Homework 3.1

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3.1-1 the key is to prove that $c_1(f(n)+g(n)) \leq max(f(n), g(n)) \leq c_2(f(n)+g(n))$ for all $n > n_0$

first step: if $max(f(n), g(n)) = f(n)$, then $g(n) \leq f(n)$

let $c_1 = 1/2$, then $1/2f(n) + 1/2g(n) \leq f(n)$

next: let $c_2 = 1$, then $f(n) \leq f(n) + g(n)$

finish

3.1-2 the key is to prove that $c_1(n^b) \leq (n+a)^b \leq c_2(n^b)$ for all $n > n_0$

note: processing is to prove the inequality is correct when $n > n_0$ not for any real integer n

$$\begin{aligned} n + a &\geq n - |a| \\ &\geq 1/2n, \quad when \quad |a| \leq 1/2n \end{aligned}$$

and

$$\begin{aligned} n + a &\leq n + |a| \\ &\leq 3/2n, \quad when \quad |a| \leq 1/2n \end{aligned}$$

so,

$1/2n < n + a < 3/2n, \quad when \quad |a| \leq 1/2n$

because $b > 0$,

$(1/2)^b n^b < (n+a)^b < (3/2)^b n^b$

let $c_1 = 1/2, c_2 = 3/2$, finish

3.1-3 The best running time not always occurs, and we want to know the average running time and the worst time, these are more important.

3.1-4 $2^{n+1} = 2 * 2^n = O(2^n)$

$2^{2n} = 4^n = O(4^n)$

3.1-5 use the definition

3.1-6 below

3.1-7 $o(g(n)) \bigcup \omega(g(n))$