FX mean reversion strategies

A statistical trading framework

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In this paper we employ a well-known statistical model for investigating mean reversion properties of financial variables. Since mean reversion is a general definition that can be used to describe different properties, in this study we refer specifically to the possibility of identifying long/short baskets of assets, belonging to the same market, that exhibit large oscillations over short time horizons and where investors can take a position on.

While the formalism is very general and can be applied to any asset class, in this paper we focus specifically on the G10 FX market and will postpone to a future paper a full cross-asset analysis, most notably considering Equity indices.

In the first section of this paper, we present the vector autoregressive formalism for trading basket mean reversion. We present the general properties of the theoretical baskets in the limit of zero trading costs and a few specific features of mean-reverting trading systems.

In the second section, we address the crucial point of how to deal with transaction costs in a high-frequency mean reversion strategy; in fact, costs tend to play a large role due to the high turnover of the positions. We present a set of original techniques which allow minimising the impact of costs, while preserving the potential of the trades.

The third section shows how to aggregate strategies capturing mean reversion properties over different frequencies, and the diversification benefit that, due to the low mutual correlation of the different strategies, can correspondingly be extracted.

In the Appendix, we present an overview of different statistical models that can describe mean-reverting properties of financial assets; for different models, we establish a link between mean reversion properties and the shape of the statistical volatility curves.

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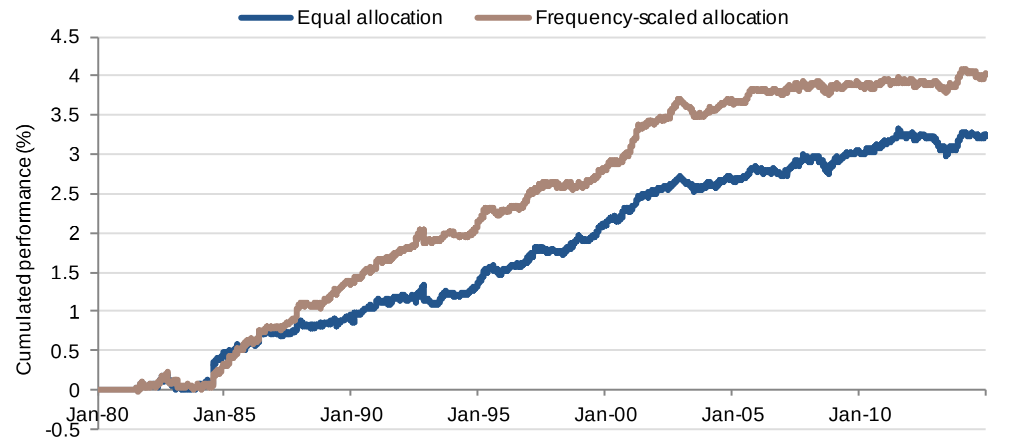
Aggregation of FX mean-reverting strategies defined over different frequencies



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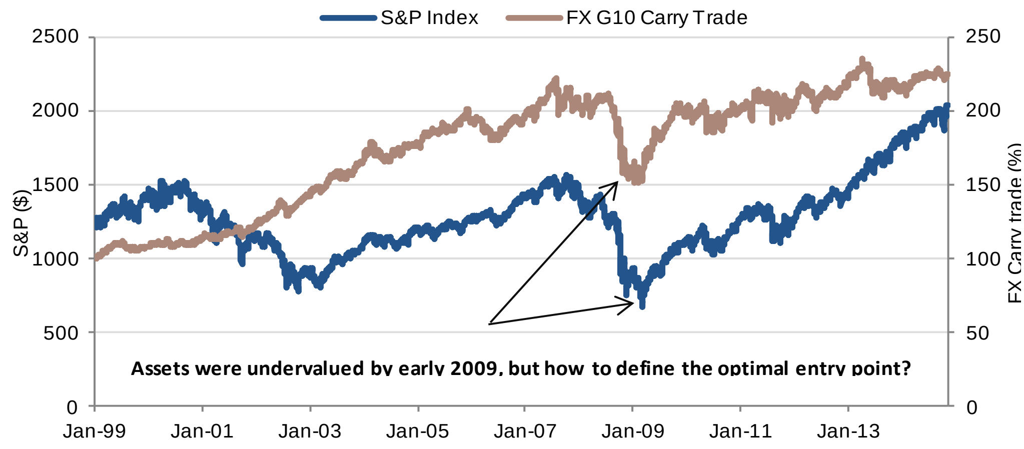
Executive summary

Mean reversion strategies represent the core of “alpha” generation, decorrelated with more mainstream strategies and long-only positions, thus making them a key ingredient within a diversified portfolio of alternative, risk premia strategies. A philosophical debate usually arises when comparing *time* and *space* mean reversion: while in the former case one tries to define trading regimes for a single asset, in the latter case one identifies, at any point in time, optimal long/short baskets whose large oscillations can be taken a position on. The potentially large impact of trend components for each asset makes “time mean reversion” a difficult task and in any case leaves the single asset trades exposed to a large amount of directional risk.

A crucial aspect on mean reversion systems concerns the typical time frame of the trades. The most natural intuition behind a mean reversion strategy regards the idea of buying assets that have fallen too much in a stressed environment and waiting for their recovery, which naturally sets a natural time scale of a few weeks or months and introduces the idea of a fair value that asset price should converge to. Medium-term macroeconomics models (fair valuations PPP- or REER-based for the FX market) are precisely aimed at achieving this task. By looking at the chart below, one can find that, having bought cheap assets like stocks or FX Carry in early 2009, one would have obtained outstanding returns since then, although arguably setting the right time for entering the trades would have been a difficult task.

Several assets were cheap after the 2008 crisis: how to define the optimal entry point?



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In this piece we consider a framework for trading space mean reversion via suitably chosen baskets, based on a statistical model and focused over short time periods. The framework relies entirely on the latest price action, without assuming any medium-term price targets.

The paper is organised as follows: in the first section we present the general framework employed for identifying basket mean reversion and show the properties of a set of mean-reverting baskets, defined for different frequencies, optimal in the limit of zero costs. The second section deals with the optimised strategies in the presence of costs, and reviews the techniques used for reducing the impact of costs on the traded portfolios. The third section shows how to aggregate strategies defined over different frequencies, and the corresponding diversification benefit that can be extracted. In the Appendix, we review a number of models commonly used for describing the mean reversion properties of financial assets.

A general framework for mean-reverting portfolios

In this section we introduce the framework used for identifying mean-reverting portfolios; while the formalism is absolutely general and could be applied to any market, numerical simulations will be performed on FX data. We define the optimal portfolios in the limit of zero costs and the issues deriving from the impact of costs on such high-frequency trades.

Mean-reverting baskets and VAR models for the return series

We introduce a statistical framework which allows identifying mean reversion properties in a given asset class, by defining long/short baskets of variables that exhibit strong oscillations. We will investigate the mean reversion properties of the return time series of N-assets, by generalising the single-asset autoregressive AR(1) formalism which was already the subject of a 2013 paper studying the statistical properties of a tradable quadratic payoff. In the Appendix, we describe a wider spectrum of models which are commonly used for modelling the mean-reverting properties of financial variables.

In the VAR(1) model, the vector of returns $\boldsymbol{r}_t$, corresponding to N assets $\boldsymbol{S}_t$ at observation time t and measured over a fixed time horizon of L days (daily to monthly), follow the equation:

$$\boldsymbol{r}_t = \boldsymbol{A}\boldsymbol{r}_{t-1} + \boldsymbol{c} + \boldsymbol{\varepsilon}_t \qquad (\boldsymbol{A} \text{ is a } NxN \text{ matrix})$$

For the $N = 1$, AR(1) case, a negative coefficient of the autoregressive matrix $\boldsymbol{A}$ implies a mean reverting dynamics for the returns. However, for a single asset, it will be always difficult to detect significant empirical discrepancies from 0 for the value of the coefficient. The typical coefficients for AR(1) dynamics on different asset classes are typically between -0.1 and 0.1, depending on the variable and the setting of the estimation[1]; this was tested in the previous paper, investigating the statistical properties of a payoff trading variance defined over different frequencies. These low values can hardly permit building a profitable model for a single asset, as taken separately, relying on the serial correlation properties only.

For N-assets, in general the matrix $\boldsymbol{A}$ is non diagonal, which means that all the assets are coupled in their dynamics. However, it is possible to find linear combinations of assets which minimise the corresponding matrix AR coefficient, in a way that this multi-d auto regressive coefficient is much more negative than that of each single asset taken separately. This observation could trigger the idea of trading the portfolios corresponding to the most negative eigenvalues in the return space, as it was done in an earlier paper testing cointegration properties in a VAR(1) framework for the prices[2]. For this reason, it is appealing to consider baskets characterised by a large number of assets, as this can lead to more negative eigenvalues and therefore more negative mean-reverting properties.

If one puts aside the issue that a real matrix does not always have real eigenvalues/vectors (one has in general to consider complex values), one can get

$$\boldsymbol{A} = \boldsymbol{\Lambda}\boldsymbol{D}\boldsymbol{\Lambda}^{-1} \;\; ; \quad \boldsymbol{\Lambda}^{-1}\boldsymbol{r}_t = \boldsymbol{D}\boldsymbol{\Lambda}^{-1}\boldsymbol{r}_{t-1} + \boldsymbol{\Lambda}^{-1}(\boldsymbol{c} + \boldsymbol{\varepsilon}_t) \quad (\boldsymbol{\Lambda}, \boldsymbol{D} \text{ are } NxN \text{ matrices})$$

[1] In other asset classes, for instance Equities, where mean-reverting properties are more evident than in FX, the estimated AR(1) coefficient would be higher in absolute value

[2] As a technical remark, the VAR(1) model on the returns considered in this paper is equivalent to a VAR(2) model for the log prices. The Appendix covers some technical details as regards to the comparison of different time series models one can use for assessing mean-reverting properties in financial time series

$\boldsymbol{D}$ is a diagonal matrix, featuring on its diagonal entries the eigenvalues $\boldsymbol{\lambda}$ of the matrix $\boldsymbol{A}$ ($\mathrm{D}_{i,i} = \lambda_i$). The matrix $\boldsymbol{\Lambda}^{-1}$ defines a set of N portfolios $\boldsymbol{W}^i$ ($\mathrm{W}^i{}_j = \Lambda^{-1}{}_{i,j}, i,j = 1 \cdots N$) in the N-asset space, with price at time $V^i{}_t = (\boldsymbol{W}^i)^T \cdot \boldsymbol{S}_t$. The returns of the i-th portfolio $r^i{}_t = \ln\left(\frac{V^i{}_t}{V^i{}_{t-L}}\right)$ follow the equations:

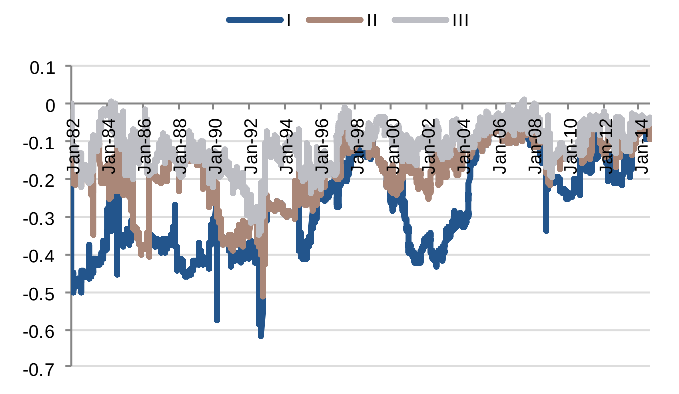
$$r^i{}_t = \lambda_i \, r^i{}_{t-1} + c^i + \epsilon^i{}_t \qquad \text{with } c^i = (\boldsymbol{\Lambda}^{-1}\boldsymbol{c})_i \, ; \, \epsilon^i{}_t = (\boldsymbol{\Lambda}^{-1}\boldsymbol{\varepsilon}_t)_i$$

The $\boldsymbol{W}^i$ portfolios follow each an independent dynamics in the time domain driven by a (scalar) AR(1) model and can be sorted based on the value of the corresponding eigenvalue, from the most negative (most mean reverting) to the most positive (most trend-following); for the model to be stationary, all the eigenvalues of the matrix A must be less than one in absolute value.

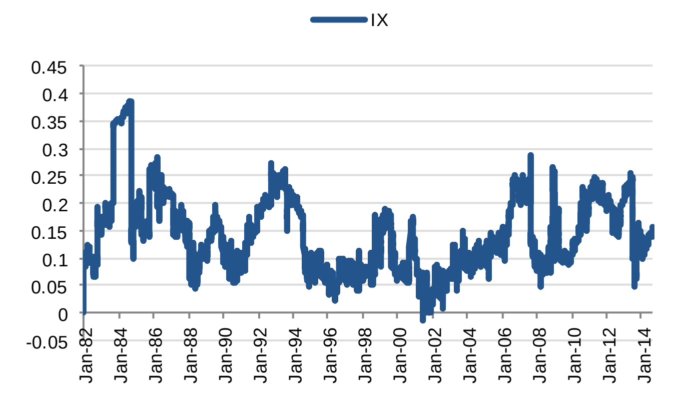
Throughout this paper, we will refer to the portfolios $\boldsymbol{W}^i$, corresponding at any time t, to the i-th eigenvalue of the $\boldsymbol{A}$ matrix, as the eigenstates of the VAR(1) model. It is important to recall that, given that the eigenvalues and eigenvectors will change with time, so will the definition of such ranked eigenstates portfolios; the compositions of the eigenstates will change with time in a way that, at any time t, the cross-sectional ranking in the eigenvalues space is preserved. We will show later that trading the eigenstates will not be optimal in the presence of non zero costs; we will show later how to express the optimal portfolios in the presence of costs as a function of the eigenstates.

