ROB530 - HW5

50036744
Kuan-Ting Lee

1.

Motion model

$$
f: \begin{cases}
x_{k+1} = x_k - \dfrac{\hat{v}}{\hat{\omega}} \sin(\theta_k) + \dfrac{\hat{v}}{\hat{\omega}} \sin(\theta_k + \hat{\omega} \Delta t) \\
y_{k+1} = y_k + \dfrac{\hat{v}}{\hat{\omega}} \cos(\theta_k) - \dfrac{\hat{v}}{\hat{\omega}} \cos(\theta_k + \hat{\omega} \Delta t) \\
\theta_{k+1} = \theta_k + \hat{\omega} \Delta t + \hat{\gamma} \Delta t
\end{cases}
$$

$$
\begin{cases}
\hat{v} = v + \varepsilon_v, \quad \varepsilon_v \sim N(0, \alpha_1 v^2 + \alpha_2 \omega^2) \\
\hat{\omega} = \omega + \varepsilon_\omega, \quad \varepsilon_\omega \sim N(0, \alpha_3 v^2 + \alpha_4 \omega^2) \\
\hat{\gamma} = \varepsilon_\gamma, \quad \varepsilon_\gamma \sim N(0, \alpha_5 v^2 + \alpha_6 \omega^2)
\end{cases}
$$

Sensor model

$$
h: \quad z_k = \begin{bmatrix} \operatorname{atan2}(m_y - y_k, m_x - x_k) - \theta_k \\ \sqrt{(m_y - y_k)^2 + (m_x - x_k)^2} \end{bmatrix} + q_k, \quad q_k \sim N(0, Q_k)
$$

1. (A) [EKF]

prediction $\begin{cases} \mu_k^- = f(u_k, \mu_{k-1}) \\ \Sigma_k^- = F_k \Sigma_{k-1} F_k^T + W_k Q W_k^T \end{cases}$, where

$$F_k = \frac{\partial f}{\partial x}\bigg|_{x=\mu_{k-1}} = \begin{bmatrix} 1 & 0 & -\frac{v}{\omega}\cos\theta + \frac{v}{\omega}\cos(\theta + \hat{\omega}\Delta t) \\ 0 & 1 & -\frac{v}{\omega}\sin\theta + \frac{v}{\omega}\sin(\theta + \hat{\omega}\Delta t) \\ 0 & 0 & 1 \end{bmatrix}$$

$$W_k = \frac{\partial f}{\partial u}\bigg|_{x=\mu_{k-1}} = \begin{bmatrix} -\frac{1}{\omega}\sin\theta + \frac{1}{\omega}\sin(\theta+\omega\Delta t) & \frac{v}{\omega^2}\sin\theta - \frac{v}{\omega^2}\sin(\theta+\omega\Delta t) & 0 \\ \frac{1}{\omega}\cos\theta - \frac{1}{\omega}\cos(\theta+\omega\Delta t) & -\frac{v}{\omega^2}\cos\theta + \frac{v}{\omega^2}\cos(\theta+\omega\Delta t) & 0 \\ 0 & \Delta t & \Delta t \end{bmatrix}$$

$$V_k = z_k - h(\mu_k^-)$$

$$S_k = H_k \Sigma_k^- H_k^T + V_k R_k V_k^T$$

$$H_k = \frac{\partial h}{\partial x}\bigg|_{x=\mu_{k-1}} = \begin{bmatrix} (m_y - y)/z_2{}^2 & -(m_x - x)/z_2{}^2 & -1 \\ -(m_x - x)/z_2{}^2 & -(m_y - y)/z_2{}^2 & 0 \end{bmatrix}$$

, where $z_k = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$$K_k = \Sigma_k^- H_k^T S_k^{-1}$$

$$\mu_k = \mu_k^- + K_k V_k$$

$$\Sigma_k = (I - K_k H_k)\Sigma_k^-$$

1. (B) [UKF]

$\mathcal{X}_{k-1} \leftarrow$ Compute the set of $2n+1$ sigma points using μ_{k-1}, Σ_{k-1}

