Homework 3.

(Due Apr. 9)

Problem 11 (Exercise 3.20)**.** Suppose X is a complete metric space, and $\{G_n\}$ is a sequence of dense open subsets of X. Prove Baire's theorem, namely, that $\cap_{n=1}^{\infty} G_n$ is not empty.

Problem 12 (Exercise 3.23)**.** Suppose $\{p_n\}$ and $\{q_n\}$ are Cauchy sequences in a metric space X. Show that the sequence $\{d(p_n, q_n)\}$ converges.

Problem 13 (Exercise 4.3)**.** If f is a continuous mapping of a metric space X into a metric space Y, prove that

$$f(\overline{E}) \subset \overline{f(E)}$$

for every set $E \subset X$. Show, by an example, that $f(\overline{E})$ can be a proper subset of $\overline{f(E)}$.

Problem 14 (Exercise 4.4)**.** Let f and g be continuous mappings of a metric space X into a metric space Y, and let E be a dense subset of X. Prove that $f(E)$ is dense in $f(X)$. If $g(p) = f(p)$ for all $p \in E$, prove that $g(p) = f(p)$ for all $p \in X$.

Problem 15 (Exercise 4.9)**.** Show that the requirement in the definition of uniform continuity can be rephrased as follows, in terms of diameters of sets: To every $\epsilon > 0$ there exists a $\delta > 0$ such that $\operatorname{diam} f(E) < \epsilon$ for all $E \subset X$ with $\operatorname{diam} E < \delta$.