On the Instability of Fractional Reserve Banking*

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Abstract

This paper develops a dynamic general equilibrium model to study the (in)stability of the fractional reserve banking system. The model shows that the fractional reserve banking system can endanger stability in that equilibrium is more prone to exhibit endogenous cyclic, chaotic, and stochastic dynamics under lower reserve requirements, although it can increase welfare in the steady-state. Introducing endogenous unsecured credit to the baseline model does not change the main results. This paper also provides empirical evidence that is consistent with the prediction of the model.

JEL Classification Codes: E42, E51, G21
Keywords: Money, Banking, Instability, Volatility

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> Motivated partly by a desire to avoid such [excessive] price-level fluctuations and possible Wicksellian price-level indeterminacy, quantity theorists have advocated legal restrictions on private intermediation. ... Thus, for example, Friedman (1959, p. 21) ... has advocated 100 percent reserves against bank liabilities called demand deposit. **Sargent and Wallace (1982)**

1 Introduction

In 2018, Switzerland had a referendum of 100% reserve banking, which was rejected with 75.72% of objecting voters. The referendum was aiming for making money safe from crisis by constructing full-reserve banking.[1] The debate on whether a fractional reserve banking system is inherently unstable has been an important policy discussion from a long time ago. Prominent examples are Peel's Banking Act of 1844 and the Chicago plan of banking reform with a 100% reserve requirement proposed by Irving Fisher, Paul Douglas, and others in 1939. Later, Friedman (1959) supported this banking reform, whereas Becker (1956) took the opposite position of supporting free banking with 0% reserve requirement.[2] Whereas the debates on the instability of fractional reserve banking started a long time ago, the debate has never stopped.

Also, in the theoretical side, there have been claims that fractional reserve banking is an important cause of boom-bust cycles (e.g., Fisher, 1935; Von Mises, 1953; Minsky, 1957; Minsky, 1970). The claims are based on the idea that, under fractional reserve banking, banks create excess credit that results in boom and bust. Although Fisher (1935) considers many other factors of business cycles including fractional reserve banking, Von Mises (1953) and Minsky (1957) see this as a root cause of boom-bust cycles.

This paper answers the following questions: (i) Can fractional reserve banking be inherently volatile even if we shut down the stochastic component of the economy? (ii) If so, under what condition can fractional reserve banking generate endogenous cycles without the presence of exogenous shock and changes in fundamentals? Previously works on banking instability mostly have focused on bank runs following the seminal

[1] The official title of the referendum was *the Swiss federal popular initiative "for crisis-safe money: money creation by the National Bank only! (Sovereign Money Initiative)"* and also titled as *"debt-free money"*.

[2] Sargent (2011) provides a novel review of the historical debates between narrow banking and free banking as tensions between stability versus efficiency.

model by Diamond and Dybvig (1983). This work, however, focuses on the volatility of real balances.

To assess the claim that fractional reserve banking causes business cycles, this paper constructs a model of money and banking that captures the role of fractional reserve banking. In the model, each agent faces an idiosyncratic liquidity shock. Banks accept deposits and extend loans to provide risk-sharing among the depositors whereas the bank's lending is constrained by the reserve requirement.

In the model, the real balance of money is determined by two factors: storage value and liquidity premium. Today's storage value is increasing in the future value of money. However, the liquidity premium, the marginal value of its liquidity function, is decreasing if the money becomes more abundant. When the liquidity premium dominates storage value, the economy can exhibit endogenous fluctuations. Fractional reserve banking amplifies the liquidity premium because it allows the bank to create inside money through lending. Due to this amplified liquidity premium, the fractional reserve banking system is more prone to endogenous cycles.

In the baseline model, lowering the reserve requirement increases welfare in the steady state. However, lowering the reserve requirements can induce two-period cycles as well as three-period cycles, which implies the existence of periodic cycles of all order and chaotic dynamics. This also implies it can induce sunspot cycles. This result holds in the extended model with unsecured credit. The model also can deliver a self-fulling bubble burst. It is worth to note that full reserve requirement does not exclude the possibility of endogenous cycles. However, the economy will be more susceptible to cycles with lower reserve requirement.[3]

The results are different from the argument that fluctuations due to exogenous shocks can be amplified by fractional reserve banking. The endogenous cycles arises even if we shut down the stochastic component of the economy. In contrast to the results on the endogenous cycles due to the financial system, most business cycle analyses in modern macroeconomics are based on locally stable models with exogenous shocks. These models see that the fluctuations come from exogenous shocks that disturb the dynamic system, and the effects of exogenous shocks shrink over time as the system goes back to its balanced path or steady-state. While there has been a lot of work on the role of financial friction in the business cycles including Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), and Gertler and Karadi (2011), the financial

[3] Gu, Monnet, Nosal and Wright (2019) show that introducing banks to the economy could induce instability in various settings which is in line this result.

sectors only play an amplification role rather than act as a source of the cycle. This paper, however, focuses on whether the fractional reserve banking system can endanger stability given that equilibrium is more prone to exhibit endogenous fluctuations.

To evaluate the main prediction from the theory that fractional reserve banking induces excess volatility, I test the association between the required reserves ratio and real inside money volatility using cointegrating regression. A significant negative relationship between the two variables are found, and the results are robust to different measures of inflation and different frequency of time series. Both theoretical and empirical evidence indicate a link between the reserve requirement and the (in)stability.

Related Literature This paper builds on Berentsen, Camera and Waller (2007), who introduce financial intermediaries with enforcement technology to the Lagos and Wright (2005) framework. The approach to introduce unsecured credit to the monetary economy is related to Lotz and Zhang (2016) and Gu, Mattesini and Wright (2016) who incorporate the agents' default decision, similar to Kehoe and Levine (1993).

This paper is related to the large literature on fractional reserve banking. Freeman and Huffman (1991) and Freeman and Kydland (2000) develop general equilibrium models that explicitly capture the role of fractional reserve banking. Using those models, they explain the observed relationships between key macroeconomic variables over business cycles. Chari and Phelan (2014) study the condition under which fractional reserve banking can be socially useful by preventing bank runs in the cash-in-advance framework. For the recent study, Andolfatto, Berentsen and Martin (2020) integrate Diamond (1997) into the Lagos and Wright (2005) framework and provide a model in which fractional reserve banking emerges endogenously and a central bank can prevent bank panic as a lender of last resort. Whereas many of previous works on instability focus on bank runs, this paper studies a different type of instability in the sense that fractional reserve banking induces endogenous monetary cycles.

This paper is also related to the large literature on endogenous fluctuations, chaotic dynamics, and indeterminacy that have been surveyed by Brock (1988), Baumol and Benhabib (1989), Boldrin and Woodford (1990), Scheinkman and Woodford (1994) and Benhabib and Farmer (1999). For a model of bilateral trade, Gu, Mattesini, Monnet and Wright (2013) show that credit markets can be susceptible to endogenous fluctuations due to limited commitment. Gu et al. (2019) show that introducing financial intermediaries to an economy can engender instability in four distinct setups that capture various functions of banking. The model in this paper is closely related to Gu et al. (2019), whereas the model here is extended to incorporate fractional reserve banking.

The rest of the paper is organized as follows. Section 2 constructs the baseline search-theoretic monetary model. Section 3 provides main results. Section 4 introduces unsecured credit. Section 5 discusses the empirical evaluation of the model. Section 6 concludes.

2 Model

The model is based on Lagos and Wright (2005) with a financial intermediary as in Berentsen et al. (2007). Time is discrete and infinite. In each period, three markets convene sequentially. First, a centralized financial market (FM), followed by a decentralized goods market (DM), and finally a centralized goods market (CM). The FM and CM are frictionless. The DM is subject to search frictions, anonymity, and limited commitment. Therefore, a medium of exchange is needed to execute trades.

There is a continuum of agents who produce and consume perishable goods. At the beginning of the FM, a preference shock is realized: With probability σ, an agent will be a buyer in the following DM and with probability $1-\sigma$, she will be a seller. Agents discount their utility each period by β. Within-period utilty is represented by

$$\mathcal{U} = U(X) - H + u(q) - c(q),$$

where X is the CM consumption, H is the CM disutility from production, and q is the DM consumption. As standard U', u', $c' > 0$, U'', $u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. One unit of H produces one unit of X in the CM. The efficient consumption in CM and DM is X^* and q^* that solve $U'(X^*) = 1$ and $u'(q^*) = c'(q^*)$, respectively.

A representative bank accepts deposits and lends loans. The banking market is perfectly competitive. The bank can enforce the repayment of loans at no cost. Last, there is a central bank that controls the money supply M_t. Let γ be the growth rate of the money stock. Changes in money supply are accomplished by lump-sum transfer if $\gamma > 0$ and by lump-sum tax if $\gamma < 0$.