$w^- \leftarrow$ Compute the set of $2n+1$ weights

$$\mu_k^- = \sum_{i=0}^{2n} w_i^- f(u_k, \mathcal{X}_{k-1,i})$$

$$\Sigma_k^- = \sum_{i=0}^{2n} w_i^- \left(f(u_k, \mathcal{X}_{k-1,i}) - \mu_k^-\right)\left(f(u_k, \mathcal{X}_{k-1,i}) - \mu_k^-\right)^T + Q_k$$

How to generate $\mathcal{X}_k$ & W $\longrightarrow$, where $x_i = \begin{cases} \mu & , i=0 \\ \mu + l_i & , i = 1 \dots n \\ \mu - l_{i-n} & , i = n+1 \dots 2n \end{cases}$

$$w_i = \begin{cases} \frac{k}{n+k} & , i=0 \\ \frac{1}{2(n+k)} & , i = 1 \dots 2n \end{cases}$$

l_i is the i-th column of $\sqrt{(n+k)}\,L$ and $\Sigma = LL^T$ can be decomposed using Cholesky decomposition, n is the dimension of the state, k is a user-defined parameter that is set to 2 in our case.

$\mathcal{X}_k^- \leftarrow$ Compute the set of $2n+1$ sigma points using μ_k^- and Σ_k^-

$w \leftarrow$ Compute the set of $2n+1$ weights

$$z_k^- = \sum_{i=0}^{2n} w_i h(\mathcal{X}_{k,i}^-) \quad \text{(predicted measurement)}$$

$$v_k = z_k - z_k^- \quad \text{(innovation)}$$

$$S_k \leftarrow \sum_{i=0}^{2n} w_i \left(h(\mathcal{X}_{k,i}^-) - z_k^-\right)\left(h(\mathcal{X}_{k,i}^-) - z_k^-\right)^T + R_k \quad \text{(innovation cov)}$$

$$\Sigma_k^{xz} \leftarrow \sum_{i=0}^{2n} w_k^{[i]} (x_{k,i}^- - \mu_k^-)(h(x_{k,i}^-) - z_k^-)^T$$ (state and measurement cross)

$$K_k \leftarrow \Sigma_k^{xz} S_k^{-1}$$ (filter gain)

$$\mu_k \leftarrow \mu_k^- + K_k v_k$$ (corrected mean)

$$\Sigma_k \leftarrow \Sigma_k^- - K_k S_k K_k^T$$ (corrected covariance)

return μ_k, Σ_k

1. (c) PF

Input: particles $X_{k-1} = \{x_{k-1}^i, \tilde{w}_{k-1}^i\}_{i=1}^n$

action u_k

measurement z_k

resampling threshold n_t

$X_k \leftarrow \emptyset$

for each $x_{k-1}^i \in X_{k-1}$ do

draw $x_k^i \sim p(x_k | x_{k-1}^i, u_k)$

$w_k^i \leftarrow \tilde{w}_{k-1}^i \, p(z_k | x_k^i)$

$W_{total} \leftarrow \sum_{i=1}^n w_k^i$

$X_k \leftarrow X_k \cup \{x_k^i, w_k^i / w_{total}\}_{i=1}^n$

$n_{eff} \leftarrow 1 / \sum_{i=1}^n (\tilde{w}_k^i)^2$

if $n_{eff} < n_t$ then

$X_k \leftarrow$ resample using X_k

return X_k

Motion model:

Add motion noise to the predicted motion using cholesky decomposition of motion covariance times standard normal distribution.

Sensor model:

Calculate likelihood of sensor measurement by applying multivariate normal pdf on the innovation based on distribution of the sensor noise.

Resampling:

Resample when n_{eff} is less than the threshold.

