smets and Wouters, 2007, Christiano, Eichenbaum and Trabandt, 2016) or 3 month treasury bill rate (e.g., Ireland, 2011) as measure of monetary policy while those models don't fit to the money demand. This section checks the robustness of the main quantitative results by refitting the model to the data using different measure of monetary policy target: commercial paper rate, federal funds rate.

Table 10: Parametrizations with different measure of monetary policy

Interest/Inflation rate	3 Month T-bill		CP		Federal Funds	
	Data	Model	Data	Model	Data	Model
Targets						
avg. retail markup	1.384	1.384	1.384	1.384	1.384	1.388
avg. C/Y	0.044	0.044	0.044	0.044	0.044	0.043
avg. R/Y	0.014	0.017	0.014	0.017	0.014	0.017
avg. C/D	0.529	0.520	0.529	0.520	0.529	0.512
avg. UC/DM	0.387	0.370	0.387	0.370	0.387	0.371
avg. Π/Y	0.0016	0.0011	0.0016	0.0011	0.0016	0.0011
semi-elasticity of C/Y	-3.716	-3.724	-3.713	-3.712	-3.020	-3.719

Note: C, R, DM, UC, Y denote currency in circulation, reserves, DM transactions, unsecured credit and nominal GDP, respectively.

Table 10 compares how model moments are changing depending on different parameterization using different measures of monetary policy. The table shows that the results are not very sensitive since changing measure of monetary policy provides similar moments that can be matched with target moments. Figure 15 compares the model fits of model-generated series under different parameterization. The model-generated series also show similar patterns for M1 Money multiplier, excess reserve ratio and currency deposit ratio except for the exercise using inflation rate.

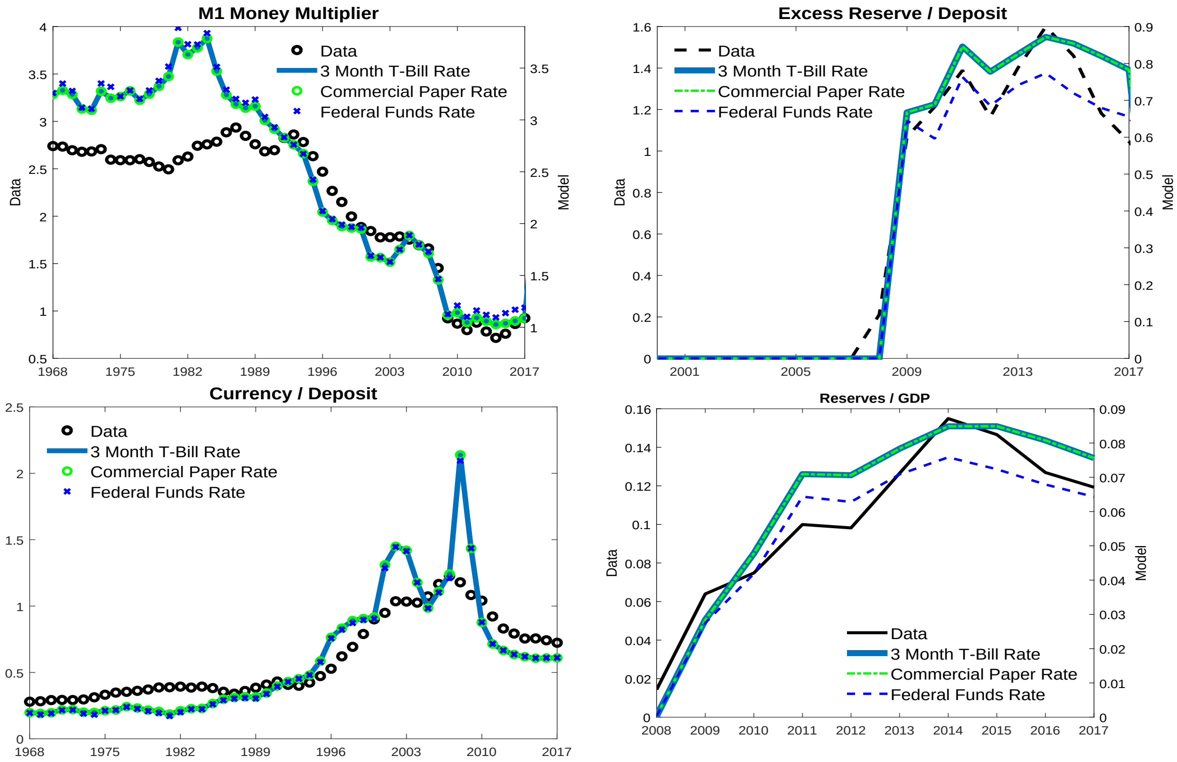

Figure 15: Model fit with different measure of monetary policy

C.2 Robustness II: Other Specifications

This section summarizes alternative parameterization results. For robustness, I examine how results are sensitive with respect to different parameter for curvature parameter for deposit operating cost while keeping other parameters are same. For Model 1, I set $\gamma(\tilde{d}) = A\tilde{d}^{1.15}$ while I set $\gamma(\tilde{d}) = A\tilde{d}^{1.25}$ in the Model 2.