The two charts below display the time series of the three most negative eigenvalues (left-hand chart) and most positive eigenvalue (right-hand chart), corresponding to the basket of G10 currencies vs the USD, for the daily return series and for a fixed estimation length of the model of 500 working days. What we see from the chart is that the largest (in absolute value) eigenvalues for the basket could highlight significant serial correlation properties for suitably selected combinations of assets.

Time series of first three eigenvalues



Time series of largest positive eigenvalue



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By putting aside for a while the issue of the constant terms appearing in the equations, one can simulate the performance where such portfolios are traded. While trading the baskets in the presence of transaction costs will require a set of rules for avoiding that the potential of the trades is reduced by costs, the model in its simple form only depends on two main parameters. One is the frequency of the returns (as discussed, in the equations above the returns $\boldsymbol{r}_t$ are measured over L days), by changing which one can test for mean reversion over different time scales. The second parameter is the time length over which the model is estimated: it is well known that one has to establish a trade-off between reducing the noise of the estimation of the matrices, which would favour a longer time window for estimating the models,[3] and coming up with a reactive model, which does not rely too much on past regimes. These aspects will be treated in detail in the following sections.

Trading mean-reverting portfolios for zero costs

Having defined optimal portfolios for trading mean reversion, one needs to specify how to implement the trades. In this subsection, for simplicity we will consider the daily frequency only ($L = 1$), and will describe the trading rules for lower frequencies of the returns in the next sections. We anticipate that, when trading frequencies different from 1/daily ($L \neq 1$), at any point in time the resulting position will result from the aggregation of the positions entered by each of the daily models defined over fixed time horizons.

For defining the traded positions, we proceed as follows: every day, we start by estimating the VAR(1) model over a specified estimation length, which will define a set of mean-reverting and trend-following portfolios, depending on the sign of the eigenvalues, referred to as eigenstates. For deciding the actual trades, for each portfolio one looks at the past return of the eigenstate and at the sign of the eigenvalue, which allows fixing the portfolio to be traded. To give an example, if one has found the portfolio X to display mean-reverting features, this will lead typically to oscillating positive and negative returns: for trading these properties, one would have to stay long X after a negative performance and long $-X$ after a positive performance. For a trend-following portfolio, on the other hand, one would buy the portfolio after a positive return and short it after a negative return.

In the VAR(1) formalism defined in the previous subsection, one can see that, other than on the past return, the model relies on an estimated constant for each asset. While this appears to be rather small compared to the volatility of the portfolios, nonetheless it has to be considered for defining the expected value of a given portfolio: the effect of the constant terms in the returns will be discussed in the next section when defining the actual portfolios in the presence of costs.

One can simulate the performance for all the eigenstates from the most negative to the most positive in terms of eigenvalues. As discussed, these “ranked” eigenvalues will change over time, so that the weights of the optimal portfolios will change with time. On a daily basis, one computes the PnL generated by the current position, readjusts the portfolio and pays costs on the difference between current and new positions: given that each model is estimated in sample and traded out of sample, such simulated performance should be a reliable indicator of the out-of-sample performance of the strategy.

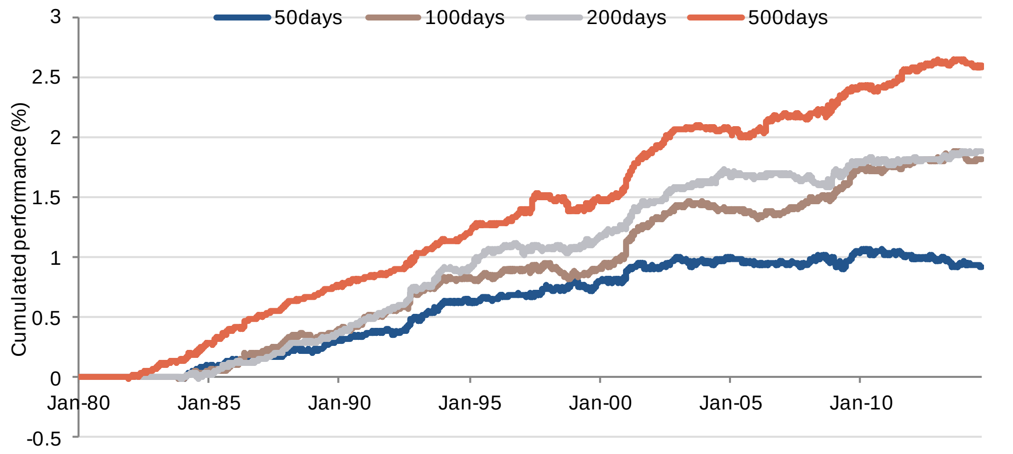
Before imposing volatility targeting, we start by assuming a basic rule that, for each trading day, fixes the value of the sum of the squares of the positions on each asset. In formulas, if $R_{t,j}$ is the matrix of the positions taken by the model at time t (the aggregated position will be the sum of all open positions), for the asset j, and with a L day holding period of each trade, one imposes the constraint that

$$(\sum_{j=1}^{N} R_{t,j}{}^2) = \frac{1}{L}\ ;\ \sum_{i=1}^{L}(\sum_{j=1}^{N} R_{t+1-i,j}{}^2) = 1$$

[3] See “Theory of Financial Risk and Derivatives Pricing”, J-P Bouchaud, M Potters (Cambridge University press, 2003)

In the case of the G10 FX portfolio with 9 currencies vs the USD, one gets the following performance as a function of the estimation length, for the daily trading strategy corresponding to the most negative eigenvalue and zero costs. Throughout this paper, when showing the simulated performance of the systems, the y-axis will always represent the cumulative % performance as a function of a constant capital (assumed to be 1$).

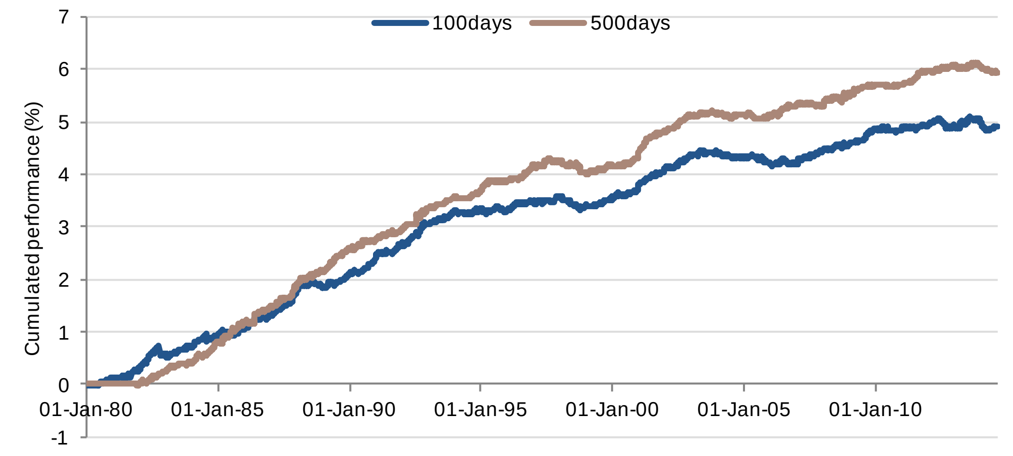
Simulated performance of the first eigenstate as a function of the estimation length (zero cost)



Source: SG Cross Asset Research/Cross Asset Quant

In general, volatility targeting rules will be important, as they allow controlling the risk of the strategy and to mix strategies corresponding to different frequencies (see next section). The next chart displays the performance of the strategy above after imposing a 10% volatility target. It is confirmed that a longer estimation period leads to a smoother backtest.

Impact of volatility scaling on the trades



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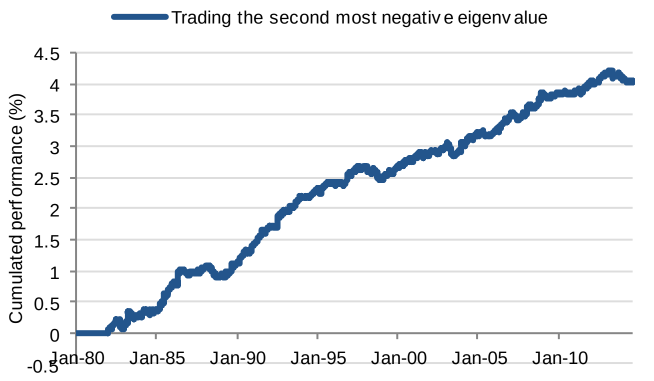
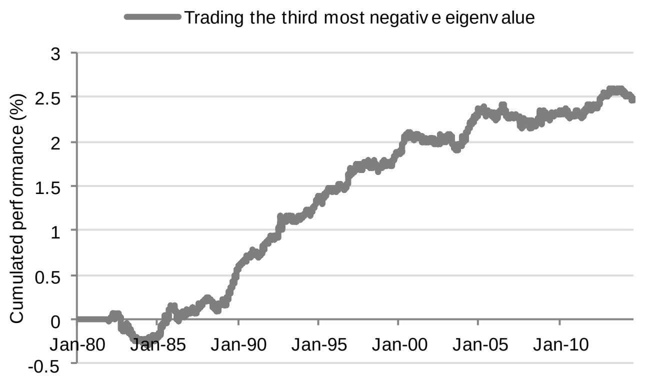
Having assessed, earlier in the section, that the second and third largest eigenvalues are negative, and so still refer to mean-reverting properties, one can simulate the performance of the portfolios corresponding to such eigenstates. As we can see in the next two charts, the performance would be slightly less appealing than for the “first” eigenstate, because of the weaker mean-reverting properties (i.e., smaller eigenvalues in absolute values). Still, the possibility of rotating across a set of valid strategies will be crucial for reducing the impact of trading costs, as discussed in the next section.

Simulated performance of second best strategy (zero costs)



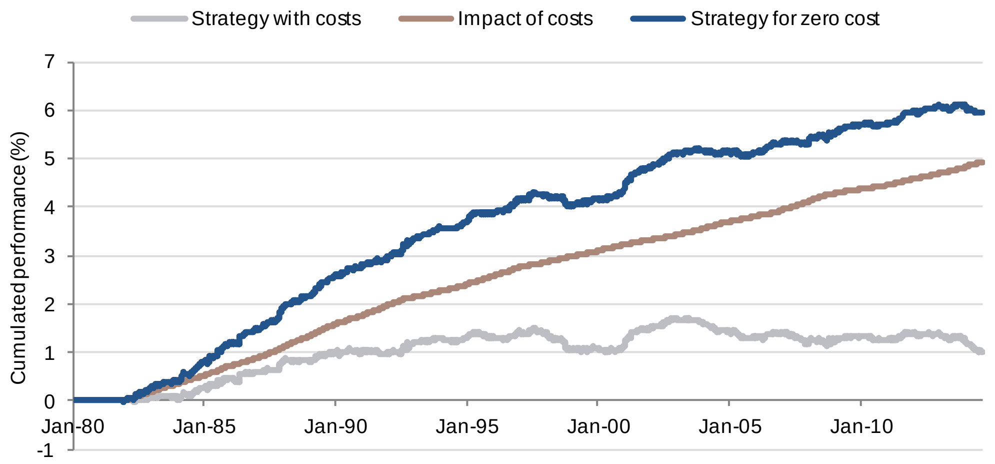
Simulated performance of third best strategy (zero costs)



Source: SG Cross Asset Research/Cross Asset Quant

It is important to emphasise that the impact of costs would be large on daily trading. If one considers the trading system corresponding to the volatility scaled position (10% target volatility), for daily frequency, one would get the following time series after taking into account costs. The impact of costs for this strategy is overwhelming and would eat up most of the performance. The strategy in the limit of zero costs yields 16.6% per year, with a vol of 9.9% and a Sharpe of 1.68. The strategy with costs has an average return of 2.7% with a vol of 9.5% and a Sharpe of 0.3. With such results in mind, one might conclude that this whole framework is a mere theoretical exercise with no practical implications, due to the high impact of costs on the performance of the strategy.

