

# MAS241 - Analysis I

Quiz 3 - May 7, 2019

Student ID:

Name:

Correct answer - 5 points

No answer - 2 points

Wrong answer - 0 points

In the following questions, every function is a real-valued function defined on a subset of  $\mathbb{R}$ .

- |  | T                        | F                        |
|--|--------------------------|--------------------------|
| 1. If $\lim_{x \rightarrow p} f(x) = q$ , then $\lim_{n \rightarrow \infty} f(p_n) = q$ for every sequence $\{p_n\}$ such that $\lim_{n \rightarrow \infty} p_n = p$ .<br>(False. Consider the sequence $p_n = p$ .) | <input type="checkbox"/> | <input type="checkbox"/> |
| 2. A function $f$ is continuous if and only if $f^{-1}(E)$ is closed for every closed set $E \subset \mathbb{R}$ .<br>(True. See Corollary of Theorem 4.8.)  | <input type="checkbox"/> | <input type="checkbox"/> |
| 3. Any uniformly continuous function is bounded.<br>(False. Consider $f(x) = x$ on $\mathbb{R}$ .)   | <input type="checkbox"/> | <input type="checkbox"/> |
| 4. If $f$ is continuous and $E$ is a connected subset of $\mathbb{R}$ , then $f(E)$ is connected.<br>(True. See Theorem 4.22.)   | <input type="checkbox"/> | <input type="checkbox"/> |
| 5. There exists a monotonically increasing function on $(a, b)$ that is discontinuous at infinitely many points.<br>(True. See Remark 4.31.)   | <input type="checkbox"/> | <input type="checkbox"/> |
| 6. If $f$ is monotonically increasing and differentiable in $(a, b)$ , then $f'(x) > 0$ for all $x \in (a, b)$ .<br>(False. Consider $f(x) = x^3$ on $(-1, 1)$ .)  | <input type="checkbox"/> | <input type="checkbox"/> |
| 7. If $f$ has a local maximum at $x$ , then $f'(x) = 0$ .<br>(False. The derivative may not exist at the local maximum.)   | <input type="checkbox"/> | <input type="checkbox"/> |
| 8. If $ f(x) - f(y)  \leq (x - y)^2$ for any $x, y \in \mathbb{R}$ , then $f$ is constant.<br>(True. It can be shown that $f'(x) = 0$ for any $x \in \mathbb{R}$ .)  | <input type="checkbox"/> | <input type="checkbox"/> |
| 9. If $f$ is differentiable on $[a, b]$ , then $f'$ cannot have discontinuities of the first kind on $(a, b)$ .<br>(True. See Corollary of Theorem 5.12.)  | <input type="checkbox"/> | <input type="checkbox"/> |
| 10. If $f$ is defined on $(a, b)$ and $f^{(n)}(x)$ exists for a point $x \in (a, b)$ , then $f^{(n-1)}(t)$ exists in some neighborhood of $x$ .<br>(True. See the remark below Definition 5.14.)                     | <input type="checkbox"/> | <input type="checkbox"/> |