15 Nov. 2018 (Part 2)

Real Analysis

Let $(X, \mathbb{R}, \|\cdot\|)$ be a normed space. Because we will always assume $\mathcal{F} = \mathbb{R}$, we will simply write $(X, \|\cdot\|)$.

<u>Recall:</u> (a), $\forall x, y \in X$, $d(x,y) := \|x-y\|$

(b) $x \in X$ and $S \subset X$, (S subset)

$$d(x, S) := \inf_{y \in S} d(x,y) := \inf_{y \in S} \|x-y\|$$

$\square$

* <u>Remark:</u> $d(x, S) = 0 \iff \forall \varepsilon > 0, \exists y \in S, \|x-y\| < \varepsilon.$

$d(x, S) > 0 \iff \exists \varepsilon > 0, \forall y \in S, \|x-y\| \geq \varepsilon$

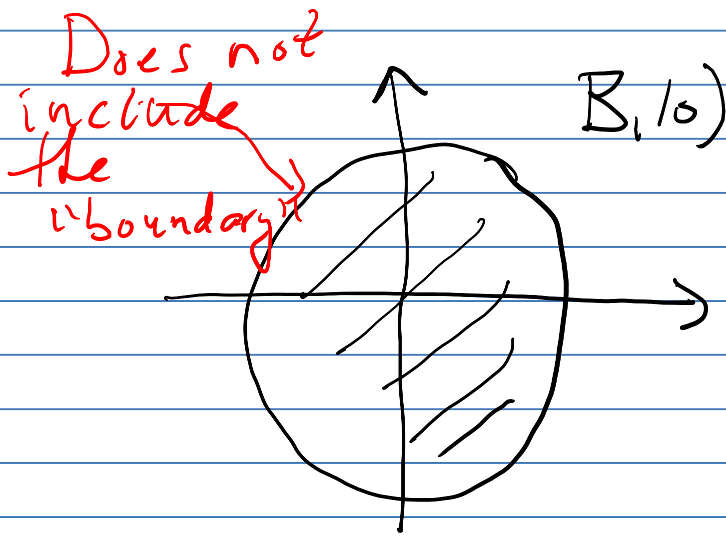
Open Sets and Closed Sets

Def. Let $x_0 \in X$, and let $a \in \mathbb{R}$, $a > 0$. Then the open ball of radius a about x_0 is

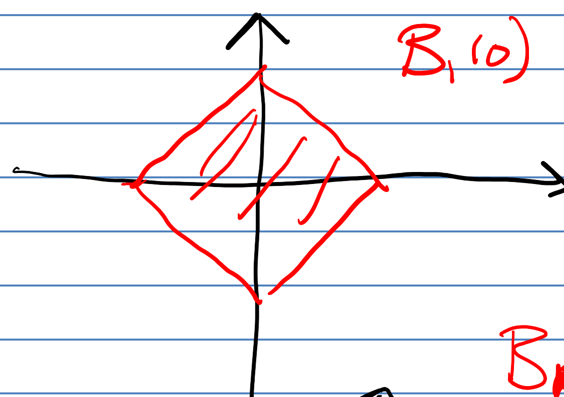
$$B_a(x_0) = \{ x \in X \mid \|x - x_0\| < a \}$$

Examples

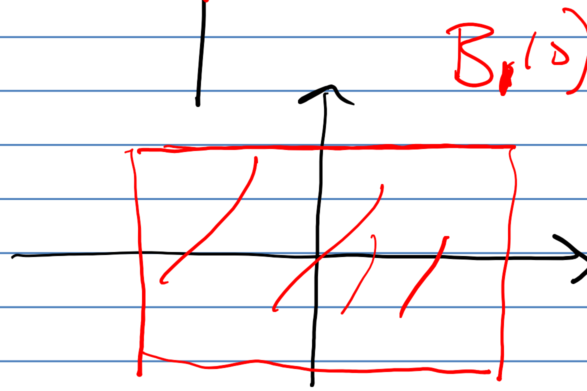
$(\mathbb{R}^2, \|\cdot\|_2)$



$(\mathbb{R}^2, \|\cdot\|_1)$



$(\mathbb{R}^2, \|\cdot\|_\infty)$

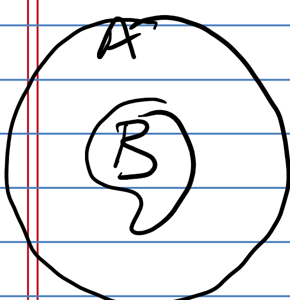


$$B \subset A \iff B \cap (\sim A) = \emptyset$$
$$B \subset \sim A \iff B \cap A = \emptyset$$

Remark $x \in X$, $S \subset X$ a subset.

a) $d(x,S) = 0 \iff \forall \varepsilon > 0,\ \exists y \in S,\ \|x-y\| < \varepsilon$
$\iff \forall \varepsilon > 0,\ B_\varepsilon(x) \cap S \neq \emptyset$

b) $d(x,S) > 0 \iff \exists \varepsilon > 0,\ B_\varepsilon(x) \cap S = \emptyset$
$\iff \exists \varepsilon > 0,\ B_\varepsilon(x) \subset \sim S$



$\sim A$ $\qquad B \subset A \iff B \cap \sim A = \emptyset$

$d(x,S) > 0$