2.1 Agent's Problem

Let W_t, G_t, and V_t denote the agent's value function in the CM, FM, and DM, respectively, in period t. There are two payment instruments for the DM transaction: fiat money (outside money) and loan from the bank (inside money). I will allow the agents to use unsecured credit as a means of payment in next section. An agent entering the CM with nominal balance m_t, deposit d_t, and loan ℓ_t, solves the following problem:

$$\begin{aligned} W_t(m_t, d_t, \ell_t) = & \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta G_{t+1}(\hat{m}_{t+1}) \\ \text{s.t. } \phi_t \hat{m}_{t+1} + X_t = & H_t + T_t + \phi_t m_t + (1 + i_{d,t})\phi_t d_t - (1 + i_{l,t})\phi_t \ell_t, \end{aligned} \tag{1}$$

where T_t is the lump-sum transfer (or tax if it is negative), $i_{d,t}$ is the deposit interest rate, $i_{l,t}$ is the loan interest rate, ϕ is the price of money in units of the CM goods, and $\hat{m}_{t+1}$ is the money balance carried to the FM where a bank takes deposits and makes loans. The first-order conditions (FOCs) are $U'(X_t) = 1$ and

$$\phi_t = \beta G'_{t+1}(\hat{m}_{t+1}), \tag{2}$$

where $G'_{t+1}(\hat{m}_{t+1})$ is the marginal value of an additional units of money taken into the FM of period $t + 1$. The envelope conditions are

$$\frac{\partial W_t}{\partial m_t} = \phi_t, \qquad \frac{\partial W_t}{\partial d_t} = \phi_t(1 + i_{d,t}), \qquad \frac{\partial W_t}{\partial \ell_t} = -\phi_t(1 + i_{l,t}),$$

implying W_t is linear in m_t, d_t, and ℓ_t.

The value function of an agent at the beginning of FM is

$$G_t(m) = \sigma G_{b,t}(m) + (1 - \sigma)G_{s,t}(m), \tag{3}$$

where $G_{j\in\{b,s\},t}$ is the value function of type j agent in the FM. Agents choose their deposit balance d_j and loan ℓ_j based on the realization of their types in the following DM. The value function $G_{j,t}$ can be written as

$$G_{j,t}(m) = \max_{d_{j,t}, \ell_{j,t}} V_{j,t}(m - d_{j,t} + \ell_{j,t}, d_{j,t}, \ell_{j,t}) \quad \text{s.t} \quad d_{j,t} \leq m, \tag{4}$$

where $V_{j,t}$ is value function of type j agent in the DM. The FOCs are

$$\frac{\partial V_{j,t}}{\partial \ell_{j,t}} \leq 0 \tag{5}$$

$$\frac{\partial V_{j,t}}{\partial d_{j,t}} - \lambda_d \leq 0 \tag{6}$$

where λ_d is the Lagrange multiplier for $d_{j,t} \leq m$.

The terms of trade in the DM are determined by an abstract mechanism that was studied in Gu and Wright (2016). The buyer must pay $p = v(q)$ to the seller to get q. As shown in Gu and Wright (2016), the bilateral trade satisfies following

$$p = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{if } z \geq p^* \end{cases} \qquad q = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{if } z \geq p^*, \end{cases} \tag{7}$$

where $v(q)$ is some payment function satisfying $v'(q) > 0$ and $v(0) = 0$, p^* is the payment required to get efficient consumption q^*, and z is the sum of outside money and inside money, $\phi(m - d + \ell)$, held by the buyer.

With probability α, a buyer meets a seller in the DM while a seller meets a buyer with probability α_s. Since the CM value function is linear, the DM value function for the buyer can be written as

$$V_{b,t}(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}) = \alpha[u(q_t) - p_t] + W(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}),$$

where $p_t \leq \phi_t(m_t - d_{b,t} + \ell_{b,t})$. Assuming interior solution, differentiating $V_{b,t}$ yields

$$\frac{\partial V_{b,t}}{\partial m} = \phi_t[\alpha\lambda(q_t) + 1], \qquad \frac{\partial V_{b,t}}{\partial d} = \phi_t[-\alpha\lambda(q_t) + i_{d,t}], \qquad \frac{\partial V_{b,t}}{\partial \ell} = \phi_t[\alpha\lambda(q_t) - i_{l,t}],$$

where $\lambda(q) = u'(q)/v'(q) - 1$ if $p^* > z$ and $\lambda(q) = 0$ if $z \geq p^*$. Combining the buyer's FOCs in the FM and the derivatives of V_b yields

$$\partial G_b/\partial d_b = \phi i_d - \phi\alpha\lambda(q) - \lambda_d \leq 0, = 0 \text{ iff } d_b > 0 \tag{8}$$

$$\partial G_b/\partial \ell_b = -\phi i_l + \phi\alpha\lambda(q) \leq 0, = 0 \text{ iff } \ell_b > 0. \tag{9}$$

A seller's DM value function is

$$V_{s,t}(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}) = \alpha_s[p_t - c(q_t)] + W_t(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}).$$

Differentiating $V_{s,t}$ yields

$$\frac{\partial V_{s,t}}{\partial m_t} = \phi_t, \qquad \frac{\partial V_{s,t}}{\partial d} = \phi_t(1+i_{d,t}), \qquad \frac{\partial V_{s,t}}{\partial \ell} = -\phi_t(1+i_{l,t}).$$

Similar to the buyer's case, combining the seller's FOCs in the FM and the first-order derivatives of V_s yields

$$\partial G_s/\partial d_s = \phi_t i_d - \lambda_d \leq 0, = 0 \text{ iff } d_s > 0 \tag{10}$$

$$\partial G_s/\partial \ell_s = -\phi_t i_l \leq 0, = 0 \text{ iff } \ell_s > 0. \tag{11}$$

2.2 Bank's Problem

A representative bank accepts deposits d and makes loans ℓ. The depositors are paid with a nominal interest rate i_d by the bank, and the borrowers need to repay their borrowing with a nominal interest rate i_l. The central bank sets reserve requirement χ. The representative bank solves the following profit maximization problem.

$$\max_{d,\ell} \quad (i_l \ell - i_d d) \quad s.t. \quad \chi \ell \leq d \tag{12}$$

The FOCs for the bank's problem are

$$0 = i_l - \lambda_L \tag{13}$$

$$0 = -i_d + \lambda_L/\chi, \tag{14}$$

where λ_L is the Lagrange multiplier with respect to the bank's lending constraint. For $\lambda_L > 0$, we have

$$i_l = \chi i_d \tag{15}$$

while $\lambda_L = 0$ implies $i_d = i_l = 0$. Given the bank's problem and the agent's problem, we can define an equilibrium as follows:

Definition 1. *Given (γ, χ), an equilibrium consists of the sequences of prices $\{\phi_t, i_{l,t}, i_{d,t}\}_{t=0}^{\infty}$, real balances $\{m_t, \ell_{b,t}, \ell_{s,t}, d_{b,t}, d_{s,t}\}_{t=0}^{\infty}$, and allocations $\{q_t, X_t, \ell_t\}_{t=0}^{\infty}$ satisfying the following:*

- *Agents solve CM and FM problems: (1) and (4)*
- *A representative bank solves its profit maximization problem: (12)*

- *Markets clear in every period:*

  1. *Deposit Market:* $\sigma d_{b,t} + (1-\sigma)d_{s,t} = d_t$
  2. *Loan Market:* $\sigma \ell_{b,t} + (1-\sigma)\ell_{s,t} = \ell_t$
  3. *Money Market:* $m_t = M_t$

The next step is to characterize the equilibrium. For $m > 0$ and $\lambda_d > 0$, we have $\partial G_b/\partial d_b < \partial G_s/\partial d_s = 0$ since

$$0 = i_d - \lambda_d/\phi > i_d - \lambda_d/\phi - \alpha\lambda(q) \tag{16}$$

$$0 = -\phi i_l + \phi\alpha\lambda(q) > -\phi i_l \tag{17}$$

implying $i_l = \alpha\lambda(q)$, $\ell_b = m(1-\sigma)/\sigma\chi > 0$, $\ell_s = 0$, $\ell = (1-\sigma)m/\chi$, $d_s = m$, and $(1-\sigma)m = d$. FOCs from buyers' and sellers' problems yield

$$i_l = \alpha\lambda(q), \qquad \frac{\lambda_d}{\phi} = [-\alpha\lambda(q) + i_d].$$

We can rewrite value functions in the FM as follows:

$$G_b(m) = \alpha[u(q) - p] + W(m + \ell_b, 0, \ell_b) \tag{18}$$

$$G_s(m) = \alpha_s[p - c(q)] + W(m - d_s, d_s, 0) \tag{19}$$

where $q = v^{-1}(p)$ and $p = \min\{p^*, \phi(m + \ell_b)\}$. Derivatives of FM value functions $G'_{j,t}(m)$ are

$$G'_{b,t}(m) = \phi_t - \phi_t i_{l,t}\frac{1-\sigma}{\sigma\chi} + \phi_t\frac{1-\sigma+\sigma\chi}{\sigma\chi}\alpha\lambda(q_t) \tag{20}$$

$$G'_{s,t}(m) = \phi_t + \phi_t i_{d,t} \tag{21}$$

for $j \in \{b, s\}$. Since $G'_t(m_t) = \sigma G'_{b,t}(m_t) + (1-\sigma)G'_{s,t}(m_t)$, we have the following expression:

$$G'_t(m_t) = \phi_t\frac{1-\sigma+\sigma\chi}{\chi}\alpha\lambda(q_t) + \phi_t + \phi_t(1-\sigma)\left(i_{d,t} - \frac{i_{l,t}}{\chi}\right).$$