Simulated performance of the first eigenstate performance in presence of costs



Source: SG Cross Asset Research/Cross Asset Quant

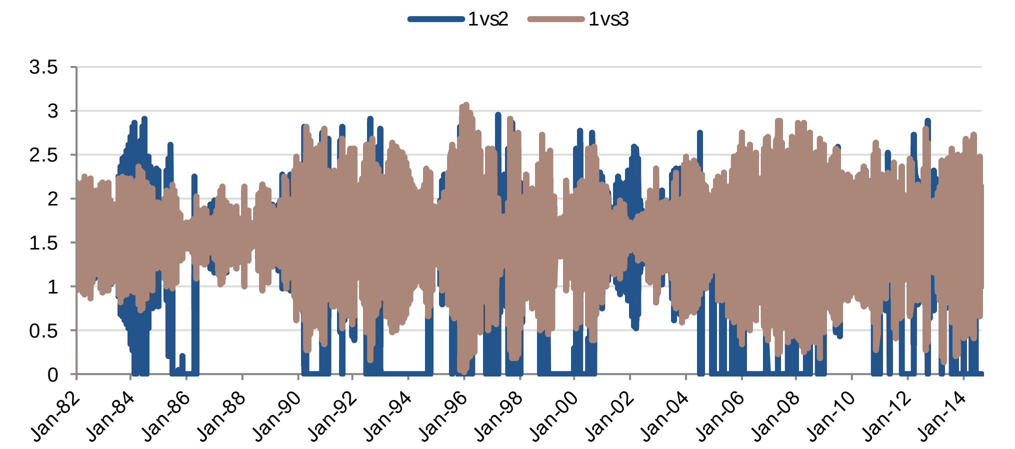
However, fortunately there are a few possibilities to overcome this issue, and the rest of this paper will be specifically dedicated to this aspect. The first possibility regards observing mean reversion over lower frequencies, i.e. $L > 1$, by using weekly or monthly returns. Quite structurally, the impact of costs decreases by reducing the frequency of the trades, although one might also expect that the serial correlations might be more evident at the high frequency scale. In other words, one has to find a trade-off between the potential of the trades and the impact of costs.

The second possibility that we will investigate in this paper concerns the use of some flexibility in defining the traded portfolios. We have already shown that, for a G10 FX basket in the zero cost limit, there would be a set of strategies displaying a long-term attractive profile.

In fact, by disposing of a set of eigenstates in the N-dimensional space, one could consider the portion of the multi-dimensional space defined by the linear combination of this set of eigenvectors, where each eigenvector is assigned a non-negative weight. This will give some additional freedom as far as the weighting in the space defined by the N assets if compared to the strict adherence to the most mean-reverting eigenstate: in fact, one could think of using this flexibility as a way to reduce the impact of costs.

For this purpose, we plot in the next chart the angles (expressed in radiants) between the positions, in the N-dimensional space, corresponding to the first and the second/third best eigenstates. A large angle between different strategies is a good feature because it implies there is more flexibility to trade portfolios with good mean-reverting properties, which can be exploited for reducing the impact of costs: furthermore, wide angles imply higher diversification when aggregating different strategies in a portfolio. On the other hand, a tight angle between the eigenstates would imply less flexibility for reducing costs and less diversification benefit. We can see from the chart that, at times, the angle between the first two eigenstates is zero; this can be understood with the degeneracy of the eigenvalues, as complex roots of the characteristics polynomial come in couples with equal real part: on the other hand, the first and third eigenstates are never degenerate in this example (this could happen only if a complex eigenvalue had multiplicity>1), which explains why it's important to consider three and not the two best eigenstates, as this largely increases the flexibility in the N-dimensional space.

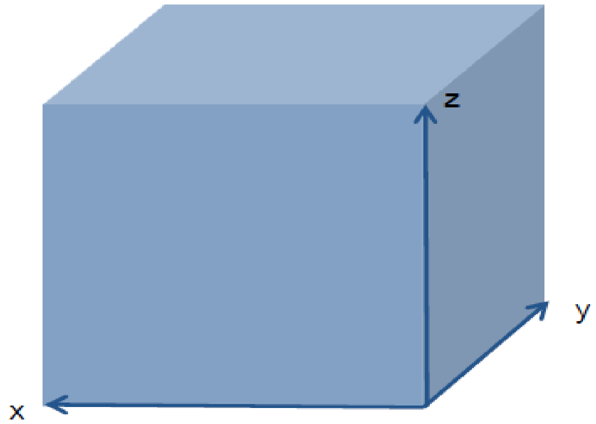
Angle (in radiants) between first and second/third eigenstates



Source: SG Cross Asset Research/Cross Asset Quant

We try to clarify the statements above by considering a limit case, displayed in the chart below. We assume that there are three assets, corresponding to the three axes in the chart, and three "good" strategies: strategy 1 is long asset 1, strategy 2 asset 2 and strategy 3 asset 3. Naturally, for this very particular case, the allowed region of the 3-d space which was consistent with the three strategies would be; x>0, y>0, z>0, namely the first octant of the 3-d space, which represents 1/8 of the volume of the whole 3d space.

Graphical representation of the allowed region in the 3-d space for three orthogonal eigenstates



Source: SG Cross Asset Research/Cross Asset Quant

In the more general case, the allowed region in the N-asset space consistent with M-eigenstates $\boldsymbol{W}^i$ (in general one has $M < N$, in our specific case $M = 3$, $N = 9$) would be

$$\boldsymbol{P} = \sum_{i=1}^{M} a_i \boldsymbol{W}^i \; ; \; a_i \geq 0$$

$\boldsymbol{P}$ is a linear combination of the M selected eigenstates; we allow only non-negative coefficients a_i as we don't want to enter short positions on any of the M best strategies. The subscript t corresponding to the calibration time for $\boldsymbol{P}, \boldsymbol{W}^i$ is omitted for simplifying the notations. The larger the angle between the eigenstates, the more would be the flexibility in selecting an optimal strategy, combining the positions for the different eigenstates, in the N-d space. We will show later how to exploit this freedom when defining an optimal trading position with the goal of reducing the impact of transaction costs.

Discussing the impact of costs on mean reversion strategies

In this paper we have assumed constant transaction costs across the whole timespan, equal to 2bp (bid-ask) for all the currencies. In practice, one would have had higher costs in the 80s, whereas current costs would be below that level. Given that the goal of the paper is to introduce an original methodology, rather than a finalised trading system, we don't make further assumptions on the size of costs, and will defer this task to a future study. For the same reasons, given that we deal with long/short baskets held for short time periods, we have omitted the impact of FX Carry throughout this preliminary paper.

However, it is interesting to explore the mechanism of why transaction costs play a big role in a mean-reverting strategy. Let's assume one has identified, at time t, a basket with strong mean-reverting properties, over a typical scale of L days. If one has entered the position and the trade was profitable, because of the strong mean reversion properties, one would have to take the opposite position at time $t + L$: the more the basket mean reverts, the more trades are profitable but also the more the impact of costs. For a trend following system, it is the opposite: when the model is successful, positions are kept rather constant, modulo some minor risk management adjustments on the positions. This is the reason why transaction costs have to be taken into account *ab initio* when building up the model.

Defining mean-reverting portfolios over different frequencies

The results above, corresponding to the daily strategy, can be tested also on frequencies for the returns other than 1/daily ($L > 1$): in fact, one can define a set of different strategies which optimise the mean-reverting properties over different frequencies. It is well known that there is a diversification benefit when strategies with low mutual correlation are aggregated; for mean-reverting strategies, this benefit is high, as mean reversion in principle takes place over different frequencies and can imply a high turnover of trades, as opposed to the case of trend-following systems, where positions are held for sustained periods as the trend continues. Furthermore, the aggregation of different mean reversion strategies should allow a substantial reduction in the impact of costs, which play a relevant role for these systems (as an example, if the daily strategy buys an asset and the weekly strategy sells it, the netting mechanism would reduce the impact of costs). We will discuss later how the flexibility in defining mean reversion properties over different frequencies will be important for improving the performance of an aggregated portfolio.

For considering mean reversion over different frequencies υ, one just runs the VAR model as defined above on a set of data such that

$$\frac{1}{\upsilon} = L\ (days)\ ;\ \boldsymbol{r}_t = \ln\left(\frac{\boldsymbol{S}_t}{\boldsymbol{S}_{t-L}}\right)$$

In practice, one considers integer values for the parameter L and derives the corresponding frequencies υ. When trading the strategy with frequency lower than daily, we do as follows: let's consider weekly as an example. Each day of the week, we consider the time series of the weekly returns, so that one has in fact an independent model for each day of the week. Each model is estimated independently, and each day one trades based on that day's model (other than for overall risk management considerations which will be discussed later). Ultimately, the total positions at each point in time will result from the aggregation, on a rolling basis, of the positions taken by each of the models corresponding to the different days of the week (and similarly for periods different than weekly); more details on the aggregation process and on the risk-management of the portfolios are described in the next section.

Given that, for each independent model, returns are sampled every L days, L also corresponds to the natural holding period of each position taken by the model with frequency υ. In general, for a fixed frequency $\frac{1}{\upsilon} = L\ (days)$ we will estimate each model over T sample points. In general, one will need

$$Total\ estimation\ period = LT$$

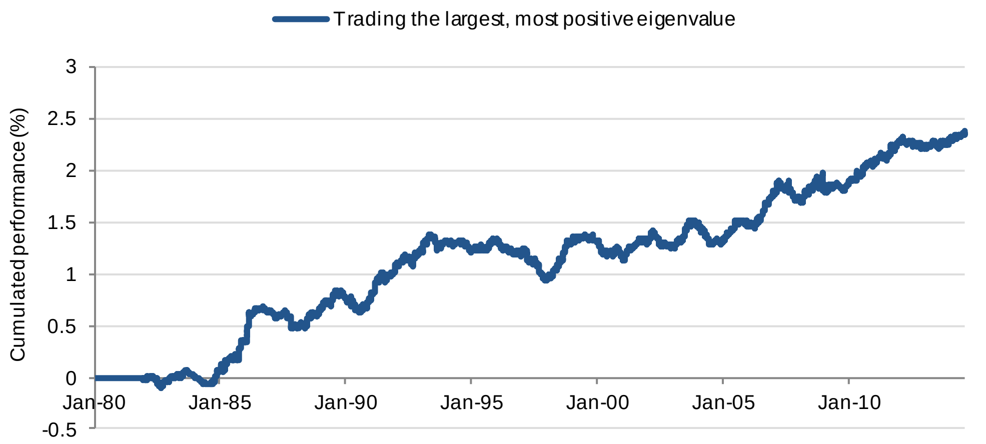
For a fixed estimation length T of each of the models, longer total estimation periods will correspond to lower frequencies. Depending on the size of the sample of the time series available (we used FX data from January 1980), the consideration above will put some constraints on the maximum estimation length for each of the models.

In the next section, we will discuss how the different positions, corresponding to the different days of the week, are aggregated, depending on the implementation of different risk management rules.

Identifying baskets with trend-following properties

The methodology that we have just presented also allows defining portfolios with strong trend-following properties, provided an eigenvalue of the VAR matrix is positive. For the case of the G10 FX basket herein investigated, the second chart of page 4 confirms empirically that the largest positive eigenvalue satisfy this requirement; it is then possible to study the simulated performance of such portfolios, displayed in the chart below (in the limit of zero costs).

Extracting trend-following behaviours out of a basket (zero trading costs)



Source: SG Cross Asset Research/Cross Asset Quant

We can see from the chart that basket trend-following properties can successfully be extracted out of the FX market by relying on the VAR methodology, although the performance measures of this strategy are lower than those for the portfolios with strongest mean-reverting properties.

It has to be stressed that, compared to mean reversion systems, trend-following trades require less frequent rebalancing (see discussion on page 5) and therefore the impact of costs should be structurally more limited. Again, in the spirit of minimising the impact of costs, such trades would add value within a diversified portfolio, especially when considering the optimisation techniques that will be implemented in this paper for reducing the impact of costs on high-frequency trades. However, the use of such trend-following strategies for maximising the diversification properties of the aggregated portfolios will not be investigated further in the context of the present paper and will be postponed to future studies.

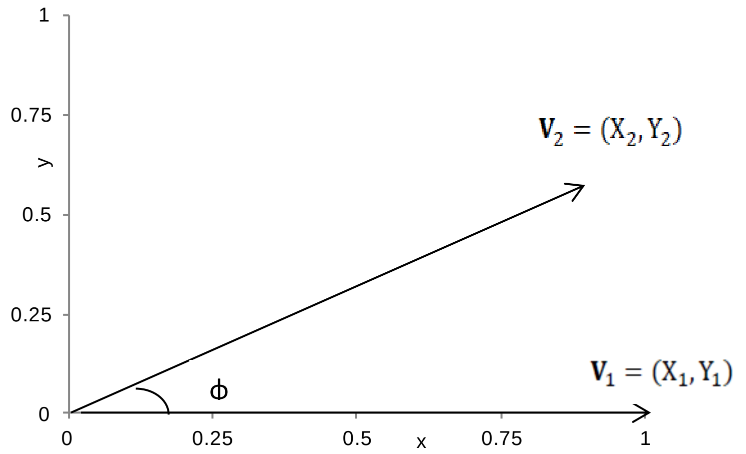
Mean-reverting baskets in presence of costs

In this section we present the rules that we have introduced for defining the optimal portfolios to be traded in the presence of costs. We will show the long-term simulated performances for such portfolios corresponding to different trading frequencies, estimation lengths and risk-management rules of the mean-reverting baskets.

Displaying the impact of costs after a trade

As minimization of trading costs will be a key element of the present paper, below we provide a simple description of the impact of costs for a trade by considering two assets, X, Y: the invested positions will be represented by 2-d vectors in the (X, Y) space. In the following chart, we display the trade from position $\boldsymbol{V}_1(X_1, V_1)$ to $\boldsymbol{V}_2(X_2, V_2)$.

Impact of costs when changing the invested position from $\boldsymbol{V}_1$ to $\boldsymbol{V}_2$



Source: SG Cross Asset Research/Cross Asset Quant

Let's consider the impact of costs corresponding to the trade, which, as done consistently in this paper, are assumed to be the same for the two assets and equal to $cost$. We also assume that the length of the two vectors remains unchanged and equal to L. We introduce the angle ϕ between the two vectors $\boldsymbol{V}_1, \boldsymbol{V}_2$.

