Money Creation and Banking: Theory and Evidence*

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First Draft: August, 2019
This Draft: October, 2025

Abstract

This paper studies the role of banks' money creation in monetary transmission. I develop a monetary-search model where demand for the monetary base and the money multiplier are endogenously determined through banks' money creation. The model and data show that reserves are not independent of interest rate policy, even with ample reserves. Furthermore, short-term rates and interest on reserves play distinct roles in monetary transmission, and downward sloping reserve demand still plays a role even in the ample reserve regime. I evaluate the theory by matching it with data, and the calibrated model can account for the evolution of reserves, excess reserves, and the money multiplier from 1968 to 2017. The results imply that the Fed's interest rate policy and balance sheet policy are not separate policies.

JEL Classification Codes: E42, E51

Keywords: Money, Banking, Interest on Reserves, Monetary Policy, Money Multiplier

*This paper is a revised version of Chapter 1 of my PhD dissertation at the University of Missouri. I am deeply indebted to my advisor, Chao Gu, for her guidance at all stages of this research project. I am grateful to my committee members, Joseph Haslag, Aaron Hedlund, and Xuemin (Sterling) Yan for helpful comments. I also thank Lukas Altermatt, Borağan Aruoba, Lonnie Hofmann, Dong Ho Kang, Duhyeong Kim, Sunham Kim, Shawn Ni, Sungmin Park, William Park, and Christian Wipf as well as seminar participants at the 2019 MVEA Annual Meeting, the 2019 KIEA Annual Meeting, the 2020 SEA Annual Meeting, the Bank of Korea, the 2021 WEAI Conference, the 2021 KER International Conference, University of Basel, and University of Mississippi for useful feedback and discussion. The views expressed in this paper are solely those of the author. All remaining errors are mine.

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> [A] model of the banking system in which currency, reserves, and deposits play distinct roles ... seems essential if one wants to consider policies like reserves requirements, interest on deposits, and other measures that affect different components of the money stock differently. **Lucas (2000)**

1. Introduction

This paper develops a theory of money and banking that articulates banks' role in money creation and its interaction with credit. The focal points are the endogenous determination of demand for base money through banking, and the central bank's control of base money to peg the short-term interest rate, which in turn influences the bank's money creation activity and other macroeconomic variables.

The central bank conducts monetary policy through interventions in the market for base money. However, most leading models of monetary policy analysis do not consider the transmission mechanism through the market for the monetary base. For instance, New Keynesian models abstract from the money market mechanism. Moreover, many models assert the independence of the quantity of reserves from interest management policy, especially since the Federal Reserve (Fed) started paying interest on reserves (IOR). In the New Monetarist framework, where monetary frictions are explicit, most models focus on monetary aggregates rather than the transmission mechanism from the market for the monetary base to the monetary aggregate through the banking sector. Instead of abstracting from the key mechanism, this paper revisits the issue of money creation to understand the role of banking in monetary transmission and to account for a number of observations within a unified framework.

Specifically, Section 2 provides or revisits four empirical observations from US data, which guide the modeling.

1. After 2008, interest on reserves is not a floor for short-term interest rates, and neither short-term interest rates nor interest on reserves are independent of the amount of reserves. These rates are related to the quantity of reserves and banks' money creation.
2. The excess reserves to deposit ratio had been close to zero until 2007. The excess reserve ratio skyrocketed when the Fed introduced interest on reserves.
3. The required reserve ratio does not have a negative relationship with the M1 money multiplier, and there were two structural breaks in the evolution of the M1 money multiplier: 1992 and 2008.
4. Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping and stable M1 money demand.

The first observation challenges the conventional approach that assumes independence between the quantity of reserves and the interest rate in the post-2008 era. Contrary to this widely used assumption, which has been a cornerstone in many unconventional monetary policy analyses, this observation implies that the quantity of reserves still plays a significant role in interest rate management. This is consistent with the empirical finding from Smith (2019). Also, while there are wide spread perception that money multiplier became irrelevant after 2008, the observation suggest that it is still related to the conduct of monetary policy.

The second observation, which was already known to many economists, is worth emphasizing again, especially in light of the first observation. This second observation used to be considered well understood. The standard interpretation is that as the Fed pays interest on reserves, the short-term rates are determined by the IOR rate via arbitrage, and banks hold reserves not because of legal restrictions but because it is profitable to do so. According to the popular argument, this allow the Fed to the increase the reserves without changing its policy rate, and the Fed increased the supply of reserves drastically by conducting quantitative easing. This standard interpretation assumes that this dramatic reserve increase was independent of IOR. This may lead to the following questions. Are reserves really independent of the IOR rate when banks hold large excess reserves to earn IOR? Given that the first observation suggests IOR and the short-term rate are still related to reserves, how can we reconcile the drastic increase in reserves with the changes in interest rate? The model in this paper addresses these questions.

The third finding contradicts the undergraduate textbook theory of the money multiplier which predicts a negative relationship between the money multiplier and the required reserves ratio. Since the textbook theory assumes zero excess reserves, the absence of a negative relationship may not be a surprising finding during the post-2008 period with excess reserves. However, there is no negative relationship during the pre-2008 period as well. Finally, the last finding suggests that considering the role of unsecured credit could be essential to understand money demand. This paper advances a theory of money and banking, which accounts for all four observations mentioned above as well as the money creation process in the US economy.

This paper builds a monetary banking model based on Lagos and Wright (2005) to understand the monetary transmission. The model features the explicit structure of monetary exchange and the role of financial intermediation in money creation. Agents can trade by using cash, transaction deposits, and unsecured credit. Banks create deposits by making loans, which can be either transaction deposits or non-transaction deposits. However, the creation of transaction deposits is constrained by reserve requirements. Given the monetary policy and credit conditions of the economy, banks determine their holdings of excess reserves and loans, and thus, the money multiplier is determined endogenously.

The model is capable of explaining how the amount of reserves is linked to interest rate management, regardless of whether banks hold excess reserves or not. Central to this framework are the distinct roles of interest on reserves and short-term interest rates, as well as the interaction between money and credit in reserve quantity determination. This approach enables the model to answer why banks are holding excess reserves when they did not before 2008. By incorporating these properties, the framework successfully explains the above four observations, both qualitatively and quantitatively.

In the model, when banks hold excess reserves, their reserve requirement constraint does not bind, and a change in reserve requirements does not change the money multiplier. Instead, the money multiplier is determined by the short-term policy rate and the interest on reserves. Unlike many recent papers that assume independence of reserve amounts from the policy rate, the model still features downward-sloping reserve demand even when the banks are holding excess reserves. Introducing and increasing interest on reserves shifts the reserve-demand curve upward and makes its lower segment more elastic while still exhibiting a downward slope. Therefore, lowering the short-term policy rate increases reserves, but the banks do not create deposit money proportionally, which lowers the money multiplier. Higher interest on reserves decreases the money multiplier because banks have a greater incentive to hold reserves and a weaker incentive to create deposit money. The interest rate on reserves and the short-term rate play distinct roles, and they jointly determine the quantity of reserves. These rates also have different impacts on the lending rate. An increase in the short-term rate raises the lending rate, while an increase in the interest on reserves lowers the lending rate.

Another ingredient of the model is unsecured credit which can substitute for other means of payment as in Gu, Mattesini and Wright (2016).[1] Better credit conditions lower transaction deposit balances as credit can substitute for money. This decrease in deposit balances leads to a lower money multiplier, regardless of whether banks hold excess reserves or not. It also reduces reserves as it decreases M1 money balances. This is important in the model because reserves are linked to the short-term rate through the money market.

Next, I quantify the model by calibrating the model and asking how it accounts for observations in Section 2 and the money creation process. Given monetary policy behaved as it did, how well can the model account for the behavior of reserves, excess reserves, and money multiplier? The analysis shows the model can explain the historical evolution of the money

[1]By modeling unsecured credit with an exogenous credit limit, this paper follows Gu et al. (2016). For other approaches to introducing credit to the monetary economy, see Sanches and Williamson (2010), Lotz and Zhang (2016), and Williamson (2016).

creation process including all the observations from Section 2. Consistent with data, the model generates zero excess reserves between 1968 and 2007, as well as massive increases in excess reserves after 2008. The model provides the counterfactual reserves to output ratio, which closely tracks its actual behavior from 1968 to 2015. It also generates drops in the money multiplier during the 1990s and 2000s without excess reserves, and its more drastic drops after 2008, which were accompanied by a massive increase in excess reserves.

The quantitative exercise shows that dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves whereas the decrease in the money multiplier before 2008 is driven by enhanced credit conditions. Contrary to previous approaches that assumed no changes in the short-term policy rate during 2009-2015 (so-called the zero-lower-bound period) and merely focused on the quantity of reserves, this study confirms that the short-term interest rate did change during that period, and its movements were directly reflected in the quantity of reserves.

Related Literature This paper contributes to three strands of literature. First, it contributes to the growing literature on unconventional monetary policy and bank reserves. Many models take the zero lower bound as given and study asset purchases financed by reserves under the assumption that reserves are independent of short-term interest rates (Curdia and Woodford, 2011, Gertler and Karadi, 2011, Lee, 2021). A prominent strand interprets quantitative easing as maturity transformation. QE effectively shortens the maturity of government debt by altering the asset composition of the public sector's balance sheet (e.g., Williamson, 2016; Bhattarai, Eggertsson and Gafarov, 2015). Another line of research seeks to estimate the "minimum ample" level of reserves and to document possible structural shifts in reserve demand (Yang, 2020; Copeland, Duffie and Yang, 2025; Afonso, Giannone, La Spada and Williams, 2022a). Within this framework, the "minimum ample" level of reserves refers to the level that is sufficiently "ample" such that marginal changes in reserve balances no longer affect short-term interest rates, effectively restoring the independence between reserves and policy rates. In contrast, this paper relaxes that independence assumption and shows that movements in reserves are systematically tied to short-term rates through an explicit reserve-demand mechanism: introducing interest on reserves shifts the reserve-demand curve outward and makes its lower segment more elastic while preserving a downward slope. Consequently, post-2008 increases in reserves can be attributed to IOR rather than to a secular structural break or to a balance-sheet policy independent of the short-term rate.

Second, this paper contributes to a large literature on inside money and money creation. Previous works capturing the explicit role of reserve requirements and money creation include Freeman (1987), Haslag and Young (1998), and Freeman and Kydland (2000). Freeman (1987) and Haslag and Young (1998) study the impact of money creation and the reserve requirements on seigniorage revenue. Freeman and Kydland (2000) develop a tractable model of the endogenous money multiplier. They show that the money-output correlation can be explained by the endogenous money supply resulting from households' choices in response to the business cycle. Recent advances in monetary economics based on a search-theoretic framework provide a deeper understanding of banking and inside money. For example, Gu, Mattesini, Monnet and Wright (2013) study the environment where banking arises endogenously, and show that banking can improve the economy by facilitating trade using inside money. Andolfatto, Berentsen and Martin (2020) integrate a model of bank and financial markets by Diamond (1997) with Lagos and Wright (2005) framework and deliver a model where the fractional reserve banking arises in the equilibrium. Altermatt (2022) studies an economy in which a bank creates inside money by extending loans to entrepreneurs, which can be used for investment. This paper contributes to the literature by constructing a model of money and banking, which establishes the conditions under which banks hold excess reserves, provides an explicit mechanism for determining the demand for reserves and the money multiplier, and explains the money creation process as observed in the data.

Third, this work relates to the literature that studies money and credit explicitly. Gu et al. (2016) show that if money is essential, the credit is irrelevant. Changes in credit conditions only crowd out real balances. This neutrality result can be overturned if one introduces the costly credit (Bethune, Choi and Wright, 2020; Wang, Wright and Liu, 2020). In this paper, there are fiat money, deposit money and unsecured credit, and the neutrality result does not hold. This is because the bank's intermediation of deposit money is costly.

This paper is organized as follows. Section 2 provides motivating evidence. Section 3 constructs the search-theoretic monetary model of money creation. Section 4 calibrates the model to quantify the theory. Section 6 concludes.

2. Motivation and Evidence

This section revisits or presents a list of observations about money creation, money demand and monetary policy, which motivates the theoretical framework developed in the next sections.

Since the Great Recession, the monetary policy environment in the U.S. has changed drastically. The Fed pays interest on reserves (IOR), banks hold large amounts of excess reserves, and the total amount of reserves has skyrocketed. Many economists have been trying to better understand the substantial changes in the conduct of monetary policy. Most of the literature focuses on the independence of the quantity of reserves from interest rate management in an ample-reserves regime (e.g., Keister, Martin and McAndrews, 2008; Bech and Klee, 2011; Curdia and Woodford, 2011; Kashyap and Stein, 2012; Cochrane, 2014; Ennis, 2018).[2]

How can one divorce the quantity of reserves from interest rate management? One popular rationale is that the interest rate paid on reserves provides a floor for the short-term interest rate that the central bank seeks to control. Then, as the central bank's target reaches the interest rate on reserves, paying interest on reserves divorces the quantity of reserves from interest rate management, and the central bank can determine the amount of reserves independently of the interest rate. This implies market rates are equal to or higher than the interest rate on reserves. The standard interpretation is that a change in the short-term rates (e.g., the federal funds rate) can be achieved by adjusting administrative rates, without a need to change the amount of reserves. Given this, some view that the main instrument of US monetary policy has changed during the low interest rate period in the post-2008 era, from an interest-rate policy to one often described as 'quantitative easing,' and treats the supply of bank reserves (or the monetary base) as an alternative instrument for monetary policy. Along with this, some consider that the money multiplier became irrelevant in this regime because the Fed pays IOR and the banks hold large excess reserves. However, I argue that none of the above arguments hold in the data. In the following, Observation 1 shows that the data do not align with the rationale above.

Observation 1. After 2008, interest on reserves is not a floor for short-term interest rates, and neither short-term interest rates nor interest on reserves are independent of the amount of reserves. These rates are related to the quantity of reserves and banks' money creation.

Figure 1 plots short-term interest rates, target range, ON RRP rate, and interest on reserves. It shows that the interest on reserves has been higher than the short-term rates and has been equal to the upper limit of the target range. This is different from the popular modeling approach which assumes the interest on reserves as lower bound. For example, Curdia and Woodford (2011) assume that a central bank can control both the short-term rate and interest on reserves, subject to the constraint that interest on reserves cannot exceed the short-term rate. This discrepancy between data and the modeling approach can be found even in recent literature, e.g., Bianchi and Bigio (2022).

[2]Many works on the central bank's large-scale asset purchases could also be cited here. Since the central bank purchases securities from financial intermediaries by paying with reserves, the independence of the quantity of reserves from interest rate management implies the independence of the amount of asset purchases from interest rate management and vice versa.

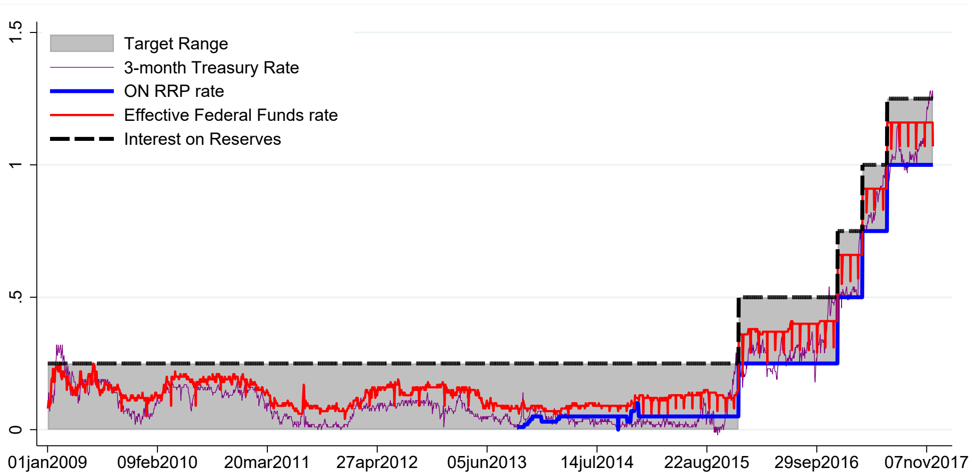


Figure 1: Interest on reserves, ON RRP and target range

Instead of IOR rate, the overnight reverse repurchase (ON RRP) facility is typically credited for maintaining the floor of federal funds rate. As shown in Figure 1, the lower bound of the target range has been equal to the ON RRP rate. Introduced in 2013 to help the Fed maintain its target range, the ON RRP facility provides an alternative to holding reserves.[3] Financial intermediaries can conduct reverse repo transactions with the Fed at predetermined rates, reducing reserve levels in the banking system. The Federal Reserve Bank of New York (NY Fed) publishes data on ON RRP operations, including those predating the facility.[4] This implies that maintaining the federal funds rate above the lower bound of the target range is achieved by adjusting the amount of reserves. Similarly, keeping the rate below the upper bound of the target range is also achieved by changing the supply of reserves.

Even the Fed still changes supply of money to keep the rates in the control bounds, does short-term rate exhibit any relationship with reserves in the data in the post-2008 era? The left panel of Figure 2 shows the reserves to deposit ratio and the spread between the federal funds rate and the interest on reserves.[5] Their opposite movements are evident, which is consistent with controlling interest rates by adjusting reserve amounts. This relationship is not solely driven by changes in the short-term rate, as both of short-term rate and IOR rates have changed over time, suggesting that interest on reserves plays a distinct role simultaneously. This pattern supports the view that the quantity of reserves still plays a role in interest rate management.

[3]Appendix A provides a further discussion on interest on reserves and the overnight reverse repurchase facility.

[4]https://www.newyorkfed.org/markets/desk-operations/reverse-repo

[5]This observation may not be new to some readers. It has been documented in earlier drafts of this paper since 2019 and later by Lopez-Salido and Vissing-Jorgensen (2023) and Lagos and Navarro (2023) as well.

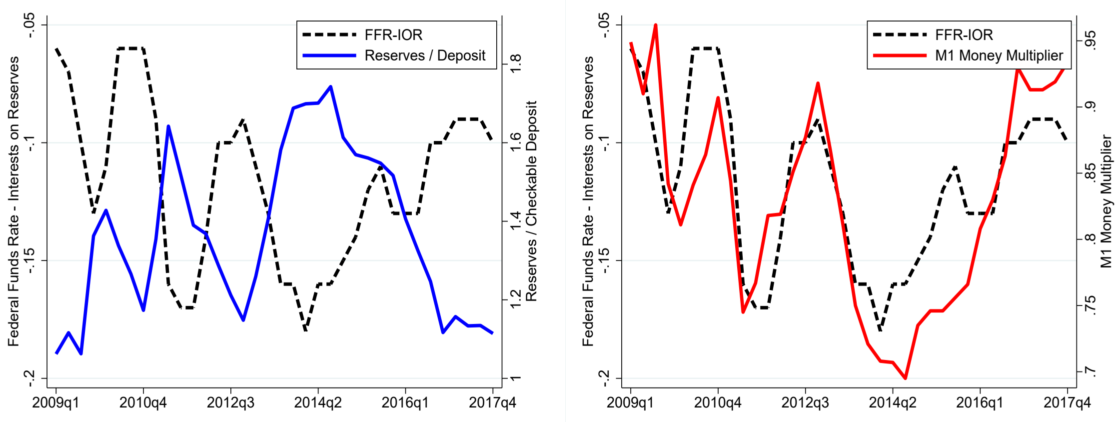


Figure 2: US excess reserves and M1 multiplier in the post-2008 period

There are some models acknowledging that the short-term rate and reserves could exhibit a negative relationship (Piazzesi, Rogers and Schneider, 2019 and Bianchi and Bigio, 2022). But they still assume the IOR rate as a floor and assume independence of reserves with respect to IOR by considering that the interest on reserves and the central bank's balance sheet size are independent instruments. The literature still sees that the central bank can set the short-term rates (e.g. 3-month treasury rate or the federal funds rate) without changing the amount of reserves but only by changing the IOR rate. However, the data in Figure 2 suggest that the IOR rate also could play a role in determining the equilibrium reserve amount.

In addition, the right panel of Figure 2 shows that the M1 money multiplier moves together with the interest rate spread. This suggests that monetary policy in the post-2008 period could be closely linked to banks' money creation activities—an area that had received little attention in previous decades. For example, Cochrane (2014) argued that the money multiplier is meaningless as the Fed implemented IOR; however, this argument is not necessarily true. While different mechanisms may be at play compared to the past, the multiplier still reflects the extent to which banks create deposit money (through lending) relative to their reserve holdings.

Observation 2. The excess reserves to deposit ratio had been zero until 2007. The excess reserve ratio skyrocketed as the Fed introduced the interest on reserve.

Figure 3 plots the excess reserves to deposit ratio from 1968Q1 to 2015Q4. Before the 2008 Great Recession, the ratio remained at zero. However, it rose drastically after the recession and exceeded the value of 1, implying that banks have held more excess reserves

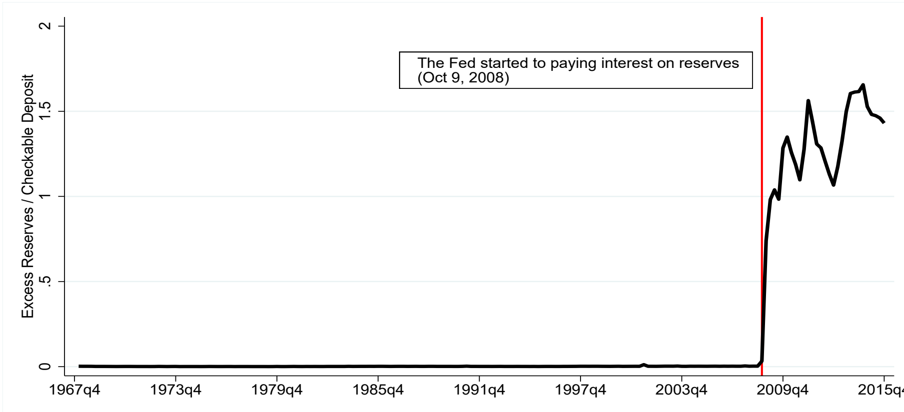


Figure 3: Excess reserves ratio

than the amount of checking account balances they issued. Figure 3 also shows that the dramatic increase in excess reserves coincided with the Fed's introduction of the interest on reserves.

This observation is well known to economists and used to be believed that this is well understood. However, the common understanding relies on some independence of reserves with respect to policy rate, either IOR rate or short-term interest rate. One popular way to understand this is as follows. As the Fed pays interest on reserves, the short-term rates are determined by the IOR rate via arbitrage, and the banks hold reserves not because of legal restrictions but because it is profitable to do so. Some economists and commentators have interpreted the Fed's quantitative easing as a massive increase in reserves without changing policy rates, and take it as evidence that reserve supply can serve as an independent monetary policy tool.

However, the first observation implies that this common understanding may not be true because it assumes that reserves are independent of the policy rate(s), while the first observation shows that changes in reserves are actually closely linked to the short-term rate and IOR rate. Furthermore, when banks maintain large excess reserves to earn IOR, this already implies that reserve levels could depend on IOR rates.

Taken together, these observations lead to the following questions: Why are banks holding such a large amount of excess reserves when they didn't before 2008? If this is due to the IOR paid to banks, does paying IOR make banks hold excess reserves regardless of its rate level? Does the short-term rate affect whether banks hold excess reserves or not? If reserves are not independent of the IOR rate and the short-term rate, can changes in these two interest rates quantitatively explain the massive increase in reserves around 2008 as well as the changes in reserves in the post-2008 period? How can we articulate the relationship between reserve demand, short-term rate, and IOR in one framework where the short-term rate and the IOR rate play different roles? How could all these be related to the money multiplier? After all, do all these details matter for monetary transmission? These questions are not addressed in many monetary models today. By leveraging a theory and its quantitative analysis, Sections 3 and 4 answer these questions.

Observation 3. The required reserve ratio and the money multiplier do not exhibit a negative correlation, and there were two structural breaks in the evolution of the money multiplier: 1992 and 2008.

The top-left panel of Figure 4 plots the US M1 multiplier over time. While the money multiplier decreased drastically after 2008, the declining trend had already begun in the early 1990s. An increase in required reserves did not accompany this decrease in the money multiplier. The bottom-left panel of Figure 4 plots the M1 multiplier against the required reserves.[6] It shows that the required reserve ratio and the money multiplier do not exhibit a negative correlation. (The correlations are 0.6881 for 1968Q1-2015Q4 and 0.8065 for 1968Q1-2007Q4.)

The left panels of Figure 4 show that the US M1 money multiplier has been decreasing since 1992. However, the declining trends before and after 2008 are different. As the right panels of Figure 4 show, the decline during 1992-2007 is accompanied by a huge increase in the ratio of currency to deposit, whereas the decline after 2008 is accompanied by a huge drop in the ratio of currency to deposit. It is also worth noting that the M1 multiplier has been lower than 1 since 2009, which contradicts the textbook theory of money creation.

The difference in the declining trends of the M1 money multiplier is further illustrated through structural breaks. The bottom-left panel of Figure 4 displays two structural breaks in the relationship between the M1 multiplier and the required reserve ratio: one in 1992Q3 and another in 2008Q4. In addition, the bottom-right panel of Figure 4 shows a structural break in the relationship between the M1 multiplier and the currency deposit ratio, which occurred in 2008Q4. The Chow tests for these breaks are described in more detail in Appendix F. The structural break of 2008Q4 coincided with the dramatic increase in excess reserves, which is illustrated in Observation 2.