Using (15) and FOCs of buyers' and sellers' decision yield

$$G'_t(m_t) = \phi_t \left[\frac{1-\sigma+\sigma\chi}{\chi} \alpha\lambda(q_t) + 1 \right]. \tag{22}$$

Combine equations (2) and (22), and use equilibrium condition $m_{t+1} = M_{t+1}$ to get

$$\phi_t = \begin{cases} \phi_{t+1}\beta \left[\frac{1-\sigma+\sigma\chi}{\chi} \alpha\lambda \circ v^{-1}(z_{t+1}) + 1 \right] & \text{if } z_{t+1} < p^* \\ \phi_{t+1}\beta & \text{if } z_{t+1} \geq p^*, \end{cases} \tag{23}$$

where $z_{t+1} = \phi_{t+1}M_{t+1}(1-\sigma+\sigma\chi)/\sigma\chi$. Here, z_{t+1} is buyer's available liquidity for the DM trade which is a sum of the buyer's money holding and the loan from the bank. Then multiplying both sides of (23) by $M_t(1-\sigma+\sigma\chi)/\sigma\chi$ allows us to reduce the equilibrium condition into one difference equation of real balances z:

$$z_t = f(z_{t+1}) \equiv \frac{z_{t+1}}{1+i} \left[\frac{1-\sigma+\sigma\chi}{\chi} \alpha L(z_{t+1}) + 1 \right], \tag{24}$$

where $(1+i) \equiv \gamma/\beta$ and $L(z) \equiv \lambda \circ v^{-1}(z)$ is the liquidity premium.[4]

3 Results

This section establishes key results on the instability of banking. Consider a stationary equilibrium, which is a fixed point that satisfies $z = f(z)$. There always exists an non-monetary equilibrium with $z = 0$. Given $i \in [0, \bar{\iota})$ and $\chi \in (0, 1]$, where $\bar{\iota} = \alpha(1-\sigma+\sigma\chi)L(0)/\chi$, an unique stationary monetary equilibrium exists and satisfies

$$\chi i = (1-\sigma+\sigma\chi)\alpha L(z_s),$$

where $z_s = v(q_s)$.[5] Since $L'(z) < 0$ (Gu and Wright, 2016), the following result holds:

Proposition 1. *In the stationary equilibrium, lowering the nominal interest rate or lowering reserve requirement increases DM consumption.*

The dynamics of monetary equilibrium is characterized by equation (24). Following

[4] In the stationary equilibrium, $i = \gamma/\beta - 1$ is the nominal interest rate.

[5] Nash and Kalai bargaining provides simple examples for $\bar{\iota}$. Under the Inada condition $u'(0) = \infty$, with Kalai, $\bar{\iota} = \theta\alpha(1-\sigma+\sigma\chi)/\chi(1-\theta)$; whereas with Nash bargaining, $\bar{\iota} = \infty$.

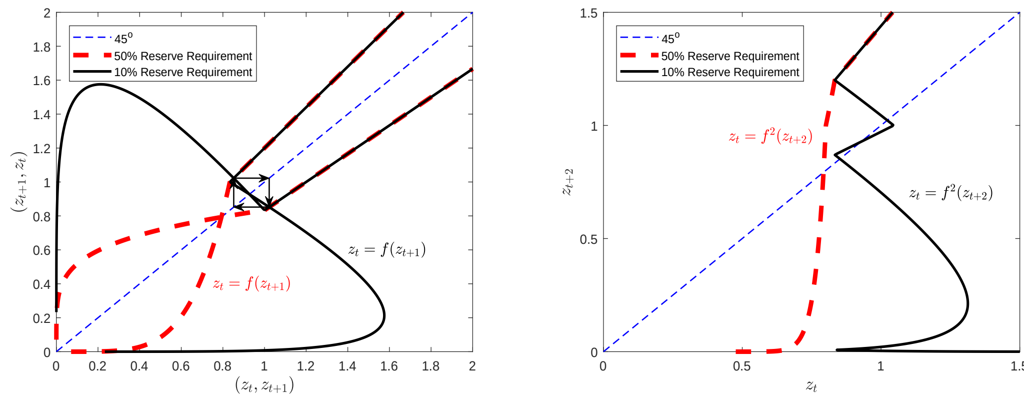

Figure 1: A Two-period Cycle under Fractional Reserve Banking

the standard textbook method (Azariadis, 1993), we can show that if $f'(z_s) < -1$, there exists a two-period cycle with $z_1 < z_s < z_2$.

Now consider a special case with some assumptions on $u(q)$ and $c(q)$. Assume the buyer makes a take-it-or-leave-it offer to the seller. Then the condition for two-period cycles, $f'(z_s) < -1$, holds when χ is lower than some threshold. The results can be expressed as follows:

Proposition 2. *Assume $-qu''(q)/u' = \eta$ and $c(q) = q$. If $\chi \in (0, \chi_m)$, where*

$$\chi_m \equiv \frac{\alpha\eta(1-\sigma)}{\eta(1-\alpha\sigma) + (2-\eta)(1+i)}, \tag{25}$$

then $f'(z_s) < -1$.

Proof. See Appendix A. ■

Proposition 2 implies that if the reserve requirement is lower than χ_m, the monetary economy exhibits endogenous cycles. Whereas condition (25) is written in terms of χ, this condition can be written in terms of i, as follows:

$$0 < i < \frac{\eta[\alpha(1-\sigma) - \chi(1-\alpha\sigma)]}{\chi(2-\eta)} \tag{26}$$

Condition (26) implies that if $\eta < 2$, lowering either χ or i can induce cycles.

To interpret the results, recall $f(z_{t+1})$ from equation (24). The first term, $z_{t+1}/(1+i)$ on the right-hand side, reflects the store of value, which is monotonically increasing in z_{t+1}. The second term $(1-\sigma+\sigma\chi)\alpha L(z_{t+1})/\chi + 1$, reflecting the liquidity premium, is

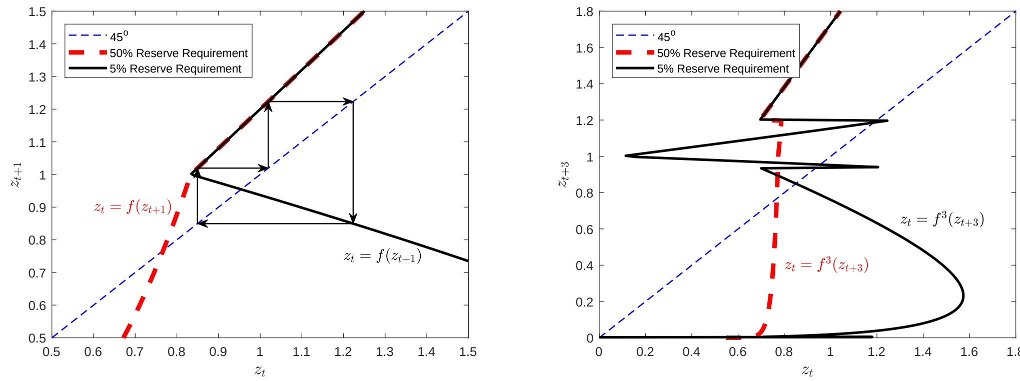

Figure 2: A Three-period Cycle under Fractional Reserve Banking

decreasing in z_{t+1}. Because $f'(z_{t+1})$ depends on both terms, $f(z_{t+1})$ is nonmonotone in general. If the liquidity premium dominates the storage value, we can have $f'(\cdot) < -1$, which is a standard condition for the existence of cyclic equilibria. Lowering the reserve requirement amplifies the liquidity premium because it allows the bank to create more liquidity through lending. This amplification of liquidity generates endogenous cycles.

In addition to the condition for two-period cycles, the next result provides the condition for three-period cycles under the general trading mechanism. The existence of three period-cycles implies cycles of all orders as well as chaotic dynamics (see Sharkovskii, 1964 and Li and Yorke, 1975).

Proposition 3 (**Three-period Monetary Cycle and Chaos**). *There exists a three-period cycle with $z_1 < z_2 < z_3$ if $\chi \in (0, \hat{\chi}_m)$, where*

$$\hat{\chi}_m \equiv \frac{(1-\sigma)\alpha L\left(\frac{p^*}{1+i}\right)}{(1+i)^3 - 1 - \sigma\alpha L\left(\frac{p^*}{1+i}\right)}.$$

Proof. See Appendix A. ■

In any periodic cycle, the liquidity constraint binds, $z_t < p^*$, at some point over the cycle.