1. (D) [In EKF]

Prediction

$$\begin{cases} X_{k+1} = X_k \exp(\xi_k^{\wedge} \Delta t) \\ \Sigma_{k+1} = \Sigma_k + Ad_{X_k} W_k Ad_{X_k}^T \end{cases}$$

, where $X_k = \begin{bmatrix} R_k & p_k \\ 0 & 1 \end{bmatrix}$, $\xi_k^{\wedge} = \begin{bmatrix} \omega_k^{\wedge} & v_k \\ 0 & 0 \end{bmatrix}$

, $\xi_k = \begin{pmatrix} v_1 \\ v_2 \\ \omega \end{pmatrix}$, $\xi_k^{\wedge} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v_1 + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} v_2 + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \omega$

, $Ad_{X_k} \cdot \xi_k = X_k \left(\sum_{i=1}^{3} \xi_i G_i \right) X_k^{-1}$

$$= \begin{bmatrix} R \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \omega \begin{pmatrix} t_2 \\ -t_1 \end{pmatrix} \\ \omega \end{bmatrix}$$

$$\Rightarrow Ad_{X_k} = \begin{bmatrix} R & \begin{matrix} t_2 \\ -t_1 \end{matrix} \\ 0 & 1 \end{bmatrix}$$

Correction

$$\begin{cases} \bar{X}_k^{+} = \exp(L_k (\bar{X}_k Y_k - b)) X_k \\ \bar{P}_k^{+} = (I - L_k H) P_k (I - L_k H)^T + L_k \bar{N} L_k^T \end{cases}$$

, where $H\xi = -\xi^{\wedge} b$

$$H\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} = -\begin{bmatrix}0 & -\xi_3 & \xi_1\\ \xi_3 & 0 & \xi_2\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}m_x\\ m_y\\ 1\end{bmatrix}$$

$$= \begin{bmatrix}-\xi_1 + m_y\xi_3\\ -\xi_2 - m_x\xi_3\\ 0\end{bmatrix}$$

$$\Rightarrow H = \begin{bmatrix}-1 & 0 & m_y\\ 0 & -1 & -m_x\\ 0 & 0 & 0\end{bmatrix}$$

Since we have measurement of two landmarks at the same time, I stack H together and remove their last row in order to achieve full rank in later calculations.

$$H_{stack} = \begin{bmatrix}H_1\\ H_2\end{bmatrix} = \begin{bmatrix}-1 & 0 & m_1y\\ 0 & -1 & -m_1x\\ -1 & 0 & m_2y\\ 0 & -1 & -m_2x\end{bmatrix}$$

N is also stacked in my implementation

$$N_{stack} = \begin{bmatrix}N[0:2,0:2] & 0\\ 0 & N[0:2,0:2]\end{bmatrix}$$

$$\bar{N_k} = \bar{X_{t_k}}\, Cov(V_k)\, \bar{X_{t_k}}^T$$

$$S = H P^r_{t_k} H^T + \bar{N_k}$$

$$L_{t_k} = P^r_{t_k} H^T S^{-1}$$

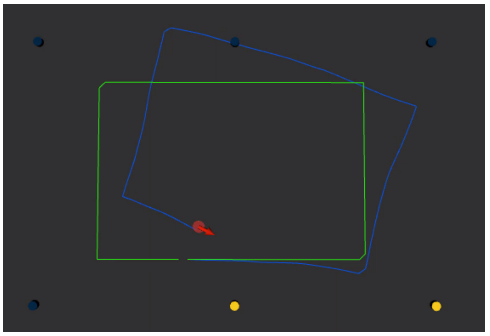
Correction

$$\overline{X_{t_k}}^{+} = \exp\left(L_{t_k}\left(\overline{X_{t_k}}\, Y_{t_k} - b\right)\right) X_{t_k}^{-}$$

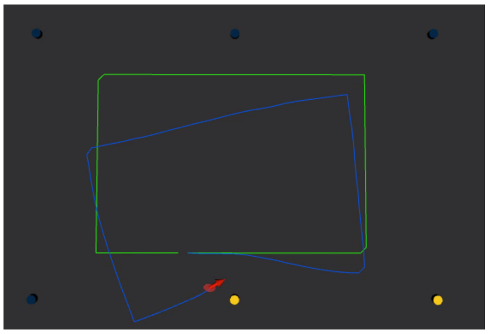
$$\overline{P_{t_k}}^{r+} = (I - L_{t_k}H)\, P^{r}_{t_k}\, (I - L_{t_k}H)^{T} + L_{t_k}\, \overline{N_k}\, L_{t_k}^{T}$$

