Abstract Linear Algebra What are scalars? What are vectors? What is a linear combination? And what is linear independence and dependence? 3-1R is the canonical example!

Definition 2-1

A field consists of a set, denoted by \mathcal{F} , of elements called *scalars* and two operations called addition "+" and multiplication ":"; the two operations are defined over \mathcal{F} such that they satisfy the following conditions:

- 1. To every pair of elements α and β in \mathcal{F} , there correspond an element $\alpha + \beta$ in \mathcal{F} called the sum of α and β , and an element $\alpha \cdot \beta$ or $\alpha\beta$ in \mathcal{F} , called the product of α and β .
- **2.** Addition and multiplication are respectively commutative: For any α , β in \mathscr{F} ,

$$\alpha + \beta = \beta + \alpha$$
 $\alpha \cdot \beta = \beta \cdot \alpha$

3. Addition and multiplication are respectively associative: For any α , β , γ in \mathcal{F} ,

$$(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma) \qquad (\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$$

4. Multiplication is distributive with respect to addition: For any α , β , γ in \mathcal{F} ,

$$\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$$

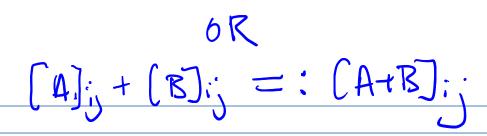
- 5. \mathscr{F} contains an element, denoted by 0, and an element, denoted by 1, such that $\alpha + 0 = \alpha$, $1 \cdot \alpha = \alpha$ for every α in \mathscr{F} .
- 6. To every α in \mathcal{F} , there is an element β in \mathcal{F} such that $\alpha + \beta = 0$. The element β is called the *additive inverse*.
- 7. To every α in \mathscr{F} which is not the element 0, there is an element γ in \mathscr{F} such that $\alpha \cdot \gamma = 1$. The element γ is called the *multiplicative inverse*.

Field you must check all seven axioms (super boring. To show something is not a field, you only need to show that one of the axioms fails #xamples (rational numbers) Von- examples Z ; ntegers (Fails #7)

(Fails #5, #6, #7) c) A = Zx2 red matrix. (Faily #2 and)

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For us, TR and C will be our main fields. We will do some think " Went to define vectors Please have the following example in mind. Let A and B be 2x3 real matrices. $[A+B]_{ij} \triangleq [A_{ij} + [B]_{ij}$ Definition x:=y Means y is already understood, and I am defining www.PrintablePaper.net A+B); = A); + (B);

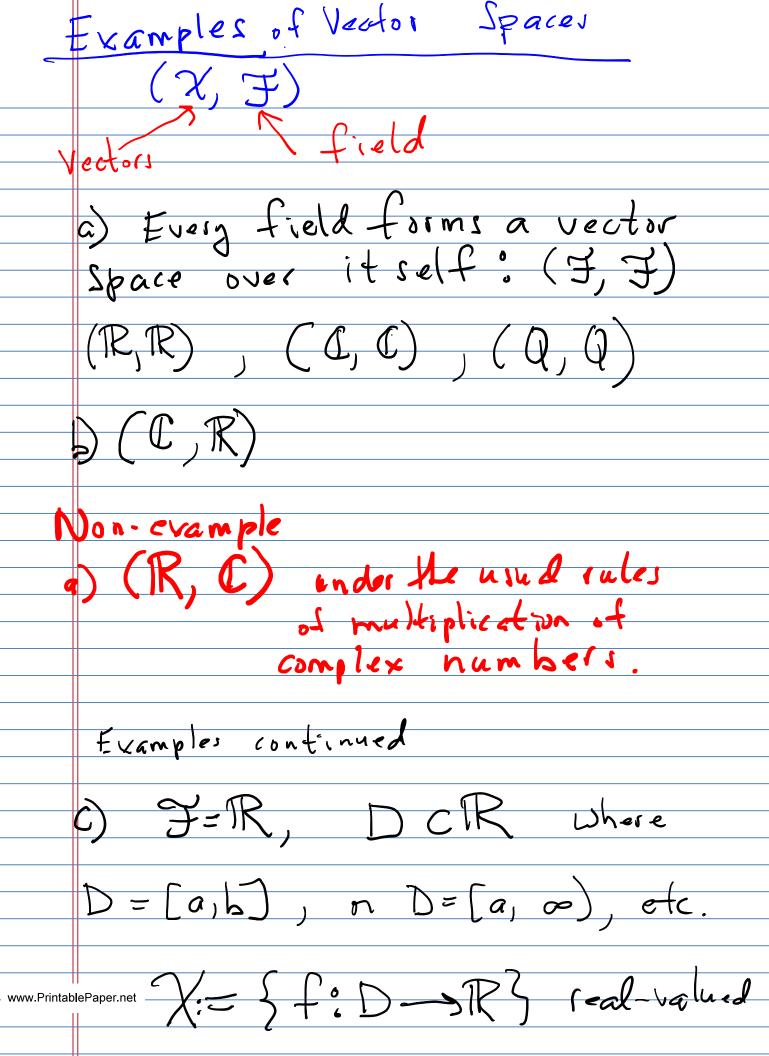


les example to have in mind R?