Table 11 compares how model moments are changing depending on different parameterization. The table shows that the results are not very sensitive since changing the curvature parameter and provides similar moments that can be matched with target moments. Figure 16 compares the model fits of model-generated series under different parameterization. The model-generated series also show similar patterns for M1 Money multiplier, excess reserve ratio and currency deposit ratio.

Table 11: Alternative parametrizations

	Data	Baseline	Model 1	Model 2
External Parameters				
a		1.2	1.15	1.25
σ_3		0.69	0.69	0.69
Calibration targets				
avg. retail markup	1.384	1.384	1.384	1.384
avg. C/Y	0.044	0.044	0.044	0.044
avg. R/Y	0.014	0.017	0.017	0.017
semi-elasticity of C/Y	-3.713	-3.712	-3.712	-3.712
avg. C/D	0.529	0.520	0.520	0.520
avg. UC/DM	0.387	0.370	0.370	0.370
avg. Π/Y	0.0016	0.0011	0.0011	0.0011

Note: C, R, DM, UC, Y denote currency in circulation, reserves, DM transactions, unsecured credit and nominal GDP, respectively.

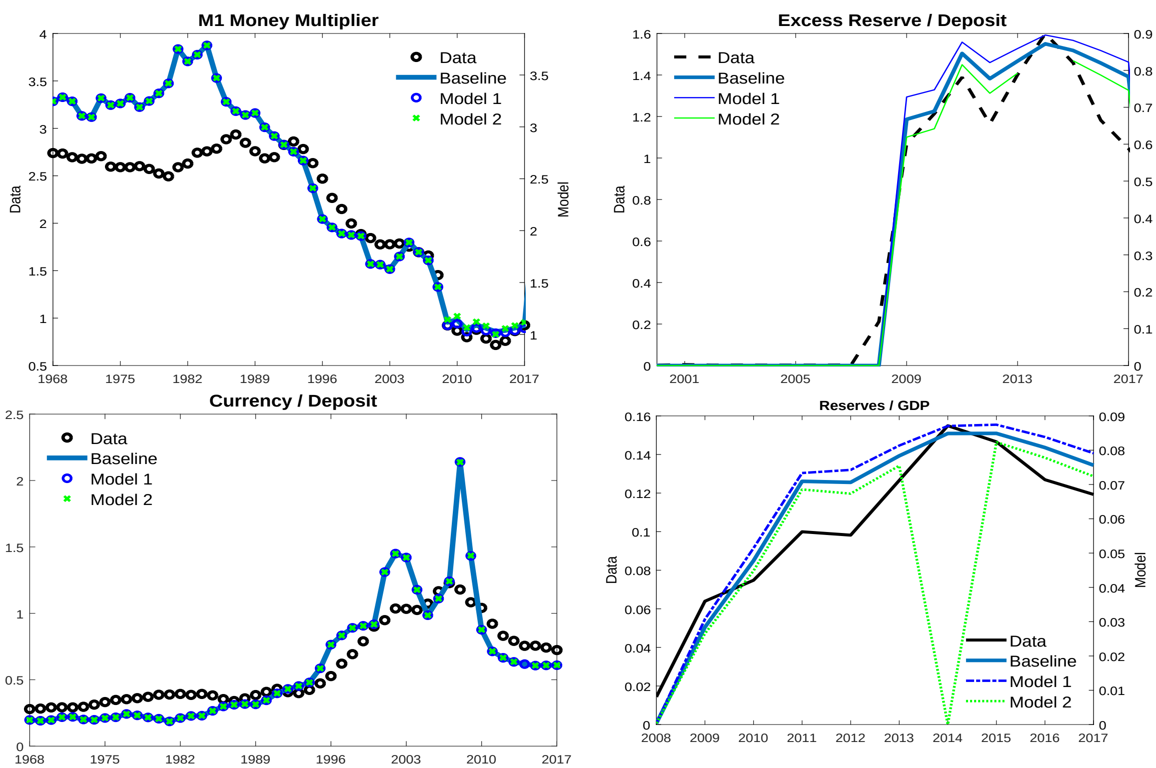

Figure 16: Model fit with different specifications