Corollary 1 (**Binding Liquidity Constraint**). *In any n-period cycle, the liquidity constraint binds, $z_t < p^*$, at least one periodic point over the cycle.*

Proof. See Appendix A. ■

The model can also generate sunspot cycles. Consider a Markov sunspot variable $S_t \in \{1, 2\}$. This sunspot variable is not related to fundamentals but may affect equilibrium. Let $\Pr(S_{t+1} = 1|S_t = 1) = \zeta_1$ and $\Pr(S_{t+1} = 2|S_t = 2) = \zeta_2$. The sunspot is realized in the FM. Let W_t^S be the CM value function in state S in period t, then

$$W_t^S(m_t, d_t, \ell_t) = \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta \left[\zeta_s G_{t+1}^S(\hat{m}_{t+1}) + (1-\zeta_s) G_{t+1}^{-S}(\hat{m}_{t+1})\right]$$
$$\text{s.t. } \phi_t^S \hat{m}_{t+1} + X_t = H_t + T_t + \phi_t^S m_t + (1 + i_{d,t})\phi_t^S d_t - (1 + i_{l,t})\phi_t^S \ell_t.$$

The FOC can be written as

$$-\phi_t^S + \beta\zeta_s G_{t+1}'^S(\hat{m}_{t+1}) + \beta(1-\zeta_s) G_{t+1}'^{-S}(\hat{m}_{t+1}) = 0. \tag{27}$$

Solving the FM problem results in

$$G_{t+1}'^S(m_{t+1}^S) = \phi_{t+1}^S \left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}^S) + 1\right]. \tag{28}$$

We substitute (28) into (27) and multiply the both sides by $(1 - \sigma + \sigma\chi)M_{t+1}/(\sigma\chi)$ to get

$$\begin{aligned} z_t^S &= \frac{\zeta_s z_{t+1}^S}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}^S) + 1\right] + \frac{(1-\zeta_s) z_{t+1}^{-S}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}^{-S}) + 1\right] \\ &= \zeta_s f(z_{t+1}^S) + (1-\zeta_s) f(z_{t+1}^{-S}) \end{aligned} \tag{29}$$

where $z_t^S = (1 - \sigma + \sigma\chi)\phi_t^S m_t/(\sigma\chi)$. We define a sunspot equilibrium as follows:

Definition 2 (**Proper Sunspot Equilibrium**). *A proper sunspot equilibrium consists of the sequences of real balances* $\{z_t^S\}_{t=0,S=1,2}^{\infty}$ *where* $z_1 \neq z_2$ *and probabilities* (ζ_1, ζ_2)*, solving (29) for all* t*.*

Similar to the baseline model, one can show the existence of a sunspot equilibrium when $f'(z_s) < -1$. If $f'(z_s) < -1$, there exists (ζ_1, ζ_2), $\zeta_1 + \zeta_2 < 1$, such that the economy has a proper sunspot equilibrium in the neighborhood of z_s (again see Azariadis, 1993 for the textbook treatment).

In addition to the condition for the cycles, the model also features the equilibria where real balance increases above the steady state until certain time, T, and crashes to zero. Consider a sequence of real balance $\{z_t\}_{t=0}^{\infty}$ with $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > z_s$ (bubble)

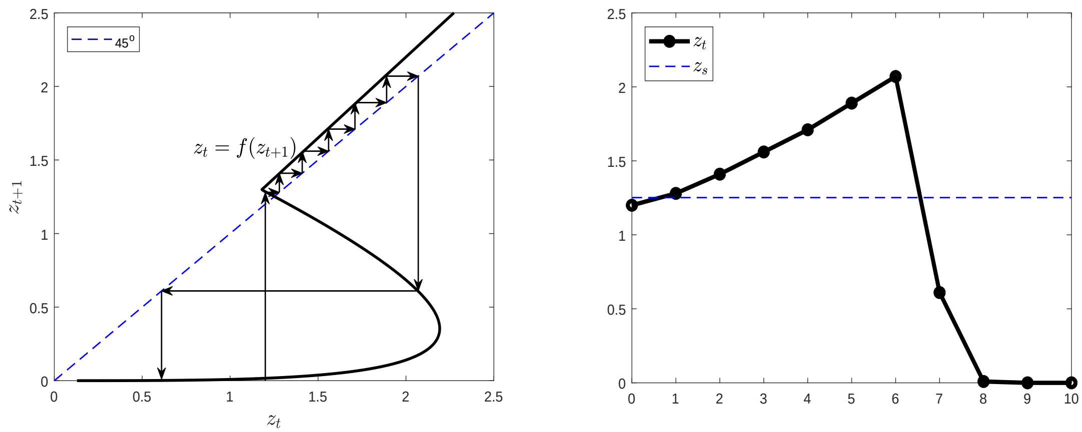

Figure 3: Self-Fulfilling Bubble and Burst Equilibria

that crashes to 0 (burst) as $t \to \infty$, where $T \geq 1$ and $z_T > z_0$. We refer to this equilibrium as a self-fulfilling bubble and burst equilibria:

Definition 3 (**Self-Fulfilling Bubble and Burst Equilibria**). *For initial real balance $z_0 > 0$, a self-fulfilling bubble and burst is a sequence of $\{z_t\}_{t=0}^{\infty}$ satisfying*

$$z_t = \frac{z_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1})+1\right] \tag{30}$$

and $0 < z_s < z_T$, $\lim_{t\to\infty} z_t = 0$, $z_T = \max\{z_t\}_{t=0}^{\infty}$ with $T \geq 1$.

The next step is to check under which condition this type of equilibria can occur. To simplify the analysis, I assume the buyer makes a take-it-or-leave-it offer to the seller. When $z_s > \bar{z}$, where $\bar{z}$ solves $f'(\bar{z}) = 0$, there exist multiple equilibria. Then, if $f(\bar{z}) \geq q^*$, the self-fulfilling bubble and burst equilibria exist. Lowering the reserve requirement can induce this type of equilibria. The following proposition summarizes the results.

Proposition 4 (**Existence of Self-Fulfilling Bubble and Burst Equilibria**). *Assume $-qu''(q)/u' = \eta$ and $c(q) = q$. There exist self-fulfilling bubble and burst equilibria, if*

$$0 < \chi < \min\left\{\frac{(1-\sigma)\alpha\eta(1+i)}{(1-\eta)^2 q^* + (1+i)[(1-\eta)(3+i-\eta)-\alpha\sigma\eta]}, \frac{(1-\sigma)\alpha\eta}{2+i(2-\eta)-\alpha\sigma\eta}\right\}$$

Proof. See Appendix A. ∎

4 Endogenous Credit Limits

Consider an alternative payment instrument in the DM - unsecured credit. The buyer can pay for DM goods using unsecured credit that will be redeemed to the seller in the following CM and she can borrow up to her debt limit, $\bar{b}_t$. For simplicity, I assume that the buyer makes a take-it-or-leave-it offer to the seller in the DM, which means the buyer maximizes her surplus subject to the seller's participation constraint. The DM cost function is $c(q) = q$. Suppose the buyer have issued b_t units of unsecured debt in the previous DM trade. The CM value function is

$$\begin{aligned} W_t(m_t, d_t, \ell_t, -b_t) = & \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta G_{t+1}(\hat{m}_{t+1}) \\ \text{s.t. } \phi_t \hat{m}_{t+1} + X_t = & H_t + T_t + \phi_t m_t + (1 + i_{d,t})\phi_t d_t - (1 + i_{l,t})\phi_t \ell_t - b_t, \end{aligned} \tag{31}$$

which is the same as before except that the agent needs to pay or collect the debt. The agent's FM problem is identical to the previous section. Then, $1-\sigma$ fraction of agents will deposit $\hat{m}_{t+1}$, and σ fraction of agents will borrow loan from the bank. The DM value function is

$$V_t^b(m_t + \ell_t, 0, \ell_t) = \alpha[u(q_t) - q_t] + W_t(m_t + \ell_t, 0, \ell_t, 0),$$

where $q_t = \min\{q^*, \bar{b}_t + \phi_t(m_t + \ell_t)\}$. Given $\bar{b}_t$, solving equilibrium yields

$$z_t = \begin{cases} \dfrac{z_{t+1}}{1+i} \left\{ \dfrac{1-\sigma+\sigma\chi}{\chi} \alpha \left[u'(z_{t+1} + \bar{b}_{t+1}) - 1 \right] + 1 \right\} & \text{if } z_{t+1} + \bar{b}_{t+1} < q^* \\ \dfrac{z_{t+1}}{1+i} & \text{if } z_{t+1} + \bar{b}_{t+1} \geq q^*, \end{cases} \tag{32}$$

where $z_{t+1} = (1 - \sigma + \sigma\chi)\phi_{t+1} M_{t+1}/(\sigma\chi)$.