The absence of a negative correlation suggests that the evolution of the money multiplier

[6] The required reserve ratio presented in Figure 4 is computed by (Required Reserves)/(Total Checkable Deposits). The legal reserve requirement for net transaction accounts was 10% from April 2, 1992, to March 25, 2020, but some banks are subject to lower requirements or exempt depending on the size of their liabilities. These criteria changed 27 times from the 1st quarter of 1992 to the last quarter of 2019. From March 2020, all the required reserve ratios have become zero. See Feinman (1993) and https://www.federalreserve.gov/monetarypolicy/reservereq.htm for more details on the historical evolution of the reserve requirement policy of the United States.

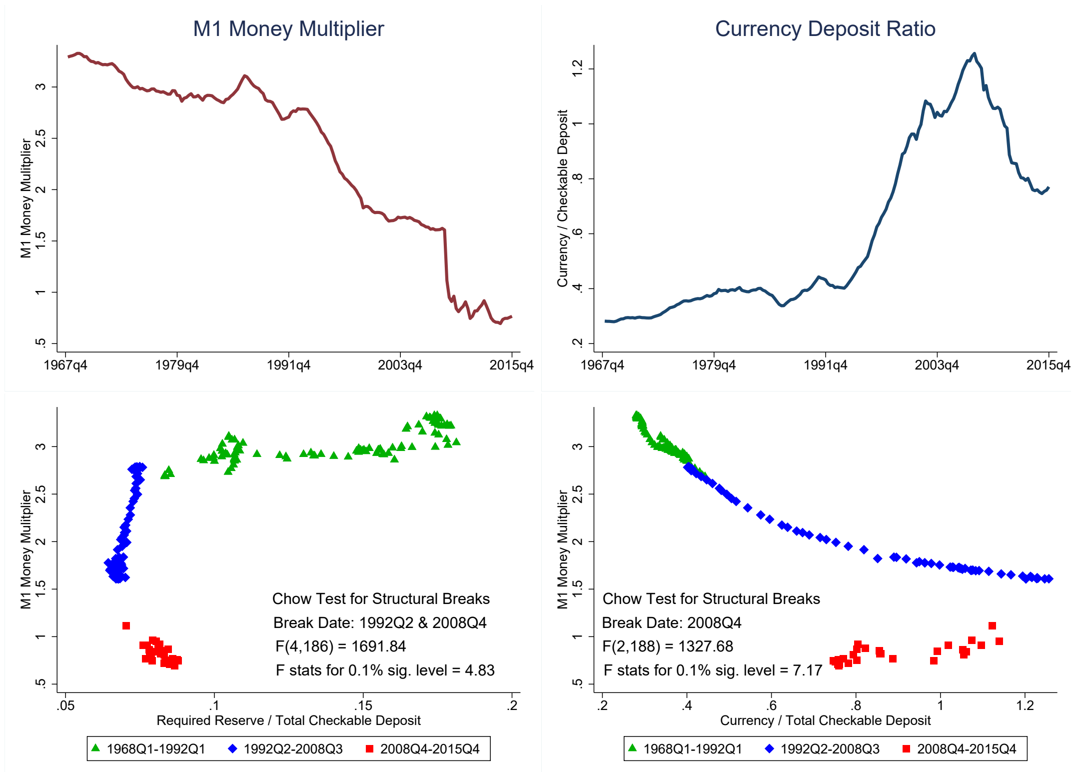


Figure 4: Money multiplier, currency/deposit ratio and required reserve ratio

Chow tests for structural breaks are implemented. The bottom-left panel reports a test statistic with the null hypothesis of no structural breaks in 1992Q2 and 2008Q4 and the bottom-right panel reports a test statistic with the null hypothesis of no structural break in 2008Q4. Sample periods are 1968Q1-2015Q4. Appendix E contains details of the Chow tests.

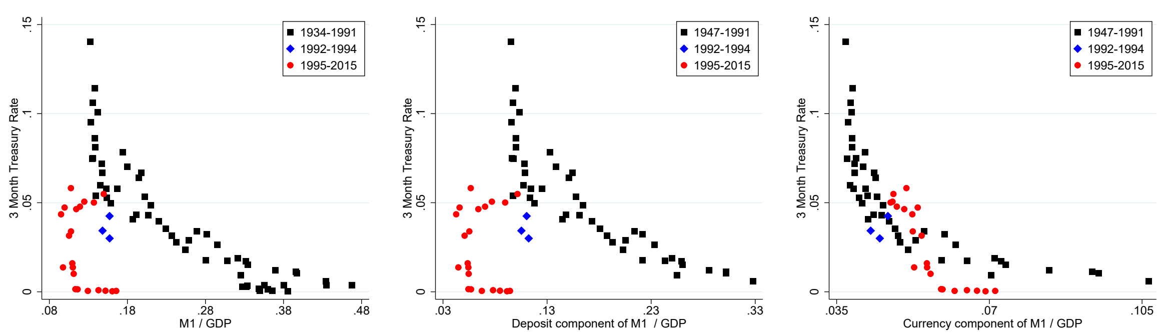


Figure 5: US Money demand for M1 and its components

can be more complicated than the textbook explanation. In the below, Observation 4 suggests that better credit conditions could be a potential explanation.

Observation 4. Adding unsecured credit into the money demand equation as a regressor recovers the downward-sloping, stable M1 money demand.

Table 1: Cointegration regressions and tests

Dependent Variable:	$ln(m_t)$			$ln(d_t)$		
	OLS	OLS	CCR	OLS	OLS	CCR
	(1)	(2)	(3)	(4)	(5)	(6)
r_t	1.600***	−2.298**	−2.022**	4.928***	−3.259*	−3.499**
	(0.419)	(0.885)	(0.818)	(1.123)	(1.198)	(1.721)
$ln(uc_t)$		−0.322***	−0.341***		−0.677***	−0.642***
		(0.042)	(0.069)		(0.082)	(0.122)
$adjR^2$	0.109	0.416	0.494	0.230	0.520	0.584
Observation	112	112	112	112	112	112
Johansen $r = 0$	12.73	44.01		16.13	51.11	
1% CV	24.60	41.07		24.60	41.07	
Johansen $r = 1$	4.15	18.66		4.75	22.19	
1% CV	12.97	24.60		12.97	24.60	
Weak Exogeneity Test (H_0: $\alpha_3 = 0$)						
SUR: $\chi^2(1)$	–	2.39		–	2.23	
VAR: $\chi^2(1)$	–	1.93		–	1.79	
VEC: $\chi^2(1)$	–	3.76		–	1.95	

Notes: Columns (1)-(2) and (4)-(5) report OLS estimates and columns (3) and (6) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on the Bartlett kernel and lag 2. For (1)-(2) and (4)-(5), Newey-West standard errors with lag 2 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported: trace test statistics and critical values (CVs). Appendix F contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

Observation 3, discussed earlier, identified two structural breaks in 1992 and 2008. To obtain a better understanding of the 1992 break, M1 can be decomposed into its deposit and currency components, as illustrated in Figure 5. The figure plots the ratio of M1 and its components to GDP against the 3-month Treasury Bill rate, revealing a breakdown in M1 around 1992 that coincided with the structural break observed in Figure 4.[7] As noted by Lucas and Nicolini (2015), this breakdown was caused by the deposit component; in contrast, the currency component displays a stable downward-sloping demand.

If an increased availability of consumer credit crowds out deposits but not cash, it implies that once we account for the substitution effect of the newly available consumer credit, there should still be a negative relationship between the real money balance and the interest rate.

Following Cagan (1956), and Ireland (2009), I relate the natural logarithm of m, the ratio of money balances to income, to the short-term nominal interest rate, denoted by r. I also regress r on the natural logarithm of d, the ratio of deposit balances to income.

$$ln(m_t) = \beta_0 + \beta_1 r_t + u_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + u_t$$

[7] One may think this is due to the introduction of retail sweep accounts and Automatic Transfer Service (ATS) in the 1990s. However, ATS was introduced in 1994 and the break of M1 occurred in 1992 which was before 1994. In addition to that, using sweep adjusted M1, Kejriwal, Perron and Yu (2022) also found a similar structural break. See Appendix B for more discussion.

In addition to the above specifications, to capture the impact of the improved availability of consumer credit that can substitute the deposit, I add a logarithm of uc, the ratio of unsecured credit to income as another regressor as follows.[8]

$$ln(m_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + u_t, \quad ln(d_t) = \beta_0 + \beta_1 r_t + \beta_2 ln(uc_t) + u_t$$

I focus on the post-1980 period, until the arrival of the Great Recession. In Table 1, columns (1) and (4) report the estimates without unsecured credit, and columns (2)-(3) and (5)-(6) report the estimates with unsecured credit. The Johansen tests in columns (1) and (4) fail to reject the null hypothesis of no cointegration, and ordinary least squares (OLS) estimates from columns (1) and (4) both report positive coefficients on r_t that contradict the conventional notion of money demand: the stable downward-sloping relationship between real balances and the interest rate.

In columns (2)-(3) and (5)-(6), however, the Johansen tests reject their null hypothesis of no cointegration (rank 0) and fail to reject the null hypothesis of cointegration (rank 1) both at a 99 percent confidence level, suggesting that there exists a stable long-run relationship among real money balances, the interest rate, and unsecured credit. To estimate the cointegration relationship, I implement the canonical cointegrating regression (CCR), proposed by Park (1992), in columns (3) and (6).[9] The estimated coefficients on r_t and $ln(uc_t)$ both are negative and significantly different from zero. Thus, using the cointegrating regressions and tests, I document the evidence that once one accounts for the substitution effect of consumer credit, there still exists a stable negative relationship between real money balances and the interest rate. This substitution effect is a potential explanation for the decline of the money multiplier before 2008.

These findings suggest that a desirable monetary model for studying monetary transmission should have the following properties. First, the model should be capable of explaining how the amount of reserves is linked to interest rate management, regardless of whether banks hold excess reserves or not. Second, the model should feature the distinct roles of interest on reserves and nominal interest rates. Third, the model should be able to answer why banks are holding excess reserves, whereas they did not before 2008. Lastly, the model needs to capture the interaction between money and credit. In the following sections, by incorporating these four properties, I develop a theoretical model of the money creation process which is consistent with the above four observations, both qualitatively and quantitatively.

[8] Following Krueger and Perri (2006), I use revolving consumer credit.

[9] The OLS estimates would likely be biased given the non-stationarity of the data and long-run variances from the cointegration relationship. Columns (2) and (5) report OLS estimates for the same variables just for comparison.

Addressing endogeneity of the long-run coefficient on unsecured credit A natural concern is that the estimated long-run relation might reflect endogeneity associated with $\ln(uc_t)$, the logarithm of unsecured credit to output ratio. While the use of real money balances and interest rates in the money demand equation is well established in the empirical literature and monetary theory, the inclusion of credit variables is less conventional. Thus, the observed association with $\ln(uc_t)$ might reflect potential endogeneity. For instance, changes in $\ln(uc_t)$ may be correlated with broader macroeconomic forces and might not be independent of changes in r_t or other variables. Specifically, Krueger and Perri (2006) construct a model with an endogenous credit limit which could evolve in response to higher income risks. Additionally, due to the complex interactions between money and credit, $\ln(uc_t)$ could also be correlated with movements in $\ln(m_t)$.

Two points mitigate this concern. First, weak exogeneity of $\ln(uc_t)$ implies that estimation and inference for the long-run parameters β can be conducted using only the conditional model (i.e., without relying on the short-run dynamics of $\Delta\ln(uc_t)$). Second, CCR (Park, 1992) eliminates the nuisance parameter problem in OLS cointegrating regressions by correcting endogeneity and serial correlation using long-run covariance matrix estimates. This yields asymptotically efficient and normally distributed estimators, enabling standard Wald inference on long-run coefficients, as long as the cointegrating relationship is true.

Consider the following system of equations in Vector Error Correction (VEC) representation:

$$\Delta y_t = \alpha(\beta' y_{t-1} - \beta_0) + \sum_{i=1}^{p-1} \Gamma_i, \Delta y_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma), \tag{1}$$

where $\alpha = [\alpha_0, \alpha_1, \alpha_2]'$, $\beta' = [1, -\beta_1, -\beta_2]$, and $y_t = [\ln(m_t), r_t, \ln(uc_t)]'$. Here, $\Gamma_i \in \mathbb{R}^{3\times 3}$ is a matrix of short-run dynamics coefficients. When deposit balance is used instead of M1 balance, the vector y_t is $[\ln(d_t), r_t, \ln(uc_t)]'$. When $\alpha_2 = 0$, weak exogeneity holds for $\ln(uc_t)$, making the estimation of β (particularly β_2) asymptotically independent of the short-run equation for $\Delta\ln(uc_t)$. The weak exogeneity of $\alpha_2 = 0$ ensures that there is no asymptotic bias in β_2 through feedback from $\Delta\ln(uc_t)$ dynamics.

I test the weak exogeneity of $\ln(uc_t)$ in two separate cointegration relationships for money demand: one with $\ln(m_t)$, and another with $\ln(d_t)$. For each relationship, I use three approaches: (1) Seemingly Unrelated Regression (SUR) with CCR-based u_{t-1} and a Wald test, (2) Vector Autoregression (VAR) in differences with CCR-based u_{t-1} and a Wald test, and (3) VEC model-based likelihood ratio (LR) tests. For the cointegration relationship involving $\ln(m_t)$, all three tests fail to reject weak exogeneity at the 5% level, though the LR test is significance at the 10% level. For the relationship involving $\ln(d_t)$, all three tests fail to reject weak exogeneity even at the 10% level. This suggests no asymptotic bias in β_2 from feedback via $\Delta \ln(uc_t)$ dynamics.

Weak exogeneity for $ln(uc_t)$ implies that it influences the system without adjusting to restore equilibrium. While the tests suggest that weak exogeneity holds, it is important to note that this is a conditional property assuming the model is correctly specified overall. The weak exogeneity of $\ln(uc_t)$ may genuinely reflect its lack of involvement in the long-run error correction process, or it may result from the presence of historically exogenous shocks (such as legislative or regulatory interventions, or increase in the volatility of idiosyncratic income shocks as in Krueger and Perri (2006)) that affected unsecured credit. The first is consistent with the historical introduction of several acts that have shaped the evolution of US unsecured credit as in Figure 6. Another possibility is the presence of omitted variables in the short-run dynamics; however, these typically do not alter the long-run characterization.

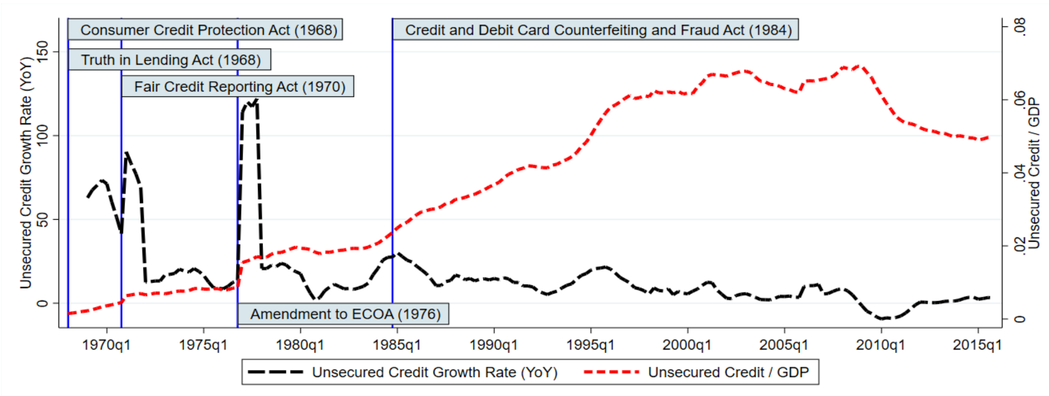


Figure 6: Institutional changes and unsecured credit growth

It is worth noting that cointegration does not imply causality. Even absent endogeneity within the cointegrating vector, the estimated coefficients should be interpreted as a stable long-run association, not a causal effect. However, weak exogeneity of $\ln(uc_t)$ indicates that $\ln(uc_t)$ does not participate in error-correction adjustment and therefore functions as a driving variable in the long run within the dynamic system (1), rather than an adjusting one.

A digression on the relevant measures of money

One may think that the structural break in 1992 found in Figures 4 and 5 can be attributed to the relaxation of bank deposit regulation in the 1990s that stimulated financial innovations such as retail sweep accounts or money market accounts (e.g., VanHoose and Humphrey, 2001, Teles and Zhou, 2005, Lucas and Nicolini, 2015, Berentsen, Huber and Marchesiani, 2015). Based on this idea, some previous works have used alternative measures of M1 as monetary aggregates: (1) "M1 adjusted for retail sweeps" (M1S, hereafter) or (2) "M1 plus money market deposit account" (M1J, hereafter) which was proposed by Lucas and Nicolini (2015).[10]

The rationale for using M1S is that the introduction of the automatic transfer system (ATS) in the early 1990s created highly liquid transaction balances outside M1. The ATS enables convenient transfer from money market deposit accounts (MMDAs) to the retail sweep accounts. The retail sweep account is a transaction deposit, a part of M1, whereas the MMDA is a saving deposit that is classified as non-M1 M2. The MMDA had been subject to the restriction on the number of convenient transactions due to Regulation D, similar to other saving deposits.[11]

The claim is that this may result in the appearance of highly liquid transaction balances within instruments outside M1 because the introduction of ATS may enable people to use MMDAs as liquid deposits. However, it is worth noting that the ATS was introduced to commercial banks in 1994 while the structural break of M1 from Section 2 occurred in 1992. Consistent with this, the difference between M1S and M1 was zero before 1994 (See Appendix B). Using M1S, Berentsen et al. (2015) and Kejriwal et al. (2022) also found the structural break in the early 1990s, suggesting retail sweep consideration cannot explain the break in the early 1990s.

The use of M1J is also based on a similar rationale, and it simply adds MMDA to the standard measure of M1, which recovers the stable long run money demand (see Lucas and Nicolini (2015) for more discussion). While the claim is that MMDAs became highly liquid transaction balances within instruments outside M1, MMDAs were still in M2, and M2 shows similar patterns to those observed in M1 and a structural break at the same time (See Appendix C for a structural break of M2). If the structural break of M1 in the early 1990s is due to MMDAs, M2 would not have the same structural break because MMDAs were still in M2. Thus, the money market accounts consideration cannot reconcile observations 3 and 4.[12]

[10]Lucas and Nicolini (2015) named their measure New M1, but this paper will label it as M1J as in Bethune et al. (2020)

[11]The MMDA was introduced in the early 1980s after the US Congress permitted its creation as of December 1982. Due to Regulation D, no more than six convenient transactions using the MMDA could be made per statement period. As of March 2020, the Federal Reserve removed this restriction.

[12]There are many people who confuse money market deposit accounts with money market funds in discussions of M1J, such as Veracierto (2018) and Krishnamurthy and Li (2023). MMDAs are bank deposit accounts insured by the Federal Deposit Insurance Corporation. Money market funds, on the other hand, are mutual funds that invest in money market securities. The rationale behind M1S and M1J is that MMDAs, which are savings deposits, have become as liquid as transaction deposits, unlike other savings deposits. This approach differs from the idea of including money market funds and other money-like short-term instruments in monetary aggregates. Benati, Lucas Jr, Nicolini and Weber (2021) clearly distinguishes between MMDAs and money market funds. Please note that retail sweeps are deposit-to-deposit sweeps, which are different from money market mutual fund sweeps.

3. Model

The model constructed here extends the standard monetary search model (Lagos and Wright, 2005) by introducing fractional reserve banking, unsecured credit, and capital accumulation. Time is discrete and two markets convene sequentially in each time period: (1) a decentralized market (DM, hereafter), where buyers and sellers meet and trade bilaterally, followed by (2) a frictionless centralized market (CM, hereafter), where agents work, consume, and adjust their balances. The DM trade features imperfect record-keeping and limited commitment. Due to these two frictions, some means of payment are needed in DM trades.

There is a unit mass of households who discount their utility each period by β. The preferences of the households for each period are

$$\mathcal{U} = U(C) - \zeta H + u(q) - c(q)$$

where C is the CM consumption, $\zeta > 0$ is a parameter, H is the CM disutility from production, and q is the DM consumption. As standard, assume U', u', $c' > 0$, U'', $u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. Furthermore, I assume that the CM utility has a constant relative risk aversion equal to 1, i.e., $-C\frac{U''(C)}{U'(C)} = 1$. Consumption goods are perishable. The efficient quantity of DM consumption in the DM is denoted by q^* which solves $u'(q^*) = c'(q^*)$. At the beginning of the DM, each household receives a preference shock such that they can be either buyer or a seller. A household will be a buyer with probability ν, while with probability $1 - \nu$ a household is a seller.

There is a representative firm. During the CM young firms are born, and they become old and die in the next CM. Firms maximize their profit by producing CM consumption goods by hiring labor and using capital as inputs. The production technology is given by $F(K, N)$ where K is the capital input and N is the labor input. The production function $F(K, N)$ satisfies $F_K, F_N > 0$, $F_{KK}, F_{NN} < 0$ and constant returns to scale. Given a constant returns to scale technology, we can define $f(k) \equiv F(k, 1) = F(K, N)/N$ where $k = K/N$. Capital is depreciated by δ every period. Assume that firms are anonymous in the CM, and they cannot commit to honour intertemporal promises. Therefore, a firm needs a medium of exchange to purchase capital goods. To finance investments, firms can borrow from banks. There exists a capital producer whose technology can transform CM consumption goods into capital goods with cost $\Phi(\cdot)$.

There are measure n of active banks that will be endogenously determined by a free entry condition in equilibrium. In the CM, banks make portfolio choices for reserves, loans, transaction deposits, and non-transaction deposits. Banks extend loans to firms by creating deposits that households can use as a means of payment to trade goods in the DM. Loans are paid back with interest i_ℓ. Enforcing repayment is costly. The cost function is described by $\eta(\tilde{\ell})$, where $\tilde{\ell}$ is the amount of loans in real terms, $\eta', \eta'' > 0$, and $\eta(0) = 0$. Managing reserve balances also incurs a cost.[13,14] The cost is represented by a cost function $\gamma(\tilde{r})$, where $\tilde{r}$ is the amount of reserves in real terms, $\gamma', \gamma'' > 0$, and $\gamma(0) = 0$. Each unit of reserves earns a nominal interest rate of i_r. Banks are subject to a reserve requirement: for transaction deposits recorded as liabilities on their balance sheet, a fraction must be held as reserves. Specifically, a bank must hold at least $\chi\tilde{d}$ as reserves, where χ is the reserve requirement and $\tilde{d}$ is the real balance of transaction deposits.

There are three types of DM meetings depending on payment methods that sellers accept: DM1, DM2, and DM3. In DM1, there is no record-keeping device, and the seller can only recognize cash. In DM2, the seller accepts transaction deposits and unsecured credit. In DM3, she accepts cash, transaction deposits and unsecured credit. The buyer can trade using unsecured credit with credit limit $\bar{b}$ as the trading is monitored imperfectly.[15] The buyer's probability of a type j meeting is σ_j, while the seller's probability of a type j meeting is $\frac{\sigma_j \nu}{1-\nu}$.

The central bank controls the base money supply B_t in the CM. Let μ denote the base money growth rate. Then, changes in the quantity of base money can be written as

$$\mu_{t+1} B_t = B_{t+1} - B_t,$$

The base money is held in two ways: (1) M_t as currency in circulation, i.e., outside money held by households; (2) R_t as reserves held by banks. Thus,

$$B_t = M_t + R_t.$$

The central bank can control the base money supply in two ways. First, it can conduct a

[13] The empirical studies support the existence of costs for expanding reserves. For recent empirical evidence on reserve-related balance sheet costs for FDIC-insured institutions, see Banegas and Tase (2020) and Afonso, Cipriani and La Spada (2022b). For models with reserve-related balance sheet costs, please see Afonso, Armenter and Lester (2019), Duffie and Krishnamurthy (2016).

[14] While empirical works have documented reserve balance related costs, in practice, banks also get clearing and settlement benefits from holding reserves. In the US, ACH uses Federal Reserve accounts. FedACH uses them directly and the Clearing House's Electronic Payments Network (EPN) uses them via the Fed's National Settlement Service, and CHIPS (Clearing House Interbank Payments System) must be prefunded via reserve accounts, with final settlement over the Fedwire Funds Service. The reserves supply the intraday liquidity that allows this process. Empirical evidence on reserve related balance sheet costs may suggest that the marginal cost exceeds the marginal benefit at equilibrium.

[15] The acceptance of different means of payment can be endogenized as in Lester, Postlewaite and Wright (2012) or Lotz and Zhang (2016) but here we assume the types of meetings are exogenously given. Lester et al. (2012) endogenize the meeting types by allowing sellers' costly *ex ante* choice to acquire the technology for recognizing the certain type of assets. Similarly, Lotz and Zhang (2016) study the environment with costly record-keeping technology where sellers must invest in a record-keeping technology to accept credit.

lump-sum transfer or collect a lump-sum tax in the CM. Second, it can increase the money supply by paying interest on reserves, i_r. Let T represents a lump-sum transfer (or tax if it is negative). The central bank's constraint is

$$\mu_{t+1}\phi_{t+1}B_t = \phi_{t+1}(B_{t+1} - B_t) = T_{t+1} + i_{r,t+1}\phi_{t+1}R_{t+1},$$

where ϕ is the price of money in terms of the CM consumption good.