Definition 2-2

A linear space over a field \mathcal{F} , denoted by $(\mathcal{X}, \mathcal{F})$, consists of a set, denoted by \mathcal{X} , of elements called *vectors*, a field \mathcal{F} , and two operations called *vector addition* and *scalar multiplication*. The two operations are defined over \mathcal{X} and \mathcal{F} such that they satisfy all the following conditions:

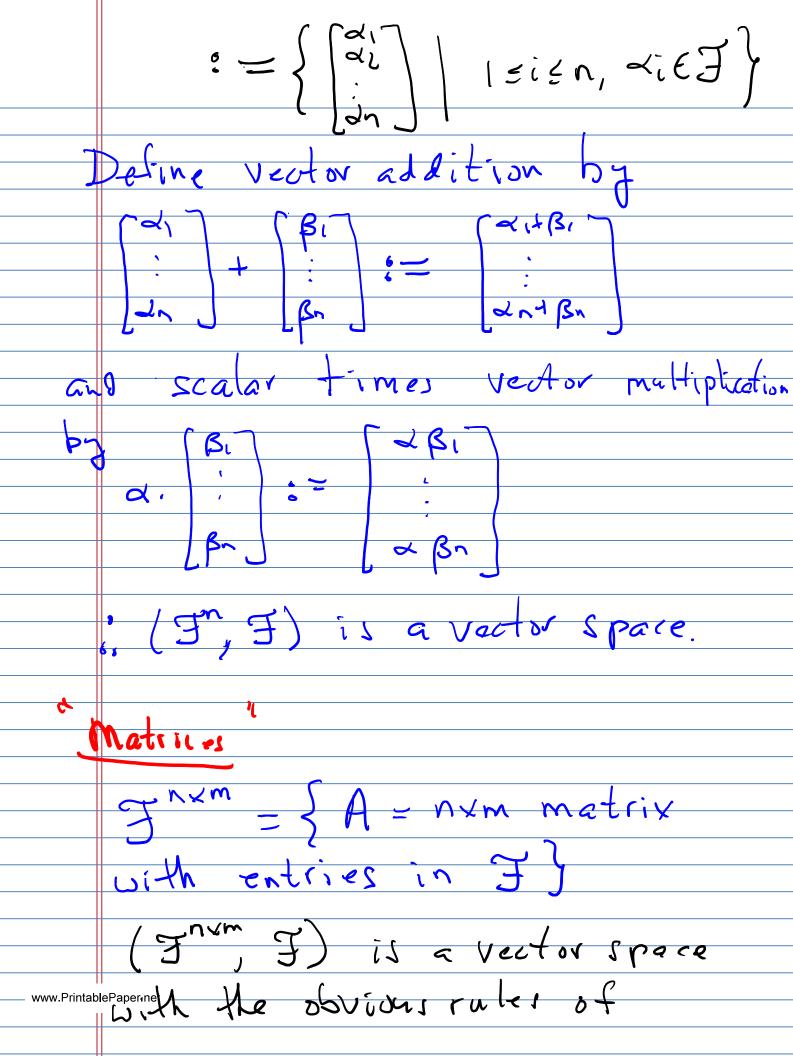
- 1. To every pair of vectors \mathbf{x}_1 and \mathbf{x}_2 in \mathcal{X} , there corresponds a vector $\mathbf{x}_1 + \mathbf{x}_2$ in \mathcal{X} , called the sum of \mathbf{x}_1 and \mathbf{x}_2 .
- 2. Addition is commutative: For any x_1 , x_2 in \mathcal{X} , $x_1 + x_2 = x_2 + x_1$.
- 3. Addition is associative: For any x_1 , x_2 , and x_3 in \mathcal{X} , $(x_1 + x_2) + x_3 = x_1 + (x_2 + x_3)$.
- 4. \mathcal{X} contains a vector, denoted by 0, such that 0 + x = x for every x in \mathcal{X} . The vector 0 is called the zero vector or the origin.
- 5. To every x in \mathcal{X} , there is a vector $\bar{\mathbf{x}}$ in \mathcal{X} , such that $\mathbf{x} + \bar{\mathbf{x}} = \mathbf{0}$.
- 6. To every α in \mathcal{F} , and every \mathbf{x} in \mathcal{X} , there corresponds a vector $\alpha \mathbf{x}$ in \mathcal{X} called the scalar product of α and \mathbf{x} .
- 7. Scalar multiplication is associative: For any α , β in \mathcal{F} and any x in \mathcal{X} , $\alpha(\beta x) = (\alpha \beta)x$.
- 8. Scalar multiplication is distributive with respect to vector addition: For any α in \mathscr{F} and any \mathbf{x}_1 , \mathbf{x}_2 in \mathscr{X} , $\alpha(\mathbf{x}_1 + \mathbf{x}_2) = \alpha \mathbf{x}_1 + \alpha \mathbf{x}_2$.
- 9. Scalar multiplication is distributive with respect to scalar addition: For any α , β in \mathcal{F} and any \mathbf{x} in \mathcal{X} , $(\alpha + \beta)\mathbf{x} = \alpha\mathbf{x} + \beta\mathbf{x}$.
- 10. For any x in \mathcal{X} , 1x = x, where 1 is the element 1 in \mathcal{F} .



functions from D to R. HteD, fateRes f: D-IR3 Define Vector addition and scalar times vector multiplication a) Vf,geX define f+g by y + eD, (f + g)(t) = f(t) + g(t)b) 4 fex, 42 e F define 2.f by 4teD, (4.f)(t) = 4.f(t)Fact: (X, F) is a vector space. To show this, you clock all 10 of the axioms o (Boring) Woll do 48 as an example www.PrintablePaper.net $\forall d \in \mathcal{F} = \mathbb{R}, \forall f, g \in \mathcal{X} \quad \forall (f+g) = \forall f+g$

mothod: Show LHS=RHS (lefthand side = right hand side) Let tED [< (f+g)] (t) := < [f+g](t) = x. f(+)+ g(+) = ~fet) + ~g(t) LHS [xf+2g](t):= [xf)(t)+[2g](t) = ~ fal + ~ glt) RUJ RHS = LUS More Vector Spares Let I be a field. Define In: set of n-tuples of elements of F

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