NA 568 Mobile Robotics: Methods & Algorithms
Winter 2022 – Homework 4 – Lie Group & InEKF

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This is a reminder that no late HW is accepted. We drop your lowest grade from HW 1-6. It is perfectly fine to drop a zero as a HW grade. We are using Gradescope for turning in HW; see relevant information on the course Canvas site.

This problem set counts for about 7% of your course grade. You are encouraged to talk at the conceptual level with other students, but you must complete all work individually and may not share any non-trivial code or solution steps. See the syllabus for the full collaboration policy.

Submission Instructions

Your assignment must be received by 11:55 pm on Friday, February 18 (Anywhere on Earth Time). This corresponds to 6:55 AM on February 19 in Eastern Time. This is selected out of fairness to all our students, including those who take the course remotely. You are to upload your assignment directly to the Gradescope website as two attachments:

1. A `.tar.gz` or `.zip` file *containing a directory* named after your uniqname with the structure shown below.

   ```
   alincoln_hw4.tgz:
   alincoln_hw4/
   alincoln_hw4/task3.m
   ```

   Or a Jupyter notebook per task using Python or Julia kernels for your programming. Follow the same naming convention for the notebooks .

2. A PDF with the written portion of your write-up. Scanned versions of hand-written documents, converted to PDFs, are perfectly acceptable. No other formats (e.g., .doc) are acceptable. Your PDF file should adhere to the following naming convention: `alincoln_hw4.pdf`.

1 Rigid Body Transformation (15 points)

1.1 Problem (4 points)

What is the main difference between a right and left Invariant EKF? Which types of measurements should be used for left and right Invariant EKFs, respectively?

1.2 Problem (4 points)

Write the Lie algebra of SO(3) and SE(3), both in vector and matrix form. Explain the physical meaning behind the dimension of each Lie algebra.

1.3 Problem (4 points)

Write out the continuous-time constant velocity motion model for an object moving in SO(3) and SE(3), respectively. Assuming a zero-order hold and a sampling time Δt for the input, write an integration rule for each model.

1.4 Problem (3 points)

Show that:

$$\begin{bmatrix} \mathbf{R}^\top & -\mathbf{R}^\top \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$$

is the inverse of a homogeneous rigid body transformation $\begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$.

2 Probability and Uncertainty Propagation (25 points)

Suppose you are using a Right-Invariant EKF to perform robot localization in 3D and the ground truth is given in coordinates using x, y, z for position and roll, pitch, yaw for the orientation with respect to x, y, and z axes, respectively. The Right-Invariant EKF error and covariance are in the Lie algebra of SE(3). Derive a model to map the error and its covariance from $\mathfrak{se}(3)$ to $\mathbb{R}^6$ (x, y, z, roll, pitch, yaw). Show work and provide necessary equations for mapping the error and covariance from the Lie algebra to the Cartesian space.

Hint: Let $\xi \in \mathfrak{se}(3)$ be the RI-EKF error which is distributed as $\mathcal{N}(\xi, \Sigma_\xi)$. The mapping from $\mathfrak{se}(3)$ to $\mathbb{R}^6$ has the following steps:

$$\xi \mapsto \eta \in \mathrm{SE}(3) \mapsto e \in \mathbb{R}^6$$

In particular,

$$\eta = \exp_m(\xi) = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

which then we can extract the Cartesian coordinates as

$$x = t_1, \quad y = t_2, \quad z = t_3,$$

and roll, pitch, yaw angles can be extracted, for example, using `eulZYX = rotm2eul(rotm)` in MATLAB.

Figure 1: VN-100 Rugged Inertial Measurement Unit and Attitude Heading Reference System (IMU/AHRS). https://www.navtechgps.com/vn_100_rugged_inertial_measurement_unit_and_attitude_heading_reference_system_imuahrs

3 Attitude and Heading Reference System (AHRS) (60 points)

In this problem, you will implement a Right-Invariant EKF (RI-EKF) on SO(3) to estimate the sensor frame orientation wrt the world frame. An Inertial Measurement Unit (IMU), shown in Figure 1, is a highly useful sensor that consists of a gyroscope, an accelerometer, and often a magnetometer as well as a barometer (to measure the pressure). The gyroscope measures the angular velocity in the body/sensor frame. The accelerometer measures linear acceleration in the body frame. The magnetometer measures the local magnetic field, hence, providing an absolute reference for the heading angle. An AHRS is an onboard system that consists of an IMU and often an EKF to estimate the sensor orientation by fusing the mentioned measurements.

In this problem, we only recorded the raw gyroscope and accelerometer measurements from a real VN-100 IMU. We also recorded the estimated orientation provided by the embedded EKF developed by the manufacturer. Since the estimated orientation is highly optimized for this sensor and is accurate (uses all onboard sensors and factory calibration/filtering of the signals), we use it as a proxy for the ground truth. For more information about the sensor, refer to its user manual and data sheet.

The process model of the RI-EKF will integrate gyroscope measurements. Let $R_t \in \mathrm{SO}(3)$ be the state variable that shows the sensor frsme orientation wrt the world frame. The dynamics is

$$\dot{R}_t = R_t(\tilde{\omega}_t - w_t^g)^\wedge,$$

where $\tilde{\omega}_t$ is the gyroscope reading and w_t^g is gyroscope noise modeled as a zero-mean white Gaussian process. The correction model uses the accelerometer as follows.

$$Y_{t_k} = R_{t_k}^\mathsf{T} g + V_{t_k},$$

where g is the constant gravity vector (used as a reference) and V_{t_k} is a vector of Gaussian noise. Here $Y_{t_k} = a_{t_k}$, where a_{t_k} is the provided linear acceleration measurement. This observation model is clearly right-invariant.

A. Derive all equations required for the implementation of the RI-EKF. Show work and include all steps in your written report. (20 points)

B. Implement the RI-EKF and run the filter using provided IMU data. Generate three Plots using the estimated orientation by converting the rotation matrix to roll, pitch, and yaw angles. For each plot, include the estimated angle and the ground truth angle against time. (40 points)

Provided Data Format

The raw gyroscope and accelerometer measurements, and ground truth orientation are provided in .mat and .csv files. Both files contain the same data. In the provided data,

- `a` is the accelerometer readings.
- `omega` is the gyroscope readings.
- `dt` is the time difference (delta time) at each step in seconds.
- `g` is the gravity vector.
- `euler_gt` is the ground truth orientation as ZYX Euler angles.

Note: We don't provide code template for this homework. You may use existing examples (`Canvas > Files > code_examples > MATLAB` or `Canvas > Files > code_examples > Python`) as you see fit.