3.1. The CM Problem

Let $W(m, d, s, b)$ denote the CM value function where m is the cash holding, d is the transaction deposit balance, s is the non-transaction deposit balance, and $b > 0$ is the unsecured debt owed to the seller from the previous DM (or unsecured loans to the buyer from the previous DM, if $b < 0$). All the state variables are in unit of the current CM consumption good. The CM problem is

$$\begin{aligned} W(m_t, d_t, s_t, b_t) = &\max_{C_t, H_t, \hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}} U(C_t) - \zeta H_t + \beta V(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \\ \text{s.t. } &\frac{\phi_t}{\phi_{t+1}}(\hat{m}_{t+1} + \hat{d}_{t+1} + \hat{s}_{t+1}) + C_t = w_t H_t + m_t + (1 + i_{d,t})d_t + (1 + i_{s,t})s_t - b_t + T_t \end{aligned}$$

where $\hat{m}_{t+1}$, $\hat{d}_{t+1}$, and $\hat{s}_{t+1}$ is the cash holding, transaction deposit balance, and non-transaction deposit balance, respectively, carried to the next DM, and w is the real wage. The first-order conditions (FOCs) are

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_m(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, \ = \ \text{if } \hat{m}_{t+1} > 0 \tag{2}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_d(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, \ = \ \text{if } \hat{d}_{t+1} > 0 \tag{3}$$

$$-\frac{\phi_t}{\phi_{t+1}}\frac{\zeta}{w_t} + \beta V_s(\hat{m}_{t+1}, \hat{d}_{t+1}, \hat{s}_{t+1}) \leq 0, \ = \ \text{if } \hat{s}_{t+1} > 0 \tag{4}$$

$$-\frac{\zeta}{w_t} + U'(C_t) = 0. \tag{5}$$

The first term on the left-hand side (LHS) of equation (2) is the marginal cost of acquiring cash. The second term is the discounted marginal value of carrying cash to the following DM. Therefore, the choice of $\hat{m}_{t+1} > 0$ equates the marginal cost and the marginal return on cash. A similar interpretation applies to equations (3) and (4) for the decision on deposits.

The envelope conditions for $W(m, d, s, b)$ are

$$W_d(m_t, d_t, s_t, b_t) = (1 + i_{d,t})\frac{\zeta}{w_t}, \quad W_m(m_t, d_t, s_t, b_t) = \frac{\zeta}{w_t}$$
$$W_s(m_t, d_t, s_t, b_t) = (1 + i_{s,t})\frac{\zeta}{w_t}, \quad W_b(m_t, d_t, s_t, b_t) = -\frac{\zeta}{w_t}$$

which implies $W(m_t, d_t, s_t, b_t)$ is linear in m, d, s, and b. This linearity allows us to write

$$W(m_t, d_t, s_t, b_t) = \frac{\zeta}{w_t}\{m_t + (1 + i_{d,t})d_t + (1 + i_{s,t})s_t\} - \frac{\zeta}{w_t}b_t + W(0, 0, 0, 0, 0).$$

3.2. The DM Problem

In the DM, the buyer and seller trade bilaterally. Let q_j and p_j be the DM consumption and payment in a type-j DM meeting. The bilateral trade is characterized by (p_j, q_j). This trade is subject to $p_j \leq z_j$ where z_j is the total liquidity of the buyer in a type-j meeting. The liquidity position for each type of buyer is

$$z_1 = m \tag{6}$$
$$z_2 = d(1 + i_d) + \bar{b} \tag{7}$$
$$z_3 = m + d(1 + i_d) + \bar{b} \tag{8}$$

The DM terms of trade are determined by Kalai (1977)'s proportional bargaining. Kalai bargaining solves the following problem:

$$\max_{p,q} u(q) - \frac{\zeta}{w}p \quad \textit{s.t} \quad u(q) - \frac{\zeta}{w}p = \theta\left[u(q) - c(q)\right] \text{ and } p_j \leq z_j$$

where $\theta \in [0, 1]$ denotes the buyers' bargaining power. The payment, p, can be expressed as $p = v(q)w/\zeta = \{(1 - \theta)u(q) + \theta c(q)\}w/\zeta$. Define *liquidity premium*, $\lambda(q)$, as follows:

$$\lambda(q) = \frac{u'(q)}{v'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{(1 - \theta)u'(q) + \theta c'(q)}$$

where $\lambda(q) > 0$ and $\lambda'(q) < 0$ for $q < q^*$ and $\lambda(q^*) = 0$. When $z_j \geq p^*$, the buyer has sufficient liquidity to purchase efficient DM output q^*. In this case, the payment to the seller is $p^* = v(q^*)w/\zeta$.

The value function of a household at the beginning of DM is

$$V(m, d, s) = \nu V^B(m, d, s) + (1 - \nu)V^S(m, d, s)$$

where $V^B(m,d,s)$ and $V^S(m,d,s)$ denote the value function for a buyer and a seller, respectively. By using the linearity of W, we can write a DM value function for a seller as follows:

$$V^S(m,d,s) = \sum_{j=1}^{3}\left\{\frac{\sigma_j \nu}{1-\nu}\left[\frac{\zeta}{w}p_j - c(q_j)\right]\right\} + W(m,d,s,0)$$

and the value function of a buyer in the DM is

$$V^B(m,d,s) = \sum_{j=1}^{3}\left\{\sigma_j\left[u(q_j) - \frac{\zeta}{w}p_j\right]\right\} + W(m,d,s,0)$$

where $p_j \leq z_j$. The second term on the right-hand side (RHS) is the continuation value when there is no trade. The rest of the RHS is the surplus from the DM trade. The DM payments are constrained by $p_j \leq z_j$. For compact notation, define inflation rate as $\pi_{t+1} \equiv \phi_t/\phi_{t+1} - 1$. Assuming interior, differentiating V and substituting its derivatives into the FOCs from the CM problem yields

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_1\lambda(q_{1,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})] \tag{9}$$

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})[1 + \nu\sigma_2\lambda(q_{2,t+1}) + \nu\sigma_3\lambda(q_{3,t+1})](1+i_{d,t+1}) \tag{10}$$

$$(1+\pi_{t+1})U'(C_t) = \beta U'(C_{t+1})(1+i_{s,t+1}) \tag{11}$$

where $q_{j,t+1} = \min\{q^*, v^{-1}(\zeta z_{j,t+1}/w_{t+1})\}$ and $\lambda(q^*) = 0$.

3.3. The Bank's Problem

The banking sector is perfectly competitive and banks take the interest rates as given: lending rate $i_{\ell,t}$, transaction deposit rate $i_{d,t}$, non-transaction deposit rate $i_{s,t}$ and interest on reserves $i_{r,t}$. The bank maximizes its profit by choosing $\{\tilde{\ell}_t, \tilde{r}_t, \tilde{d}_t, \tilde{s}_t\}$ subject to its balance sheet identity constraint and reserve requirement constraint, where $\tilde{\ell}_t$ is lending, $\tilde{r}_t$ is reserve balance, $\tilde{d}_t$ the transaction deposit issuance, and $\tilde{s}_t$ the non-transaction deposit issuance, respectively, denoted in real terms:

$$\begin{gathered}\max_{\tilde{r}_t,\tilde{d}_t,\tilde{\ell}_t,\tilde{s}_t} \quad (1+i_{\ell,t})\tilde{\ell}_t + (1+i_{r,t})\tilde{r}_t - (1+i_{d,t})\tilde{d}_t - (1+i_{s,t})\tilde{s}_t - \gamma(\tilde{r}_t) - \eta(\tilde{\ell}_t) \\ \text{subject to} \quad \tilde{\ell}_t + \tilde{r}_t = \tilde{d}_t + \tilde{s}_t \quad \text{and} \quad \tilde{r}_t \geq \chi\tilde{d}_t\end{gathered} \tag{12}$$

In the first constraint, balance sheet identity, the LHS represents the value of assets such as reserves and loans, and the RHS represents the value of liabilities such as transaction deposits and non-transaction deposits. The second constraint is the reserve requirement.

Let $\lambda_{\chi,t}$ denote the Lagrange multiplier for the reserve requirement constraint. Assuming interior, the FOCs for the bank's problem can be written as

$$0 = i_{\ell,t} - i_{s,t} - \eta'(\ell_t) \tag{13}$$
$$0 = i_{r,t} - i_{s,t} - \gamma'(\tilde{r}_t) + \lambda_{\chi,t} \tag{14}$$
$$0 = i_{s,t} - i_{d,t} - \chi\lambda_{\chi,t}. \tag{15}$$

The bank's *ex post* profit equals to the entry cost, κ

$$(1 + i_{\ell,t})\tilde{\ell}_t + (1 + i_{r,t})\tilde{r}_t - (1 + i_{d,t})\tilde{d}_t - (1 + i_{s,t})\tilde{s}_t - \gamma(\tilde{r}_t) - \eta(\tilde{\ell}_t) = \kappa. \tag{16}$$

Suppose there are active banks i.e., $n > 0$. Consider two cases. In the first case, the reserve requirement constraint is binding, i.e., $\lambda_{\chi,t} > 0$. In the second case, the reserve requirement constraint is loose, i.e., $\lambda_{\chi,t} = 0$. We call the first case a "scarce-reserves case," and the second an "ample-reserves case."

The Scarce-Reserves Case Consider the case where the bank does not have enough reserves. It needs to acquire reserves to issue more transaction deposits, which implies a binding constraint. With $\lambda_{\chi,t} > 0$, the bank's FOCs (13)-(15) give

$$i_{d,t} = (1 - \chi)i_{s,t} + \chi i_{r,t} - \chi\gamma'(\tilde{r}_t) \tag{17}$$
$$i_{\ell,t} = i_{s,t} + \eta'(\tilde{\ell}_t). \tag{18}$$

The Ample-Reserves Case Consider the case where the bank has sufficient reserves. Its reserve requirement constraint does not bind, $\lambda_{\chi,t} = 0$. Then the three FOCs for the bank's problem become

$$i_{d,t} = i_{s,t} \tag{19}$$
$$i_{r,t} = i_{s,t} + \gamma'(\tilde{r}_t) \tag{20}$$
$$i_{\ell,t} = i_{s,t} + \eta'(\tilde{\ell}_t). \tag{21}$$

The key difference between these two cases is that in the scarce-reserve case, banks only hold required reserves as the constraint binds. In contrast, in the ample-reserve case, banks can hold excess reserves in addition to required reserves because the reserve requirement constraint is no longer binding.

3.4. The Firm and Capital Producer

A representative firm maximizes its profit by producing CM consumption goods and using its capital K_t and hiring labor N_t as inputs. In the CM of $t-1$, the firm borrows funds L_t from banks, and purchases capital goods K_t using the funds. A firm purchases capital from a perfectly competitive capital producing firm at the end of period $t-1$. This capital is used in production at t and its undepreciated $(1-\delta)K_t$ part is resold to a capital producer once the production is over. The firm's problem can be written as follows:

$$\max_{N_t,K_t,L_t} L_t - Q_{t-1}K_t + \beta\left[F(K_t,N_t) - w_tN_t + Q_t(1-\delta)K_t - (1+i_{\ell,t})\frac{L_t}{1+\pi_t}\right]$$

subject to $L_t = Q_{t-1}K_t$, where Q_t is price of capital in terms of CM consumption good at period t. The firm's problem gives

$$F_N(K_t,N_t) = w_t, \quad F_K(K_t,N_t) = Q_{t-1}\frac{1+i_{\ell,t}}{1+\pi_t} - Q_t(1-\delta). \tag{22}$$

The capital law of motion is given as:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

A capital producer can transform CM consumption goods into capital goods with cost $\Phi(I_t)$. Formally, a capital producer solves the following profit-maximization problem:

$$\max_{I_t} Q_tI_t - \Phi(I_t)$$

which gives $Q_t = \Phi'(I_t)$. Assuming a linear cost function, $\Phi(I_t) = I_t$, gives $Q_t = 1$ for all t. Given above results, define the real lending rate:

$$\rho_t \equiv \frac{1+i_{\ell,t}}{1+\pi_t} - 1$$

Then we have the following equilibrium condition for the real lending rate and the marginal product of capital as follows:

$$F_K(K_t,N_t) = \rho_t + \delta \tag{23}$$

3.5. Equilibrium

In the equilibrium, the resource constraint for CM consumption goods and labor market clearing condition are satisfied.

$$C_t + K_{t+1} = F(K_t, N_t) + (1-\delta)K_t, \text{ and } N_t = H_t \tag{24}$$

The money market clearing conditions are given as below

$$\phi_{t+1}M_{t+1} = m_{t+1}, \quad \phi_{t+1}R_{t+1} = n_{t+1}\tilde{r}_{t+1}, \quad \text{and} \quad B_{t+1} = M_{t+1} + R_{t+1} \tag{25}$$

and market clearing condition for lending and deposits are satisfied.

$$L_{t+1} = n_{t+1}\tilde{\ell}_{t+1}, \quad \text{and} \quad d_{t+1} = n_{t+1}\tilde{d}_{t+1} \tag{26}$$

Given agents' optimal choices and market clearing conditions, we define a monetary equilibrium as follows:

Definition 1. *Given a sequence of monetary policy $\{B_t, i_{rt}, \chi_t\}_{t=1}^{\infty}$ and credit condition $\{\bar{b}_t\}_{t=1}^{\infty}$, and initial conditions (K_0, B_0), a monetary equilibrium is a sequence of quantities $\{K_t, N_t, m_t, d_t, s_t, r_t\}_{t=1}^{\infty}$, prices $\{\phi_t, i_{\ell t}, i_{d,t}, w_t, \rho_t\}_{t=1}^{\infty}$, and measures of active banks, $\{n_t\}_{t=1}^{\infty}$ that satisfies:*

1. *The Euler equations (9)-(11), and transversality conditions:*

$$\lim_{t\to\infty} \beta^t K_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t m_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t d_t = \lim_{t\to\infty} \beta^t U'(C_t)\phi_t s_t = 0$$

2. *Optimality conditions of banks and the firm, (13)-(16) and (22);*

3. *Market clearing (24)-(26), and $\phi_t B_t > 0$.*

As standard, the short-term policy rate i_t is given by the Fisher equation

$$1 + i_t = (1 + \pi_{t+1})\frac{U'(C_t)}{\beta U'(C_{t+1})} \tag{27}$$

implying $i_t = i_{s,t+1}$.

Similar to Sargent and Wallace (1975) and Gu, Han and Wright (2020), a central bank can peg i_t by letting B_{t+1} evolve endogenously as long as the base money is valued, $\phi_{t+1}B_{t+1} > 0$. Here, monetary policy implementation is different from what is assumed in the previous literature. For example, New Keynesian models simply assume that the central bank can determine interest rates. In the other monetary models with fiat money, the central bank implements the monetary policy by controlling the aggregate money supply, or by controlling the growth rate of aggregate money supply. In this model, the central bank can set interest rate by controlling the supply of base money which eventually influences the supply of monetary aggregate, and other macroeconomic variables. Here, for the central bank's monetary policy implementation, it is crucial to have monopoly power over the supply of base money which is the sum of reserves and currency in circulation.

Given this environment, we have the following results:

Proposition 1. *When the central bank does not pay interest on reserves i.e., $i_{r,t+1} = 0$, banks do not hold excess reserves i.e., $\tilde{r}_{t+1} = \chi \tilde{d}_{t+1}$. When banks hold excess reserves, $\partial \tilde{\ell}_{t+1}/\partial i_t > 0$, $\partial \tilde{\ell}_{t+1}/\partial i_{r,t+1} < 0$, $\partial \tilde{r}_{t+1}/\partial i_t < 0$, $\partial \tilde{r}_{t+1}/\partial i_{r,t+1} > 0$, $\partial i_{\ell,t+1}/\partial i_t > 0$, and $\partial i_{\ell,t+1}/\partial i_{r,t+1} < 0$.*

Proposition 1 says that paying interest on reserves is necessary to have banks hold excess reserves. It also states that changes in the short-term policy rate and the changes in interest on reserves have different effects on the banking sector when banks are holding excess reserves. For example, an increase in the short-term policy rate reduces each bank's reserve balances, while an increase in the interest on reserves raises the bank's reserve balances. This is because an increase in the short-term policy rate reduces returns on reserves while an increase in the interest on reserves raises returns on reserves. Also, an increase in the short-term policy rate raises the lending rate, whereas an increase in the interest on reserves lowers the lending rate.

Stationary Monetary Equilibrium In the remaining of this section, I focus on a symmetric stationary monetary equilibrium in which the agents make the same decisions and all real variables are constant over. Given that $\phi_t/\phi_{t+1} = B_{t+1}/B_t = M_{t+1}/M_t = 1 + \mu$, the net inflation rate, π, is equal to the currency growth rate, μ, in the stationary monetary equilibrium. By the Fisher equation, $1 + i = (1 + \mu)/\beta$.[16] This leads to the following definitions:

Definition 2 (Stationary Monetary Equilibrium). *Given monetary policy, i, i_r, and χ and credit limit $\bar{b}$, a stationary monetary equilibrium consists of real balances, (m, r, d, s, ℓ), allocation (q_1, q_2, q_3, C, K, N), the measure of banks n, and prices (i_d, i_ℓ), satisfying Definition 1 except for initial conditions.*

[16]Note that $i \geq 0$ is necessary for the existence of equilibrium. Whereas the lower bound of the nominal interest rate is zero in this setting, one can relax this constraint by introducing liquid assets or costs associated with holding currency. Rocheteau, Wright and Xiao (2018b) and Lee (2016) show that liquid assets can bear negative nominal yields. By incorporating costs associated with holding currency, Kim (2024) studies how introducing a non-traditional reserve policy, such as a non-par exchange rate between cash and reserves, can reduce the lower bound on nominal interest rates.

Definition 3. *The stationary monetary equilibrium is a scarce-reserves equilibrium when $\tilde{r} = \chi\tilde{d}$ and an ample-reserves equilibrium, when $\tilde{r} > \chi\tilde{d}$, respectively.*

Given the above definitions, we have the following result.

Proposition 2. *Given $(i, \chi, \bar{b})$: (i) $\exists!$ ample-reserves equilibrium if and only if $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$; (ii) $\exists$ scarce-reserves equilibrium if and only if $i_r \leq \bar{\iota}_r$; and the thresholds satisfy $\bar{\Delta} = \gamma'(\underline{r})$ and $\bar{\iota}_r = \gamma'(\bar{r}) + i$ where $\underline{r}$ solves $\kappa = \gamma'(\underline{r})\underline{r} - \gamma(\underline{r})$ and $(\bar{r}, \bar{\ell}, \bar{K}, \bar{N}, \bar{C}, \bar{n})$ solves*

$$F_K(\bar{K}, \bar{N}) = \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\bar{\ell})}{\beta(1+i)}, \qquad \max\left\{0, \frac{\chi\left\{v(q^*)F_N(\bar{K}, \bar{N})/\zeta - \bar{b}\right\}}{\bar{n}}\right\} = (1+i)\bar{r},$$

$\eta'(\bar{\ell})\bar{\ell} + \gamma'(\bar{r})\bar{r} - \gamma(\bar{r}) - \eta(\bar{\ell}) = \kappa$, $\bar{C} + \delta\bar{K} = F(\bar{K}, \bar{N})$, $U'(\bar{C}) = \zeta/F_N(\bar{K}, \bar{N})$, and $\bar{K} = \bar{n}\bar{\ell}$.

When the central bank does not pays interest on reserves or pays low interest such that $i_r \leq \bar{\iota}_r$, banks only hold required reserves because the opportunity cost of holding reserves is higher than its benefit. When the central bank pays interest on reserves such that $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$, banks always hold positive amount of excess reserve balances that satisfies $\gamma'(\tilde{r}) = i_r - i$ and $\tilde{r} > \chi\tilde{d}$.[17] In this case, banks hold a large amount of reserves because the benefit of holding reserves outweighs its opportunity cost of holding reserves.

Having solved the stationary monetary equilibrium, I proceed to establish the results on the money multiplier. Define the monetary aggregate as $\mathcal{M} \equiv m + d$ and the money multiplier as $\xi \equiv \mathcal{M}/(\phi B)$, then we have the following results.

Proposition 3. *In the ample-reserves equilibrium, the money multiplier is increasing in i and decreasing in i_r, i.e, $\partial\xi/\partial i > 0$ and $\partial\xi/\partial i_r < 0$.*

Proposition 3 shows that the money multiplier is increasing in short-term policy rate while it is decreasing in interest on reserves, which is consistent with the observation illustrated in Figure 2. This result is intuitive. In the ample-reserves equilibrium, banks hold reserves because holding reserves itself is profitable regardless of the reserve requirement. The higher interest on reserves decreases the money multiplier because the banks have more incentive to hold reserves and less incentive to create transaction deposits. Increasing the short-term policy rate decreases reserves, but the banks do not create transaction deposits proportionally, which lowers the money multiplier. These are new findings compared to the literature.

To get more analytical results, I assume that the following restriction is satisfied in any equilibrium[18]:

Assumption 1. $1 > \gamma'(\tilde{r}) + \gamma''(\tilde{r})\tilde{r}$.

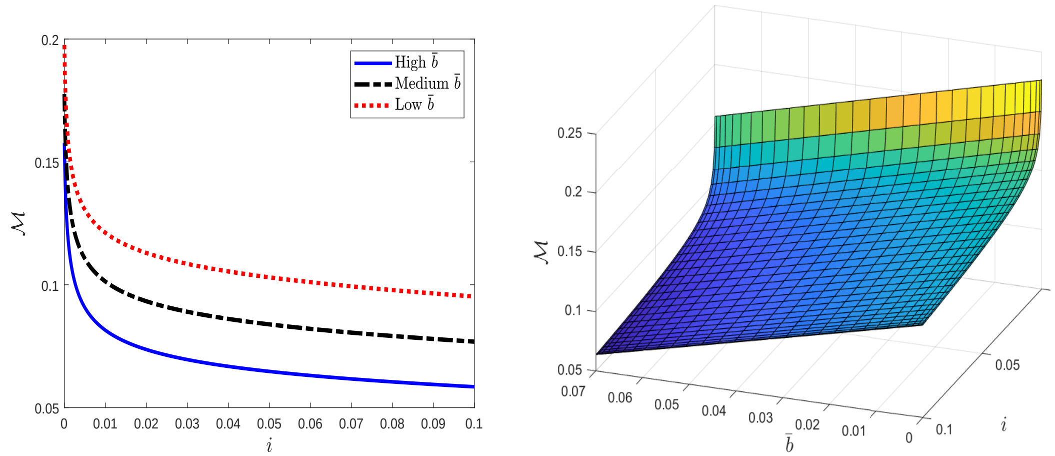


Figure 7: Money Demand under Different Credit Limit

For each type of equilibrium, the following results are proved in Appendix D.

Proposition 4. *Suppose $p^* > \bar{b}$. In the ample-reserves equilibrium, $\partial i_d/\partial \bar{b} = 0$ and $\partial d/\partial \bar{b} < 0$. In the scarce-reserves equilibrium, when the equilibrium is unique, $\partial i_d/\partial \bar{b} > 0$ and $\partial d/\partial \bar{b} < 0$ if $\bar{b}$ or σ_3 is small.*

Proposition 4 shows two results: (1) a better credit condition increases the deposit rate in the scarce-reserve equilibrium but not in the ample-reserve equilibrium; and (2) an increase in $\bar{b}$ can reduce d in both types of equilibria. One implication for the deposit rate changes is that neutrality between money and credit does not hold in the scarce-reserve equilibrium.[19] This is because the use of transaction deposit money incurs costs for operating banks. An increase in the credit limit lowers the cost of operating deposits by reducing real balances, which increases DM2 and DM3 trades through the increase in the deposit rate. In contrast to the scarce-reserve equilibrium, changes in credit conditions do not impact the deposit rate in the ample-reserve equilibrium; they merely crowd out real balances. Consistent with observation 4 from Section 2, the second result implies that an increase in $\bar{b}$ can reduce d, which can eventually reduce the real balances of the monetary aggregate, $\mathcal{M}$. Figure 7 shows some examples.

As can be seen from the results above, the model can successfully address the mechanism illustrated in Section 1 and 2. We can interpret the breakdown of the money demand relationship in the pre-2008 economy as a result of improved availability of consumer credit under the scarce-reserve equilibrium. For the post-2008 period, after the Fed started paying

[17]When $i_r > \bar{\Delta} + i$, no equilibrium exists with active banks ($n > 0$), but an equilibrium without active banks ($n = 0$) exists. We'll focus solely on the equilibrium with an active banking sector.

[18]This assumption implies the convex cost function $\gamma(\cdot)$ is not too convex.

[19]For more discussion on the neutrality of money and credit, see Gu et al. (2016) and Wang et al. (2020).

interest on reserves, the economy shifted to the ample-reserves equilibrium. The model suggests that the changes in the money multiplier and the excess reserves during the post-2008 period are the results of the Fed's management of two interest rates: the nominal policy rate and the interest on reserves.

We now look into DM trades more closely. From (9) and (10), we have

$$\frac{i_t}{\nu} = \sigma_1 \lambda(q_1) + \sigma_3 \lambda(q_3) \tag{28}$$

$$\left\{ \frac{1+i}{1+i_d} - 1 \right\} \frac{1}{\nu} = \sigma_2 \lambda(q_2) + \sigma_3 \lambda(q_3). \tag{29}$$

where $i \geq i_d$ and $i \geq 0$. It is straightforward to see that DM1 and DM3 consumptions are efficient, $q_1 = q_3 = q^*$, when $i_t = 0$, i.e. the Friedman rule applies. However, if the central bank pays sufficient interest on reserves, we have $i_d = i$ which gives efficient consumptions in DM3 as well as in DM2 even when $i > 0$. This result can be formally summarized in the following proposition.

