MAS241 - Analysis I

Quiz 4 - May 23, 2019

Student ID: Name:

Correct answer - 5 points
No answer - 2 points
Wring answer - 0 points

In the questions 1-5, f is a real-valued function defined on $[a, b]$ and α is monotonically increasing on $[a, b] \subset \mathbb{R}$. In the questions 6-10, $\{f_n\}$ is a sequence of real-valued functions defined on $[0, 1]$.

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1.	In the Riemann integral, the upper integral and the lower integral are defined for every bounded function f. (True.)	☐	☐
2.	If f is bounded, then $f \in \mathscr{R}(\alpha)$. (False.)	☐	☐
3.	If f is monotonic, then $f \in \mathscr{R}(\alpha)$. (False.)	☐	☐
4.	If f is bounded, $a < s < b$, and $\alpha(x) = I(x - s)$ where I the unit step function, then $\int_a^b f\, d\alpha = f(x)$. (False. The integral is not defined if f is not continuous at s.)	☐	☐
5.	The function F defined by $F(x) = \int_a^x f(t)dt$ is differentiable if f is Riemann integrable. (False. It is not differentiable if f is not continuous.)	☐	☐
6.	Suppose $\lvert f_n(x)\rvert \leq M_n$ for all $x \in [0, 1]$. Then, $\sum f_n$ converges uniformly if and only if $\sum M_n$ converges. (False. It is not a sufficient condition.)	☐	☐
7.	If $f_n \to f$ uniformly, then f is continuous on $[0, 1]$ (False.)	☐	☐
8.	If a sequence of continuous functions converge to a continuous function, then the convergence is uniform. (False.)	☐	☐
9.	The space $C([0, 1])$ is a complete metric space with the supremum norm $\lVert \cdot \rVert$. (True.)	☐	☐
10.	If $\{f_n\}$ is differentiable and $\{f_n'\}$ converges uniformly on $[0, 1]$, then $\{f_n\}$ converges uniformly on $[0, 1]$. (False. Consider $f_n \equiv n$.)	☐	☐