The total cost an investor pays on the two assets after trading is:

$$Cost\ paid = cost * |X_1 - X_2| + cost * |Y_1 - Y_2| = cost * L|1 - \cos(\phi)| \ + cost * L|sin\,(\phi)|$$

For small angles one gets the following expression:

$$Cost\ paid = 2 * cost * L * |\phi|$$

So we see that, at least for small angles, the total cost paid on the trade is proportional to the angle between the positions. This reasoning can be easily extended to the case of N-assets. Minimizing the angle between the traded positions will be one of our main ingredients in cost optimization as introduced in the next section.

Reducing the impact of costs/use of trading rules

As discussed, the eigenstates portfolios $\boldsymbol{W}^{i}{}_{t}$ described above would work well in the limit of zero costs, but would suffer in the presence of non-zero costs due to the high frequency at which they are rebalanced. In the following, we will distinguish between the linear combination, at time t, $\mathbf{P}_t$ of the eigenstates portfolios $\boldsymbol{W}^{i}{}_{t}$ and the actual, traded portfolio $\boldsymbol{\Pi}_t$. For simplicity, we assume that the holding period of each trade is one day ($L = 1$). For reducing the impact of costs, the following rules are considered for defining the actual traded portfolios:

- One considers a new optimal position $\mathbf{P}_{t+1}$ which is a linear combination of the three best eigenstates (as described on page 9). In fact, one looks for a constrained optimisation for defining the linear combination of the three best eigenstates such that this minimises the angle in the multi-d positions space. The minimum angle, as we have just displayed, will reduce the impact of costs. The constraints imposed are that all three coefficients have to be positive (one considers only a portion of the multi-d space), and that the first eigenstate needs to have a weight of at least 50% (with the two remaining weights summing up to at most 50%).

- If the new optimal portfolio forms an angle with the old position which is below a critical value, one does not trade (i.e., one keeps the old position $\boldsymbol{\Pi}_{t+1} = \boldsymbol{\Pi}_t$). This critical value for the angle has been fixed to be $\pi/8$.

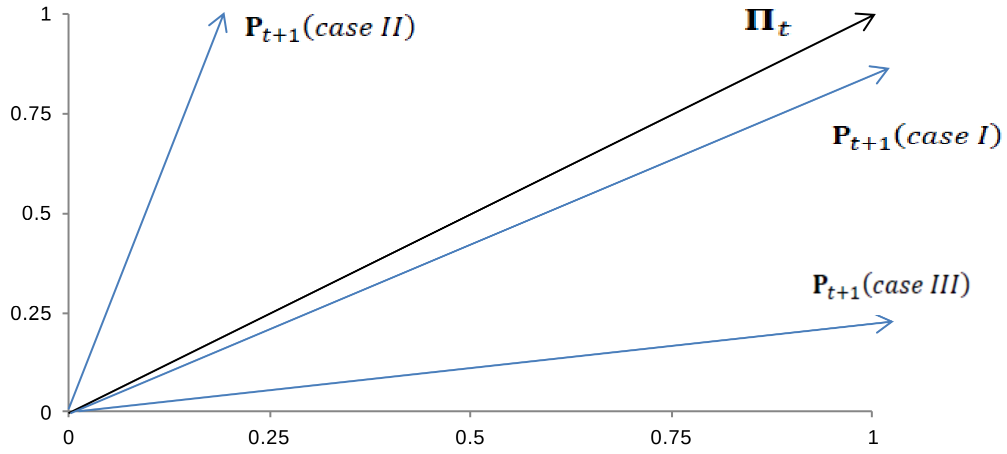
- For each new optimal portfolio, one can compute an expected return, based on the weights of the constrained optimisation, on the values of the eigenvalues and on the past returns. One can compare this return with the impact of costs for rebalancing the portfolio: if trading still leaves a residual positive expected value, after costs are taken into account, one trades; otherwise, one keeps the old position in order to reduce the impact of costs.

- Risk management rules are enforced (see next section for more details).

- As a final condition, in some cases the vol scaling rules will impose some “scaling” (dilation/contraction) of the portfolios. If one finds that this scaling operation reduces performance because of the impact of costs, one does not do the volatility scaling.

As a graphical representation of our rules, in the following chart we display three different examples which correspond to the decisions of trading or not. The old position the model is taking at time t is $\boldsymbol{\Pi}_t$. At time $t+1$, we consider three different outcomes for the optimal position $\mathbf{P}_{t+1}$. In case I, the angle between the old position and the new optimal position is less than the critical value $\pi/8$, so we don’t trade: $\boldsymbol{\Pi}_{t+1} = \boldsymbol{\Pi}_t$. In case II, the expected value of the trade is negative so we don’t trade: $\boldsymbol{\Pi}_{t+1} = \boldsymbol{\Pi}_t$. In case III, the angle is above the critical level and the expected value of the trade is positive, so we trade: $\boldsymbol{\Pi}_{t+1} = \mathbf{P}_{t+1}$.

A 2-d graphical representation of our rules for minimising the impact of trading costs



Source: SG Cross Asset Research/Cross Asset Quant

Basically, the rules we have considered have the goal of reducing the frequency at which portfolios are rebalanced: one keeps the position unchanged unless the adjustments in the portfolio add enough value to justify the trade.

Risk management rules

Here we explain how volatility-scaling rules can be applied to such portfolios, especially in the presence of frequencies different from daily. For simplicity, we start from the 1/daily frequency model ($L = 1$): at any given day one re-estimates the model and gets a set of optimal positions. One can compute the in-sample volatility of such fixed portfolios and impose the constraint that these "fixed" optimal portfolios have an in-sample volatility equal to a given target (let's say 10% here). The day after, one computes the PnL and adjusts the weights based on the in-sample volatility targeting of the new optimal positions.

Let's consider now the weekly model as an example (the reasoning would be the same for any other frequency). As discussed, every day of the week corresponds to a different model, on which one can impose the constraint of having an in-sample volatility equal to a given target. However, when entering positions every day (each with a natural holding period of one week in this case), one has to decide how to aggregate the models for the different days of the week together. If one has built the volatility scaled models for each day of the week, the volatility of the strategy which aggregates the different models for the different days of the week will depend on the mutual correlations of their PnLs, which might be difficult to estimate and in some cases introduce a degree of instability to the system.

For this reason, we will implement two sets of volatility scaling rules: in the first, one ensures that for each estimated daily model the in-sample volatility matches the target value, with no constraints on the volatility of the aggregated portfolio. We will refer to this technique as "local volatility scaling". In the second case, which we will call "global volatility scaling", one imposes a further constraint for ensuring that the volatility of the aggregated model, combining the different daily positions, is equal to a given target. Several technical aspects which would require additional clarifications will be described in more detail in the next two subsections, summarising the results corresponding to the global and local volatility scaling risk management rules; such technical aspects will prove important when considering the aggregation of the strategies for different frequencies as done later on in the paper.

Results with local volatility scaling

We present the results for the local volatility scaling rule, which, as discussed, is imposed in-sample on each daily model. In the next two charts, we display the Sharpe ratio, with and without trading costs, as a function of the frequency and of the estimation period (measured in number of days) of the strategies; the highest value of the Sharpe ratio is highlighted in the tables. When showing the results for zero trading costs, in the first chart below, we can see that the model successfully captures mean-reversion properties from daily to by-weekly scales; the Sharpe ratios are all positive in this range and reach values up to 1.4 for the daily mean-reversion case.

Sharpe ratios for zero cost

		Estimation length			
		100	250	350	500
1/frequency	1	1.09	1.38	1.37	**1.40**
	2	1.09	1.08	1.36	1.23
	3	0.54	0.57	1.01	1.13
	4	0.96	1.21	0.76	0.75
	5	1.24	1.21	1.06	0.94
	6	0.40	0.63	0.42	1.18
	7	1.23	0.27	0.48	0.29
	8	0.38	0.17	0.59	0.73
	9	0.43	0.54	0.71	0.58
	10	0.14	-0.60	0.13	0.59
	15	0.23	0.01	-0.24	0.65
	20	-0.21	-0.14	-0.39	N/A

Sharpe ratios with cost

		Estimation length			
		100	250	350	500
1/frequency	1	0.20	0.66	0.75	0.83
	2	0.59	0.68	0.97	0.84
	3	0.28	0.32	0.75	0.85
	4	0.70	0.99	0.53	0.56
	5	1.03	1.02	0.87	0.78
	6	0.24	0.51	0.29	1.03
	7	**1.11**	0.15	0.39	0.17
	8	0.27	0.07	0.48	0.62
	9	0.34	0.44	0.61	0.48
	10	0.05	-0.68	0.04	0.50
	15	0.17	-0.04	-0.29	0.61
	20	-0.25	-0.17	-0.42	N/A

Source: SG Cross Asset Research/Cross Asset Quant

As we see in the right-hand chart above, the impact of cost is the largest for the highest frequency strategies and, overall, the strategy remains successful up to the two-week holding period scale. Due to the important impact of costs, we will show later the benefit of netting positions when diversifying across different frequencies. The N/A appearing in the bottom-right corner in the tables are due to the fact that the burn-out period required for estimating the model in this case is longer than the available timespan for the data.

Volatility of the strategies (with trading costs)

		Estimation length			
		100	250	350	500
1/frequency	1	11.08%	10.78%	11.42%	11.58%
	2	5.98%	6.20%	6.05%	5.53%
	3	5.37%	4.33%	3.95%	3.46%
	4	3.05%	2.90%	2.83%	3.18%
	5	2.44%	2.18%	2.21%	2.36%
	6	2.12%	2.38%	2.23%	1.82%
	7	1.97%	1.80%	2.18%	1.54%
	8	1.73%	1.61%	1.40%	1.40%
	9	1.64%	1.34%	1.21%	1.24%
	10	1.28%	1.19%	1.09%	1.15%
	15	0.88%	0.92%	0.96%	1.00%
	20	0.82%	0.79%	0.72%	N/A

Max Drawdown of the strategies (with trading costs)

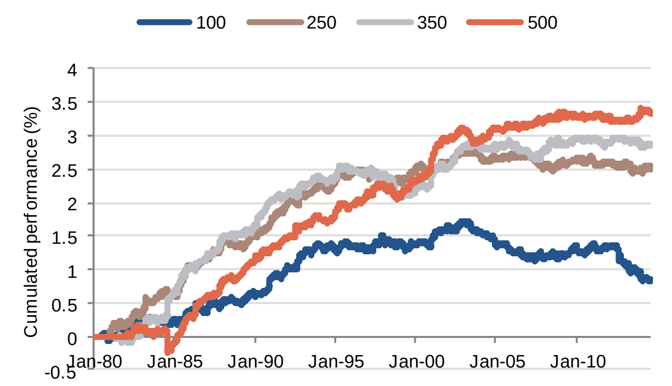
		Estimation length			
		100	250	350	500
1/frequency	1	99.29%	36.74%	48.72%	44.12%
	2	43.99%	42.45%	48.61%	37.13%
	3	43.10%	30.55%	32.88%	20.62%
	4	24.79%	18.01%	24.77%	25.73%
	5	15.14%	13.03%	17.84%	14.24%
	6	20.84%	24.37%	17.92%	13.47%
	7	11.97%	18.79%	23.31%	20.65%
	8	23.78%	22.81%	11.36%	8.83%
	9	18.53%	17.15%	9.27%	13.99%
	10	16.37%	26.63%	8.67%	10.00%
	15	9.77%	9.19%	14.20%	9.58%
	20	23.21%	11.01%	8.80%	N/A

Source: SG Cross Asset Research/Cross Asset Quant

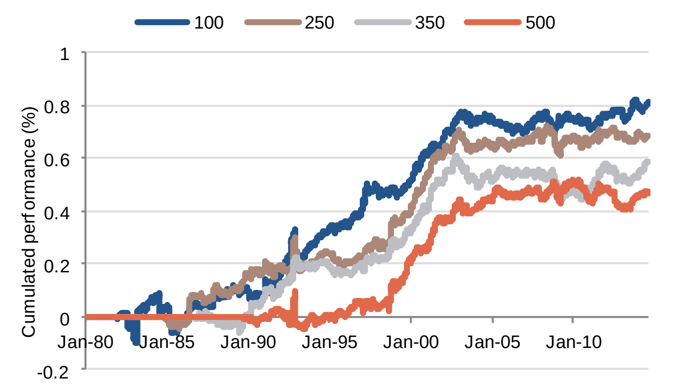
In the two charts above, we display the volatility and the maximum drawdown of the strategies as a function of frequency and estimation length parameters. As shown, the volatility of the strategies deviates markedly from the 10% target when the holding period of each trade is longer than one day ($L > 1$). For the strategy with frequency $\upsilon\,(\frac{1}{\upsilon} = L\,(days))$, an equally-weighted $\frac{1}{L}$ allocation to the L different risk-targeted positions taken every day would result in a volatility of the portfolio less than the common target volatility if the mutual correlation of the different daily strategies is less than 100%, and this is what we empirically observe in practice.

In the next two charts, we display the performance of the daily and weekly strategies, including cost, as a function of the length of the estimation length parameter. The two models have a different sensitivity on the length of the estimation period parameter. For both models, the best set of parameters are associated with Sharpe ratios around 1.