Proposition 5. *Let the short-term policy rate be positive $i > 0$. Then the DM consumptions in DM2 and DM3 are efficient $q_2 = q_3 = q^*$ when $i_r \geq \bar{\iota}_r$.*

The intuition behind the efficient DM consumptions when $i_r > \bar{\iota}_r$ is straightforward. In many monetary models, a higher inflation or interest rate increases the opportunity cost of holding money. In the environment where money is valued as a medium of exchange, having less liquidity in the economy because of the opportunity cost of holding money is inefficient. However, the interest on reserves provides a proportional return. If this return is properly distributed across households, it eliminates the inefficiency arising from the opportunity cost of holding money, leading to efficient consumption in DM2 and DM3. Therefore, when the central bank pays sufficient interest on reserves, the DM2 and DM3 meeting consumptions can be efficient even though the economy is not under the Friedman rule.

In addition to that, if credit limit $\bar{b}$ is sufficiently high, DM2 and DM3 consumptions also can be efficient even though $i > 0$. Appendix D verifies the following:

Proposition 6. *The threshold $\bar{\iota}_r$ is decreasing in $\bar{b}$. When $\bar{b} \geq p^*$, $\bar{\iota}_r = i$, $d = 0$ and $q_2 = q_3 = q^*$.*

Proposition 6 simply states that if the credit limit is high enough, it results in efficient consumption in both DM2 and DM3 trades. As $\bar{b} \to p^{*-}$, the household's transaction deposit balance d converges to 0. This is reminiscent of a result by Gu et al. (2016): if credit is easy, money is irrelevant; if credit is tight, money is essential, but credit becomes irrelevant. One difference is that even though credit is easy (i.e., $\bar{b} \geq p^*$) the household always holds cash $m > 0$ as long as $i < \nu\sigma_1\lambda(0)$ because the household only can trade using cash in the

DM1 meeting. Since the better credit condition lowers transaction deposit balance, required reserves also shrink accordingly, leading to decreases in $\bar{\iota}_r$.

One can also check the interest rate pass-through of the monetary policy. Its pass-through depends on the type of equilibrium.

Proposition 7. *(i) In the ample-reserve equilibrium, $\partial i_\ell/\partial i > 0$, $\partial i_\ell/\partial i_r < 0$, $\partial i_d/\partial i = 1 > 0$, $\partial i_d/\partial i_r = 0$, and $\partial \rho/\partial i_r < 0$ but $\partial \rho/\partial i$ is ambiguous. (ii) In the scarce-reserve equilibrium, when the stationary monetary equilibrium is unique, $\partial i_\ell/\partial i_r < 0$, and $\partial \rho/\partial i_r < 0$ if $\bar{b}$ or σ_3 is small. When σ_3 is small, $\partial i_\ell/\partial i > 0$, and $\partial i_d/\partial i > 0$ but $\partial \rho/\partial i$ is ambiguous.*

Proposition 7 tells us that the monetary policy rates pass through the lending rate and deposit rate. Similar to Proposition 1, changes in the short-term policy rate and the changes in interest on reserves have different effects. In both types of equilibrium, the lending rate is strictly increasing in the short-term policy rate but is strictly decreasing in interest on reserves. In the scarce-reserves equilibrium, the deposit rate is strictly increasing in the short-term policy rate and the interest on reserves. The pass-through from the short-term policy to the real lending rate is ambiguous. However, the real lending rate is strictly decreasing in interest on reserves. As the marginal product of capital is determined by the real lending rate, $F_K(K, N) = \rho + \delta$, the central bank can stimulate the economy by exploiting the pass-through of monetary policy to interest rates.

Monetary Transmission and Breaking Neoclassical Dichotomy In the classical frictionless monetary models, output and the real interest rate are determined independently of monetary policy. In other words, monetary policy is neutral with respect to those real variables. Here, monetary policy could influence the real variable such as investment. It is worth discussing the difference in monetary transmission channels from the previous literature. In the textbook by Galí (2015), Chapter 1 shows that monetary policy is neutral in the classical frictionless monetary models, and Chapter 3, discusses how the presence of sticky prices makes monetary policy non-neutral.[20,21] In contrast to those approaches, in this model, monetary policy could influence the real variable without the presence of sticky prices.

[20]The term monetary neutrality is often used differently in the literature. In New Keynesian literature, monetary neutrality implies that changes in short-term policy rates and money supply both do not have impacts on real variables. In contrast, as discussed in Head, Liu, Menzio and Wright (2012), in the New Monetarist models, although money is not superneutral, since real effects result from changes in nominal interest rates, inflation, or money growth rate, money is neutral because changes in aggregate money supply do not have real effects on the real variable.

[21]It is worth mentioning that presence of sticky prices does not necessarily make monetary policy non-neutral. Head et al. (2012) provides a monetary search model where the price stickiness emerges endogenously in contrast to the models imposing price stickiness exogenously. While Head et al. (2012) explains price stickiness and micro-level price level changes which can match with microdata, money is neutral in their model.

To inspect the mechanism, recall the Equation (23), $F_K(K_{t+1}, N_{t+1}) = \rho_{t+1} + \delta$. A decrease in the real lending rate unambiguously lowers the marginal production of capital. To see the anatomy of what constitutes marginal production of capital, rewrite the marginal product of capital as below:

$$\underbrace{F_K(K_{t+1}, N_{t+1})}_{\text{marginal product of capital}} = \underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})} - 1 + \delta}_{\text{standard neoclassical term}} \quad + \quad \underbrace{\frac{\eta'(\tilde{\ell}_{t+1})}{1 + \pi_{t+1}}}_{\text{bank's marginal cost of lending}} \tag{30}$$

What distinguishes this model from the neoclassical growth model is the last term of the above equation. The banks' enforcement cost provides a wedge to finance the investments. For example, as shown in Proposition 1, an increase in interest on reserves lowers $\tilde{\ell}_{t+1}$ when the banks hold excess reserves. This reduces the marginal cost of financing investment and influences the real lending rate through general equilibrium impact.

This channel is different from other micro-founded monetary models with capital. Aruoba and Wright (2003) is the one of first papers that introduced the neoclassical growth model to the Lagos and Wright (2005) environment. As pointed out by Waller (2003), it features a strong neoclassical dichotomy, meaning the outcomes in the DM and the CM can be solved independently. Later Aruoba, Waller and Wright (2011) and Waller (2011) break this dichotomy by introducing the role of capital in the DM where capital accumulation lowers the cost of producing DM goods. The other way to break down the dichotomy is to introduce pledgeable capital, allowing more credit trade across agents by holding more capital. (e.g.,Venkateswaran and Wright, 2013 and Gu, Jiang and Wang, 2019) Here, what breaks down the neoclassical dichotomy is the limited commitment problem between firms and households.

4. Quantitative Analysis

The above section has developed a model of money creation analyzing monetary transmission. The model is tractable, and analytical results can be established. In this section, I calibrate the model and evaluate the model quantitatively.

4.1. Calibration

The model period length is set to one year. The utility functions for the DM and the CM are $u(q) = B[(q+\varsigma)^{1-\varphi} - \varsigma^{1-\varphi}]/(1-\varphi)$ and $U(C) = \log(C)$. The cost function for the DM is $c(q) = q$. The production function takes the form of a standard Cobb-Douglas function, $F(K, N) = K^{\alpha}H^{1-\alpha}$. The enforcement cost for lending is assumed to be quadratic, $\eta(\tilde{\ell}) = \Psi\tilde{\ell}^2$, and the balance sheet cost for managing reserves balances takes the form, $\gamma(\tilde{r}) = G\tilde{r}^g$.

The calibration period is 1968–2007. The baseline nominal interest rate is $i = 0.0593$, the average 3-month treasury rate, and the baseline required reserve ratio is $\chi = 0.1111$, the average of the ratio between required reserves and total checkable deposits. Since the Federal Reserve did not pay interest on reserve before October 2008, I set $i_r = 0$ as the baseline.

Some parameters are directly pinned down. The discount factor β is set to match a 3% real interest rate. The capital share in CM output is set to $\alpha = 1/3$ as the standard, and the capital depreciation rate is matched with $I/K = \delta = 0.0825$. To ensure that changes in credit conditions do not affect currency holdings, σ_3 has been set to 0, based on the stable downward sloping currency demand illustrated in Figure 8. The fraction of DM2 meetings, σ_2, has been set to 0.689 so that the equilibrium percentage of unsecured credit users matches 68.9%, which is the average percentage of US households holding at least one credit card from 1970 to 2007, based on the Survey of Consumer Finances. For simplicity, the bargaining power has been set to $\theta = 1$, which implies the buyer makes a take-it-or-leave-it offer to the seller in the DM. The probability of being a buyer has been normalized to $\nu = 0.5$. The parameter ς has been introduced in $u(q)$ merely to ensure that $u(0) = 0$, and has been set to $\varsigma = 0.0001$, as in Aruoba et al. (2011). The curvature parameter of $\gamma(\cdot)$ has been set to $g = 1.5$ to ensure that $\gamma(\cdot)$ is less convex than $\eta(\cdot)$.

The remaining 8 parameters $(\zeta, \kappa, \sigma_1, \bar{b}, G, B, \varphi, \Psi)$ are set to match the following 8 targets: (i) the standard measure of work as a fraction of discretionary time, $H = 1/3$; (ii) the average nominal lending rate, $i_\ell = 0.0862$; (iii) the currency output ratio, $M/PY = 0.0443$; (iv) the reserves output ratio, $R/PY = 0.0121$; (v) the unsecured credit output ratio, 0.0347; (vi) the capital output ratio, $K/Y = 2.1896$; (vii) the elasticity of currency demand to the

Table 2: Model parametrization

Parameter	Description	Value	Target Description	Target	Model
			External Parameters		
δ	depreciation rate	0.0825	investment/capital, I/K		
α	capital share in F	0.3333	labor's share of income, $2/3$		
β	discount factor	0.9709	real interest rate, 3%		
σ_2	DM2 matching prob.	0.6890	share of credit meeting		
ς	parameter of $u(\cdot)$	0.0001	Aruoba et al. (2011)		
θ	bargaining power	1	take-it-or-leave-it offer		
ν	prob. of being a buyer	0.5	normalization		
			Internal Parameters		
ζ	coeff. on labor supply	2.4949	labor supply, H	0.3333	0.3321
σ_1	DM1 matching prob.	0.0006	currency/output, M/PY	0.0443	0.0438
G	parameter of $\gamma(\cdot)$	0.0018	reserves/output, R/PY	0.0121	0.0117
$\bar{b}$	credit condition	0.0555	unsecured credit/output	0.0347	0.0347
κ	bank entry cost	0.0122	lending rate, i_ℓ	0.0862	0.0883
B	parameter of $u(\cdot)$	0.0156	capital/output, K/Y	2.1896	2.1957
φ	parameter of $u(\cdot)$	3.3144	elast. of M/PY to i	-0.1948	-0.2893
Ψ	parameter of $\eta(\cdot)$	0.0172	semi-elast. of i_ℓ to i	11.3673	11.2972

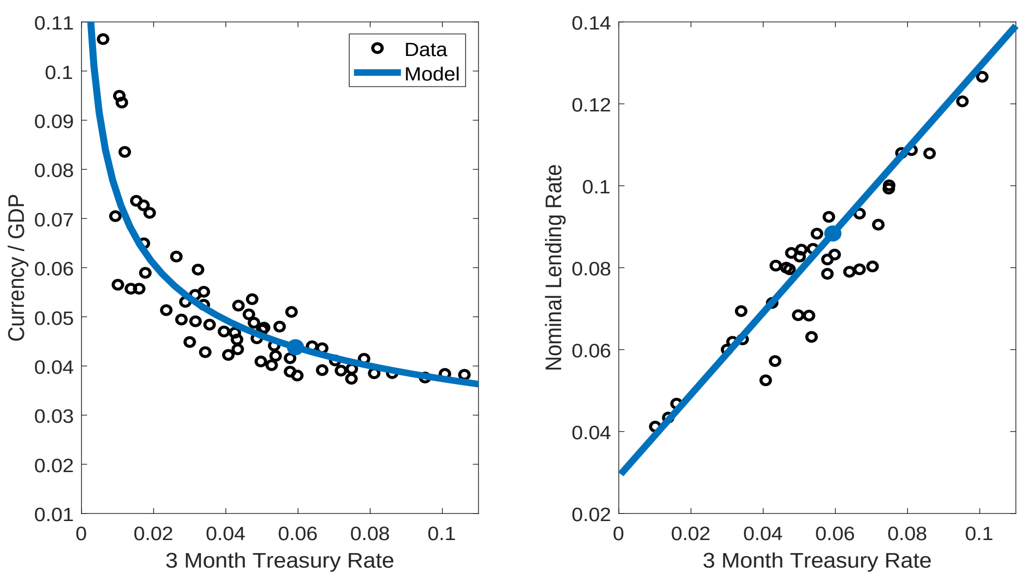


Figure 8: Money Demand for Currency and Interest rate Pass-through

nominal interest rate, -0.1948; (viii) the semi-elasticity of the nominal lending rate to the nominal interest rate, 11.3673. The targets are computed based on 1968-2007 data.

All of the targets in the model, except the elasticity of currency demand and the semi-elasticity of lending rate, are directly computed using straightforward formulas given the baseline nominal interest rate and the required reserve ratio. Similar to Aruoba et al. (2011), the elasticity of currency demand is computed using changes in money demand when the interest rate changes from $i - 0.05$ to $i + 0.05$. The semi-elasticity of lending rate is also computed using changes in the nominal lending rate when the interest rate changes from $i - 0.05$ to $i + 0.05$. The calibrated parameters and the targets are summarized in Table 2, and the calibrated money demand of currency and lending rate pass-through are shown in Figure 8. The model fits the targeted moments well.

4.2. Benchmark Result

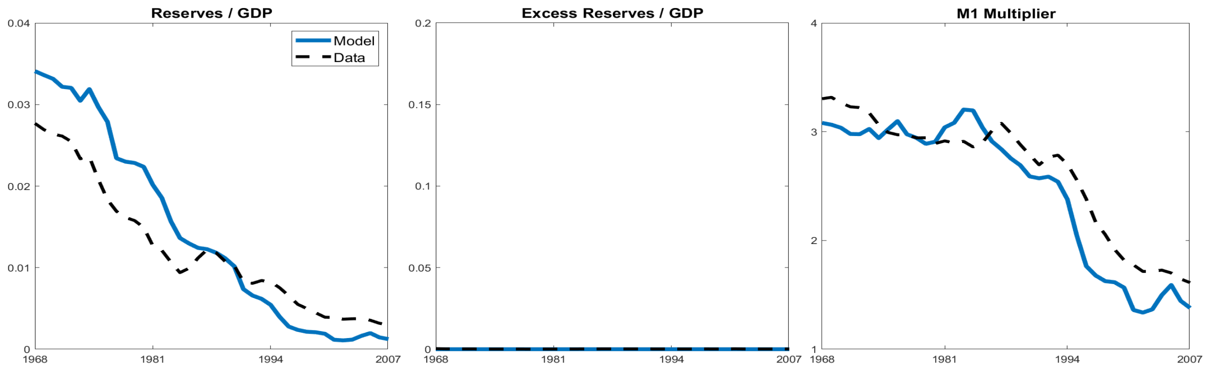
This section explores how well the model can account for the low-frequency behavior of reserves and the money creation process, assuming the only driving forces are monetary policy, (i, i_r, χ), and credit conditions, $\bar{b}$. The monetary policy variables are from data. The unsecured credit limit $\bar{b}$ is computed using the unsecured credit to output ratio, i.e., solving below

$$\frac{\sigma_2 \bar{b}}{AF(K,N) + \sum_{j=1}^{2} \sigma_j z_j} = \frac{\text{Unsecured Credit}}{\text{Nominal Output}}.$$

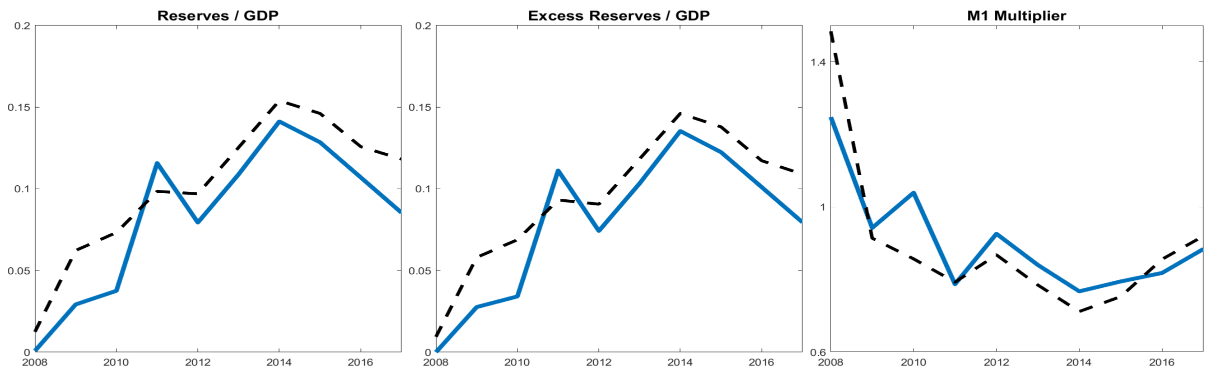
For policy variables, χ, i, and i_r are variables the central bank can determine. In this exercise, i and i_r are treated separately. The rate i_r is an administrative rate paid on reserves by the central bank, whereas i is not only a policy rate but also a market rate determined at equilibrium by controlling the supply of base money. I treat i and i_r as distinct policy variables to examine their separate roles in monetary policy, unlike much of the literature that assumes i is pinned down by i_r through arbitrage. As shown in Section 2, the weak exogeneity tests indicate that $\bar{b}$ influences the system (1) without adjusting to restore long-run equilibrium. This suggests that it is not involved in the long-run error correction process, and it functions more as a driving variable within the system in the long-run. This aligns with treating $\bar{b}$ as one of the driving forces at low frequency (long-run).

Using the calibrated parameters, I compute the stationary equilibrium for given monetary policy data (i, i_r, χ) and the unsecured credit-to-output ratio for each year. I refer to this factual simulation as the benchmark simulation. For monetary policy variables, I use the 3-month Treasury rate, interest on excess reserves,[22] and the required reserves to total check-

[22]While the Federal Reserve announced interest on excess and required reserves separately, it paid the same rate for both. As of March 2020, these were unified as "Interest Rate on Reserve Balances" after the reserve



(a) 1968-2007



(b) 2008-2017

Figure 9: Model vs. Data

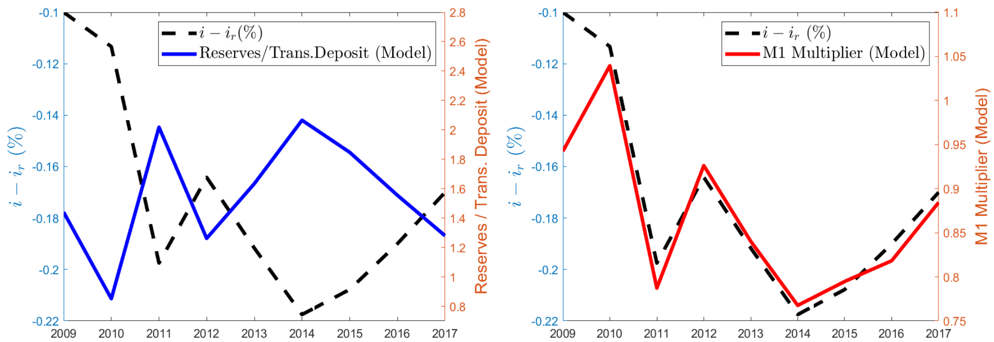
able deposit ratio. The benchmark simulation computes the reserves-to-output ratio, excess reserves-to-output ratio, and M1 multiplier. It fits the data well and successfully reproduces four empirical observations discussed in Section 2. Details follow below.

Reserves and Money Multiplier Figure 9 compares the model and data from 1968 to 2017. The top-left panel and the bottom-left panel show reserves to output ratios from 1968 to 2007 and from 2008 to 2017, respectively. The model generates the series of quantity of reserves, excess reserves and money multiplier, which matches the movement in the data. This result contradicts the view that the Fed drastically increased the reserve supply without changing short-term rates, because in the model this is mostly driven by changes in the interest rate. We will get into more details of tranmision mechaism on this in the next sections.

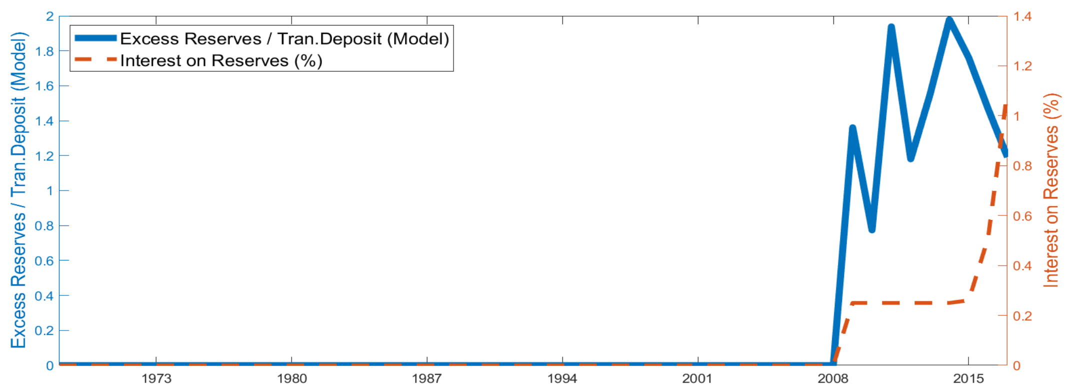
The top-middle panel and the bottom-middle panel of the Figure show excess reserves as a fraction of output from 1968 to 2007 and from 2008 to 2017, respectively. The model also successfully generates zero excess reserves during 1968-2007 and the huge increase

requirement was reduced to zero.

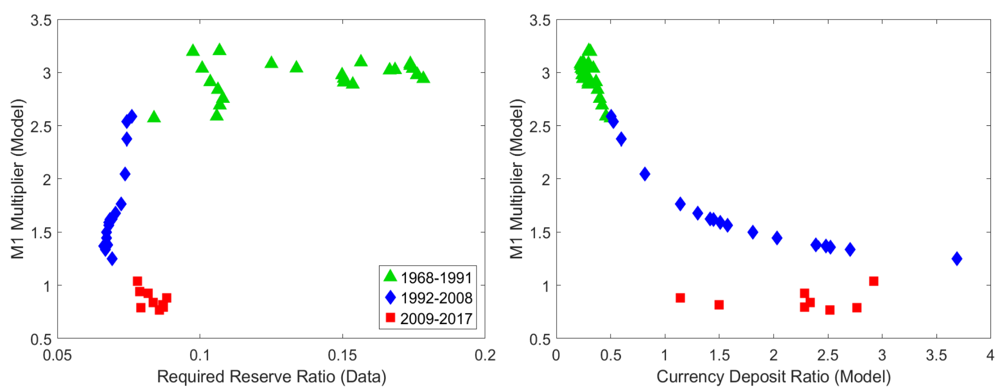
during 2008-2017. The top-right panel and the bottom-right panel of Figure 9 show the M1 money multiplier. The model also generates very similar patterns. The money multiplier decreased during 1968-2007 without excess reserves. During 2008-2017, the money multiplier decreased to lower than one. The benchmark simulation of the model fits data well.



(a) Reserves Deposit ratio and M1 multiplier: Model



(b) Excess reserves ratio (Model)



(c) Money multiplier, currency/deposit ratio and required reserve ratio: Model

Figure 10: Additional results: Benchmark Simulation

Table 3: Money demand and Credit: Model vs. Data

Dependent Variable: $ln(m_t)$	OLS				CCR	
	Data	Model	Data	Model	Data	Model
	(1)	(2)	(3)	(4)	(5)	(6)
r_t	1.600***	10.462	−2.298**	−1.807	−2.022**	−0.845
	(0.419)		(0.885)		(0.818)	
$ln(uc_t)$			−0.322***	−1.014	−0.341***	−0.999
			(0.042)		(0.069)	
$adjR^2$	0.109	0.567	0.416	0.919	0.494	0.608

Notes: Columns (1)-(4) report OLS estimates and columns (5) and (6) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on Bartlett kernel and lag 2. For (1) and (3) Newey-West standard errors with lag 2 are reported in parentheses. Intercepts are included but not reported.

Matching Observations We now examine whether this benchmark result reproduces the observed patterns discussed in Section 2. The simulation successfully replicates Observations 1 and 2 regarding reserves. Figure 10a displays the evident opposite movements of the reserve-to-deposit ratio with respect to the spread between the short-term policy rate and interest on reserves, consistent with Figure 2. As shown in Proposition 3, a higher interest rate on reserves increases banks' incentive to hold reserves but reduces the money multiplier since banks do not create transaction deposits proportionally. The model also reproduces the pattern where excess reserves remained at zero until interest on reserves was introduced, after which they skyrocketed (Figure 10b).