Task 2

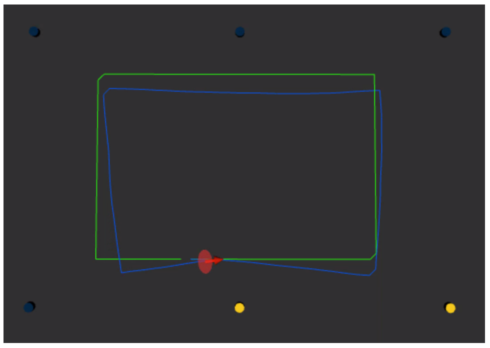
EKF



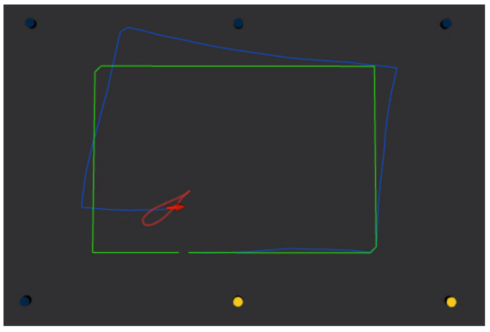
UKF



PF

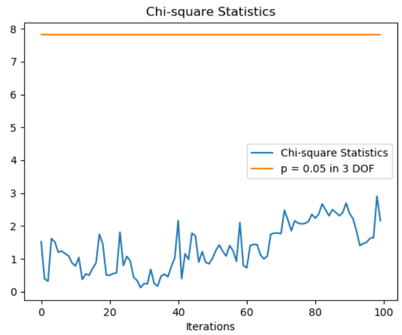


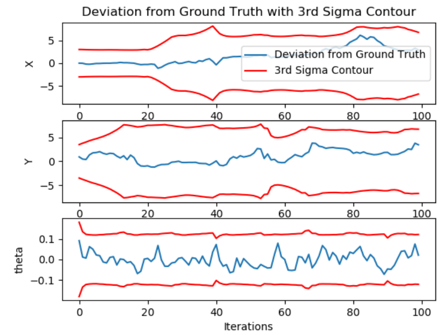
InEKF



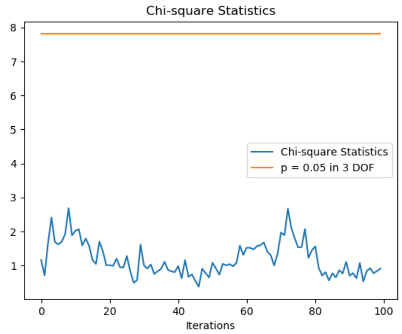
Bonus

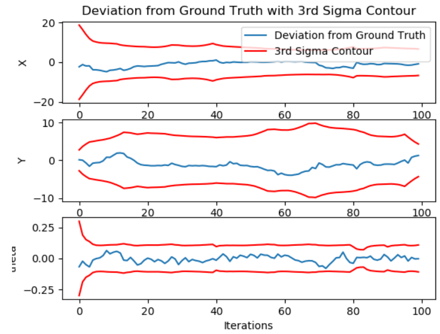
EKF





UKF





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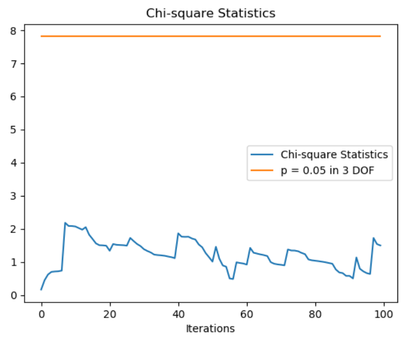

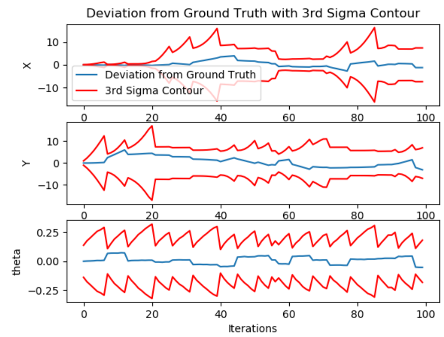

InEKF