MAS241 - Analysis I

Quiz 2 - April 4, 2019

Student ID: Name:

Correct answer - 5 points
No answer - 2 points
Wrong answer - 0 points

In the following questions, every sequence is a sequence of real numbers.

		T	F
1.	If $\{p_n\}$ converges to p and p', then $p' = p$. (True. See Theorem 3.2.)	☐	☐
2.	There exists a convergent sequence whose subsequences have different limits. (False.)	☐	☐
3.	Every bounded sequence contains a convergent subsequence. (True. See Theorem 3.6.)	☐	☐
4.	If E_n is a sequence of sets such that $E_n \supset E_{n+1}$ and if $\lim_{n\to\infty} \operatorname{diam} E_n = 0$, then $\bigcap_{n=1}^{\infty} E_n = \emptyset$. (False. See Theorem 3.10.)	☐	☐
5.	Every Cauchy sequence converges. (True. See Theorem 3.11.)	☐	☐
6.	If $a_n \geq b_n$ for all $n = 1, 2, \ldots$ and if $\sum b_n$ diverges, then $\sum a_n$ diverges. (False. Consider $a_n = 0$ and $b_n = -1$.)	☐	☐
7.	If $\sum a_n$ converges and $a_n \geq 0$, then $\sum \frac{\sqrt{a_n}}{n}$ converges. (True. Check $\sqrt{a_n}/n \leq a_n + n^{-2}$.)	☐	☐
8.	There exists a convergent series $\sum a_n$ such that $\limsup_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$. (True. See Example 3.35.)	☐	☐
9.	The radius of convergence of the power series $\sum \frac{z^n}{\sqrt{n}}$ is 1. (True.)	☐	☐
10.	If $\lvert a_1 \rvert \geq \lvert a_2 \rvert \geq \cdots$, and if $a_{2k-1} \geq 0$ and $a_{2k} \leq 0$ for $k = 1, 2, \ldots$, then $\sum a_n$ converges. (False. Consider $a_n = (-1)^{n+1}$.)	☐	☐