The model successfully captures Observations 3 and 4 regarding structural changes and money demand. Figure 10c shows the persistent decrease in the money multiplier since 1992 that was not accompanied by a decrease in the required reserve ratio but rather by an increase in the currency-to-deposit ratio. The model also reproduces the post-2008 decrease in the money multiplier without an accompanying increase in the currency-to-deposit ratio. Finally, based on the benchmark simulation series, Table 3 shows that, in the model, unsecured credit consideration recovers the downward-sloping money demand, as in the data.[23] Column (2) shows that the regressing only on interest rate gives positive estimates. Columns (4) and (6) show that adding the unsecured-credit-to-output ratio restores the expected negative relationship. The benchmark simulation reproduces the observed patterns discussed in Section 2 well.

[23]To compare the result from Table 1, the model part of Table 3 uses annualized quarterly series instead of annual data.

[24]The monetary base and currency are sourced from 'H.6 Money Stock Measures', published by the Federal Reserve. These data can be found in the Federal Reserve Economic Data (FRED) database under the series BOGMBASE and CURRSL. Additionally, data on required reserves, excess reserves, and vault cash surplus are obtained from 'H.3 Aggregate Reserves of Depository Institutions and the Monetary Base', also published by the Federal Reserve. These series are compiled in the FRED under the series EXCRESNS, EXCSRESNS

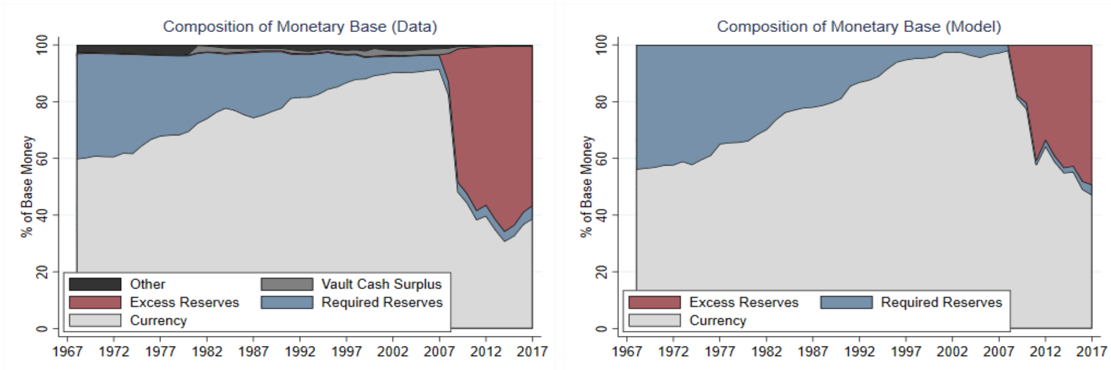


Figure 11: Composition of monetary base: data vs. model[24]

Composition of Monetary Base The model also generates the composition of the monetary base over time. Figure 11 compares the composition of the monetary base between the data and the model. The model successfully captures the changes in each component of the monetary base - currency, required reserves, and excess reserves - both before and after 2008. The currency portion consistently increased from 1968 to 2007 and then drastically decreased as the portion of excess reserves significantly increased. The portion of required reserves has consistently decreased. It is worth emphasizing that the share of currency in the monetary base has been substantial. Until 2008, its share had increased from 60% to 90%. After 2008, its share has hovered around 40%. Even though the share of currency has been reduced, it still accounts for a large portion of the central bank's balance sheet. This implies that if one wants to consider the central bank's balance sheet as an important channel of monetary policy, incorporating the currency in the analysis might be necessary.

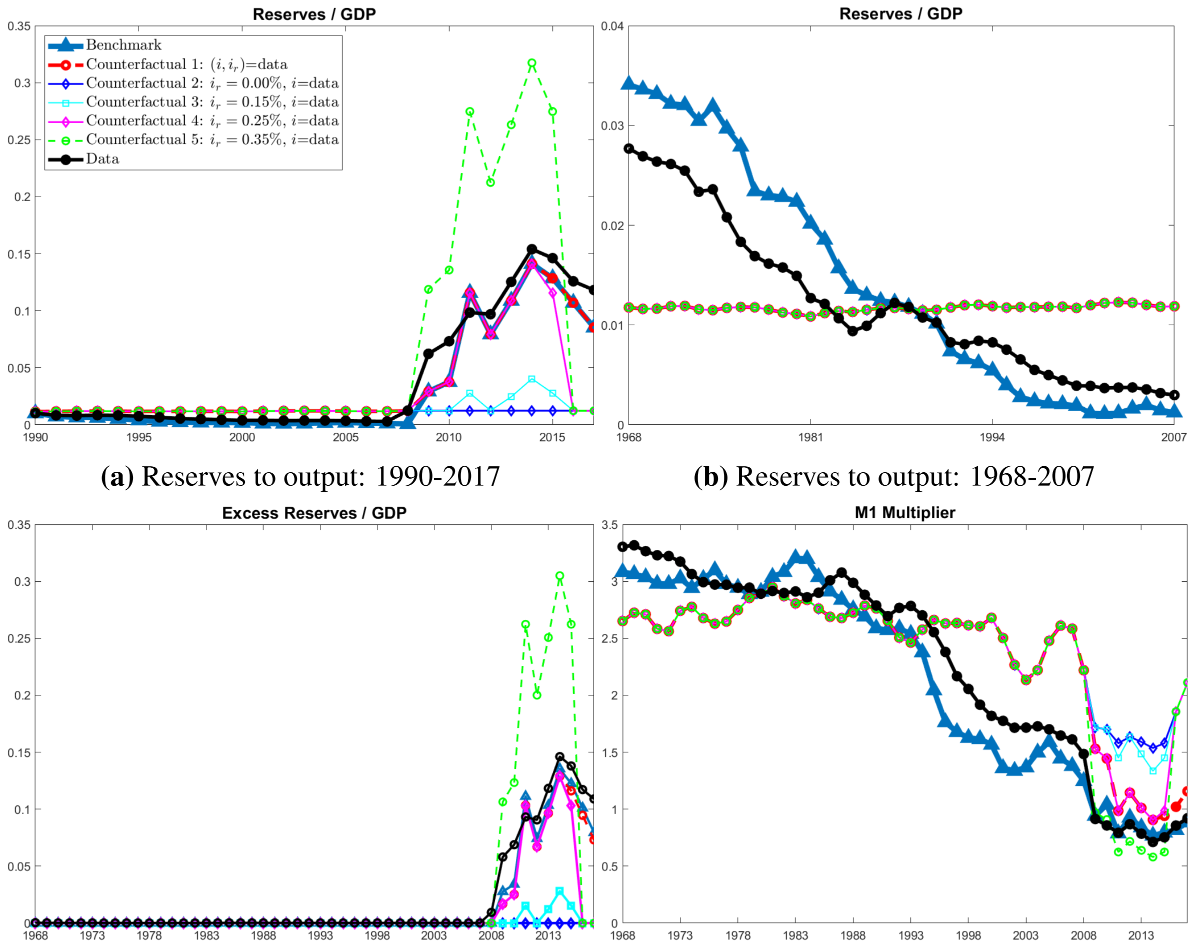
4.3. Effects of Interest on Reserves

First, to identify how the IOR contributed to the changes observed in each variable during the benchmark simulation, this section presents a series of counterfactual experiments. The first counterfactual re-computes the stationary equilibrium using two monetary policy variables

Table 4: Summary of Counterfactual Experiments

Counterfactual	Description
1	$b = 0.0555$, $\chi = 0.11$, (i, i_r) at data values.
2	$\bar{b} = 0.0555$, $\chi = 0.11$, $i_r = 0.00\%$, i at data values.
3	$\bar{b} = 0.0555$, $\chi = 0.11$, $i_r = 0.15\%$, i at data values.
4	$\bar{b} = 0.0555$, $\chi = 0.11$, $i_r = 0.25\%$, i at data values.
5	$\bar{b} = 0.0555$, $\chi = 0.11$, $i_r = 0.35\%$, i at data values.

REQRESNS, and VAULTSUR. The category 'Other' represents the monetary base that does not fall into any of these specified categories.



(a) Reserves to output: 1990-2017 **(b)** Reserves to output: 1968-2007

(c) Excess reserves to output: 1968-2017 **(d)** M1 multiplier: 1968-2007

Figure 12: Data and Counterfactual Examples

(i, i_r) set equal to their data values, while setting $\bar{b} = 0.0555$ and $\chi = 0.11$. The remaining counterfactuals maintain the fixed baseline values for $\bar{b}$ and χ, set the short-term rates (i) at their data values over time, and then examine the isolated effects of interest on reserves (i_r) at levels: 0.00%, 0.15%, 0.25%, and 0.35% (see Table 4).

Figure 12b shows the reserves-to-GDP ratio for 1990–2017. Similar to the benchmark simulation, Counterfactual 1 reproduces post-2008 reserve series that closely track the data both in levels and patterns. This suggests that changes in reserves during these periods cannot be attributed solely to balance sheet policy operating independently of interest rate policy. For example, in the literature, the sharp increase in reserves from 2008 to 2014 is often interpreted as the outcome of the Fed's balance sheet expansion independent of the policy rate. The model, however, reproduces a sizeable increase in reserves by adjusting only (i, i_r), and it fits the data well. This finding directly challenges the prevailing consensus in the literature. How much of the change in reserves can be explained by movements in the short-term rate? Counterfactuals 2–5 indicate that variations in the nominal short-term rate account for a large share of changes in the quantity of reserves. Moreover, as i_r rises from 0% to 0.35%, the model generates reserve paths that diverge substantially from the observed data, highlighting the sensitivity of reserves to policy rate changes.

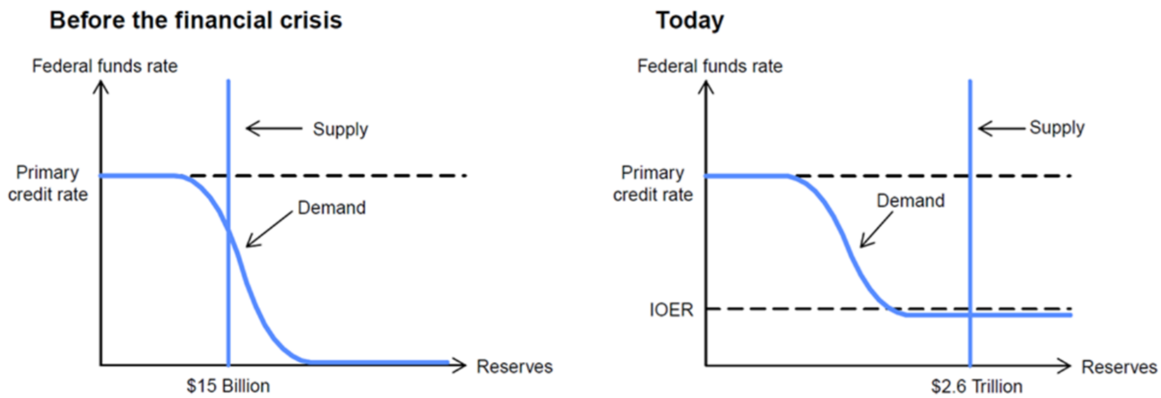
Figure 12a shows the reserves-to-GDP ratio for 1968–2007. This timeframe provides a useful comparison, as it covers an era when the Fed did not pay interest on reserves. Paying IOR and changing IOR do not produce noticeable effects on reserve balances in this period. However, the quantity of reserves mostly depends on credit conditions, reserve requirements, and the short-term policy rate. This channel will be revisited in Section 4.4. Figure 12c shows the excess reserves-to-output ratio for 1968–2017. This demonstrates that whether the economy operates under an ample reserves regime depends jointly on i and i_r. This result is already established in Proposition 2. Given i, paying interest on reserves above $\bar{\imath}_r$ yields an ample reserves regime; however, here $\bar{\imath}_r$ decreases as i decreases. Therefore, the central bank can implement monetary policy in the ample reserves regime by paying i_r and setting i low enough. Once the economy is in the ample reserves regime, banks are willing to hold more excess reserves under higher i_r, and real reserve balances are determined via downward-sloping reserve demand.

Figure 12d shows the M1 money multiplier for 1968–2017. While the M1 multiplier has decreased since the early 1990s in both the data and the benchmark simulation of Section 4.2, the results differ when $\bar{b}$ is fixed: the multiplier does not drop starting from the early 1990s; instead, it drops when IOR is paid. When IOR is not paid, the multiplier remains above 1.5, though it decreases to around 1.5 when the central bank pays IOR. Paying IOR at higher rates further decreases the M1 multiplier.

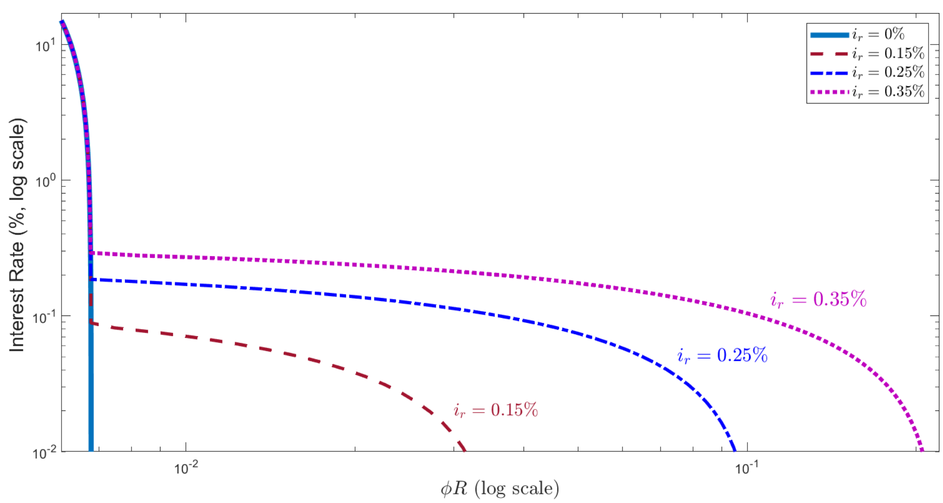
Reserve Demand and Interest on Reserve: Comparison with other Approach How was the model able to generate such results? To illustrate the mechanism of this model, we compare a popular understanding and this model's framework of reserve demand.

Figure 13 compares the model's features with a popular understanding of the policy implementation framework. Figure 13a, taken from Ihrig, Meade and Weinbach (2015), illustrates the current popular understanding of policy implementation: Before 2008, the Fed changed the supply of reserves to set the short-term interest rate at its target level. After 2008, the Fed pays IOR. As long as the Fed supplies a sufficient level of reserves, the short-term rate is determined by arbitrage, which pins down the short-term rate around the IOR rate. The popular interpretation is that the central bank's balance sheet became a new instrument for monetary policy as reserves became independent of short-term rates.

This paper provides a different understanding of monetary transmission. In figure 13b,



(a) Popular Understanding of the Policy Implementation Framework (Ihrig et al., 2015)

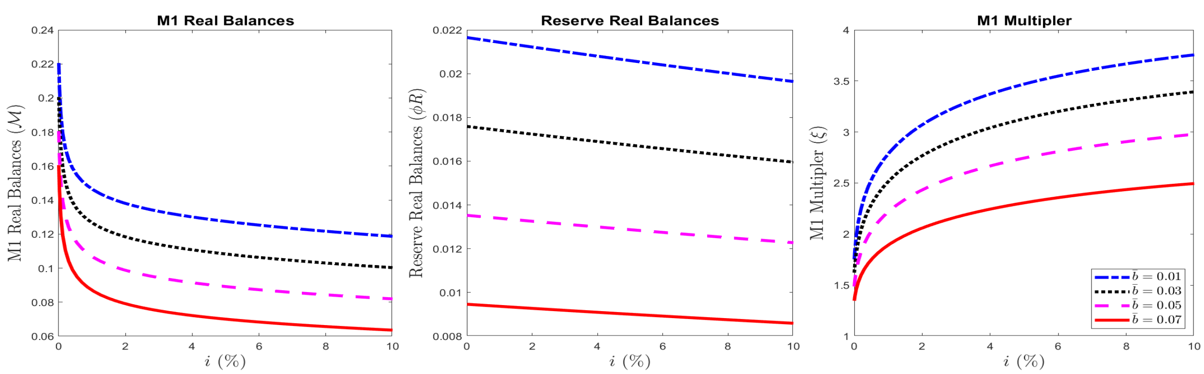


(b) This Paper's Framework: Reserves Demand under Different Interest on Reserves

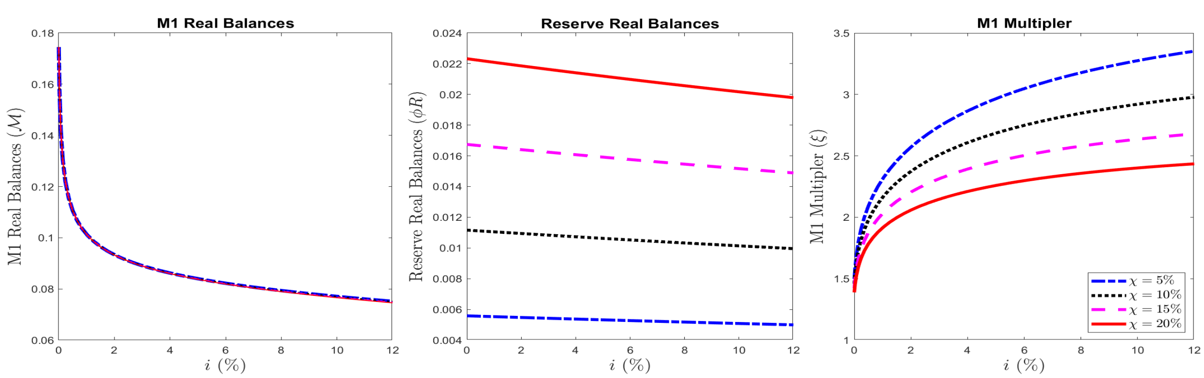
Figure 13: Common Understanding vs. This Paper's Framework

(real) reserve demand is still downward sloping. Paying IOR increases reserve demand, causing it to shift to the right, and makes the lower part of reserve demand more elastic. In this framework, the tremendous amount of reserves in the post-2008 period compared to the small amount of reserves during the pre-2008 period can be quantitatively explained based on interest rates changes, which has been shown in Section 4. This approach does not require the assumption of independence between equilibrium prices (short-term policy rates) and quantities (monetary base or reserves). It is worth to note that even under this framework, in the ample reserve regime, the central bank still can change the policy rate by changing the IOR rate without changing the quantity of reserves, as reserve demand shifts with respect to IOR changes.

4.4. Effects of Credit Condition and Reserve Requirement



(a) Effects of Credit Condition



(b) Effects of Reserve Requirement

Figure 14: Effects of Credit Condition and Reserve Requirement

Figure 14 shows the effects of the credit condition parameter $\bar{b}$ and the reserve requirement χ when the central bank pays no interest on reserves. The three panels in each row correspond to the M1 real balances ($\mathcal{M}$), reserve real balances (ϕR), and the M1 multiplier (ξ), respectively.

As shown in Proposition 4, Figure 14a illustrates how credit conditions influence monetary aggregates. When $\bar{b}$ increases, households rely less on deposit money for transactions. As a result, M1 real balances decrease. Because reserves are held as a fixed proportion of deposits due to the reserve requirement, reserve real balances also fall accordingly. This pattern reflects that more credit crowds out deposits via substitution, while the demand for currency remains largely unaffected. Consequently, the M1 money multiplier decreases as $\bar{b}$ rises, as shown in the right panel.

Figure 14b depicts the effects of varying the reserve requirement χ. Unlike the credit condition, higher χ does not substantially change the M1 real balances. However, reserve real balances rise proportionally with the increase in χ, as banks are required to hold a larger fraction of deposits as reserves. This increase in reserve holdings lowers the M1 multiplier. The right panel shows a clear downward shift of the multiplier curve as χ rises. This result mirrors the textbook relationship where a higher reserve ratio reduces the money multiplier.

Taken together, the comparative statics in Figure 14 highlight two distinct mechanisms. Improvement in credit conditions (higher $\bar{b}$) reduces the need for deposit-based transactions, contracting M1 and reserves simultaneously, while tighter reserve regulations (higher χ) mainly alter the composition of reserves without significantly affecting total money demand. Both mechanisms lower the M1 multiplier, but for different reasons: one through the substitution between credit and deposits, and the other through regulatory constraints on deposit expansion.

5. Implications for Monetary Transmission

5.1. Reserves Are Not So Ample After All?

On September 17, 2019, the target policy rate range of the Fed was 2.00-2.25 percent and the interest on reserves (IOR) rate was at 2.10%. On that day, the effective federal funds rate was 2.30%, with some federal funds transactions made above 4%. Secured Overnight Financing Rates (SOFR) spiked even higher, in some cases exceeding 9%. This money market disruption was resolved only after the New York Fed injected \$75 billion into the money market. This episode occurred during the period of quantitative tightening. Copeland et al. (2025) provides empirical evidence showing that the level of reserve balances played a critical role during this money market disruption.

This event may seem to be the direct opposite of the current popular framework, because short-term rates were hiked when the Fed was reducing the supply of reserves. How has the literature reconciled this observation with the framework that assumes reserves are independent of policy implementation? The popular view is that quantitative tightening pushed reserves close to the minimum ample level. Within the framework of Figure 13a, the concept of the minimum ample level of reserves is central to quantitative tightening because the Fed wants to maintain a large balance of reserves aiming to implement monetary policy in an "ample reserves" framework. For example, FOMC (2022) noted:

> "[T]he Committee intends to maintain securities holdings in amounts needed to implement monetary policy efficiently and effectively in its ample reserves regime."

For this reason, many economists have attempted to estimate the minimum ample level and documented possible structural changes in reserve demand (Yang, 2020, Copeland et al., 2025, and Afonso et al., 2022a). Several speeches of the Federal Reserve Banks' presidents also recognize the structural increases in reserve demand as well as increases in the minimum ample level of reserves (Bostic, 2019 and Logan, 2023).

However, this paper takes a different approach. The post-2008 reserve demand does not require any structural change beyond the introduction of IOR to explain the historical patterns. This is because the model still features a downward-sloping reserve demand even though paying interest on reserves shifts the demand curve outward and makes its lower segment more elastic. This mechanism could quantitatively explain the observed increases in reserves after 2008, unlike the current consensus based on the independence assumption on short-term rate and reserves. The increase in reserve demand could be explained by the changes in IOR rather than secular and permanent increases in reserve demand.

5.2. Monetary Base, Money Creation, and Monetary Policy

The central bank implements its interest-rate target by managing the monetary base. This is consistent with the conventional notion of monetary policy. Recall Romer (2000):

> "[The] appropriate concept of money is unambiguously high-powered money. Here M is not a variable the central bank is targeting, but rather one it is manipulating to make interest rates behave in the way it desires. This is an excellent description of high-powered money. Moreover, for high-powered money, the assumption that the opportunity cost of holding money is the nominal rate is appropriate. In addition, the assumption that the central bank can control the money stock is a much better approximation for high-powered money than for broader measures of the money stock."

This still holds in the conduct of US monetary policy in the post-2008 era. As explained in early sections, in the repo market disruption of 2019, the short-term rate became a lot higher than the IOR and the target range, and it was resolved after the Fed injected $75 billion into the money market. Keeping the short-term rate below the upper bound is achieved by supplying reserves. The ON RRP facility or ON RRP operation absorb reserves from the money market to keep the short-term rate above the lower bound of the target range.

However, the recent literature does not seem to align with this understanding of monetary policy. Many macroeconomic models got rid of the money demand channel of monetary transmission, especially after the Fed introduced interest on reserves.[25] This paper show that paying interest on reserves does not make the quantity of reserves independent of interest-rate policy: in this paper's framework and in the data, neither the short-term market rate nor IOR is independent of the quantity of reserves. The model therefore does not require any independence assumption between equilibrium prices (short-term policy rates) and quantities (the monetary base or reserves).

Given credit conditions, the model delivers a one-to-one mapping from a set of policy instruments to the quantity of currency, total reserves, excess reserves, and the money multiplier. This approach does not assume that the central bank directly fixes broad monetary aggregates, nor does it posit an abstract ability to set interest rates 'out of nothing.' Instead, it provides an explicit implementation mechanism: the central bank sets short-term policy rates by managing the monetary base, which in turn influences monetary aggregates and other macroeconomic variables.

This paper considers reserve requirement policy and interest on reserves, extending beyond a sole focus on the short-term rate. These policies affect different components of the money supply in distinct ways. As Lucas (2000) noted, to understand the effects of these policies, a model that features different roles of currency, reserves, and deposits is essential. Ignoring this aspect may lead to inadequate interpretation of policy effects, such as assuming that reserves are independent of the short-term policy rate and the IOR rate under the ample reserve regime.

This perspective also reframes so-called unconventional policies. Quantitative easing is not an asset-purchase program independent of the short-rate operating target. Rather, it combines two rates (IOR and the short-term policy rate) which are closely linked to balance-sheet quantity. Changes in reserves and short-term rates are jointly determined along a downward-sloping reserve demand curve, before and after 2008.[26] As short-term rates are determined

[25] Since Woodford (1998), many models (especially New Keynesian models) abstract away from money demand using the cashless limit result. After 2008, many models introduced reserves into the models but with the assumption of their independence from the short-term rate, at least in an ample reserve regime.