Daily PnL as a function of the estimation length parameter



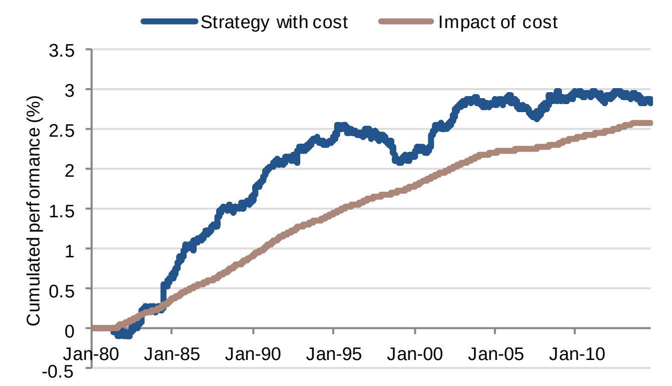
Weekly PnL as a function of the estimation length parameter



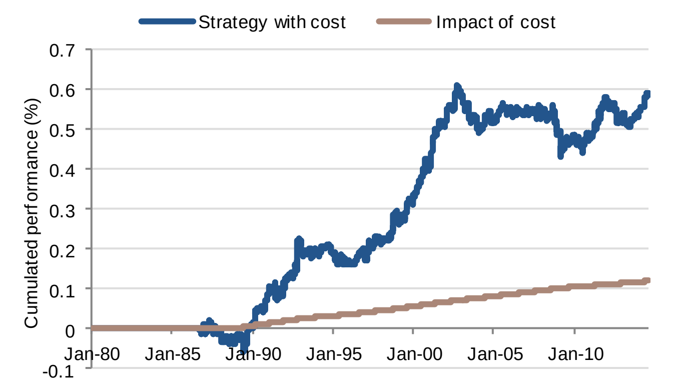
Source: SG Cross Asset Research/Cross Asset Quant

One can see a flattening of the performance over the past few years, but it's also the case that in practice the impact of costs would be less than it is assumed here. For instance, by looking at the daily vol scaled strategy in the first chart below, with an estimation length of 350 days, the performance of the strategy is flattish over the past few years, but that is mainly due to trading costs.

Impact of costs on the daily strategy



Impact of costs on the weekly strategy



Source: SG Cross Asset Research/Cross Asset Quant

By looking at the weekly strategy, in the second chart above, one finds a similar picture, in that the impact of costs grows (roughly linearly) with time and plays a big role when the returns of strategy for zero costs are comparable to the scale of costs. By comparing the two charts above one can see that the impact of the costs is, overall, less important for the weekly than for the daily strategy, but also that the potential of the strategy is reduced for lower frequencies.

The table below displays the mutual correlation of the strategies, for different frequencies, for a fixed estimation length of the strategies (350 days), with global volatility scaling and in the limit of zero costs. The very important observation is that the correlation of the strategies is very limited which, as commented before, should give a large performance boost for an aggregated strategy.

Correlation of the strategies for different frequencies (as a function of the holding period parameter $L = 1/\upsilon$)

	1	2	3	4	5	6	7	8	9	10	15	20
1	100.0%	9.5%	-10.0%	5.1%	1.0%	1.8%	0.0%	0.8%	-0.5%	-0.1%	-0.1%	0.0%
2	9.5%	100.0%	19.4%	19.6%	17.7%	23.5%	21.9%	0.3%	2.5%	2.6%	3.7%	0.9%
3	-10.0%	19.4%	100.0%	33.3%	20.1%	33.7%	31.8%	-2.0%	4.1%	4.7%	1.4%	2.4%
4	5.1%	19.6%	33.3%	100.0%	42.6%	45.4%	38.5%	10.7%	10.4%	11.8%	8.0%	5.3%
5	1.0%	17.7%	20.1%	42.6%	100.0%	40.1%	28.1%	27.0%	19.8%	19.0%	12.8%	4.8%
6	1.8%	23.5%	33.7%	45.4%	40.1%	100.0%	70.7%	23.0%	25.6%	24.9%	12.1%	6.0%
7	0.0%	21.9%	31.8%	38.5%	28.1%	70.7%	100.0%	25.4%	29.6%	27.0%	11.8%	5.8%
8	0.8%	0.3%	-2.0%	10.7%	27.0%	23.0%	25.4%	100.0%	43.9%	42.3%	19.8%	6.2%
9	-0.5%	2.5%	4.1%	10.4%	19.8%	25.6%	29.6%	43.9%	100.0%	48.0%	24.0%	6.7%
10	-0.1%	2.6%	4.7%	11.8%	19.0%	24.9%	27.0%	42.3%	48.0%	100.0%	36.3%	21.4%
15	-0.1%	3.7%	1.4%	8.0%	12.8%	12.1%	11.8%	19.8%	24.0%	36.3%	100.0%	41.0%
20	0.0%	0.9%	2.4%	5.3%	4.8%	6.0%	5.8%	6.2%	6.7%	21.4%	41.0%	100.0%

Source: SG Cross Asset Research/Cross Asset Quant

In the table above, we display the correlation of the strategies in the limit of zero costs; the impact of costs on each of the strategies, if traded independently, would naturally increase their mutual correlation. However, the netting mechanism resulting from the aggregation of different strategies will allow paying costs only after aggregating the trades, which would boost the diversification mechanism as suggested by the table above.

Results with global volatility scaling

As we have seen in the previous section, the local volatility scaling mechanism does not ensure that the volatility of the strategy for frequencies different from 1/daily ($L > 1$) is fixed to a given target; however, controlling the volatility of the aggregated position is an important risk-management feature to be imposed on invested portfolios.

In this section, we impose this additional constraint, which we refer to as global volatility scaling; this is done by scaling the daily positions in a way that the resulting portfolio, aggregating the different daily models, reaches the target volatility. While the local volatility-scaling rule fixes the in-sample volatility on each of the daily models, independently from each other, the global volatility scaling rule involves calculating the actual PnL generated after aggregating the daily positions, and rescaling them accordingly. The PnL generated by the trades will depend on the mutual correlation of the strategies whose positions were taken on different days.

The two charts below display the Sharpe ratios of the strategy for different frequencies and estimation length parameters. As before, we see that the impact of costs is highest for the high-frequency strategies, whereas for frequencies lower than 1/weekly the impact is relatively modest. The Sharpe ratios obtained with this risk-management rule are comparable to those obtained with the local volatility scaling approach reported before: the best values of the Sharpe ratios, usually around or higher than 1, are obtained for frequencies higher than 1/weekly.

Sharpe ratios zero cost

1/frequency	Estimation length: 100	250	350	500
1	0.98	1.22	1.29	**1.66**
2	0.86	1.13	1.27	0.73
3	0.59	0.44	0.78	0.50
4	1.17	0.83	0.20	0.03
5	0.92	0.66	0.41	-0.04
6	0.50	0.59	0.42	0.27
7	0.75	0.02	-0.25	-0.25
8	0.34	-0.31	0.23	0.93
9	0.28	-0.30	0.03	0.48
10	0.00	-0.58	0.01	1.15
15	-0.54	-1.07	0.40	2.23
20	-0.30	0.24	2.27	N/A

Sharpe ratios with cost

1/frequency	Estimation length: 100	250	350	500
1	0.04	0.50	0.67	1.03
2	0.48	0.83	1.02	0.50
3	0.44	0.29	0.66	0.38
4	1.02	0.73	0.12	-0.04
5	0.81	0.58	0.34	-0.09
6	0.42	0.55	0.36	0.23
7	0.69	-0.02	-0.30	-0.28
8	0.29	-0.34	0.20	0.91
9	0.24	-0.33	-0.01	0.46
10	-0.04	-0.61	-0.01	1.14
15	-0.56	-1.09	0.38	**2.22**
20	-0.32	0.23	2.26	N/A

Source: SG Cross Asset Research/Cross Asset Quant

The left-hand chart below displays the volatility of the strategy, with costs, by implementing the global volatility scaling rule. We can see that the (out-of-sample) realised volatility of the strategies is generally very close to the target value of 10%. The slight discrepancy between target and actual volatilities can be justified via the fact that the former is imposed in-sample whereas the latter is measured out-of-sample and by the fact that the trading rules, imposed to reduce the impact of costs (described on page 12), can lead to a modest deviation from the perfectly vol-targeted portfolios. In the second chart, we report the values of the maximum drawdown as a function of the (constant) capital. We see that, except for the high frequency strategies, the maximum drawdowns reach high values, which would make it difficult trading such strategies on a standalone basis.

Volatility of the strategies (with costs)

1/frequency	Estimation length: 100	250	350	500
1	10.22%	10.31%	10.98%	9.89%
2	11.06%	10.22%	10.50%	10.40%
3	16.27%	10.53%	10.81%	10.23%
4	9.90%	10.92%	12.33%	11.43%
5	10.27%	10.25%	10.48%	11.85%
6	11.25%	12.64%	9.48%	10.46%
7	11.01%	11.12%	8.98%	9.61%
8	10.65%	10.86%	9.82%	10.21%
9	12.59%	10.38%	8.48%	10.23%
10	10.59%	10.46%	10.17%	9.89%
15	10.53%	10.44%	10.51%	7.24%
20	10.16%	9.50%	9.61%	N/A

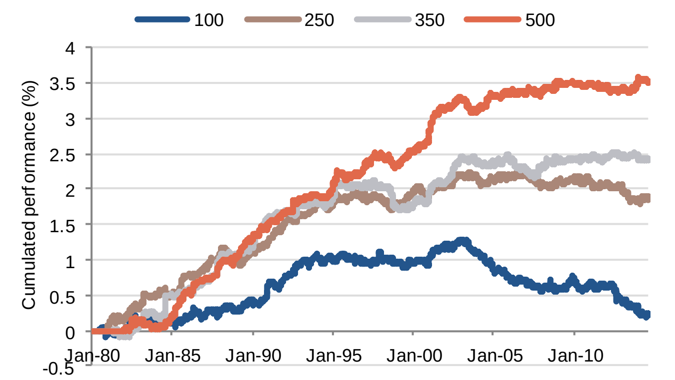
Max Drawdown of the strategies (with costs)

1/frequency	Estimation length: 100	250	350	500
1	118.18%	44.05%	41.11%	24.30%
2	78.76%	69.64%	78.44%	63.81%
3	128.72%	74.69%	68.87%	61.75%
4	46.03%	81.92%	114.82%	108.20%
5	50.84%	101.32%	85.69%	161.24%
6	95.65%	142.91%	73.26%	131.24%
7	99.97%	156.65%	147.94%	163.43%
8	77.48%	154.24%	90.49%	95.22%
9	104.31%	167.20%	149.86%	92.67%
10	179.01%	217.53%	166.69%	54.45%
15	329.72%	280.86%	150.17%	48.22%
20	352.95%	215.95%	73.47%	N/A

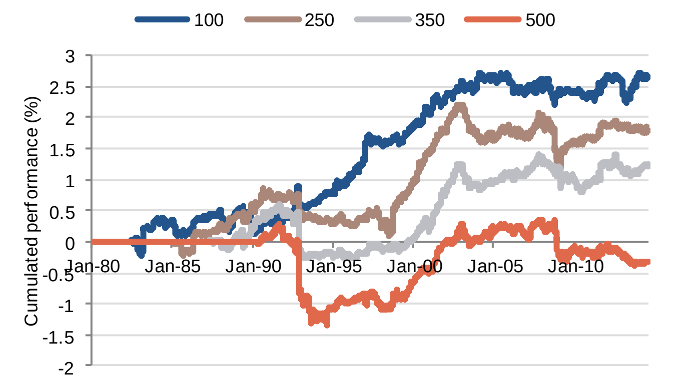
Source: SG Cross Asset Research/Cross Asset Quant

In the next two charts we display the total PnL generated by the daily and weekly strategies as a function of the estimation length parameter. For the daily strategy, we see that the performance of the system increases with the length of estimation, and for the weekly strategy, it is the opposite. However, we recall that estimating the weekly model with a length of estimation of 500 days would correspond to a total timespan of about 10 years, which appears excessively long. We will come back to this point later when discussing the aggregation of the strategies with different frequencies; the very low correlation between the two strategies, as described in the past section, should support their combination in a diversified system.

Daily strategies with global vol scaling



Weekly strategies with global vol scaling



Source: SG Cross Asset Research/Cross Asset Quant

A few technical aspects related to the implementation of the risk-management rules require additional clarifications. For instance, as discussed earlier, a difference between local and global volatility scaling rules is that, in the latter, the volatility target is imposed on the out-of-sample PnL generated by the positions: this explains why the Sharpe ratio of the 1/daily frequency ($L = 1$) strategy is slightly different for the two risk-management rules, although the difference is not huge. Also, in general one can use a different time window for estimating the volatility of the realised PnL of the strategy compared to that used for estimating the VAR models, which can further motivate a modest mismatch in the two different volatility estimates.

Another aspect concerns the fact that, as mentioned, when imposing the global-volatility scaling condition, we introduce a sensitivity to the correlation between the PnL generated by the different daily models; this quantity can be estimated by using different statistical prescriptions, some of which are better suited than others for forecasting purposes. Furthermore, such variables might be unstable over time and subject to statistical noise, and could turn out to be correlated with the performance of the aggregated strategy itself. In other words, when considering the global vol-scaling condition, the sensitivity on this correlation parameter might introduce an additional source of instability in the composition of the optimal portfolios and for these reasons we have decided to show the results of the model with and without the latter risk-management rule being implemented.