The buyer is captured with probability μ if she reneges. The punishment for a defaulter is permanent exclusion from the DM trade. The value of autarky is $\underline{\mathrm{W}}(0, 0, 0, 0) = \{U(X^*) - X^* + T\}/(1 - \beta)$. The incentive condition for voluntary repayment is

$$\underbrace{-b_t + W_t(0, 0, 0, 0)}_{\text{value of honoring debts}} \geq \underbrace{(1 - \mu)W_t(0, 0, 0, 0) + \mu \underline{\mathrm{W}}(0, 0, 0, 0)}_{\text{value of not honoring debts}}.$$

One can write the debt limit $\bar{b}_t$ as $b_t \leq \bar{b}_t \equiv \mu W_t(0, 0, 0) - \mu \underline{\mathrm{W}}(0, 0, 0)$. Recall the CM value function. Using the solution of FM, we can rewrite the buyer's CM value function as

$$W_t(0,0,0,0) = U(X^*) - X^* + T_t + \beta W_{t+1}(0,0,0,0)$$
$$+ \max_{\hat{m}_{t+1}}\{-\phi_t \hat{m}_{t+1} + \beta\alpha\sigma[u(q_{t+1}) - q_{t+1}] + \beta\phi_{t+1}\hat{m}_{t+1}\},$$

where $q_{t+1} = \min\{q^*, \bar{b}_{t+1} + \phi_{t+1}(1-\sigma+\sigma\chi)\hat{m}_{t+1}/(\sigma\chi)\}$. In the equilibrium, substituting $W_t(0,0,0) = \bar{b}_t/\mu + \underline{\mathrm{W}}(0,0,0)$ yields

$$\frac{\bar{b}_t}{\mu} = -\phi_t M_{t+1} + \beta\alpha\sigma[u(z_{t+1} + \bar{b}_{t+1}) - z_{t+1} - \bar{b}_{t+1}] + \frac{\beta\bar{b}_{t+1}}{\mu} + \beta\phi_{t+1}M_{t+1},$$

where M_{t+1} and z_{t+1} solve (32). Rearranging terms yields

$$\bar{b}_t = \begin{cases} \beta\bar{b}_{t+1} + \dfrac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(z_{t+1} + \bar{b}_{t+1}) & \text{if } z_{t+1} + \bar{b}_{t+1} < q^* \\ \beta\bar{b}_{t+1} + \dfrac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(q^*) & \text{if } z_{t+1} + \bar{b}_{t+1} \geq q^*, \end{cases} \tag{33}$$

where $S(z_{t+1} + \bar{b}_{t+1}) \equiv [u(z_{t+1} + \bar{b}_{t+1}) - z_{t+1} - \bar{b}_{t+1}]$ is the buyer's trade surplus. The equilibrium can be collapsed into a dynamic system satisfying (32)-(33).

The stationary equilibrium falls into one of the three cases: the pure money equilibrium,[6] the pure credit equilibrium, and the money-credit equilibrium. The debt limit at the stationary equilibrium, $\bar{b}$, is a fixed point satisfying $\bar{b} = \Omega(\bar{b})$ where

$$\Omega(\bar{b}) = \begin{cases} \dfrac{\mu\sigma\alpha}{r}[u(\tilde{q}) - \tilde{q}] - \dfrac{i\mu\sigma\chi}{1-\sigma+\sigma\chi}[\tilde{q} - \bar{b}] & \text{if } \tilde{q} > \bar{b} \geq 0 \\ \dfrac{\mu\sigma\alpha}{r}[u(\bar{b}) - \bar{b}] & \text{if } q^* > \bar{b} \geq \tilde{q} \\ \dfrac{\mu\sigma\alpha}{r}[u(q^*) - q^*] & \text{if } \bar{b} \geq q^* \end{cases} \tag{34}$$

where $\tilde{q}$ solves $u'(\tilde{q}) = 1 + i\chi/[\alpha(1-\sigma+\sigma\chi)]$. The DM consumption q_s is determined by $q_s = \min\{q^*, \max\{\tilde{q}, \bar{b}\}\}$. Money and credit coexist if and only if $0 < \mu < \min\{1, \tilde{\mu}\}$, where $\tilde{\mu} \equiv \sigma\{i\chi[(1-\sigma+\sigma\chi)/\tilde{q} - 1] + (\alpha/r)(1-\sigma+\sigma\chi)^2[u(\tilde{q})/\tilde{q} - 1]\}$, because they coexist when $0 < \bar{b} < \tilde{q}$. The DM consumption is decreasing in i in the monetary equilibrium.

Consider the dynamics of equilibria where money and credit coexist. I claim the

[6] If no one can capture the buyer after she reneges, $\mu = 0$, the unsecured credit is not feasible. In this case, the equilibrium will be the pure money equilibrium.

main results from Section 3 - lowering the reserve requirement can induce endogenous cycles - still hold even after unsecured credit is introduced. One can establish the conditions for two-period cycles, three-period cycles, and chaotic dynamics. For compact notation, let $\iota \equiv \max\{i, r\}$ and $w_j \equiv z_j + \bar{b}_j$. The following proposition summarizes the results.

Proposition 5 (**Monetary Cycles with Unsecured Credit**). *There exists a two-period cycle of money and credit with $w_1 < q^* < w_2$ if $\chi \in (0, \chi_c)$, where*

$$\chi_c \equiv \frac{(1-\sigma)\alpha\left[u'\left(\frac{q^*}{1+\iota}\right)-1\right]}{(1+i)^2 - 1 - \sigma\alpha\left[u'\left(\frac{q^*}{1+\iota}\right)-1\right]}.$$

There exists a three-period cycle of money and credit with $w_1 < q^* < w_2 < w_3$, if $\chi \in (0, \hat{\chi}_c)$, where

$$\hat{\chi}_c \equiv \frac{(1-\sigma)\alpha\left[u'\left(\frac{q^*}{1+\iota}\right)-1\right]}{(1+i)^3 - 1 - \sigma\alpha\left[u'\left(\frac{q^*}{1+\iota}\right)-1\right]}.$$

Proof. See Appendix A. ■

5 Empirical Evaluation: Inside Money Volatility

In the previous sections, the theoretical results show that lowering the required reserve ratio can induce instability. To evaluate the model prediction, this section examines whether the required reserve ratio is associated with the cyclical volatility of the real balance of the inside money.

Following Jaimovich and Siu (2009) and Carvalho and Gabaix (2013), I measure the cyclical volatility in quarter t as the standard deviation of a filtered log real total checkable deposit during a 41-quarter (10-year) window centered around quarter t. Total checkable deposits are from the H.6 Money Stock Measures release published by the Federal Reserve Board and converted to real value using the Consumer Price Index (CPI). Seasonally adjusted series are used to smooth the seasonal fluctuation. I adopt the Hodrick-Prescott (HP) filter with a 1600 smoothing parameter as standard. To construct an annual series, quarterly observations are averaged for each year. The sample period is from 1960:I to 2018:IV so that there are annual series from 1965 to 2013. To check whether the results are sensitive to different measures of price level, I also use the core CPI, the Personal Consumption Expenditures (PCE), and the core PCE to transform the total checkable deposit into real value.

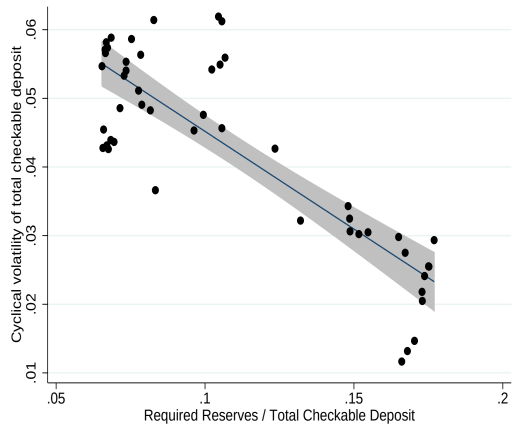

Figure 4: Scatter Plot for Inside Money Volatility and Required Reserve Ratio

The legal reserve requirement for the demand deposits has been 10% since April 2, 1992. However, the Federal Reserve imposes different reserve requirements depending on the size of a bank's liability. These criteria have changed over time. For example, during 1992:Q1-2019:Q4, this changed 27 times. To consider these changes, I divide the required reserves by total checkable deposits to compute the required reserve ratio.

Figure 4 presents a scatter plot of the cyclical volatility of the real inside money balance and the required reserve ratio. Column (1) of Table 1 reports its regression estimates with Newey-West standard errors. The plot and estimates show a negative relationship between the cyclical volatility of the real inside money balance and the required reserve ratio with statistically significant regression coefficients. However, this result can be driven by a spurious regression. Table 2 provides unit root test results for federal funds rates, required reserve ratio, and cyclical volatility of inside money. Both Augmented Dickey-Fuller test and Phillips-Perron test fail to reject the null hypothesis of unit root for these series, whereas they reject the null hypothesis of unit root at their first difference. In addition to that, the Johansen cointegration test in Column (1), suggest that there is no cointegration relationship between two variables. So it is hard to rule out that Column (1)'s results are driven by a spurious regression.