[26] It is worth noting that the Federal Reserve now supplies and absorbs reserves largely through standing

by the downward-sloping reserve demand curve, QE can be interpreted as the coordinated use of two separate tools: IOR and reserve quantities (alongside asset-composition choices that affect term premia).

6. Concluding Remarks

This paper develops a monetary-search model with fractional reserve banking and unsecured credit, and studies the role of money creation in monetary transmission. In the fractional reserve banking system, money is created when banks make loans. The bank's inside money creation, however, can be constrained by the reserve requirement and the reserves. Whether the reserve requirement constraint binds or not is endogenously determined by the banks' profit maximization.

Banks hold excess reserves when the central bank pays sufficiently high interest on reserves. In this case, the money multiplier and the quantity of the reserve depend on the short-term policy rate and the interest on reserves rather than the reserve requirement. In contrast to previous works, these two interest rates play distinct roles, and the quantity of reserves is not independent of interest rate management. Furthermore, their impact on the lending rate are different: the lending rate increases with the short-term interest rate, while it decreases with interest on reserves. Quantitative analysis can generate simulated data that resemble the actual data. This paper provides evidence from the model and the data that suggests that the dramatic changes in the money multiplier after 2008 are mainly driven by the introduction of the interest on reserves.

This work can be extended in various ways. Although I focus on the centralized market for the reserves with homogeneous banks, in reality, the market for reserves is a decentralized interbank market and banks have different portfolios. Therefore, one can further investigate how much the market structure and heterogeneity matter for the transmission of monetary policy (e.g., Afonso and Lagos, 2015; Armenter and Lester, 2017; Afonso, Armenter and Lester, 2019). Second, I assume that bank assets are composed of loans and reserves. But commercial banks' assets are composed of securities, loans, and reserves. Extending the model to incorporate banks' portfolio choices and analyzing the role of investment, financial regulation, and monetary policy can open up other research avenues. (e.g., Rocheteau, Wright and Zhang, 2018a).

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facilities and their administered rates—such as the overnight reverse repo (ON RRP) facility, the standing repo facility (SRF), and the discount window. This approach simplifies daily implementation relative to the pre-2008 system, which "requires forecasting the many exogenous factors that affect the amount of bank reserves outstanding, and then engaging in open market operations on a near-daily basis to keep reserves at a level consistent with the FOMC's target range" (Dudley, 2018).

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APPENDIX

A. Interest on Reserves and Floor System?

This section discusses details about the interest on reserves and the floor system. The Federal Reserve started paying interest on reserves (IOR) in Oct 2008. In contrast to the idea that paying interest on reserves provides a floor, Figure 15 shows that interest on reserves has been equal to the upper bound of the Fed's target range. Instead, the interest rate of the Overnight Reverse Repurchase (ON RRP) has been equal to the lower bound of the target rate. To put it simply, the interest on reserves serves as the upper bound of the target range, while the ON RRP rate acts as the lower bound of the target range for most of the period. The short-term policy rate is tightly controlled within this target range.

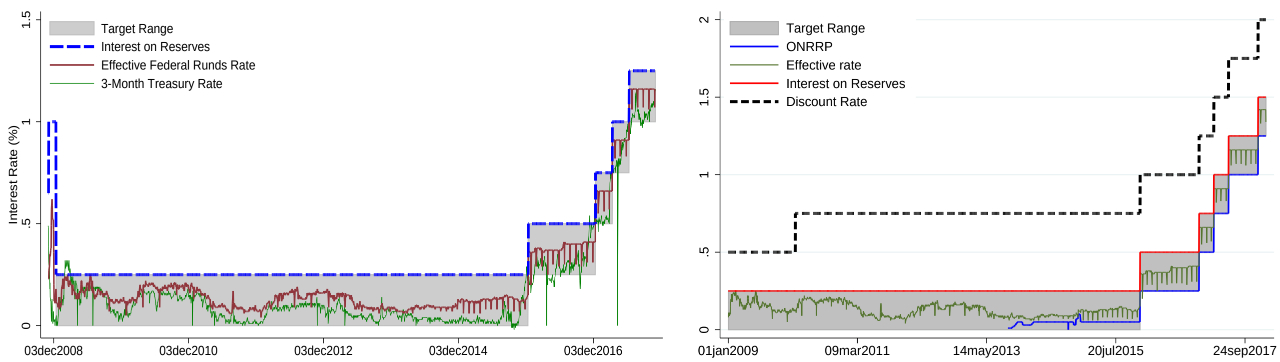


Figure 15: Interest on reserves, ON RRP and target range

Given that the federal funds market is a market for unsecured bilateral interbank lending of reserves, a lower federal funds rate than the IOR may seem counterintuitive. This discrepancy is due to the institutional framework. While both depository institutions (DIs) and government-sponsored enterprises (GSEs) can trade in the federal funds market, only DIs are eligible to earn IOR, not GSEs. The GSEs, such as the Federal Home Loan Banks, can earn arbitrage profits by borrowing from the repo market and lending to DIs in the federal funds market. As DIs can earn interest on reserves and the GSEs earn the arbitrage profit, the federal funds rates are usually determined as below IOR.

Armenter and Lester (2017) present a model of interbank trade in which GSEs' lending to DIs that earn IOR determines the federal funds rates. Since GSEs also participate in the repo market, the ON RRP facility's interest rate serves as an alternative for financial institutions not eligible for the IOR, establishing a lower bound on the federal funds rate. See Afonso, Entz, LeSueur et al. (2013a) and Afonso, Entz, LeSueur et al. (2013b) for more discussion on the trade in the federal funds market.

B. Retail Sweep

One may think that the structural break in 1992 found in Section 2 can be attributed to the relaxation of bank deposit regulation in the 1990s that stimulated financial innovations such as retail sweep accounts in the 1990s (e.g., VanHoose and Humphrey, 2001, Teles and Zhou, 2005, Lucas and Nicolini, 2015, Berentsen, Huber and Marchesiani, 2015). Base on this idea, some previous works have used an alternative measure of M1 as monetary aggregates, "M1 Adjusted for Retail Sweeps" (M1S, hereafter).

The rationale for using M1S is that the introduction of the automatic transfer system (ATS) in the early 1990s made highly liquid transaction balances outside M1. The ATS enables convenient transfer from money market deposit account (MMDA) to sweep account. The sweep account is a transaction deposit, whereas the MMDA is a saving deposit that is classified as Non-M1 M2. While the MMDA pays higher deposit rates than other saving deposits, same as other saving deposits, the MMDA had been subject to the restriction on the number of convenient transactions due to Regulation D.[27] Thus, the introduction of ATS may enable people to use MMDA as a liquid deposit. The claim is that this may result in the appearance of highly liquid transaction balances within instruments outside M1. Cynamon, Dutkowsky and Jones (2006) discuss the benefits of using M1S instead of M1 as a measure of money supply.

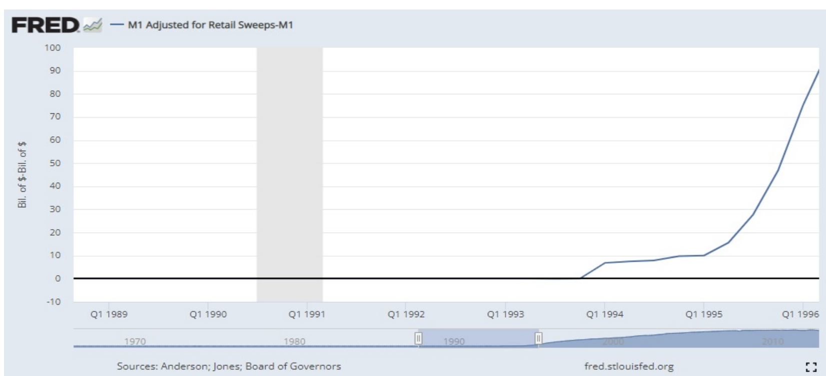


Figure 16: Difference between M1 adjusted for retail sweep and M1

However, it is worth noting that the ATS was introduced to commercial banks in 1994 while the structural break of M1 from Section 2 occurred in 1992. Consistent with this, Figure 16 shows that the difference between M1S and M1 was zero before 1994. Using M1S, Berentsen et al. (2015) and Kejriwal et al. (2022) also found the structural break of M1 in the early 1990s, suggesting retail sweep consideration cannot explain the break in the early 1990s.

[27]The MMDA was introduced in the early 1980s after the US Congress permitted its creation as of December 1982. Due to Regulation D, no more than six convenient transactions using the MMDA could be made per statement period. As of March 2020, the Federal Reserve removed this restriction.

C. M2 and Its Structural Break

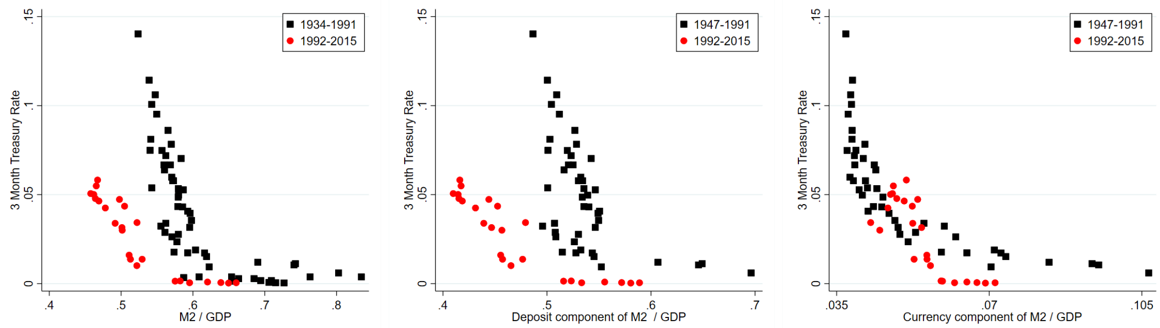


Figure 17: US Money demand for M2 and its components

Similar to M1, M2 can also be decomposed into its deposit and currency components. Figure 17 plots the ratio of M2 and its components to GDP against the 3-month Treasury Bill rate, revealing a breakdown in M2 around 1992 that coincided with the structural break observed in Figures 4 and 5. This breakdown was caused by the deposit component; in contrast, the currency component displays a stable downward-sloping demand.

To see whether the unsecured credit can account for this breakdown in M2, I repeated the analysis of Table 1 using M2 instead of M1. Table 5 reports the results. Again, I focus on the post-1980 period, until the arrival of the Great Recession. In columns (2) and (4), the Johansen tests reject their null of no cointegration at the 99 percent confidence level, suggesting there exists a stable relationship between M2 real money balances, interest rates, and real balances of unsecured credit. The canonical cointegrating regression estimates in columns (2) and (4) show that the estimated coefficients on r_t and $ln(uc_t)$ are both negative and significantly different from zero.

Thus, using the cointegrating regressions and tests, I document evidence that once we account for the substitution effect of consumer credit, there still exists a stable negative relationship between M2 real balances and the interest rates. Table 6 provides the unit root test results for the M2 to output ratio and the deposit component of the M2 to output ratio. For both variables, the unit root tests fail to reject the null hypothesis of non-stationarity, while their first differences reject the null hypothesis of non-stationarity at the 1% significance level. All series are demeaned before implementing the unit root test.

Table 5: Cointegration regressions and tests

Dependent Variable:	$ln(m_t)$			$ln(d_t)$		
	OLS	OLS	CCR	OLS	OLS	CCR
	(1)	(2)	(3)	(4)	(5)	(6)
r_t	0.848***	−1.633***	−1.417***	1.279***	−1.762***	−1.389***
	(0.186)	(0.347)	(0.511)	(0.226)	(0.377)	(0.610)
$ln(uc_t)$		−0.205***	−0.205***		−0.251***	−0.388***
		(0.025)	(0.000)		(0.023)	(0.060)
$adjR^2$	0.130	0.657	0.423	0.198	0.718	0.479
Observation	112	112	112	112	112	112
Johansen $r = 0$	14.00	43.14		15.97	43.83	
1% CV	24.60	41.07		24.60	41.07	
Johansen $r = 1$	2.64	14.94		2.63	15.54	
1% CV	12.97	24.60		12.97	24.60	

Notes: Columns (1)-(2) and (4)-(5) report OLS estimates and columns (3) and (6) report the canonical cointegrating regression (CCR) estimates. First-stage long-run variance estimation for CCR is based on the Bartlett kernel and lag 2. For (1)-(2) and (4)-(5), Newey-West standard errors with lag 2 are reported in parentheses. Intercepts are included but not reported. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Johansen cointegration test results are reported: trace test statistics and critical values (CVs). Appendix F contains unit root tests for each series. The data are quarterly from 1980Q1 to 2007Q4.

Table 6: Unit root test

		Phillips-Perron test			
		Test statistic	1% CV	5% CV	10% CV
$ln(m)$	$Z(\rho)$	−2.626	−19.837	−13.722	−11.015
	$Z(t)$	−1.171	−3.506	−2.889	−2.579
$ln(d)$	$Z(\rho)$	−2.053	−19.837	−13.722	−11.015
	$Z(t)$	−1.053	−3.506	−2.889	−2.579
$ln(uc)$	$Z(\rho)$	−1.120	−19.837	−13.722	−11.015
	$Z(t)$	−1.717	−3.506	−2.889	−2.579
r	$Z(\rho)$	−7.721	−19.837	−13.722	−11.015
	$Z(t)$	−2.471	−3.506	−2.889	−2.579
$\Delta ln(m)$	$Z(\rho)$	−61.426***	−19.833	−13.720	−11.013
	$Z(t)$	−6.458***	−3.507	−2.889	−2.579
$\Delta ln(d)$	$Z(\rho)$	−59.193***	−19.833	−13.720	−11.013
	$Z(t)$	−6.302***	−3.507	−2.889	−2.579
$\Delta ln(uc)$	$Z(\rho)$	−41.882***	−19.833	−13.720	−11.013
	$Z(t)$	−5.098***	−3.507	−2.889	−2.579
Δr	$Z(\rho)$	−94.183***	−19.833	−13.720	−11.013
	$Z(t)$	−9.263***	−3.507	−2.889	−2.579

D. Proofs

Proof of Proposition 1. Consider the case where the reserve requirement constraint is not binding. Recall equation (20) and $i_t = i_{s,t+1}$:

$$i_{r,t+1} - i_t = \gamma'(\tilde{r}_{t+1})$$

Given $i_{r,t+1} = 0$ and $i_t \geq 0$, solving (20) yields $\tilde{r}_{t+1} \leq 0$ which could not be a equilibrium. Therefore, when the central bank does not pay interest on reserves $i_{r,t+1} = 0$ there is no equilibrium with excess reserves $\tilde{r} > \chi\tilde{d} \geq 0$.

Next, we move on to the comparative static analysis. Recall (16) and (19)-(21):

$$\begin{aligned} 0 = G^1 \equiv\ & (1 + i_{\ell,t+1})\tilde{\ell}_{t+1} + (1 + i_{r,t+1})\tilde{r}_{t+1} \\ & - (1 + i_t)(\tilde{\ell}_{t+1} + \tilde{r}_{t+1}) - \gamma(\tilde{r}_{t+1}) - \eta(\tilde{\ell}_{t+1}) - \kappa \end{aligned} \tag{31}$$

$$0 = G^2 \equiv i_{r,t+1} - i_t - \gamma'(\tilde{r}_{t+1}) \tag{32}$$

$$0 = G^3 \equiv i_{\ell,t+1} - i_t - \eta'(\tilde{\ell}_{t+1}) \tag{33}$$

To simplify the notation, drop the time subscripts. Applying the implicit function theorem yields

$$\underbrace{\begin{bmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{bmatrix}}_{\equiv \boldsymbol{G}} \begin{bmatrix} d\tilde{\ell} \\ d\tilde{r} \\ di_\ell \end{bmatrix} = - \begin{bmatrix} G^1_{i_r} di_r + G^1_i di \\ G^2_{i_r} di_r + G^2_i di \\ G^3_{i_r} di_r + G^3_i di \end{bmatrix}$$

where

$$\begin{aligned} G^1_{i_r} &= \tilde{r}, & G^1_i &= -(\tilde{\ell} + \tilde{r}), \\ G^2_{i_r} &= 1, & G^2_i &= -1, \\ G^3_{i_r} &= 0, & G^3_i &= -1. \end{aligned} \qquad |\boldsymbol{G}| = \begin{vmatrix} G^1_\ell & G^1_{\tilde{r}} & G^1_{i_\ell} \\ G^2_\ell & G^2_{\tilde{r}} & G^2_{i_\ell} \\ G^3_\ell & G^3_{\tilde{r}} & G^3_{i_\ell} \end{vmatrix} = \begin{vmatrix} 0 & 0 & \tilde{\ell} \\ 0 & -\gamma'' & 0 \\ -\eta'' & 0 & 1 \end{vmatrix} = -\eta''\gamma''\tilde{\ell} < 0$$

By Cramer's rule, we have

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} \tilde{\ell} + \tilde{r} & 0 & \tilde{\ell} \\ 1 & -\gamma'' & 0 \\ 1 & 0 & 1 \end{vmatrix} > 0, \quad \frac{\partial \tilde{\ell}}{\partial i_r} = \frac{1}{|\boldsymbol{G}|} \begin{vmatrix} -\tilde{r} & 0 & \tilde{\ell} \\ -1 & -\gamma'' & 0 \\ 0 & 0 & 1 \end{vmatrix} < 0,$$

$$\frac{\partial \tilde r}{\partial i}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}0 & \tilde\ell+\tilde r & \tilde\ell\\ 0 & 1 & 0\\ -\eta'' & 1 & 1\end{vmatrix}<0,\quad \frac{\partial \tilde r}{\partial i_r}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}0 & -\tilde r & \tilde\ell\\ 0 & -1 & 0\\ -\eta'' & 0 & 1\end{vmatrix}>0,$$

$$\frac{\partial i_\ell}{\partial i}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}0 & 0 & \tilde\ell+\tilde r\\ 0 & -\gamma'' & 1\\ -\eta'' & 0 & 1\end{vmatrix}>0,\quad \frac{\partial i_\ell}{\partial i_r}=\frac{1}{|\boldsymbol{G}|}\begin{vmatrix}0 & 0 & -\tilde r\\ 0 & -\gamma'' & -1\\ -\eta'' & 0 & 0\end{vmatrix}<0.$$

■

Proof of Proposition 2. This proof is divided into 2 parts. (i) Existence and uniqueness of the ample-reserve equilibrium; (ii) Existence of the scarce-reserve equilibrium.

Part 1 (Existence and uniqueness of the ample-reserve equilibrium) Consider the ample-reserve equilibrium. The ample-reserve equilibrium satisfies $i_d = i$, $\lambda_\chi = 0$, and $\tilde r > \chi\tilde d$. Given $i_d = i$ and $\lambda_\chi = 0$, the equilibrium solves the following system of equations.

$$0 =-\gamma'(\tilde r)+i_r-i \tag{34}$$
$$0 =-\kappa+\eta'(\tilde\ell)\tilde\ell+\gamma'(\tilde r)\tilde r-\eta(\tilde\ell)-\gamma(\tilde r) \tag{35}$$
$$0 =-\frac{1}{\beta}+1-\delta-\frac{\eta'(\tilde\ell)}{\beta(1+i)}+f'(k) \tag{36}$$
$$0 =-f(k)+f'(k)k+\zeta/U'\big(\{f(k)-\delta k\}N\big) \tag{37}$$
$$0 =-(1+i)n\tilde d+\max\big\{0,p^*-\bar b\big\} \tag{38}$$
$$0 =-kN+n\tilde\ell \tag{39}$$
$$0 =-i+\nu\sigma_1\lambda(q) \tag{40}$$

First, let's check equations (34)-(39) because q_1 is already pinned down by i. Given (i, i_r) where $i_r \ge i$, equation (34) uniquely pins down $\tilde r \ge 0$ and $\partial\tilde r/\partial i_r > 0$ because $\gamma'' > 0$. When $i_r < i$, there is no solution $\tilde r \ge 0$ satisfying equation (34). Since $\eta'' > 0$ and $\gamma'' > 0$, equation (35) uniquely pins down $\tilde\ell$ as long as $\bar\Delta + i \ge i_r$ where $\bar\Delta = \gamma'(\underline{r})$ and $\underline{r}$ solves $\kappa = \gamma'(\underline{r})\underline{r} - \gamma(\underline{r})$. When $i_r > \bar\Delta + i$, there is no solution for (35) given $\tilde r \ge 0$ satisfying (34). Similarly, since $U'' < 0$ and $f'' < 0$, equation (36) uniquely pins down k. Given $(\tilde r, \tilde\ell, k)$ from (34)-(36), it is straightforward to show that (37)-(39) give (n, N, d) uniquely. Therefore, the system of equations (34)-(39) provides a unique solution for $(\tilde r, \tilde\ell, k, n, N, d)$ when $i_r \in (i, \bar\Delta + i)$.

Next, we will check under what condition $\tilde r > \chi\tilde d$ is satisfied. Consider a case satisfying $\tilde r = \chi\tilde d$ where $i_d = i$ which is a threshold between ample-reserve case and scarce reserve case. In this case, we have $(\tilde r, \tilde\ell, k, n, N) = (\bar r, \bar\ell, \bar k, \bar n, \bar N)$ where $(\bar r, \bar\ell, \bar K, \bar N, \bar C, \bar n)$ solves

$$\eta'(\bar{\ell})\bar{\ell} + (i_r - i)\bar{r} - \gamma(\bar{r}) - \eta(\bar{\ell}) = \kappa,$$

$$F_K(\bar{K},\bar{N}) = \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\bar{\ell})}{\beta(1+i)}, \qquad \max\left\{0, \frac{\chi\left\{v(q^*)F_N(\bar{K},\bar{N})/\zeta - \bar{b}\right\}}{\bar{n}}\right\} = (1+i)\bar{r},$$

and $\bar{C} + \delta\bar{K} = F(\bar{K},\bar{N})$, $U'(\bar{C}) = \zeta/F_N(\bar{K},\bar{N})$, and $\bar{K} = \bar{n}\bar{\ell}$.

Since $\partial\tilde{r}/\partial i_r > 0$, showing $\partial\tilde{d}/\partial i_r < 0$ will suffice to show (i) part of the Proposition 2. Consider the following system of equations.

$$\begin{aligned}
0 = \tilde{G}^1 &\equiv -\gamma'(\tilde{r}) + i_r - i \\
0 = \tilde{G}^2 &\equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} + \gamma'(\tilde{r})\tilde{r} - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \\
0 = \tilde{G}^3 &\equiv -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + F_K(n\tilde{\ell}, N) \\
0 = \tilde{G}^4 &\equiv -F_N(n\tilde{\ell}, N) + \frac{\zeta}{U'(F(n\tilde{\ell}, N) - \delta n\tilde{\ell})} \\
0 = \tilde{G}^5 &\equiv -n\tilde{d} + \max\left\{\frac{v(q^*)F_N(n\tilde{\ell}, N)/\zeta - \bar{b}}{1+i}, 0\right\}
\end{aligned}$$

Applying the implicit function theorem gives the following:

$$\underbrace{\begin{bmatrix} 0 & \tilde{G}^1_r & 0 & 0 & 0 \\ 0 & \tilde{G}^2_r & \tilde{G}^2_\ell & 0 & 0 \\ \tilde{G}^3_n & 0 & \tilde{G}^3_\ell & \tilde{G}^3_N & 0 \\ \tilde{G}^4_n & 0 & \tilde{G}^4_\ell & \tilde{G}^4_N & 0 \\ \tilde{G}^5_n & 0 & \tilde{G}^5_\ell & \tilde{G}^5_N & \tilde{G}^5_d \end{bmatrix}}_{=\tilde{\boldsymbol{G}}} \begin{bmatrix} dn \\ d\tilde{r} \\ d\tilde{\ell} \\ dN \\ d\tilde{d} \end{bmatrix} = - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} di_r - \begin{bmatrix} -1 \\ 0 \\ \frac{\eta'}{\beta(1+i)^2} \\ 0 \\ -\frac{v(q^*)F_N}{\zeta(1+i)^2} \end{bmatrix} di - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{1+i} \end{bmatrix} d\bar{b} \tag{41}$$

where

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} 0 & -\gamma'' & 0 & 0 & 0 \\ 0 & \gamma''\tilde{r} & \eta''\tilde{\ell} & 0 & 0 \\ F_{KK}\tilde{\ell} & 0 & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)} + F_{KK}n & F_{KN} & 0 \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & 0 & -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} & 0 \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & 0 & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & -n \end{bmatrix}$$

Using Cramer's rule, we have the following comparative statics:

$$\frac{\partial \tilde{r}}{\partial i_r} = \frac{n\gamma''\eta''\tilde{\ell}^2}{|\tilde{\boldsymbol{G}}|} \begin{vmatrix} F_{KK}\tilde{\ell} & F_{KN} \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -F_{NN} - \frac{\zeta F_N U''}{U'^2} \end{vmatrix} > 0$$

$$\frac{\partial \tilde{d}}{\partial i_r} = \frac{\gamma''\tilde{r}}{|\tilde{\boldsymbol{G}}|} \begin{vmatrix} F_{KK}\tilde{\ell} & -\frac{\eta''}{\beta(1+i)} + F_{KK}n & F_{KN} \\ -\{F_{KN} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & -\{F_{KN} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} \end{vmatrix} < 0$$

since $(F_{KN})^2 = F_{KK}F_{NN}$, $F_{KN} = F_{NK}$ and $F_N > F_{NK}K$. Given $\partial\tilde{r}/\partial i_r > 0$ $\partial\tilde{d}/\partial i_r < 0$, we can conclude that given $(i, \chi, \bar{b})$, $\exists!$ ample-reserves equilibrium if and only if $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$.