Summary of trading and risk management rules

Here we summarise the main steps towards the construction of the mean-reverting portfolios as presented in this paper.

Let's assume that, at time t, we have entered the positions $\mathbf{\Pi}_t$. If the holding period of each trade is L days, the rebalancing of the position entered at time t will occur at $t + L$.

Then, at time $t + L$:

- We calibrate the VAR(1) model (over a period T) and obtain a set of eigenstate portfolios ${\boldsymbol{W}^i}_{t+L}$ corresponding to the strongest mean-reverting properties. We select three such eigenstates $(i = 1 \cdots 3)$ corresponding to the strongest mean-reverting properties (most negative eigenvalues)

- We consider the linear combination of the selected portfolios $\mathbf{P}_{t+L} = \sum_{i=1}^{3} a_i \, {\boldsymbol{W}^i}_{t+L}$ (with positive coefficients $a_i > 0$), such that the angle with the position taken by the model at the previous trading time t, $\mathbf{\Pi}_t$, is minimised. We minimise the angle with the goal of reducing the impact of costs. If the angle is below a critical value of $\pi/8$, we keep the position unchanged $\mathbf{\Pi}_{t+L} = \mathbf{\Pi}_t$

- For deciding the actual position $\mathbf{\Pi}_{t+L}$ taken at time $t + L$, we compute the expected value of the trade, taking into account trading costs. If the expected value of the trade is positive, $\mathbf{\Pi}_{t+L} = \mathbf{P}_{t+L}$, otherwise $\mathbf{\Pi}_{t+L} = \mathbf{\Pi}_t$.

- Further risk management conditions will be imposed on $\mathbf{\Pi}_{t+L}$

For risk-management, we have considered two distinct rules:

- Local volatility scaling: for a L day holding period of each trade, we consider an equal allocation (with $1/L$ weight each) to the L models defined over the past L days. Each of the daily models is vol targeted. The resulting portfolio aggregating the L models is not vol targeted, as the correlation of the different daily models is in general less than 100%

- Global volatility scaling: the position $\mathbf{\Pi}_{t+L}$ is further scaled to ensure that the portfolio aggregating the positions entered over the past L days reaches the target volatility.

Aggregating strategies over different frequencies

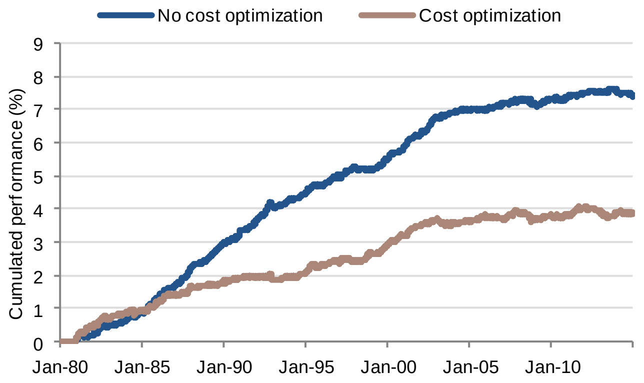
In this section we consider the aggregation of the mean reversion strategies defined over different frequencies, which we have described in the previous sections of this paper, into a combined diversified portfolio. While a full description of the portfolio construction techniques required for aggregating the strategies would require a more extensive treatment, especially as far as the issue of minimising the trading costs for the aggregated strategy, a few general features can be inferred from the study. In the final subsection we investigate the link between the performance of a typical FX mean reversion strategy and the dynamics of global markets volatility variables. Also, FX mean reversion is compared with two other strategies (Carry trade and momentum) which are commonly traded in the FX space.

Aggregating strategies with different frequencies

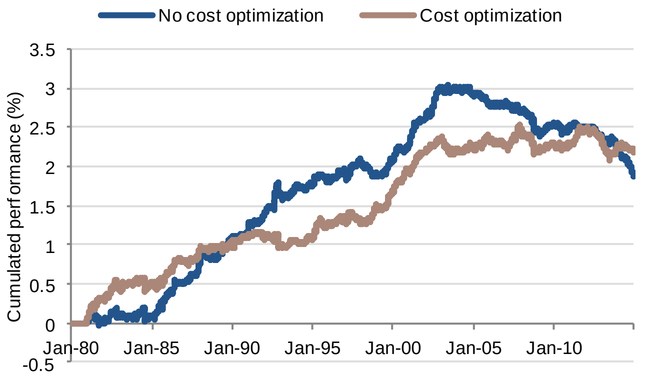
The starting point for the aggregation of the strategies defined over different frequencies is the earlier section describing the "Global volatility scaling" risk management rule; the positions of the fixed-frequency strategies will be the input for the aggregated positions, so it makes sense to ensure that a common level of risk (i.e., volatility) is associated to each of them, before the actual allocation process is implemented. In the following, we will consider different allocation of risk, rebalanced daily, to the volatility-scaled input states; a further volatility targeting (with volatility calculated over a 1yr rolling period) will be imposed at the end of the aggregation process on the resulting aggregated portfolio.

In the next two charts, we compare the performance of the diversified strategies with and without the risk-optimisation techniques described in the previous section; for the non cost-optimised strategy, we basically trade the vol-scaled first eigenstates portfolios described earlier on. In both cases, we start by considering an equal allocation to all the strategies with 1/frequency L parameter ranging from 1 to 10 (and the four estimation lengths from 100 to 500 days).

Aggregated portfolio of mean reversion strategies for zero costs



Aggregated portfolio of mean reversion strategies with costs



Source: SG Cross Asset Research/Cross Asset Quant

The first chart above displays the performance of the strategies in the limit of zero costs: we see that the portfolios trading only the largest eigenstates outperform significantly the cost-optimised portfolios; the Sharpe ratios are 1.80 and 0.96 respectively. However, when taking costs into account, the picture is reversed, with the portfolio of cost-optimised strategies delivering the best results (Sharpe ratios of 0.47 vs 0.55), with a much more reduced impact of costs on the overall performance of the strategy.

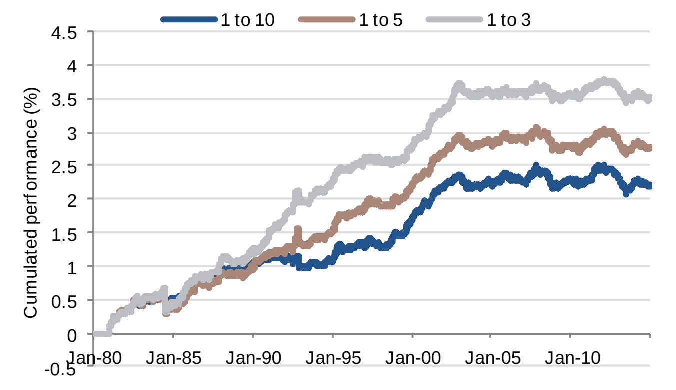
The observation above is significant, as it confirms that the cost-optimisation technique is indeed effective for optimising the composition of the invested portfolios in a way that the impact of costs is minimal. It is also important to stress that, in the charts above, the cost optimisation technique has been applied to the input states (i.e., the fixed frequency strategies with global vol scaling): this is clearly sub-optimal, as one could think of first aggregating the fixed-frequency strategies (corresponding to the first eigenstates) and later applying the cost optimisation technique on such portfolios. This optimisation in the multi-dimensional strategies space would be more cumbersome from the numerical point of view but, as shown from the comparison of the first and second charts above, should further improve the potential of the diversified portfolios.

Optimisation rules for the aggregated portfolios

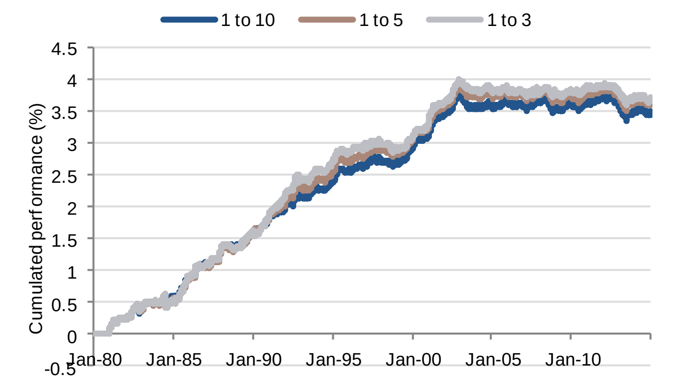
In this section we consider a few possible allocation schemes for the fixed-frequency, cost-optimised strategies, by taking into account transaction costs.

In the first chart below, we start by assessing the equal allocation to a set of different ranges of frequencies (1/frequency 1 to 3, 1 to 5, 1 to 10). We see that the best performances are usually achieved for the highest frequency strategies, which suggests that in an allocation scheme, it could make sense to attribute a higher weight to higher frequency strategies. The results of the chart could also suggest the interest of investigating the application of the technique to intra-day data.

Equal allocation to different frequencies



Frequency-scaled allocation to different frequencies



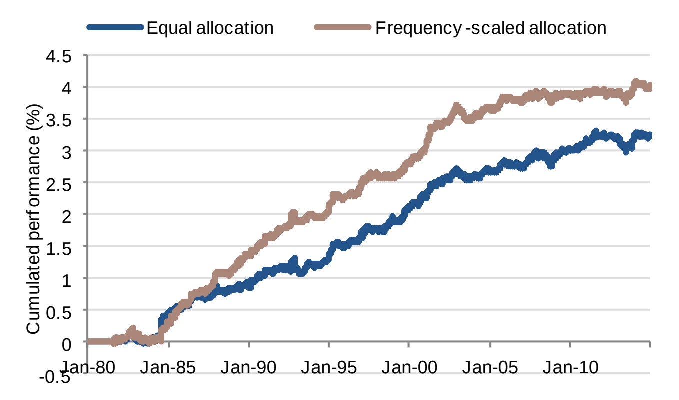
Source: SG Cross Asset Research/Cross Asset Quant

The second chart above introduces an allocation to the strategies via a relative weight which is proportional to the frequency of each strategy: by suppressing the strategies with a lower frequency, we see that the performance of the system increases and converges to that allocating risk to the higher frequency strategies only.

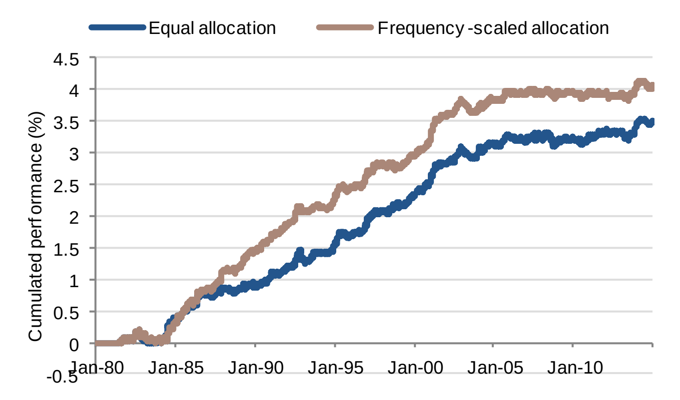
When trading the strategy in practice, it would make sense to introduce a dynamic selection of the optimal frequencies and time estimation parameters, despite having extensively assessed that the sensitivity of the strategy on such parameters is not dramatic.

In the first chart below, we display the performance of the strategy which allocates risk to all frequencies, with the optimal time length parameter selected for each frequency. Again, we see that the introduction of an allocation proportional to the frequency of each strategy further boosts the performance of the aggregated system (the Sharpe ratios are 0.78 and 1.05 for the equal and frequency-scaled allocations, respectively).

Picking the best time sample parameters for each frequency



Selecting the best frequencies/time sample parameters



Source: SG Cross Asset Research/Cross Asset Quant

In the second chart above, we go one step further and consider the aggregation of only the best 5 frequencies (1/frequency equal to 1,2,4,5,8); the added value of the frequency-dependent allocation scheme, over the equally-weighted scheme, is confirmed (Sharpe ratios are 1.07 for the former and 0.86 for the latter). In all cases, such optimisation techniques would improve the performance of the frequency-diversified portfolios. In general, we see that the frequency-diversified strategy allows for a sensible reduction of the maximum drawdown of the diversified portfolios.

We stress again that one could imagine obtaining better results after performing the cost-optimisation algorithm after (and not before) aggregating the different frequencies. Also, the implemented volatility-scaling at the end of the aggregation process might not be optimal from a reduction of costs perspective. We plan to address these points in a future paper, and to discuss a dynamic selection of the optimal parameters/strategies into the diversified portfolio over time.

General properties of the diversified portfolios

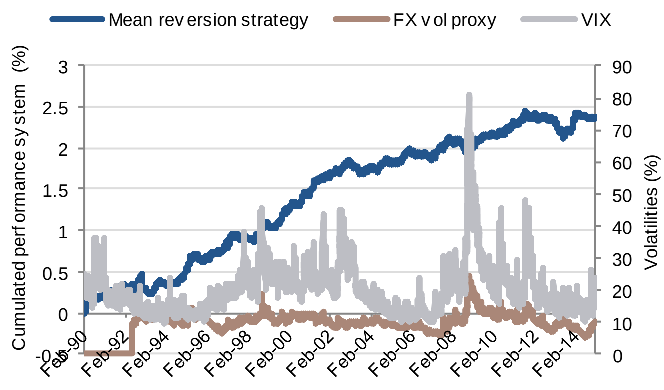
It is worth comparing the performance of the FX mean reversion strategy detailed above with other cross-asset indicators representative of markets dynamics; as a benchmark for the mean reversion strategy we consider the version equally allocating to the strategies of different frequencies, with optimal estimation length parameters.

