Homework 2.

(Due Mar. 28)

Problem 6 (Exercise 2.14). Give an example of an open cover of the segment $(0,1)$ which has no finite subcover.

Problem 7 (Exercise 2.18). Is there a nonempty perfect set in $\mathbb{R}^1$ which contains no rational number?

Problem 8 (Exercise 2.20). Are closures and interiors of connected sets always connected?

Problem 9 (Exercise 3.3). If $s_1 = \sqrt{2}$, and

$$s_{n+1} = \sqrt{2 + \sqrt{s_n}} \qquad (n = 1, 2, 3, \dots),$$

prove that $\{s_n\}$ converges, and that $s_n < 2$ for $n = 1, 2, 3, \dots$.

Problem 10 (Exercise 3.7). Prove that the convergence of $\sum a_n$ implies the convergence of

$$\sum \frac{\sqrt{a_n}}{n}$$

if $a_n \geq 0$.