To overcome this spurious relationship issue, I adopt the cointegrating regression with an additional variable, the federal funds rate. Column (2) of Table 1 provides Johansen cointegration test results for federal funds rates, required reserves, and cyclical volatility of inside money. The trace test suggests a cointegration relationship among these three variables, which is consistent with the theoretical result: The instability

Table 1: Effect of Required Reserve Ratio

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: σ_t^{Roll}	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
χ	-0.283^{***}	-0.245^{***}	-0.267^{***}	-0.221^{***}	-0.306^{***}	-0.227^{***}	-0.307^{***}	-0.220^{***}
	(0.027)	(0.002)	(0.027)	(0.003)	(0.029)	(0.004)	(0.027)	(0.005)
`ffr`		-0.109^{***}		-0.125^{***}		-0.187^{***}		-0.207^{***}
		(0.002)		(0.003)		(0.004)		(0.004)
Constant	0.074^{***}	0.074^{***}	0.070^{***}	0.071^{***}	0.074^{***}	0.075^{***}	0.073^{***}	0.073^{***}
	(0.003)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)
Obs.	49	49	49	49	49	49	49	49
$adjR^2$	0.700	0.621	0.728	0.648	0.740	0.650	0.764	0.665
$\lambda_{trace}(r=0)$	**9.807**	35.688	**9.120**	35.145	**9.109**	35.367	**8.593**	35.028
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	3.324	**10.682**	2.839	**10.065**	2.723	**9.894**	2.417	**9.345**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), (3), (5) and (7), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2), (4), (6), and (8), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; χ denotes the required reserve ratio, `ffr` denotes federal funds rates and σ_t^{Roll} denotes the cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 2: Unit Root Tests

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`		-6.766	-1.704	-2.362
χ		-1.492	-1.173	-1.341
σ_t^{Roll}	(CPI)	-4.708	-2.191	-2.090
σ_t^{Roll}	(Core CPI)	-4.681	-2.189	-1.978
σ_t^{Roll}	(PCE)	-4.329	-2.038	-2.047
σ_t^{Roll}	(Core PCE)	-4.076	-1.954	-1.930
Δ`ffr`		-28.373^{***}	-5.061^{***}	-6.357^{***}
$\Delta\chi$		-31.818^{***}	-4.802^{***}	-3.693^{***}
$\Delta\sigma_t^{Roll}$	(CPI)	-24.905^{***}	-3.416^{**}	-2.942^{**}
$\Delta\sigma_t^{Roll}$	(Core CPI)	-24.758^{***}	-3.509^{**}	-2.942^{**}
$\Delta\sigma_t^{Roll}$	(PCE)	-23.691^{***}	-3.330^{**}	-2.842^{*}
$\Delta\sigma_t^{Roll}$	(Core PCE)	-22.826^{***}	-3.296^{**}	-2.768^{*}

Note: `ffr` denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

depends on the reserve requirement and the interest rate. With the cointegration relationship, we may not have to worry about a spurious relationship. Columns (2) of Table 1 report the estimates for the cointegrating relationship. Because of the potential bias from long-run variance, I estimate a canonical cointegrating regression (CCR) for Column (2) of Table 1. The estimates are statistically significant with a sizeable level and consistent with the prediction from the model.

To check the sensitivity of the results, I redo all the analyses using the core CPI, the Personal Consumption Expenditures (PCE), and the core PCE to transform the total checkable deposit into real value. Columns (3), (5), and (7) of Table 1 regress required reserve ratio on the inside money volatility and report its Newey-West standard errors. They also report the trace test statistics of Johansen cointegration test between these two variables. The results are consistent with the benchmark case in Columns (1). Columns (4), (6), and (8) of Table 1 report CCR estimates regressing the required reserve ratio and federal funds rate on the inside money volatility and the trace test statistics of Johansen cointegration test between these three variables. All the results are also consistent with the benchmark case in Columns (2).

Appendix B includes more sensitivity analyses: (1) Using quarterly series instead of annual series; (2) Using time series before 2008. All the results are not sensitive with respect to different frequency and time period.

6 Conclusion

The goal of this paper is to examine the (in)stability of fractional reserve banking. To that end, this paper builds a simple monetary model of fractional reserve banking that can capture the conditions for (in)stability under different specifications. The baseline model and its extension establish the conditions for endogenous cycles and chaotic dynamics. The model also features stochastic cycles and self-fulfilling boom and bust under explicit conditions. The model shows that fractional reserve banking can endanger stability in the sense that equilibrium is more prone to exhibit cyclic, chaotic, and stochastic dynamics under lower reserve requirements. This is due to the amplified liquidity premium. This result holds in the extended model with unsecured credit. However, lowering the reserve requirement increases the welfare at the steady state.

This paper also provides some empirical evidence that is consistent with the prediction of the model. I test the association between the required reserves ratio and the real inside money volatility using cointegrating regression. I find a significant negative relationship between the two variables. Both theoretical and empirical evidence find a link between the reserve requirement policy and (in)stability.

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Appendix

Appendix A Proofs

Proof of the Existence of a Two-period Monetary Cycle. Let $f^2(z) = f \circ f(z)$. With given the unique steady state, $f(z) > z$ for $z < z_s$ and $f(z) < z$ for $z > z_s$. Because $f(z)$ is linear increasing function for $z > p^*$, there exist a $\tilde{z} > p^*$ s.t $f(\tilde{z}) > p^*$. Since $\tilde{z} > p^*$ and $f(\tilde{z}) < \tilde{z}$, $\tilde{z}$ satisfies $f^2(\tilde{z}) < f(\tilde{z}) < \tilde{z}$. We can write slope of $f^2(z)$ as follows.

$$\frac{\partial f^2(z)}{\partial z} = f'[f(z)]f'(z) = f'(z)f'(z) = [f'(z)]^2$$

which implies $\partial f^2(z)/\partial z > 1$ when $f(z) < -1$. And it is easy to show $\partial f^2(0)/\partial z > 0$. With given $i > 0$ and $\chi > 0$, there exist a (z_1, z_2), satisfying $0 < z_1 < z_s < z_2$ which are fix points for $f^2(z)$ ■

Proof of Proposition 2. When DM trade is based on take-it-or-leave-it offer from buyer to seller with $c(q) = q$ and $-qu''(q)/u' = \eta$, f' can be written as

$$f'(q) = \frac{1}{1+i}\left\{\frac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u''(q)q + u'(q) - 1\right] + 1\right\} < -1$$

Using $u''(q)q = -\eta u'(q)$ gives

$$\frac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u'(q)(1-\eta) - 1\right] + 1 < -(1+i)$$

where $u'(q) = 1 + \frac{i\chi}{\alpha(1-\sigma+\sigma\chi)}$. Substituting $u'(q)$ and rearranging terms give

$$0 < \chi < \frac{\alpha\eta(1-\sigma)}{\eta(1-\alpha\sigma) + (2-\eta)(1+i)}$$

■

Proof of Corollary 1: Let $z_1 < z_2 < ...z_n$ be the periodic points of a n-cycle. Suppose $z_j > z_s$ for all $j = 1, 2, ..n$. By the definition of a n-period cycle, $z_1 = f(z_n) < z_n$ since $f(z) < z$ for $z > z_s$.

$$z_n = f(z_{n-1}) < z_{n-1} = f(z_{n-2}) < z_{n-2}... < z_1.$$

which shows the contradiction implying at least one periodic point satisfies $z_t < z_s < p^*$. ■

Proof of Proposition 3. I divide three period cycles into two cases.
Case 1: Let there exists a three-period cycle satisfying $z_1 < z_s < p^* < z_2 < z_3$. Since $z_2, z_3 > p^*$, we have $z_2 = (1+i)z_1$, $z_3 = (1+i)z_2 = (1+i)^2 z_1$. Using (24) with $z_1 < p^*$ gives

$$\chi = \frac{(1-\sigma)\alpha L(z_1)}{(1+i)^3 - 1 - \sigma\alpha L(z_1)} \tag{35}$$

This three-period cycle should satisfy $z_1 < z_s < p^*$ and $z_2 = (1+i)z_1 > p^*$. First one can be easily shown using

$$0 = L(p^*) < L(z_s) = \frac{i}{\alpha(1-\sigma+\sigma\chi)}\chi < \frac{(1+i)^3 - 1}{\alpha(1-\sigma+\sigma\chi)}\chi = L(z_1)$$

since we have $L'(\cdot) < 0$. Because $dz_1/d\chi < 0$, the latter one, $z_1 > p^*/(1+i)$, is held when

$$0 < \chi < \frac{(1-\sigma)\alpha L\left(\frac{p^*}{1+i}\right)}{(1+i)^3 - 1 - \sigma\alpha L\left(\frac{p^*}{1+i}\right)}.$$

Case 2: Let there exists a three-period cycle satisfying $z_1 < z_2 < p^* \leq z_3$. Since