We start by assessing, in the first chart below, the performance of the strategy as a function of market volatility, for which we consider VIX and an average of FX G10 3m implied vols as representative proxies. We see that the strategy suffered in connection to the spikes in volatilities that occurred in 1998 and 2008, and also during the low-volatility markets between 2001 and 2013.

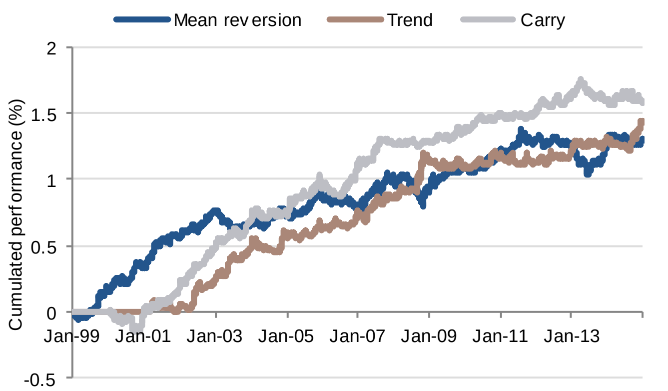
It is instructive to compute correlations between the strategy and the two volatilities considered. By considering the daily correlation between the returns of the strategy and the returns of the volatilities, the values are modestly negative (-5% VXY and -3% VIX), which means that the strategy typically slightly underperforms as the volatilities rise. However, by considering the daily correlation between the mean reversion strategy returns and volatility levels, one gets slightly positive values (+1% for the VXY and +0.4% for the VIX). This means that the level of volatilities appears to play a role, although very minor, on the performance of the strategy. The fact that a higher volatility increases the mean reversion properties of the markets had already been found in our earlier piece investigating mean reversion strategies implemented via quadratic payoffs.

It should also be stressed that cross-asset indicators are commonly used as filters for enhancing the performance of trading strategies. Based on the correlation with the volatility variables, it appears that the best environment for mean reversion is where vols are high and on a declining mode, whereas the worst could be where vols are low and on a rising mode. While these observations might suggest the potential of investigating a volatility-based filter for mean reversion systems, a more extensive analysis would be required and will not be carried out here.

FX mean reversion strategy vs Equity and FX volatilities



FX trading strategies: Mean reversion, Trend and Carry



Source: SG Cross Asset Research/Cross Asset Quant

It is also interesting to compare the performance of the mean reversion strategy with that of two other systems, namely Carry trade and momentum, which are popular in the FX space: as a benchmark for the latter we take the versions as implemented in our FX Enhanced risk premia strategy (for the Carry trade, we refer to the G10 strategy). By using data from January 1999, the (daily) correlation with the mean reversion strategy is negative (-7%) for the Trend and positive (+5%) for the Carry systems; the low, or even negative, values of correlation should indicate the large diversification benefit that should emerge by introducing mean reversion into a portfolio of FX strategies. It would also make sense to classify the strategy in terms of trading style, as it was done in our Risk premia piece of September 2013.

Conclusion

In this paper we have presented a general formalism which allows taking a position on the empirically observed mean reversion properties of financial variables. While mean reversion is a common term that is associated with a rather wide set of trading styles and statistical models, in this report we have considered the possibility of maximising the mean reversion properties of suitably chosen baskets of variables belonging to the same asset class. In other words, rather than trying to define time regimes where the mean reversion properties of a given variable dominate over its trend-following properties, here we aim to define baskets that structurally exhibit large oscillations that an investor can successfully take a position on.

While the formalism is very general and could be in principle applied to any asset class, we have carried out the numerical simulations as presented in this paper by using G10 FX variables only.

In the first section of the paper we have introduced the technical formalism employed for identifying the baskets with the strongest mean-reverting properties. We started by considering the case of zero transaction costs, and found that the methodology would lead to a successful simulated performance in the long run. However, as it is the case in general for strategies with a high turnover of positions, the impact of costs is rather high and has to be taken into account as an input for reducing the number of unnecessary transactions. In order to do so, in the second section of the paper we have considered several rules employed for optimising the composition of the baskets and the number of transactions in presence of trading costs. We have discussed extensively the question of how to implement risk management rules on strategies defined over different frequencies.

In the third section, we have described the aggregation of strategies defined over different frequencies into a diversified portfolio. Here the main message is the diversification benefit of the strategies one can benefit from, due to their low mutual correlation, and the fact that the netting of positions when aggregating strategies can lead to a reduction of the impact of costs. Similar arguments could be introduced when considering the diversification benefit brought by a mean reversion strategy to a portfolio of more mainstream systematic strategies, within a given asset class, like momentum or Carry in FX, or of long-only positions on traditional asset classes, like Equity and Bonds, within a multi-asset portfolio.

In the Appendix, we have reviewed a class of statistical models that are commonly employed for describing mean-reverting properties of financial variables, by linking, for a given asset, the mean-reversion properties and the shape of the statistical volatility curve in different models.

We plan to release a future piece where we extend the scope of the analysis. For the FX variables, it would be interesting to assess the impact of the Carry (not discussed here), the fact that transaction costs are time-varying variables and to introduce a dynamic selection of the parameters/strategies when considering a frequency-diversified portfolio. Also, given that the formalism here is very general, it would be interesting to apply that to other asset classes, most notably Equity indices.

Appendix – Mean reversion and statistical models

In this section we review a class of statistical models for describing mean-reverting features of financial variables. We will explain the difference between the mean reversion to a fair value, the cointegration approach and the one pursued here for capturing basket mean reversion over short time frames. As an example, we also describe how different models can lead to different behaviours of the statistical estimation of volatility curves. The main message is that, essentially, mean reversion is a general umbrella term which covers a set of structurally different approaches: the guidance towards the most suitable framework will mainly come from the characteristic time frame one is trying to investigate mean-reverting properties on.

Fair values and mean reversion in levels

The most natural interpretation of mean reversion regards the convergence of an asset price towards a reference fair value, which can be established as a function of global-macro variables, economic cycles, statistical models etc. A dislocation occurs when different drivers lead to a marked deviation of an asset's value from its fair value[4]; while it might be difficult to estimate the exact timing of the correction towards its fair value, fair value models ultimately assume such a correction will take place over a long enough time scale, whereas competing effects (like trends) might be dominant over shorter time periods.

As an example one can consider a multi-linear regression setting for an asset's price time series Y against a set of exogenous factors[5]

$$Y = \sum_i \beta_i X_i + \varepsilon = \bar{Y} + \varepsilon$$

Here for each asset one recovers a reference value $\bar{Y}$ for its price. One could consider other non-linear techniques which allow expressing the fair value of a variable as a function of a set of other variables $\bar{Y} = F(X_1 \cdots X_N)$. The nature of the variables (market data, macro-variables, statistical factors etc.) involved in the regression depends on the nature of the model above: for the FX market, statistical factors, rates differentials, relative prices (PPP-models), inflation (REER-, NEER-models) are commonly used. We also stress that h estimated fair values are in general time-dependent and not an intrinsic property of each asset.

The general idea of mean reversion trading concerns the possibility of buying cheap assets and selling expensive assets, using the fair value as a reference, expressed either via single-assets or baskets trades: the latter case allows for a better management of the directional risk of the trades as one can impose constraints on the betas of a basket. The general issue of such models is the control of drawdowns for the mean-reverting trades: as discussed, the market drivers which play for a deviation from the fair value act on higher frequencies than the forces that regulate the convergence to a fair value. In other words, one must be willing to accept losses over short time periods before the convergence takes place.

For instance, we run a FX fair value model based on statistical factors (FX-PCA). As shown in the table below, one can gather fair values for the different FX variables and for different time

[4] Fair values and dislocations at the cross-asset level were the subject of a recent piece of the X-asset quant team

[5] Strictly speaking, one should test that the obtained residual ε is a stationary variable for the linear regression to be meaningful

horizons: this latter parameter refers to the different time intervals over which one can expect the dislocations to correct. As of mid-May 2015, USD/JPY, AUD/USD and GBP/USD were found to be expensive, whereas EUR/USD and USD/CHF were found to be cheap.

A snapshot (as of 13 May 2015) of statistical fair values in G10 FX

Currency pair		*FX short term (1y)*			*FX long term (10Y)*		
	Market	Model	z-score	r2	Model	z-score	r2
EUR/USD	1.1243	1.1286	-0.51	99%	1.1859	-2.68	96%
USD/JPY	119.83	115.86	2.57	97%	97.25	2.39	69%
GBP/USD	1.5680	1.5418	1.98	97%	1.4597	1.13	66%
USD/CHF	0.9261	0.9198	0.33	69%	1.0217	-3.03	94%
AUD/USD	0.7988	0.7873	1.65	99%	0.7812	0.85	98%
NZD/USD	0.7414	0.7649	-2.12	94%	0.6838	1.26	77%
USD/CAD	1.2005	1.2366	-2.36	96%	1.2462	-1.16	94%
EUR/NOK	8.3778	8.5586	-1.21	98%	8.3525	0.11	92%
EUR/SEK	9.3112	9.3185	-0.07	99%	9.3444	-0.09	87%

Source: SG Cross Asset Research/Cross Asset Quant

When backtesting the model above, it is challenging to manage a proper control of the drawdown of the trades during periods where the dislocations are widening. This usually calls for a number of trading rules which, while successful for improving the results of a backtest, might introduce less clarity on the drivers of performance for the model. The other issue is that, as discussed, fair values are time dependent quantities, and there are cases where it is the fair value which converges to the market value rather than other way around.

Cointegration and VAR models for the prices

In the language of statistics, time series of asset prices follow a non stationary dynamics. If one considers the following equation for the time evolution of an asset's price

$$X_t = c + \alpha\, X_{t-1} + b\epsilon_t$$

$\Delta X_t = X_t - X_{t-1} = c + (\alpha - 1)X_{t-1} + b\epsilon_t = -\alpha'(X_t - \mu) + b\epsilon_t$ with $\alpha' = -(\alpha - 1)\,;\ \mu = c/\alpha'$

For the time series X_t to be stationary, one needs $\alpha < 1 \rightarrow \alpha' > 0$. Apart from a normalisation factor on the volatility of the noise ϵ_t, the equation above converges in the continuous limit to

$$dX_t = -\alpha'(X_t - \mu)dt + \sigma dW_t$$

which is the well known Ornstein-Uhlenbeck (OU) stochastic process (W_t is a Brownian motion). While $0 < \alpha < 1$ would describe a mean-reverting dynamics for the asset prices, empirical estimation usually find $\alpha = 1$ which is referred to as the unit root case: a unit root corresponds to the case where the variable follows a non stationary dynamics. Actual prices do not mean revert, in fact efficient market hypothesis assumes that the latest price embeds all relevant information available and price dynamics follow a Random walk.

The cointegration approach, however, allows defining linear combination of asset prices which exhibit mean-reverting features: we refer to our previous article on FX cointegration for more details. While one can consider more structured dynamics, considering higher-order lags[6], the

[6] We refer to the book "Analysis of financial time series" by Ruey S. Tsay for a more detailed treatment of unit roots of financial time series and cointegration models.

starting point for the formalism is the VAR(1) model for the prices as in the equation below: if $\boldsymbol{X}_t$ is a vector of assets in the N-dimensional space, at time t, the dynamic equation is:

$$\boldsymbol{X}_t = \boldsymbol{c} + \boldsymbol{A}\,\boldsymbol{X}_{t-1} + \boldsymbol{\epsilon}_t$$

In general, one can find a mean-reverting portfolio (with N-dimensional weights $\boldsymbol{\beta}$) such that

$$\Pi_t = \boldsymbol{\beta}^T \cdot \boldsymbol{X}_t = \sum_i \beta_i X_{t,i} = c' + \alpha\,\Pi_{t-1} + \varepsilon'\,;\; 0 < \alpha < 1$$

As it was seen for the previous univariate case, the portfolio's value Π_t now tends to mean revert around an average value $\mu = \frac{c'}{1-\alpha}$. As the value of the estimated parameter α determines the strength of mean reversion for the basket, we looked at the baskets corresponding to the lowest possible values of α for maximising the mean reversion properties of the baskets[7]. In practice, when re-estimating the VAR model as time progresses, one will obtain different values of the VAR parameters, which will lead to time-varying values for the average value of the portfolios.

In the earlier paper on the topic, we had tested the application of this technique, based on the VAR(1) model for the prices described above, for trading high frequency FX portfolios (cointegration properties can actually be tested over different frequencies, ranging from intra-day to multi-weeks periods): we relied on the assumption that, while exogenous factors could lead to a temporary deviation of a cointegrated basket from its fair value, longer term forces would play for a convergence towards the fair value. By looking at the formulas above, the greater the distance from the fair value, the stronger the mean-reverting force to it; therefore the notion of dislocation is essential for measuring the potential of a given trade in such a framework. In the paper, we discussed a number of basic measures that were necessary to define the optimal entry and exit points for each trade. The notion of z-score, which measures the deviation between market and fair value of an asset expressed in number of standard deviations, was relied upon for estimating dislocations and the potential of a given trade.