Part 2 (Existence of the scarce-reserve equilibrium): From Part 1, we already know that there is no scarce-reserve equilibrium when $i_r \in (\bar{\iota}_r, \bar{\Delta} + i)$. Hence, we only focus on that case where $i_r \in [0, \bar{\iota}_r]$. The scarce-reserve equilibrium solves the following system of equations.

$$0 = -i_d + (1-\chi)i_s + \chi i_r - \chi\gamma'(\tilde{r}) \tag{42}$$

$$0 = -i_\ell + i + \eta'(\tilde{\ell}) \tag{43}$$

$$0 = -\kappa + \eta'(\tilde{\ell})\tilde{\ell} + \gamma'(\tilde{r})\tilde{r} - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \tag{44}$$

$$0 = -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1+i)} + f'(k) \tag{45}$$

$$0 = -f(k) + f'(k)k + \zeta/U'\big(\{f(k) - \delta k\}N\big) \tag{46}$$

$$0 = -\tilde{r} + \chi\tilde{d} \tag{47}$$

$$0 = -kN + n\tilde{\ell} \tag{48}$$

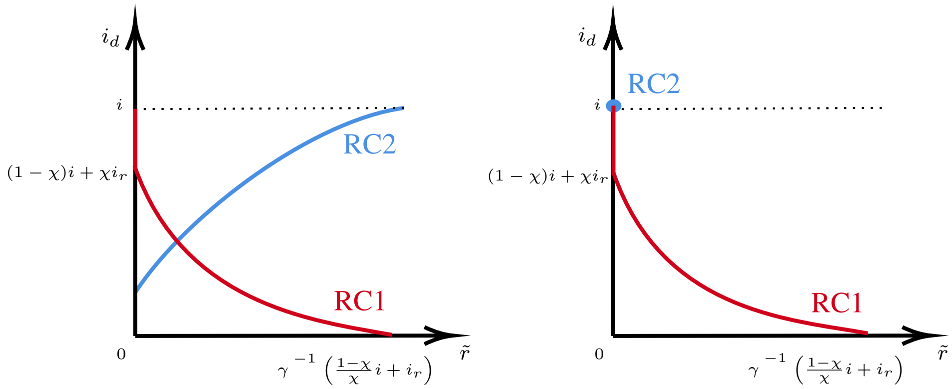
$$0 = -i + \nu\sigma_1\lambda(q_1) + \nu\sigma_3\lambda(q_3) \tag{49}$$

$$0 = -\frac{1+i}{1+i_d} + 1 + \nu\sigma_2\lambda(q_2) + \nu\sigma_3\lambda(q_3) \tag{50}$$

$$0 = -v(q_1) + \min\left\{v(q^*), \frac{\zeta m}{f(k) - f'(k)k}\right\} \tag{51}$$

$$0 = -v(q_2) + \min\left\{v(q^*), \frac{\zeta(n\tilde{d} + \bar{b})}{f(k) - f'(k)k}\right\} \tag{52}$$

$$0 = -v(q_3) + \min\left\{v(q^*), \frac{\zeta(m + n\tilde{d} + \bar{b})}{f(k) - f'(k)k}\right\} \tag{53}$$



(a) Case 1 $\quad \bar{b} < p^*$ (a) Case 2 $\quad \bar{b} \geq p^*$

Figure 18: RC1 and RC2

The equations (43)-(53) can be simplified as follows:

$$\begin{aligned}
0 =& \tilde{H}^1 \equiv -i + \nu\sigma_1\Lambda\left(\frac{\zeta}{f(k)-f'(k)k}m\right) \\
&+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k)-f'(k)k}\left[m+\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right]\right\}\right) && (54) \\
0 =& \tilde{H}^2 \equiv -\frac{1+i}{1+i_d}+1 && (55) \\
&+ \nu\sigma_2\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k)-f'(k)k}\left[\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right]\right\}\right) \\
&+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k)-f'(k)k}\left[m+\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right]\right\}\right) \\
0 =& \tilde{H}^3 \equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} - \eta(\tilde{\ell}) + \gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) && (56) \\
0 =& \tilde{H}^4 \equiv -f'(k) + \frac{1}{\beta} - 1 + \delta + \frac{\eta'(\tilde{\ell})}{\beta(1+i)} && (57) \\
0 =& \tilde{H}^5 \equiv \frac{\zeta}{f(k)-f'(k)k} - U'\left([f(k)-\delta k]N\right) && (58) \\
0 =& \tilde{H}^6 \equiv -k + \frac{n\tilde{\ell}}{N} && (59)
\end{aligned}$$

Relaxing the equality of (42), we obtain $i_d \geq (1-\chi)i_s + \chi i_r - \chi\gamma'(\tilde{r})$. From equation (42), it is given that $\tilde{r}$ decreases with i_d for $i_d \in [0, (1-\chi)i_s + \chi i_r]$. The curve that satisfies equation (42) in the $(\tilde{r}, i_d)$ space, labeled as RC1, represents the amount of real reserve balances a bank is willing to hold for a given i_d. The curve that satisfies equations (54) to (59) in the $(\tilde{r}, i_d)$ space, labeled as RC2, represents the required reserves for transaction deposit demand. This takes into account the values of i_d and the corresponding activities of households and firms. The RC1 curve is a downward-sloping single-valued function, while

RC2 can be either a single-valued or a multi-valued curve.

The RC1 curve starts at $(\tilde{r}, i_d) = (0, (1-\chi)i + \chi i_r)$, with a continuum of points where $\tilde{r} = 0$ and $i \geq i_d \geq (1-\chi)i + \chi i_r$. Along the RC1 curve, i_d weakly decreases with $\tilde{r}$ until it reaches the point $(\tilde{r}, i_d) = (\gamma^{-1}\left(\frac{1-\chi}{\chi} i + i_r\right), 0)$. In equilibrium, since $i \geq i_d$, the right end of the RC2 curve solves (54)-(59) with $i_d = i$. For a given $(i_r, \bar{b})$ with $i_d = i$, this holds when $\tilde{r} > 0$ as long as $\bar{b} < p^*$. If $\bar{b} \geq p^*$, then $\tilde{r} = 0$. At the left end of the RC2 curve, it solves (54)-(59) where $\tilde{r} = 0$ and $i \geq i_d \geq 0$. It is evident that when $\bar{b} \geq p^*$, the RC2 curve becomes a degenerate case with a single point at $(\tilde{r}, i_d) = (0, i)$. When $\bar{b} < p^*$, the left end of the RC2 curve satisfies $\tilde{r} > 0$, and $i > i_d \geq 0$. Therefore, as $\tilde{r}$ increases there is at least one point where the RC1 curve intersects the RC2 curve where the RC2 curve crosses the RC1 curve from below. Consequently, we can conclude that a scarce-reserves equilibrium exists if and only if $0 \leq i_r \leq \bar{\imath}_r$. ■

Proof of Proposition 3. Recall money multiplier ξ:

$$\xi = \frac{m+d}{\phi B} = \frac{m+d}{m+r} = \frac{m+n\tilde{d}}{m+n\tilde{r}}$$

Showing $\partial\tilde{r}/\partial i_r > 0$, $\partial\tilde{r}/\partial i < 0$, $\partial\tilde{d}/\partial i_r < 0$, and $\partial\tilde{d}/\partial i > 0$ suffices to show $\partial\xi/\partial i > 0$ and $\partial\xi/\partial i_r < 0$. Recall $\gamma'(\tilde{r}) = i_r - i$. It is straightforward to show that $\partial\tilde{r}/\partial i_r > 0$ and $\partial\tilde{r}/\partial i < 0$. Part 1 of Proposition 2 already has shown $\partial\tilde{d}/\partial i_r < 0$. To show $\partial\tilde{d}/\partial i > 0$, recall (41). Applying implicit function theorem and comparative statics gives

$$\frac{\partial\tilde{d}}{\partial i} = \frac{|\tilde{\boldsymbol{G}}_{\boldsymbol{i,d}}|}{|\tilde{\boldsymbol{G}}|}$$

where

$$\tilde{\boldsymbol{G}}_{\boldsymbol{i,d}} = \begin{bmatrix} 0 & -\gamma'' & 0 & 0 & 1 \\ 0 & \gamma''\tilde{r} & \eta''\tilde{\ell} & 0 & 0 \\ F_{KK}\tilde{\ell} & 0 & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)} + F_{KK}n & F_{KN} & -\frac{\eta'}{\beta(1+i)^2} \\ -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}\tilde{\ell} & 0 & -\{F_{NK} + \frac{\zeta U''}{U'^2}(F_K - \delta)\}n & -F_{NN} - \frac{\zeta F_N U''}{U'^2} & 0 \\ -\tilde{d} + \frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & 0 & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & \frac{v(q^*)F_N}{\zeta(1+i)^2} \end{bmatrix}.$$

Solving this yields the following expression:

$$\frac{\partial \tilde{d}}{\partial i} = \frac{|\tilde{\boldsymbol{G}}_{\boldsymbol{i},\boldsymbol{d}}|}{|\tilde{\boldsymbol{G}}|} = -\frac{\gamma''\eta''\tilde{\ell}}{|\tilde{\boldsymbol{G}}|}\begin{vmatrix} F_{KK}\tilde{\ell} & F_{KN} & -\frac{\eta'}{\beta(1+i)^2} \\ -\{F_{NK}+\frac{\zeta U''}{U'^2}(F_K-\delta)\}\tilde{\ell} & -F_{NN}-\frac{\zeta F_N U''}{U'^2} & 0 \\ -\tilde{d}+\frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} & \frac{v(q^*)F_N}{\zeta(1+i)^2} \end{vmatrix}$$

$$-\frac{\gamma''\tilde{r}}{|\tilde{\boldsymbol{G}}|}\begin{vmatrix} F_{KK}\tilde{\ell} & -\frac{\eta''(\tilde{\ell})}{\beta(1+i)}+F_{KK}n & F_{KN} \\ -\{F_{NK}+\frac{\zeta U''}{U'^2}(F_K-\delta)\}\tilde{\ell} & -\{F_{NK}+\frac{\zeta U''}{U'^2}(F_K-\delta)\}n & -F_{NN}-\frac{\zeta F_N U''}{U'^2} \\ -\tilde{d}+\frac{v(q^*)F_{KN}\tilde{\ell}}{\zeta(1+i)} & \frac{v(q^*)F_{KN}n}{\zeta(1+i)} & \frac{v(q^*)F_{NN}}{\zeta(1+i)} \end{vmatrix} > 0.$$

■

Proof of Proposition 4. This proof is divided into 2 parts: (i) Ample reserves equilibrium and (ii) scarce reserve equilibrium.

Ample reserves equilibrium In the ample reserve equilibrium, since $i_d = i$, it is easy to show that $\partial i_d/\partial \bar{b} = 0$. When $p^* > \bar{b}$ we have

$$d = \frac{v(q^*)\{f(k) - f'(k)k\}/\zeta - \bar{b}}{1+i}$$

in the ample reserves equilibrium. From comparative statics, we have

$$\frac{\partial \tilde{\ell}}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{\ell}}}|}{|\tilde{\boldsymbol{G}}|} = 0, \quad \frac{\partial n}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\boldsymbol{n}}|}{|\tilde{\boldsymbol{G}}|} = 0, \quad \frac{\partial N}{\partial \bar{b}} = \frac{|\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\boldsymbol{N}}|}{|\tilde{\boldsymbol{G}}|} = 0$$

since

$$|\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{\ell}}}| = -\frac{1}{1+i}\begin{vmatrix} 0 & \tilde{G}_r^1 & 0 & 0 \\ 0 & \tilde{G}_r^2 & 0 & 0 \\ \tilde{G}_n^3 & 0 & \tilde{G}_N^4 & 0 \\ \tilde{G}_n^4 & 0 & \tilde{G}_N^4 & 0 \end{vmatrix} = 0, \quad |\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\boldsymbol{n}}| = -\frac{1}{1+i}\begin{vmatrix} \tilde{G}_r^1 & 0 & 0 & 0 \\ \tilde{G}_r^2 & \tilde{G}_\ell^2 & 0 & 0 \\ 0 & \tilde{G}_\ell^3 & \tilde{G}_N^3 & 0 \\ 0 & \tilde{G}_\ell^4 & \tilde{G}_N^4 & 0 \end{vmatrix} = 0$$

$$|\tilde{\boldsymbol{G}}_{\bar{\boldsymbol{b}},\boldsymbol{N}}| = \frac{1}{1+i}\begin{vmatrix} 0 & \tilde{G}_r^1 & 0 & 0 \\ 0 & \tilde{G}_r^2 & \tilde{G}_\ell^2 & 0 \\ \tilde{G}_n^3 & 0 & \tilde{G}_\ell^3 & 0 \\ \tilde{G}_n^4 & 0 & \tilde{G}_\ell^4 & 0 \end{vmatrix} = 0.$$

The above comparative statics implies $\frac{\partial k}{\partial \bar{b}} = 0$ since $k = K/N = n\tilde{\ell}/N$. Then it is straightforward to show $\partial d/\partial \bar{b} < 0$ and $\partial \tilde{d}/\partial \bar{b} < 0$.

Scarce reserves equilibrium First, I define $\tilde{z}_j \equiv \frac{\zeta}{f(k)-f'(k)k}\max\{p^*, z_j\}$ to simplify notations. The scarce reserve equilibrium solves the following system of equations:

$$
\begin{aligned}
0 = J^1 \equiv& -i + \nu\sigma_1\Lambda\left(\frac{\zeta}{f(k)-f'(k)k}m\right) \\
&+ \nu\sigma_3\Lambda\left(\frac{\zeta}{f(k)-f'(k)k}\left\{m+\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right\}\right)
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
0 = J^2 \equiv& -\frac{1+i}{1+i_d} + 1 + \nu\sigma_2\Lambda\left(\frac{\zeta}{f(k)-f'(k)k}\left\{\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right\}\right) \\
&+ \nu\sigma_3\Lambda\left(\min\left\{v(q^*), \frac{\zeta}{f(k)-f'(k)k}\left[m+\bar{b}+\frac{[1+i_d]n\tilde{r}}{\chi}\right]\right\}\right)
\end{aligned}
\tag{61}
$$

$$
0 = J^3 \equiv -\kappa + \eta'(\tilde{\ell})\tilde{\ell} - \eta(\tilde{\ell}) + \gamma'(\tilde{r})\tilde{r} - \gamma(\tilde{r}) \tag{62}
$$

$$
0 = J^4 \equiv \frac{\zeta}{f(k)-f'(k)k} - U'\left([f(k)-\delta k]N\right) \tag{63}
$$

$$
0 = J^5 \equiv -k + \frac{n\tilde{\ell}}{N} \tag{64}
$$

$$
0 = J^6 \equiv -f(k) + \frac{1}{\beta} - 1 + \delta + \frac{\eta(\tilde{\ell})}{\beta(1+i)} \tag{65}
$$

where $i_d = (1-\chi)i + \chi i_r - \chi\gamma'(\tilde{r})$.

Assuming interior, applying the implicit function theorem yields

$$
\underbrace{\begin{bmatrix}
J^1_m & J^1_{\tilde{r}} & 0 & J^1_k & J^1_n & 0 \\
J^2_m & J^2_{\tilde{r}} & 0 & J^2_k & J^2_n & 0 \\
0 & J^3_{\tilde{r}} & J^3_{\tilde{\ell}} & 0 & 0 & 0 \\
0 & 0 & 0 & J^4_k & 0 & J^4_N \\
0 & 0 & J^5_{\tilde{\ell}} & J^5_k & J^5_n & J^5_N \\
0 & 0 & J^6_{\tilde{\ell}} & J^6_k & 0 & 0
\end{bmatrix}}_{\equiv \boldsymbol{J}}
\begin{bmatrix} dm \\ d\tilde{r} \\ d\tilde{\ell} \\ dk \\ dn \\ dN \end{bmatrix}
= -
\underbrace{\begin{bmatrix}
J^1_{\bar{b}} & J^1_{i_r} & J^1_i \\
J^2_{\bar{b}} & J^2_{i_r} & J^2_i \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & J^6_i
\end{bmatrix}}_{\equiv \bar{\boldsymbol{J}}}
\begin{bmatrix} d\bar{b} \\ di_r \\ di \end{bmatrix}
\tag{66}
$$

where

$J_m^1 = \frac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}$, $\quad J_m^2 = \frac{\nu\zeta}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3)$,
$J_{\tilde{r}}^1 = \frac{n\nu\zeta\{1+i_d-\tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sigma_3\Lambda'(\tilde{z}_3)$, $\quad J_r^2 = -\frac{1+i}{(1+i_d)^2}\chi\gamma''(\tilde{r}) + \frac{n\nu\zeta\{1+i_d-\tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sum_{j=2}^3\{\sigma_j\Lambda'(\tilde{z}_j)\}$,
$J_{\tilde{r}}^3 = \gamma''(\tilde{r})\tilde{r}$, $\quad J_{\tilde{\ell}}^3 = \eta''(\tilde{\ell})\tilde{\ell}$,
$J_{\tilde{\ell}}^5 = \frac{n}{N}$, $\quad J_{\tilde{\ell}}^6 = \frac{\eta''(\tilde{\ell})}{\beta(1+i)}$,
$J_k^1 = \frac{\nu\zeta f''(k)k}{\{f(k)-f'(k)k\}^2}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}$, $\quad J_k^2 = \frac{\nu\zeta f''(k)k}{\{f(k)-f'(k)k\}^2}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}$,
$J_k^4 = \frac{\zeta f''(k)k}{\{f(k)-f'(k)k\}^2} - \{f'(k)-\delta\}NU''(C)$, $\quad J_k^5 = -1$,
$J_k^6 = -f''(k)$, $\quad J_n^1 = \frac{\nu\zeta(1+i_d)\tilde{r}}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3)$,
$J_n^2 = \frac{\nu\zeta(1+i_d)\tilde{r}}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}$, $\quad J_n^5 = \frac{\tilde{\ell}}{N}$,
$J_N^4 = -\{f-\delta k\}U''(C)$, $\quad J_N^5 = -\frac{n\tilde{\ell}}{N^2}$,
$J_{\tilde{b}}^1 = \frac{\nu\zeta}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3)$, $\quad J_{\tilde{b}}^2 = \frac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}$,
$J_{i_r}^1 = \frac{\sigma_3 n\tilde{r}\nu\zeta}{f(k)-f'(k)k}\Lambda'(\tilde{z}_3)$, $\quad J_{i_r}^2 = \frac{n\tilde{r}\nu\zeta}{f(k)-f'(k)k}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}$,
$J_i^1 = -1 + \frac{(1-\chi)d\sigma_3\nu\zeta}{f-f'k}\Lambda(\tilde{z}_3)$, $\quad J_i^2 = -\frac{1}{1+i_d} + \frac{(1-\chi)d\nu\zeta}{f-f'k}\{\sigma_2\Lambda(\tilde{z}_2) + \sigma_3\Lambda(\tilde{z}_3)\} + \frac{1+i}{(1+i_d)^2}\chi$,
$J_i^6 = -\frac{\eta'(\tilde{\ell})}{\beta(1+i)^2}$.

The determinant of $\boldsymbol{J}$ is

$$
\begin{aligned}
|\boldsymbol{J}| = &- U''\gamma''\tilde{r}f''\frac{n}{N}(f-\delta k)\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{(1+i_d)d}{n} \\
&\quad\times\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
&- U''\eta''\tilde{\ell}f''\frac{\ell}{N}(f-\delta k)\frac{\nu\zeta}{f-f'k}\left\{-\frac{1+i}{(1+i_d)^2}\chi\gamma''\right\}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\} \\
&+ U''\gamma''\tilde{r}\frac{\eta''}{\beta(1+i)}\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{(1+i_d)d}{n}(f-f'k) \\
&\quad\times\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
&- U''\eta''\tilde{\ell}f''\frac{\ell}{N}(f-\delta k)[1+i_d-\tilde{r}\chi\gamma'']\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{d}{\tilde{r}} \\
&\quad\times\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
&+ U'\frac{\tilde{\ell}}{N^2}\gamma''\tilde{r}\frac{\eta''}{\beta(1+i)}\left(\frac{\nu\zeta}{f-f'k}\right)^2\frac{f''k}{f-f'k} \\
&\quad\times\Big[(1+i_d)d\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\} \\
&\qquad - \{z_2\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + (z_3-z_1)\sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + z_2\sigma_2\sigma_3\Lambda'(\tilde{z}_2)\Lambda'(\tilde{z}_3)\}\Big]
\end{aligned}
$$

Since $z_2 = z_3 - z_1 \to (1+i_d)d$ when $\bar{b} \to 0$, we have

$$\lim_{\bar{b}\to 0} |\boldsymbol{J}| < 0$$

under Assumption 1. One can also show $\lim_{\sigma_3 \to 0} |\boldsymbol{J}| < 0$ as well. Therefore, for sufficiently small $\bar{b}$ or σ_3, we have $|\boldsymbol{J}| < 0$. From comparative statics we have

$$\begin{aligned}
\frac{\partial \tilde{r}}{\partial \bar{b}} &= \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} \\
&= \frac{1}{|\boldsymbol{J}|} \underbrace{\left(\frac{\nu\zeta}{f - f'k}\right)^2}_{\oplus} \underbrace{\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_3\sigma_2\Lambda(\tilde{z}_3)\Lambda(\tilde{z}_2)\}}_{\oplus} \underbrace{\frac{f''U''\eta''\ell(f-\delta k)}{\beta(1+i)N}}_{\oplus}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial n}{\partial \bar{b}} &= \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\boldsymbol{n}}|}{|\boldsymbol{J}|} \\
&= \frac{1}{|\boldsymbol{J}|} \underbrace{\left(\frac{\nu\zeta}{f - f'k}\right)^2}_{\oplus} \underbrace{\{\sigma_1\sigma_2\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_2) + \sigma_1\sigma_3\Lambda'(\tilde{z}_1)\Lambda'(\tilde{z}_3) + \sigma_3\sigma_2\Lambda(\tilde{z}_3)\Lambda(\tilde{z}_2)\}}_{\oplus} \\
&\quad \times \underbrace{\left[\frac{(f'k - f)U''\gamma''\tilde{r}k\eta''}{\beta(1+i)} + (f-\delta k)U''f''\frac{n}{N} + \frac{-\zeta f''k\eta''k}{\{f - f'k\}^2 N\beta(1+i)}\right]}_{\oplus}
\end{aligned}$$

where

$$\boldsymbol{J}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{r}}} = \begin{bmatrix} J^1_m & -J^1_{\bar{b}} & 0 & J^1_k & J^1_n & 0 \\ J^2_m & -J^2_{\bar{b}} & 0 & J^2_k & J^2_n & 0 \\ 0 & 0 & J^3_{\tilde{\ell}} & 0 & 0 & 0 \\ 0 & 0 & 0 & J^4_k & 0 & J^4_N \\ 0 & 0 & J^5_{\tilde{\ell}} & J^5_k & J^5_n & J^5_N \\ 0 & 0 & J^6_{\tilde{\ell}} & J^6_k & 0 & 0 \end{bmatrix}, \quad \boldsymbol{J}_{\bar{\boldsymbol{b}},\boldsymbol{n}} = \begin{bmatrix} J^1_m & J^1_{\tilde{r}} & 0 & J^1_k & -J^1_{\bar{b}} & 0 \\ J^2_m & J^2_{\tilde{r}} & 0 & J^2_k & -J^2_{\bar{b}} & 0 \\ 0 & J^3_{\tilde{r}} & J^3_{\tilde{\ell}} & 0 & 0 & 0 \\ 0 & 0 & 0 & J^4_k & 0 & J^4_N \\ 0 & 0 & J^5_{\tilde{\ell}} & J^5_k & 0 & J^5_N \\ 0 & 0 & J^6_{\tilde{\ell}} & J^6_k & 0 & 0 \end{bmatrix}.$$

Then we have

$$\lim_{\bar{b}\to 0} \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\bar{b}\to 0} \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\boldsymbol{n}}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\sigma_3\to 0} \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} < 0, \quad \lim_{\sigma_3\to 0} \frac{|\boldsymbol{J}_{\bar{\boldsymbol{b}},\boldsymbol{n}}|}{|\boldsymbol{J}|} < 0$$

because $|\boldsymbol{J}_{\bar{\boldsymbol{b}},\tilde{\boldsymbol{r}}}| > 0$ and $|\boldsymbol{J}_{\bar{\boldsymbol{b}},\boldsymbol{n}}| > 0$. Since $d = n\tilde{d}/\chi$, it is easy to show that $\partial d/\partial \bar{b} < 0$ for small $\bar{b}$ or σ_3. Since $i_d = (1-\chi)i + \chi i_r - \chi\gamma'(\tilde{r})$ and $\partial \tilde{r}/\partial \bar{b} < 0$ for small $\bar{b}$ or σ_3, we have

$\partial i_d/\partial \bar{b} > 0$ for small $\bar{b}$ or σ_3. ■

Proof of Proposition 7. This proof is divided into 4 parts:

Part 1 (Comparative Statics in the Ample-Reserve Equilibrium): Consider the ample-reserves equilibrium. It can be summarized as 4 equations 4 unknowns as follows.