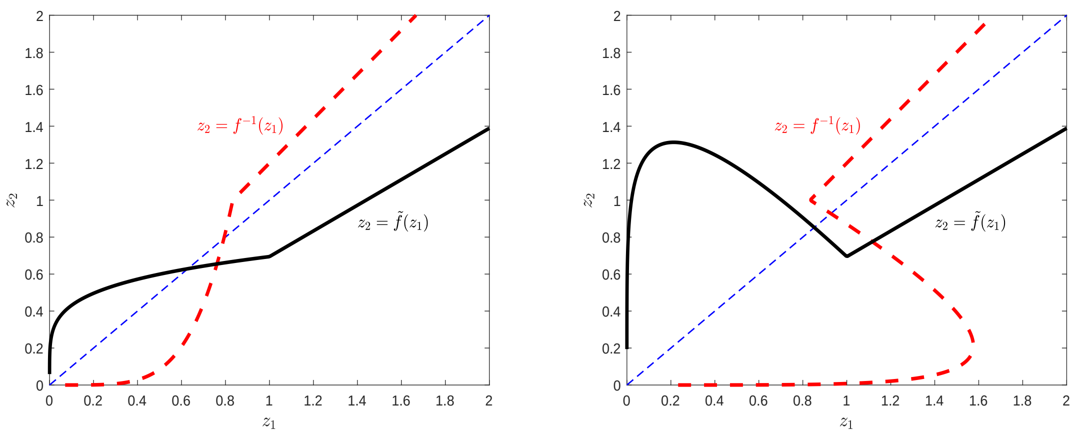

Figure 5: Intersection of $\tilde{f}(z)$ and $f(z)$

$z_3 > p^*$, we have $z_3 = z_2(1+i)$ and (z_2, z_1) solves (36)-(37).

$$z_1 = f(z_2) = \left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_2) + 1\right]\frac{z_2}{1+i} \tag{36}$$

$$z_2 \equiv \tilde{f}(z_1) = \left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_1) + 1\right]\frac{z_1}{(1+i)^2}. \tag{37}$$

These functions satisfies $f(x) > x$ for $x < z_s$, $f(x) < x$ for $x > z_s$, $\tilde{f}(x) > x$ for $x < \tilde{z}$ and $\tilde{f}(x) < x$ for $x > \tilde{z}$ where $\tilde{z}$ solves $\tilde{z} = \tilde{f}(\tilde{z})$. One can easily show $\tilde{z} < z_s$. Therefore any intersection between $z_1 = f(z_2)$ and $z_2 = \tilde{f}(z_1)$ satisfies $z_1 > z_2$ which contradicts to our initial conjecture $z_1 < z_2$. This implies there is no three-period cycles satisfying $z_1 < z_2 < p^* \leq z_3$. Therefore we can conclude that a three-period cycle exist when

$$0 < \chi < \frac{(1-\sigma)\alpha L\left(\frac{p^*}{1+i}\right)}{(1+i)^3 - 1 - \sigma\alpha L\left(\frac{p^*}{1+i}\right)}.$$

The existence of a three-cycle implies the existence of cycles of all orders and chaotic dynamics by the Sarkovskii theorem (Sharkovskii, 1964) and the Li-Yorke theorem (Li and Yorke, 1975). ■

Proof of Proposition 4. Consider $z_t = f(z_{t+1})$. If $z_s > \bar{z}$ where $\bar{z}$ solves $f'(\bar{z}) = 0$. In this case, there exist multiple equilibria. If $q^* \leq f(\bar{z})$, then there exist equilibria $\{z_t\}_{t=0}^{\infty}$ with $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > q^*$ (bubble) which crashes to 0 (burst) as $t \to \infty$, where $T \geq 1$ and $z_T > z_0$. Then there exist equilibria with bubble-burst as a self-fulfilling crisis. Conditions for this case are shown as below. Similar to Corollary 2, consider take-it-leave-it offer with $-qu''/u' = \eta$ and $c(q) = q$. Then we have following difference equation:

$$z_t = f(z_{t+1}) \begin{cases} \dfrac{z_{t+1}}{1+i}\left\{\dfrac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u'(z_{t+1}) - 1\right] + 1\right\} & \text{if } z_{t+1} < q^* \\ \dfrac{z_{t+1}}{1+i} & \text{if } z_{t+1} \geq q^* \end{cases} \tag{38}$$

Step 1: [Concavity of f, $f'' < 0$] For $z_{t+1} < q^*$ and $\eta \in (0,1)$, $f''(z_{t+1}) < 0$ since

$$\begin{aligned} f''(z) &= \frac{\alpha(1-\sigma+\sigma\chi)}{(1+i)\chi}u'' + \frac{z}{1+i}\frac{\alpha(1-\sigma+\sigma\chi)}{\chi}\alpha[-u''\eta z - u'\eta] \\ &= \frac{\alpha(1-\sigma+\sigma\chi)}{(1+i)\chi}u''[1+q-\eta q] < 0 \end{aligned}$$

Step 2: [Multiplicity i.e., $z_s > \bar{z}$ where $\bar{z}$ solves $f'(\bar{z}) = 0$] Consider the following condition.

$$f'(\bar{z}) = \frac{\alpha(1-\sigma+\sigma\chi)}{\chi}[u'(\bar{z})(1-\eta)-1] = 0$$

Since $z_s > \bar{z} \to u'(z_s) < u'(\bar{z})$, we have

$$u'(z_s) = 1 + \frac{i\chi}{\alpha(1-\sigma+\sigma\chi)} < 1 - \frac{(2+i)\chi}{\alpha(1-\sigma+\sigma\chi)} = u'(\bar{z}).$$

This can be reduced as

$$\chi < \frac{(1-\sigma)\alpha\eta}{2+i(2-\eta)-\alpha\sigma\eta}$$

Step 3: [Show $q^* \leq f(\bar{z})$] It is straightforward to show that $q^* < f(\bar{z})$ holds when

$$\chi < \frac{(1-\sigma)\alpha\eta(1+i)}{(1-\eta)^2 q^* + (1+i)[(1-\eta)(3+i-\eta)-\alpha\sigma\eta]}$$

Therefore, when

$$0 < \chi < \min\left\{\frac{(1-\sigma)\alpha\eta(1+i)}{(1-\eta)^2 q^* + (1+i)[(1-\eta)(3+i-\eta)-\alpha\sigma\eta]}, \frac{(1-\sigma)\alpha\eta}{2+i(2-\eta)-\alpha\sigma\eta}\right\}$$

there exist $\{z_t\}_{t=0}^{\infty}$ satisfying $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > q^*$ and $\lim_{t\to\infty} z_t = 0$, where $T \geq 1$ and $z_T > z_0 > q^*/(1+i)$. ■

Proof of Proposition 5. A two period cycle result is presented and three-period case will follow. Let there exists a two-period cycle satisfying $w_1 < q^* < w_2$ where $w_j = z_j + \bar{b}_j$. Since $w_2 > q^*$, we have $z_2 = (1+i)z_1$ and $\bar{b}_2 = (1+r)\bar{b}_1$ where q_1, $\bar{b}_1$, and z_1 solve

$$u'(q_1) = 1 + \chi\frac{(1+i)^2-1}{\alpha(1-\sigma+\sigma\chi)}, \quad \bar{b}_1 = [(1+r)^2-1]^{-1}\left\{\frac{i\mu\sigma\chi}{1-\sigma+\sigma\chi}\left[1-\frac{(1+i)^2}{\beta}\right]z_1 + \mu\alpha\sigma[u(q_1)-q_1]\right\}$$

and $z_1 = q_1 - \bar{b}_1$. This two-period cycle should satisfy $q_1 < q^*$ and $w_2 = (1+i)z_1 + (1+r)\bar{b}_1 > q^*$. For given $i > 0$ and $\chi > 0$, first one can be easily shown using

$$1 = u'(q^*) < u'(q_s) = 1 + \frac{i}{\alpha(1-\sigma+\sigma\chi)}\chi < 1 + \frac{(1+i)^2-1}{\alpha(1-\sigma+\sigma\chi)}\chi = u'(q_1)$$

since we have $u''(\cdot) < 0$. Now we also can check the latter using the below conditions

$$\begin{aligned}(1+r)q_1 >& (1+i)z_1 + (1+r)\bar{b}_1 = w_2 > q^* > q_1 = z_1 + \bar{b}_1 \quad \text{if } r > i \\ (1+i)q_1 >& (1+i)z_1 + (1+r)\bar{b}_1 = w_2 > q^* > q_1 = z_1 + \bar{b}_1 \quad \text{if } i > r.\end{aligned}$$