As it was discussed in more detail in the previous paper, despite the immediate appeal of the formalism, in practice one can face the problem of a widening of the dislocation of the portfolios from their fair value, which induces large drawdowns in the PnL generated by the trades. Furthermore, as commented, the average values of the portfolios are time dependent quantities which change when re-estimating the model. The two points above necessarily call for the introduction of a set of trading rules for the dynamic management of the positions which, although they might prove effective for stabilising the model's performance, could become dominant over the alpha-generating component. Furthermore, we found that the impact of costs was dominant when running the strategy out of sample.

High frequency mean reversion and VAR models for the returns

In this paper, we have considered a VAR(1) on the returns formalism for trading mean-reverting baskets strategies: this is basically the multi-asset version of a model that was used in a 2013 paper for measuring the statistical properties of a quadratic payoff, trading variance defined over different frequencies time horizons. In this section, we aim to explain the main differences compared to the other models reviewed in the Appendix.

[7] A similar approach was pursued in “Identifying small mean-reverting portfolios” by Alexandre d'Aspremont (2008), where the author maximised the basket mean-reverting properties by working in the continuous-time limit (OU)

We start by considering the single-asset, AR(1) version of the model

$$r_t = c + \alpha\, r_{t-1} + b\epsilon_t\, ;\;\; r_t = \log\left(\frac{X_t}{X_{t-1}}\right) = x_t - x_{t-1};\; x_t = \log(X_t)$$

Therefore one has

$$x_t - x_{t-1} = c + \alpha\,(x_{t-1} - x_{t-2}) + b\epsilon_t$$

$$x_t - (1 + \alpha\,)x_{t-1} + \alpha x_{t-2} = c + b\epsilon_t$$

So, this model is a AR(2) model for the log prices, with a given constraint on the AR(1) and AR(2) parameters, which are here related. The stationarity of the return series is guaranteed provided that $|\alpha| < 1$. We will see later what impact this model has, compared to the AR(1) model in prices, on the statistical shape of the volatility surface.

Contrary to the VAR(1) model for the prices, where the investigation of the multivariate case was structurally modifying the picture compared to the univariate one - allowing for defining linear combinations of assets with stationary properties - here we consider the multivariate case merely as a means for strengthening the mean reversion properties of the selected baskets compared to the single asset case. When considering the extension to the N-asset case, the conclusions above remain valid, for the i-th eigenstate with weights $\boldsymbol{W}^i$, after replacing $\alpha \rightarrow \lambda_i = D_{i,i}$ ($\boldsymbol{D}$ being the matrix of the eigenvalues of the autoregressive matrix $\boldsymbol{A}$).

In the approach that we have pursued here, we bypass the issue of the stationarity of the price time series, as we focus directly on the returns of the underlying variables, which follow a stationary dynamics. In other words, there is no notion of a reference value that the traded basket should converge to: the model just aims at successfully forecasting the next future return rather than highlighting stationary price time series. In fact, while the earlier models relied on the notion of “dislocation”, namely the temporary abnormal behaviour of an asset which moves away from a reference value, and proposed a set of trading rules that allowed benefiting from the expected correction of that dislocation, here we pursue the opposite approach: we try to define baskets which, in a normal market environment, exhibit large swings and try to forecast such oscillations.

In a way, this latter approach is less ambitious compared to the previous ones, where it was possible to introduce a target level on the price of a traded basket, but at the same time, presents several advantages which we try to explain here. The first advantage is related to the tactical nature of the approach: given that each new signal depends on the latest price action and will correspond to a single future trade, there is no need to introduce rules for defining entry and exit points for the trades. Secondly, the model is parsimonious in terms of parameters: there is a parameter related to the estimation length of the VAR model, but, contrary to the level case, there will be much less of a need of introducing trading rules, and corresponding parameters for the management of the drawdowns. The other advantage is that the model can easily investigate mean reversion properties over different frequencies: as a pure quant model, it’s no surprise that the best results are detected over relatively high frequencies, which makes the model well decorrelated with more traditional systems whose natural turnovers take place over longer time periods.

Amongst the disadvantages, the high frequency nature of the model implies a large impact of trading costs, which requires a dedicated management: however, we have come up with a number of original solutions for portfolio construction in the presence of costs that could be applied to other contexts as well. As mentioned, the very tactical nature of the model prevents the possibility of buying "cheap" assets, over medium to low frequencies, which is a feature that investors might want to find in a typical mean reversion model.

Shape of the volatility curve in autoregressive models

In this section we compare the AR(1) models for the prices and the returns as far as their implications for the shape of the volatility curves in the two models.

We have already seen that when considering the discrete AR(1) model in prices (X is a price time series)

$$X_t = c + \alpha\, X_{t-1} + b\epsilon_t$$

$\Delta X_t = X_t - X_{t-1} = c + (\alpha - 1)X_{t-1} + b\epsilon_t = -\alpha'(X_t - \mu) + b\epsilon_t$ with $\alpha' = -(\alpha - 1)\, ;\ \mu = c/\alpha'$

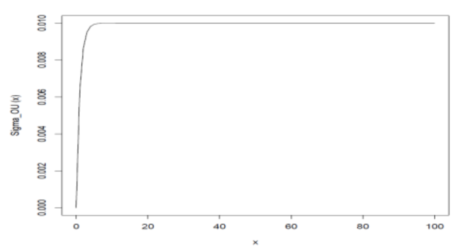
and that, for the time series X_t to be stationary, one needs $\alpha < 1 \rightarrow \alpha' > 0$. While the latter condition is not satisfied by most single-asset price time series (i.e., asset prices are typically non stationary), suitable linear combination of several asset prices can satisfy such a condition if they exhibit cointegrated properties and, in that case, the series X_t would correspond to the price of a basket of assets. We have already seen that this process converges to the Ornstein-Uhlenbeck (OU) stochastic process in the limit of continuous times, so we can use the formulas for the OU process as a proxy of the future expected variance of the process X (future distribution of the process X at time T).

In fact, X_T at maturity T is a normal variable, distributed around a mean value function of the initial value (which will converge to μ for large time T) and with a variance

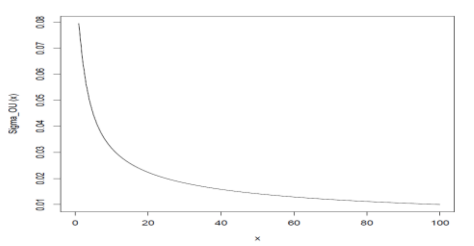
$$Var[X_T|X_0 = x_0] = \frac{\sigma^2}{2\alpha'}(1 - \exp(-2\alpha' T))$$

In the limit of large T, the variance of the process converges to a fixed value. Given the boundaries on $\alpha' > 0$ (otherwise the model would explode), the variance above as a function of the time to maturity T has an upward sloping shape for any positive α'.

Variance function in the AR(1) model for the prices



Volatility function in the AR(1) model for the prices



Source: SG Cross Asset Research/Cross Asset Quant

For log-normal processes, it is natural to introduce the volatility function as

$Vol^2{}_T T = Var[X_T|X_0 = x_0]$

which basically means that, for a fixed volatility curve, the variance of the distribution of the underlying increases linearly with time T. From the equation above, one can recover the corresponding volatility function (second chart above); this is a downward sloping function regardless of the choice of the parameter α'. To conclude, the OU process is the continuous limit of the AR(1) model in prices, and the two models lead to monotonic behaviour of the volatility function for any choice of the mean-reversion parameter in its accepted range of validity. In fact, the OU model accepts only mean reversion towards a given level for the underlying, so it does not have the flexibility of describing trend-following behaviours, which is a feature that several trading models are based on. This might suggest that the AR(1) formalism for the prices might not have enough flexibility for describing different market dynamics, reflected for instance by the different shapes of the volatility curves in different trading regimes.

We then consider the AR(1) model for the return time series, which was introduced in our 2013 paper describing the long/short payoffs trading variance over different frequencies:

$r_t = c + \alpha\, r_{t-1} + b\epsilon_t$ where $r_t = \log\left(\frac{X_t}{X_{t-1}}\right) = x_t - x_{t-1}$ where $x_t = \log(X_t)$

We have already seen that this model is a AR(2) model for the log prices, with a given constraint on the AR(1) and AR(2) parameters which are related. In this model, in order to have a stationary time series, the constraint is $-1 < \alpha < 1$; positive (negative) values of the parameter α tend to reflect trend-following (mean-reverting) dynamics for the asset over the considered time frame $t \to t+1$.

It is possible to investigate how the value of the AR(1) parameter can impact the shape of the volatility curve from a statistical point of view: we will see that, this time, the sign of α has the effect of driving the shape of the vol curve, as it was shown in the 2013 variance payoff paper.

Contrary to the case of the AR(1) model in prices, here the continuous-time limit of the model does not converge to the OU process, for which a compact solution could be obtained, but rather to a stochastic process[8] whose solution could only be found via Monte-Carlo simulations. For this reason, we investigate the properties of the volatility curve in the discrete-time, rather than continuous-time, setting, as such functions can be computed analytically.

We refer again to our long/short variance payoffs paper where the properties of the vol curves were investigated in a number of different discrete time series models. In the case of the AR(1) models for the returns, one can find analytical formulas for the (mean removed) conditional variances V_N^1 as a function of the time horizon N. By introducing a volatility term structure σ_N^1 one gets:

$$\mathbf{V_N^1 = N(\sigma_N^1)^2;\ \sigma_N^1 = \frac{\sigma}{(1-\alpha)}\sqrt{1 + \frac{\alpha^2}{N}\frac{1-\alpha^{2N}}{1-\alpha^2} - \frac{2\alpha}{N}\frac{1-\alpha^N}{1-\alpha}}}$$

$$\mathbf{V_1^1 = \sigma^2;\ V_N^1 = N\,\frac{\sigma^2}{(1-\alpha)^2},\ \sigma_N^1 = \frac{\sigma}{(1-\alpha)}\ for\ N \to \infty}$$

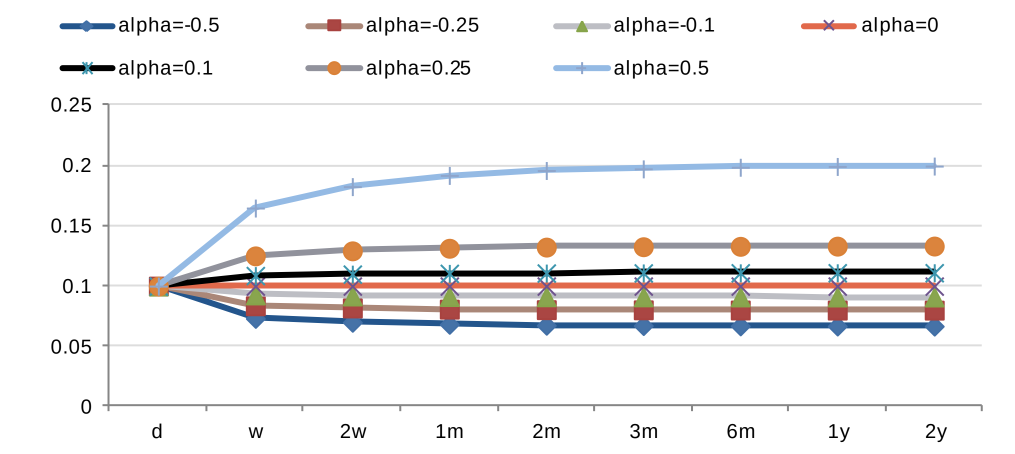
[8] See for instance “Continuous-Time Linear Models” by John H. Cochrane, 2012

One can study the shape of the volatility curve as a function of the time horizon N. One can distinguish three main scenarios:

- $\alpha = 0$. In this case returns are assumed to be a homoscedastic white noise series (with constant volatility) and the statistical term structure of volatility is flat and equal to the unconditional value of volatility
- $\alpha > 0$. In this case returns are assumed to follow a trend-following dynamics. This impacts the volatility term structure in that it becomes an upward sloping function which converges to the unconditional limit for large N.
- $\alpha < 0$. In this case returns are assumed to follow a mean-reverting dynamics. This impacts the volatility term structure in that it becomes a downward sloping function which converges to the unconditional limit for large N.

The following chart displays the shape of the volatility curve for different choices of the autoregressive parameter α.

Volatility term structure as a function of the autoregressive parameter



Source: SG Cross Asset Research/Cross Asset Quant

Therefore, we can see that the parameter of the AR(1) model in returns has a direct impact on the shape of the volatility curve and that mean-reverting behaviours tend to imply downward-sloping vol curves. Different market environments are characterised by different shapes of the vol curves, so this added flexibility introduced by the AR(1) in the returns might prove advantageous if compared to the AR(1) model in prices. In the FX market, vol curves tend to be in upward sloping (contango) mode during quiet markets and in downward sloping (backwardation) mode during stressed markets: based on the results of this section, the empirical analysis would favour the hypothesis that market participants price in higher expectations of mean-reverting properties during stressed, high-volatility markets. As we have discussed, possible extensions of the VAR(1) model, by taking into account extra lags as in a VAR(p) model, might allow identifying cointegrated baskets and, at the same time, offer more flexibility in describing empirical observation concerning financial variables.

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