$$
\begin{aligned}
0 &= G^1 \equiv -\gamma'(\tilde{r}) + i_r - i \\
0 &= G^2 \equiv -\kappa + \{1 + i + \eta'(\tilde{\ell})\}\tilde{\ell} + (1 + i_r)\tilde{r} - (1 + i)(\tilde{\ell} + \tilde{r}) - \eta(\tilde{\ell}) - \gamma(\tilde{r}) \\
0 &= G^3 \equiv -\frac{1}{\beta} + 1 - \delta - \frac{\eta'(\tilde{\ell})}{\beta(1 + i)} + f'(k) \\
0 &= G^4 \equiv -f(k) + f'(k)k + \zeta/U'(\{f(k) - \delta k\}N)
\end{aligned}
$$

where $k = K/H$. Applying the implicit function theorem yields

$$
\underbrace{\begin{bmatrix} G^1_k & G^1_N & G^1_\ell & G^1_r \\ G^2_k & G^2_N & G^2_\ell & G^2_r \\ G^3_k & G^3_N & G^3_\ell & G^3_r \\ G^4_k & G^4_N & G^4_\ell & G^4_r \end{bmatrix}}_{\equiv \boldsymbol{G}} \begin{bmatrix} dk \\ dN \\ d\tilde{\ell} \\ d\tilde{r} \end{bmatrix} = - \underbrace{\begin{bmatrix} G^1_{i_r} & G^1_i & G^1_b \\ G^2_{i_r} & G^2_i & G^2_b \\ G^3_{i_r} & G^3_i & G^3_b \\ G^4_{i_r} & G^4_i & G^4_b \end{bmatrix}}_{\equiv \bar{\boldsymbol{G}}} \begin{bmatrix} di \\ di_r \\ d\bar{b} \end{bmatrix} \tag{67}
$$

where

$$
\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 0 & -\gamma'' \\ 0 & 0 & \eta''\tilde{\ell} & 0 \\ f'' & 0 & -\frac{\eta''}{\beta(1+i)} & 0 \\ f''k - N(f' - \delta)U''\zeta/(U')^2 & -(f - \delta k)U''\zeta/(U')^2 & 0 & 0 \end{bmatrix}
$$

and

$$
\bar{\boldsymbol{G}} = \begin{bmatrix} -1 & 1 & 0 \\ -\tilde{r} & \tilde{r} & 0 \\ \frac{\eta'}{\beta(1+i)^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
$$

By Cramer's rule, we have

$$\frac{\partial \tilde{\ell}}{\partial i} = \frac{1}{|\boldsymbol{G}|}\begin{vmatrix} G_k^1 & G_N^1 & -G_i^1 & G_r^1 \\ G_k^2 & G_N^2 & -G_i^2 & G_r^2 \\ G_k^3 & G_N^3 & -G_i^3 & G_r^3 \\ G_k^4 & G_N^4 & -G_i^4 & G_r^4 \end{vmatrix} = \frac{-(f-\delta k)U''\frac{\zeta}{(U')^2}f''\gamma''\tilde{r}}{|\boldsymbol{G}|} > 0,$$

$$\frac{\partial \tilde{\ell}}{\partial i_r} = \frac{1}{|\boldsymbol{G}|}\begin{vmatrix} G_k^1 & G_N^1 & -G_{i_r}^1 & G_r^1 \\ G_k^2 & G_N^2 & -G_{i_r}^2 & G_r^2 \\ G_k^3 & G_N^3 & -G_{i_r}^3 & G_r^3 \\ G_k^4 & G_N^4 & -G_{i_r}^4 & G_r^4 \end{vmatrix} = \frac{(f-\delta k)U''\frac{\zeta}{(U')^2}f''\gamma''\tilde{r}}{|\boldsymbol{G}|} < 0$$

since

$$|\boldsymbol{G}| = -(f-\delta k)\frac{\gamma''U''}{(U')^2}\zeta f''\eta''\tilde{\ell} < 0.$$

Because $\partial\tilde{\ell}/\partial i > 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, we have $\partial i_\ell/\partial i = 1 + \eta''(\tilde{\ell})\frac{\partial\tilde{\ell}}{\partial i} > 0$. Similarly, because $\partial\tilde{\ell}/\partial i_r < 0$ and $i_\ell = i + \eta'(\tilde{\ell})$, $\partial i_\ell/\partial i_r = \eta''(\tilde{\ell})\frac{\partial\tilde{\ell}}{\partial i_r} < 0$. The real lending rate can be written as $\rho = 1/\beta - 1 + \frac{\eta'(\tilde{\ell})}{\beta(1+i)}$. It is straightforward to show $\partial\rho/\partial i_r = \frac{\partial\tilde{\ell}}{\partial i_r}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} < 0$ and $\partial\rho/\partial i = \frac{\partial\tilde{\ell}}{\partial i}\frac{\eta''(\tilde{\ell})}{\beta(1+i)} - \frac{\eta'(\tilde{\ell})}{\beta(1+i)^2} \lesseqgtr 0$. Since $i_d = i$, $\partial i_d/\partial i = 1 > 0$ and $\partial i_d/\partial i_r = 0$.

Part 2 (Comparative Statics in the Scarce-Reserve Equilibrium with respect to i_r):

$$\boldsymbol{J_{i_r,\tilde{r}}} = \begin{bmatrix} J_m^1 & -J_{i_r}^1 & 0 & J_k^1 & J_n^1 & 0 \\ J_m^2 & -J_{i_r}^2 & 0 & J_k^2 & J_n^2 & 0 \\ 0 & 0 & J_{\tilde{\ell}}^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_k^4 & 0 & J_N^4 \\ 0 & 0 & J_{\tilde{\ell}}^5 & J_k^5 & J_n^5 & J_N^5 \\ 0 & 0 & J_{\tilde{\ell}}^6 & J_k^6 & 0 & 0 \end{bmatrix}.$$

$$|\boldsymbol{J_{i_r,\tilde{r}}}| = (J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1)\underbrace{J_{\tilde{\ell}}^3 J_k^6 J_N^4 J_n^5}_{\oplus}$$

where

$$J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 =$$
$$\left(\frac{\nu\zeta}{f-f'k}\right)\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}\chi\left[\frac{1+i}{(1+i_d)^2} + \left\{\frac{(n\tilde{r}/\chi)\nu\zeta}{f-f'k}\right\}\{\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\}\right]$$
$$-n\tilde{r}\left(\frac{\nu\zeta}{f-f'k}\right)^2\{\sigma_3\Lambda'(\tilde{z}_3)\}^2.$$

Therefore, as long as

$$J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0 \tag{68}$$

holds (which we will check later), we have $\partial\tilde{r}/\partial i_r > 0$ for small $\bar{b}$ which also implies $\partial\tilde{\ell}/\partial i_r < 0$ for small $\bar{b}$. Immediate results of above results are $\partial i_\ell/\partial i_r < 0$ and $\partial\rho/\partial i_r < 0$. The next part (Part 3) verifies the condition (68).

Part 3 (Confirming $J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0$ holds when scarce-reserve equilibrium is unique): To confirm $J_m^1 J_{i_r}^2 - J_m^2 J_{i_r}^1 < 0$, recall (54)-(59) and apply implicit function theorem:

$$\underbrace{\begin{bmatrix} \tilde{H}_m^1 & \tilde{H}_{\tilde{r}}^1 & 0 & \tilde{H}_k^1 & \tilde{H}_n^1 & 0 \\ \tilde{H}_m^2 & \tilde{H}_{\tilde{r}}^2 & 0 & \tilde{H}_k^2 & \tilde{H}_n^2 & 0 \\ 0 & \tilde{H}_{\tilde{r}}^3 & \tilde{H}_{\tilde{\ell}}^3 & 0 & 0 & 0 \\ 0 & 0 & \tilde{H}_{\tilde{\ell}}^4 & \tilde{H}_k^4 & 0 & 0 \\ 0 & 0 & 0 & \tilde{H}_k^5 & 0 & \tilde{H}_N^5 \\ 0 & 0 & \tilde{H}_{\tilde{\ell}}^6 & \tilde{H}_k^6 & \tilde{H}_n^6 & \tilde{H}_N^6 \end{bmatrix}}_{\equiv\tilde{\boldsymbol{H}}} \begin{bmatrix} dm \\ d\tilde{r} \\ d\tilde{\ell} \\ dk \\ dn \\ dN \end{bmatrix} = - \begin{bmatrix} (n\tilde{r}/\chi)\nu\sigma_3\Lambda'(\tilde{z}_3) \\ \frac{1+i}{(1+i_d)^2} + (n\tilde{r}/\chi)\nu\left[\sigma_2\Lambda'(\tilde{z}_2) + \sigma_3\Lambda'(\tilde{z}_3)\right] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} di_d$$

where

$$\begin{aligned}
&H_m^1 = \tfrac{\nu\zeta}{f(k)-f'(k)k}\{\sigma_1\Lambda'(\tilde{z}_1) + \sigma_3\Lambda'(\tilde{z}_3)\}, && H_m^2 = \tfrac{\nu\zeta}{f(k)-f'(k)k}\sigma_3\Lambda'(\tilde{z}_3), \\
&H_{\tilde{r}}^1 = \tfrac{n\nu\zeta\{1+i_d-\tilde{r}\chi\gamma''(\tilde{r})\}}{\chi\{f(k)-f'(k)k\}}\sigma_3\Lambda'(\tilde{z}_3), && H_r^2 = -\tfrac{1+i}{(1+i_d)^2} + \tfrac{n\nu\zeta(1+i_d)}{\chi\{f(k)-f'(k)k\}}\textstyle\sum_{j=2}^3\{\sigma_j\Lambda'(\tilde{z}_j)\}, \\
&H_{\tilde{r}}^3 = J_{\tilde{r}}^3, && H_{\tilde{\ell}}^3 = J_{\tilde{\ell}}^3, \\
&H_{\tilde{\ell}}^5 = J_{\tilde{\ell}}^5, && H_{\tilde{\ell}}^6 = J_{\tilde{\ell}}^6, \\
&H_k^1 = J_k^1, && H_k^2 = J_k^2, \\
&H_k^4 = J_k^4, && H_k^5 = J_k^5, \\
&H_k^6 = J_k^6, && H_n^1 = J_n^1, \\
&H_n^2 = J_n^2, && H_n^5 = J_n^5, \\
&H_N^4 = J_N^4, && H_N^5 = J_N^5,
\end{aligned}$$

From Part 2 of Proposition 2, there exists at least one point where RC2 crosses RC1 curve from below. If the equilibrium is unique, the equilibrium exists where RC2 crosses RC1 curve from below which implies that the equilibrium satisfies $\partial\tilde{r}/\partial i_d > 0$. The uniqueness of scarce reserve implies $\partial \tilde{r}/\partial i_d > 0$ which also means $\frac{|\tilde{\boldsymbol{H}}_{\boldsymbol{i_d},\tilde{\boldsymbol{r}}}|}{|\tilde{\boldsymbol{H}}|} > 0$. This holds if and only if

$$\begin{aligned}
&\tilde{H}^2_m \tilde{H}^1_{i_d} - \tilde{H}^1_m \tilde{H}^2_{i_d} \\
&= (n\tilde{r}/\chi)\left(\frac{\nu\zeta}{f - f'k}\right)^2 \{\sigma_3 \Lambda'(\tilde{z}_3)\}^2 \\
&\quad - \frac{\nu\zeta}{f - f'k}\{\sigma_1 \Lambda'(\tilde{z}_1) + \sigma_3 \Lambda'(\tilde{z}_3)\}\left[\frac{1+i}{(1+i_d)^2} + \frac{(n\tilde{r}/\chi)\nu\zeta}{f - f'k}\{\sigma_1 \Lambda'(\tilde{z}_1) + \sigma_3 \Lambda'(\tilde{z}_3)\}\right] > 0
\end{aligned}$$

because

$$|\tilde{\boldsymbol{H}}| = (\tilde{H}^1_m \tilde{H}^2_r - \tilde{H}^1_r \tilde{H}^2_m)\tilde{H}^3_\ell \tilde{H}^4_k(-\tilde{H}^5_N \tilde{H}^6_n) + (\tilde{H}^1_m \tilde{H}^3_r \tilde{H}^2_k - \tilde{H}^2_m \tilde{H}^3_r \tilde{H}^1_k)\tilde{H}^3_{\tilde{\ell}}(-\tilde{H}^6_n \tilde{H}^5_N) > 0$$

and $\tilde{H}^2_m \tilde{H}^1_{i_d} - \tilde{H}^1_m \tilde{H}^2_{i_d} = -(J^1_m J^2_{i_r} - J^2_m J^1_{i_r})\chi$, we have $J^1_m J^2_{i_r} - J^2_m J^1_{i_r} < 0$ when the equilibrium is unique.

Part 4 (Comparative Statics in the Scarce-Reserve Equilibrium with respect to i): Recall (66). Using Cramer's rule, we have $\frac{\partial \tilde{r}}{\partial i} = \frac{|\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|}$ where

$$\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{r}}} = \begin{bmatrix} J^1_m & -J^1_i & 0 & J^1_k & J^1_n & 0 \\ J^2_m & -J^2_i & 0 & J^2_k & J^2_n & 0 \\ 0 & 0 & J^3_{\tilde{\ell}} & 0 & 0 & 0 \\ 0 & 0 & 0 & J^4_k & 0 & J^4_N \\ 0 & 0 & J^5_{\tilde{\ell}} & J^5_k & J^5_n & J^5_N \\ 0 & -J^6_i & J^6_{\tilde{\ell}} & J^6_k & 0 & 0 \end{bmatrix}$$

and

$$\begin{aligned}
|\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{r}}}| =& (J^1_m J^2_i - J^2_m J^1_i)J^3_{\tilde{\ell}} J^6_k J^5_n J^4_N - J^6_i J^3_\ell J^4_k (J^1_m J^2_n J^5_N - J^2_m J^1_n J^5_N) \\
&- J^6_i J^3_\ell J^4_N \{J^1_m (J^2_k J^5_n - J^2_n J^5_k) - J^2_m (J^1_k J^5_n - J^1_n J^5_k)\}.
\end{aligned}$$

Given that, one can show that

$$\lim_{\sigma_3 \to 0} \frac{|\boldsymbol{J}_{\boldsymbol{i},\tilde{\boldsymbol{r}}}|}{|\boldsymbol{J}|} < 0$$

which implies $\frac{\partial \tilde{r}}{\partial i} < 0$ and $\frac{\partial \tilde{\ell}}{\partial i} > 0$. Given these results, it is straightforward to show that $\partial i_\ell / \partial i > 0$ and $\partial i_d / \partial i > 0$ for small σ_3. ■

Proof of Proposition 5. Recall equation (28)-(29).

$$\frac{i_t}{\nu} = \sigma_1 \lambda(q_1) + \sigma_3 \lambda(q_3)$$
$$\left\{ \frac{1+i}{1+i_d} - 1 \right\} \frac{1}{\nu} = \sigma_2 \lambda(q_2) + \sigma_3 \lambda(q_3)$$

where $v(q_1) = \zeta m / \{f(k) - f'(k)k\}$, $v(q_2) = \zeta(d + \bar{b}) / \{f(k) - f'(k)k\}$, and $v(q_3) = \zeta(m + d + \bar{b}) / \{f(k) - f'(k)k\}$. When $i_r \geq \bar{\iota}_r$, we have $i_d = i$ which gives $\lambda(q_2) = \lambda(q_3)$. Therefore, it is straightforward to show $q_2 = q_3 = q^*$. ■

Proof of Proposition 6. Since $\partial \tilde{r} / \partial \bar{b} < 0$ in the ample reserve equilibrium and $\bar{\iota}_r = \gamma'(\bar{r}) + i$, $\bar{\iota}_r$ is decreasing in $\bar{b}$. When the credit limit is sufficiently large $\bar{b} > p^*$, z_2,$z_3 > p^*$ which results in $q_2 = q_3 = q^*$. Since $\bar{b} > p^*$ the households do not have any incentive to hold transaction deposits, $d = 0$ which implies $\chi d = 0$. Therefore, the reserve requirement constraint does not bind. In this case, each bank's reserve balance is determined by $\gamma'(\tilde{r}) = i_r - i$. The bank holds excess reserves as long as $i_r > i$. ■

E. Chow Test

Figure 4 includes the Chow test for structural breaks. The test result reported in the bottom-left panel of Figure 4 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{RequiredReserves/Deposit})_t \\ &+ \mathbf{1}_{t\geq 1992Q2}[\gamma_0 + \gamma_1(\text{RequiredReserves/Deposit})_t] \\ &+ \mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{RequiredReserves/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 7a reports F-statistics which are obtained by testing $\gamma_0 = \gamma_1 = \delta_0 = \delta_1 = 0$. The Chow test in the bottom-right panel of Figure 4 is implemented by estimating following regression.

$$\begin{aligned}\text{Money multiplier}_t =& \beta_0 + \beta_1(\text{Currency/Deposit})_t \\ &+ \mathbf{1}_{t\geq 2008Q4}[\delta_0 + \delta_1(\text{Currency/Deposit})_t] + \epsilon_t\end{aligned}$$

Table 7b reports F-statistics is obtained by testing $\delta_0 = \delta_1 = 0$. The regression estimates and the Chow test results are summarized at Table 7.

Table 7: Chow test for structural breaks

(a) Require reserve ratio

Dependent Variable: Money Multiplier	
RR	3.878^{***}
	(0.491)
RR $\times\ \mathbf{1}_{t\geq 1992Q2}$	125.316^{***}
	(7.887)
RR $\times\ \mathbf{1}_{t\geq 2008Q4}$	-146.440^{***}
	(8.732)
$\mathbf{1}_{t\geq 1992Q2}$	-9.417^{***}
	(0.554)
$\mathbf{1}_{t\geq 2008Q4}$	9.177^{***}
	(0.633)
Constant	2.471^{***}
	(0.067)
Obs.	192
R^2	0.977
DF for numerator	4
DF for denominator	186
F Statistic for Chow test	1691.84
F Statistic for 1% sig. level	3.42
F Statistic for 0.1% sig. level	4.83

(b) Currency deposit ratio

Dependent Variable: Money Multiplier	
CD	-1.814^{***}
	(0.057)
CD $\times\ \mathbf{1}_{t\geq 2008Q4}$	2.300^{***}
	(0.130)
$\mathbf{1}_{t\geq 2008Q4}$	-3.219^{***}
	(0.103)
Constant	3.607^{***}
	(0.031)
Obs.	192
R^2	0.976
DF for numerator	2
DF for denominator	188
F Statistic for Chow test	1327.68
F Statistic for 1% sig. level	4.72
F Statistic for 0.1% sig. level	7.17

Notes: Newy-West standard errors are in parentheses. ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. Degree of freedom is denoted by DF.

F. Unit Root Tests

Columns (3) and (6) in Table 1 includes the canonical cointegrating regression estimates and the cointegration tests. This section reports unit root tests for the series used in Columns (3) and (6). For all four variables, the unit root tests fail to reject the null hypothesis of non-stationarity while their first difference rejects the null hypothesis of non-stationarity at 1% significance level. All series are demeaned before implementing the unit root test following Elliott and Müller (2006) and Harvey, Leybourne and Taylor (2009), because the magnitude of the initial value can be problematic. Let ***, **, and * denote significance at the 1, 5, and 10 percent levels, respectively. CV denotes critical values. The data are quarterly from 1980Q1 to 2007Q4.

Table 8: Unit root test

		Phillips-Perron test			
		Test statistic	1% CV	5% CV	10% CV
$ln(m)$	$Z(\rho)$	0.602	−19.837	−13.722	−11.015
	$Z(t)$	0.315	−3.506	−2.889	−2.579
$ln(d)$	$Z(\rho)$	1.317	−19.837	−13.722	−11.015
	$Z(t)$	1.099	−3.506	−2.889	−2.579
$ln(uc)$	$Z(\rho)$	−1.120	−19.837	−13.722	−11.015
	$Z(t)$	−1.717	−3.506	−2.889	−2.579
r	$Z(\rho)$	−7.721	−19.837	−13.722	−11.015
	$Z(t)$	−2.471	−3.506	−2.889	−2.579
$\Delta ln(m)$	$Z(\rho)$	−46.597***	−19.833	−13.720	−11.013
	$Z(t)$	−5.333***	−3.507	−2.889	−2.579
$\Delta ln(d)$	$Z(\rho)$	−42.304***	−19.833	−13.720	−11.013
	$Z(t)$	−5.061***	−3.507	−2.889	−2.579
$\Delta ln(uc)$	$Z(\rho)$	−41.882***	−19.833	−13.720	−11.013
	$Z(t)$	−5.098***	−3.507	−2.889	−2.579
Δr	$Z(\rho)$	−94.183***	−19.833	−13.720	−11.013
	$Z(t)$	−9.263***	−3.507	−2.889	−2.579

G. Data Sources and Variable Definitions

The quantitative analysis uses the annual averages of the series listed below. The empirical analysis uses data from the same sources. These series are from Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis.

- M1: "M1 Money Stock" (FRED series M1SL). Before 1959, Ireland (2009)
- Currency: "Currency Component of M1" (FRED series CURRSL).
- Deposit Component of M1: M1 $-$ Currency Component of M1.
- Federal funds rates: "Effective Federal Funds Rate" (FRED series FEDFUNDS).
- Interest on reserves: "Interest Rate on Excess Reserves" (FRED series IOER) and "Interest Rate on Required Reserves" (FRED series IORR).
- 3-month treasury rate: "3-Month Treasury Bill: Secondary Market Rate" (FRED series TB3MS).
- Deposit (Total checkable deposits): "Total Checkable Deposits" (FRED series TCDSL).
- Excess reserves: "Excess Reserves of Depository Institutions" (FRED series EXCRESNS) and (FRED series EXCSRESNS).
- Excess reserve ratio: $\frac{\text{Excess reserves}}{\text{Total checkable deposits}}$.
- Required reserves: "Required Reserves of Depository Institutions" (FRED series REQRESNS).
- Required reserves ratio: $\frac{\text{Required reserves}}{\text{Total checkable deposits}}$.
- Reserves: "Total Reserves of Depository Institutions" (FRED series TOTRESNS).
- M1 money multiplier: 'M1 Money Multiplier' (FRED series MULT) from 1984-2015 and $\frac{\text{M1}}{\text{St. Louis Adjusted Monetary Base}}$ for 1968-1983, where 'St. Louis Adjusted Monetary Base' is from FRED series AMBSL.
- Unsecured credit: "Revolving Consumer Credit Owned and Securitized" (FRED series REVOLSL)
- GDP: "Gross Domestic Product" (FRED series GDP), quarterly and "Gross Domestic Product" (FRED series GDPA), annual.
- Capital stock: "Current-Cost Net Stock of Fixed Assets: Private " (FRED series K1PTOTL1ES000)
- Investment: "Gross Private Domestic Investment" (FRED series GPDI)
- Lending rate: "Bank Prime Loan Rate" (FRED series DPRIME)

G.1. Money Multiplier Data Comparison

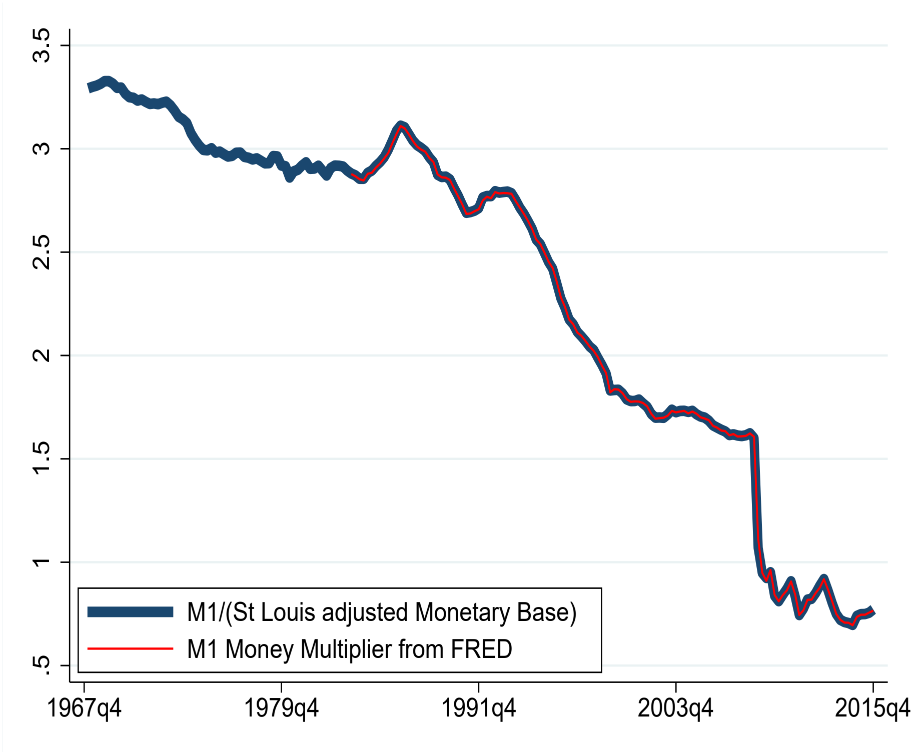


Figure 19: M1 Multiplier from FRED and the computed M1 Multiplier

To confirm that the correct series of M1 multiplier is being used, Figure 19 compares the computed M1 money multiplier with the M1 money multiplier downloaded directly from FRED (FRED series MULT). The figure shows that the author's calculation yields the same money multiplier as the one from FRED.