The sufficient conditions to have $w_2 > q^*$ is $q_1 > q^*/(1+r)$ for $r > i$ and $q_1 > q^*/(1+i)$ for $i > r$. Since we have $dq_1/d\chi < 0$, there exist a three period cycle $q_1 = w_1 < q_s < q^* < w_2 < w_3$ when

$$0 < \chi < \frac{(1-\sigma)\alpha[u'\left(\frac{q^*}{1+\iota}\right) - 1]}{(1+i)^2 - 1 - \sigma\alpha[u'\left(\frac{q^*}{1+\iota}\right) - 1]}$$

where $\iota = \max\{i, r\}$. Now, let there exists a three-period cycle satisfying $q_1 = w_1 < q_s < q^* < w_2 < w_3$ where $w_j = z_j + \bar{b}_j$. Since w_3, $w_2 > q^*$, we have $z_2 = (1+i)z_1$, $z_3 = (1+i)^2 z_1$, $\bar{b}_2 = (1+r)\bar{b}_1$ and $\bar{b}_3 = (1+r)^2\bar{b}_1$ where q_1, $\bar{b}_1$, and z_1 solve

$$u'(q_1) = 1+\chi\frac{(1+i)^3 - 1}{\alpha(1-\sigma+\sigma\chi)}, \quad \bar{b}_1 = [(1+r)^3 - 1]^{-1}\left\{\frac{i\mu\sigma\chi}{1-\sigma+\sigma\chi}\left[1 - \frac{(1+i)^2}{\beta}\right]z_1 + \mu\alpha\sigma[u(q_1) - q_1]\right\}$$

and $z_1 = q_1 - \bar{b}_1$. This three-period cycle should satisfy $q_1 < q_s < q^*$ and $w_2 = (1+i)z_1 + (1+r)\bar{b}_1 > q^*$. For given $i > 0$ and $\chi > 0$, first one can be easily shown using

$$1 = u'(q^*) < u'(q_s) = 1 + \frac{i}{\alpha(1-\sigma+\sigma\chi)}\chi < 1 + \frac{(1+i)^3 - 1}{\alpha(1-\sigma+\sigma\chi)}\chi = u'(q_1)$$

since we have $u''(\cdot) < 0$. Now we also can check the latter using below conditions

$$\begin{aligned}(1+r)q_1 >& (1+i)z_1 + (1+r)\bar{b}_1 = w_2 > q^* > q_1 = z_1 + \bar{b}_1 \quad \text{if } r > i \\ (1+i)q_1 >& (1+i)z_1 + (1+r)\bar{b}_1 = w_2 > q^* > q_1 = z_1 + \bar{b}_1 \quad \text{if } i > r.\end{aligned}$$

The sufficient conditions to have $w_2 > q^*$ is $q_1 > q^*/(1+r)$ for $r > i$ and $q_1 > q^*/(1+i)$ for $i > r$. Since we have $dq_1/d\chi < 0$, there exist a three period cycle $q_1 = w_1 < q_s < q^* < w_2 < w_3$ when

$$0 < \chi < \frac{(1-\sigma)\alpha[u'\left(\frac{q^*}{1+\iota}\right) - 1]}{(1+i)^3 - 1 - \sigma\alpha[u'\left(\frac{q^*}{1+\iota}\right) - 1]}$$

where $\iota = \max\{i, r\}$. Again, the existence of a three-cycle implies the existence of cycles of all orders and chaotic dynamics by the Sarkovskii theorem and the Li-Yorke theorem. ■

Appendix B Empirical Appendix

This section provides robustness checks for empirical results. To check the sensitivity of the results, the following results repeat all the empirical analysis using quarterly series instead of annual data. This section also provides robustness checks using time-series before 2008.

Table 3: Effect of Required Reserve Ratio:Robustness Check (Quarterly)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent variable: σ_t^{Roll}	OLS (1)	CCR (2)	OLS (3)	CCR (4)	OLS (5)	CCR (6)	OLS (7)	CCR (8)
χ	-0.282^{***}	-0.452^{***}	-0.266^{***}	-0.400^{***}	-0.305^{***}	-0.485^{***}	-0.306^{***}	-0.476^{***}
	(0.016)	(0.001)	(0.014)	(0.003)	(0.015)	(0.000)	(0.014)	(0.006)
`ffr`		-0.050^{***}		-0.058^{***}		-0.015^{***}		-0.047^{***}
		(0.000)		(0.002)		(0.000)		(0.005)
Constant	0.074^{***}	0.085^{***}	0.070^{***}	0.079^{***}	0.074^{***}	0.089^{***}	0.073^{***}	0.086^{***}
	(0.002)	(0.000)	(0.002)	(0.000)	(0.002)	(0.000)	(0.002)	(0.001)
Obs.	196	196	196	196	196	196	196	196
$adjR^2$	0.696	0.240	0.725	0.263	0.737	0.222	0.761	0.268
$\lambda_{trace}(r=0)$	**9.496**	31.950	**11.045**	33.808	**10.930**	34.481	**12.103**	35.951
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	1.677	**11.162**	1.959	**12.266**	1.938	**12.094**	1.887	**12.485**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; `ffr` denotes federal funds rates and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 4: Unit Root Tests:Robustness Check (Quarterly)

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`		-8.611	-1.956	-2.183
χ		-1.335	-1.145	-1.199
σ_t^{Roll}	(CPI)	-4.320	-2.062	-1.554
σ_t^{Roll}	(Core CPI)	-4.388	-2.201	-1.924
σ_t^{Roll}	(PCE)	-3.822	-1.946	-1.868
σ_t^{Roll}	(Core PCE)	-3.565	-1.928	-2.023
Δ`ffr`		-139.701^{***}	-10.792^{***}	-10.288^{***}
$\Delta\chi$		-163.796^{***}	-12.272^{***}	-9.909^{***}
$\Delta\sigma_t^{Roll}$	(CPI)	-23.132^{***}	-2.604^{*}	-3.576^{***}
$\Delta\sigma_t^{Roll}$	(Core CPI)	-30.423^{***}	-3.544^{***}	-4.894^{***}
$\Delta\sigma_t^{Roll}$	(PCE)	-24.507^{***}	-2.874^{*}	-4.362^{***}
$\Delta\sigma_t^{Roll}$	(Core PCE)	-28.054^{***}	-3.373^{**}	-5.138^{***}

Note: `ffr` denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 5: Effect of Required Reserve Ratio:Robustness Check (pre-2008)

Price level	CPI		Core CPI		PCE		Core PCE	
Dependent	OLS	CCR	OLS	CCR	OLS	CCR	OLS	CCR
variable: σ_t^{Roll}	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
χ	-0.266^{***}	-0.297^{***}	-0.266^{***}	-0.268^{***}	-0.307^{***}	-0.288^{***}	-0.305^{***}	-0.277^{***}
	(0.030)	(0.001)	(0.030)	(0.001)	(0.032)	(0.002)	(0.029)	(0.002)
`ffr`		-0.107^{***}		-0.124^{***}		-0.189^{***}		-0.210^{***}
		(0.001)		(0.001)		(0.002)		(0.002)
Constant	0.070^{***}	0.080^{***}	0.070^{***}	0.076^{***}	0.074^{***}	0.082^{***}	0.072^{***}	0.080^{***}
	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.000)	(0.004)	(0.002)
Obs.	43	43	43	43	43	43	43	43
$adjR^2$	0.727	0.659	0.727	0.710	0.739	0.708	0.759	0.734
$\lambda_{trace}(r=0)$	**8.373**	32.228	**7.438**	31.299	**7.661**	31.867	**6.897**	31.250
5% CV	15.41	29.68	15.41	29.68	15.41	29.68	15.41	29.68
$\lambda_{trace}(r=1)$	1.504	**9.554**	1.125	**8.428**	1.146	**8.603**	0.938	**7.693**
5% CV	3.76	15.41	3.76	15.41	3.76	15.41	3.76	15.41

Note: For (1), OLS estimates are reported, and Newey-West standard errors with lag 1 are reported in parentheses. For (2) and (3), first-stage long-run variance estimations for CCR are based on the quadratic spectral kernel and Bayesian information criterion. The bandwidth selection is based on Newey-West fixed lag, $4 \times (T/100)^{2/9}$; `ffr` denotes federal funds rates and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.

Table 6: Unit Root Tests:Robustness Check (pre-2008)

		Phillips-Perron test		ADF test
		$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`		-9.476	-2.258	-2.868^{**}
χ		-0.768	-0.660	-0.877
σ_t^{Roll}	(CPI)	-2.966	-1.738	-1.770
σ_t^{Roll}	(Core CPI)	-2.860	-1.641	-1.495
σ_t^{Roll}	(PCE)	-2.662	-1.515	-1.627
σ_t^{Roll}	(Core PCE)	-2.412	-1.371	-1.400
Δ`ffr`		-25.378^{***}	-4.773^{***}	-5.833^{***}
$\Delta\chi$		-28.208^{***}	-4.594^{***}	-3.658^{***}
$\Delta\sigma_t^{Roll}$	(CPI)	-25.627^{***}	-4.281^{***}	-3.813^{***}
$\Delta\sigma_t^{Roll}$	(Core CPI)	-25.836^{***}	-4.329^{***}	-3.764^{***}
$\Delta\sigma_t^{Roll}$	(PCE)	-24.420^{***}	-4.101^{***}	-3.594^{**}
$\Delta\sigma_t^{Roll}$	(Core PCE)	-23.848^{***}	-4.034^{***}	-3.464^{**}

Note: `ffr` denotes federal funds rates, χ denotes required reserve ratio, and σ_t^{Roll} denotes cyclical volatility of real inside money balances. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively.