Analysis 1 - Midterm

MAS241 - Spring 2019

Problem 1. *Suppose that A and B are nonempty subsets of $\mathbb{R}$. Prove or disprove the following:*

1. *(10 points) If $x \in A$ and x is an upper bound of A, then $x = \sup A$.*
2. *(10 points) If $a < b$ for all $a \in A, b \in B$, then $\sup A < \sup B$.*

Problem 2. *(30 points) Let S and T be nonempty subsets of $\mathbb{R}$. Define the Cartesian product of S and T by*

$$S \times T = \{(a, b) \in \mathbb{R}^2 : a \in S, b \in T\}.$$

Prove that $S \times T$ is compact if and only if S and T are compact.

Problem 3. *Given a series of real numbers $\sum a_n$, define the k-th partial sum s_k by $s_k = a_1 + a_2 + \cdots + a_k$. Let σ_m be the average of the first m partial sums,*

$$\sigma_m = \frac{s_1 + s_2 + \cdots + s_m}{m}.$$

If σ_m converges to a limit as $m \to \infty$, we say that the series $\sum a_n$ is Cesàro summable.

1. *(15 points) Find an example of a Cesàro summable series that does not converge.*
2. *(15 points) Prove that any convergent series of real numbers is Cesàro summable.*

Problem 4. *Let f be a real uniformly continuous function on a bounded set E in $\mathbb{R}$.*

1. *(10 points) Prove that f is bounded on E.*
2. *(10 points) Show that the conclusion is false if boundedness of E is omitted from the hypothesis.*