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| Target Date Investment Models for Investors and Money Managers.  Target-date funds  **Managing the Tradeoff between Market and Longevity Risk using a Value at Risk (VaR) Approach** | **Research Question**  “To what extent has the VaR of Target-Date Funds, for investors near retirement, shifted from prior to the 2008 financial crisis to today and what methodologies can we use to accurately evaluate VaR during this period?”  Team Members: Marcus Lee Karen Smith Yufei Zhang  FIN 285A – Computer Risk and Simulation Professor Ying Becker |

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**Introduction**

Planning for retirement is a considerable responsibility for many individuals especially with the gradual fading out of defined benefit (DB) pension plans in favor of defined contribution (DC) plans.  In its 2017 Private Pension Plans Bulletin, the Department of Labor noted that the assets of private pension plans accounted for 67% or $6.6 trillion[[1]](#footnote-1) of total assets in the industry versus 33% still managed under defined benefit plans.

Defined contribution pension plans usually take the form of 401(k) or 403(b) investments, which despite being often self-directed with voluntary participation, are often guided by a Plan Sponsor (employer designated asset manager).

This shift from DB to DC most significantly, represents a transfer of risk from the employer to the employee.  Individuals, through their plan sponsors, are faced with the daunting responsibility of managing two key types of risk: 1) Longevity Risk: the risk of outliving their savings (insufficient savings accumulation) and 2) Market Risk: the risk of losing substantial value in their investments, especially at a critical juncture such as retirement.

More specifically, the challenge for investors and/or plan sponsors is to maximize wealth accumulation and preservation. To accomplish these goals many have turned to Target-Date Funds (TDFs) (first introduced by Wells Fargo in 1994).  TDFs are diversified portfolios that employ a gradual de-risking of the portfolio via asset reallocation as time progresses.  They allow plan sponsors to maintain a riskier asset allocation earlier in an investor’s life when the focus is on capital accumulation and the tolerance for risk is higher, and gradually progress to a safer allocation focused on capital preservation, as retirement nears. This gradual asset reallocation is known as the *‘glide path’* of the TDF.

Traditional TDFs, however, have come under heavy criticism over the past decade as they have largely failed to protect investors against market risk particularly as retirement nears, when the allocation of the fund is supposed to be ‘safer’. This has occurred because whilst the portfolio de-risks in terms of asset allocation, from a risk management perspective, these funds are still very much driven by equity versus fixed-income risk.   Particularly in terms of equity market volatility, the equity allocation of these funds remains inflexible (ie. sticks to the predetermined glidepath) leading to significant drawdowns in portfolio value, with dire consequences for investors at or near retirement.

This over-exposure to equity risk, in essence, is a form of market tail-risk, to which TDFs are particularly susceptible.

**Research Objective**

In view of the issue of tail-risk characteristic of traditional Target-Date Funds our research objective is to examine the Value-at-Risk (VaR) of these funds as they near retirement. We are particularly focused on the risk-exposure of these funds which are near to their respective target dates (within a three to five-year period).  Further, we wish to contrast the VaR of these Funds prior to the financial crisis and after the financial crisis of 2008.

Our reasoning is that the VaR of TDFs which are closer to retirement should differ substantially prior to and after the financial crisis of 2008 as fund sponsors/investors suffered significant losses due to the lack of risk-diversification as opposed to portfolio diversification.  These findings should also hold true for an analysis of tail risk over the period. That is, compared to before 2008, TDFs today should have lower tail risk as investors have suffered significant losses in the past due to inadequate risk management.

*Our research question is*: To what extent has the VaR of Target-Date Funds, for investors near retirement, shifted from prior to the 2008 financial crisis to today? In other words: Has the financial crisis of 2008 led to a reallocation-of-risk across TDFs, approaching their respective target dates?

We are of the view that the findings of our research proposal bear particular significance to investors considering TDFs as a form of risk-management/mitigation and more-so in view of the prevalence of equity market volatility.  An expected outcome of our research proposal is that similar to the approach of many other types of mutual funds, risk-allocation and its measurement ought to be the driving force of TDFs as they approach their respective target dates.

Whilst the overall strategy of asset-reallocation may ‘theoretically’ fit well with the investor’s lifecycle, the ‘risk-cycle’ inherent in financial markets must also be a key concern when developing the asset allocation strategy for TDFs. In other words, there may be a need for a dynamic approach to asset allocation of TDFs, even as they approach the Target date.

**Literature Review**

The literature on Target-Date Funds emerges from the integration of the underlying approach to optimal portfolio management and ‘life-cycle’ analysis. That is, as the investor approaches different stages of the lifecycle, the risk-return profile of the investments held should shift to reflect the risk-profile of the investor. Consequently, much of the research on these funds focuses on assessing the asset allocation strategy based on the risk assumptions or risk profile of the investments.

Similar to Yoon (2010) our proposal asserts that a static approach to reallocation may be insufficient to capture the changing nature of risk.  In the paper entitled ‘Glide Path and Dynamic Asset Allocation of Target-Date Funds’ shows that the optimal way to reallocate assets in these funds, is based on the consideration of the time-varying nature of risk. Traditional ‘glide path’ analysis is based on a static risk assumption and Yoon (2010) shows that a dynamic asset allocation for these funds within a prespecified risk budget, can be achieved in consideration of the term structure of risk.

Our research proposal also facilitates a comparison of risk-performance of TDF’s over time, which can easily be extended to a comparison across Funds to evaluate Fund Managers’ effectiveness at risk management. This is the approach of Balduzzi and Reuter (2015) in their NBER working paper entitled ‘Heterogeneity in Target-Date Funds: Optimal Risk Taking or Risk Matching?’

In this paper the authors argue that funds with similar target dates across providers may still differ in terms of realized return and risk profiles.  They show firstly that Fund managers employ a form of ‘Optimal-Risk-Taking’ as a way of increasing market share, even as the fund approaches the target date.  The idea is that the greater the equity risk allocation, the greater the market share.

Secondly, they contend that there is a certain degree of ‘risk-matching’ among Plan Sponsors in selecting TDFs for investors which offsets the risks faced by investors within a certain industry to match the risk of investors as requested by the investors themselves. They contend that it is in this sense that the risk-allocation of TDFs may seem inefficient in some instances or may seem to deviate from the fundamental ‘life-cycle’ approach to asset allocation.

Whilst these and many other authors rightfully address the need for flexibility in assessing the risk-profile of investments, particularly as the TDF approaches maturity, what is unique about our proposal is the incorporation of VaR analysis of TDFs in light of financial market crises.  At the same time, in our proposal we recognize that this analysis can be extended to examine the implications of different VaR and tail-risk measurement approaches such as GARCH, ARIMA and generalized Pareto distribution (GPD), which can potentially reveal other interesting and informative findings about the relative positioning of TDFs.

We now explore these approaches through a review of our proposed methodology which follows below.

**Data**

Within our research as we examine the Value-at-Risk (VaR) of these funds as they near retirement, the data we have selected to observe are the Fidelity Freedom 2010 (FFFCX) comprising of years 2002 through 2007, and the Fidelity Freedom 2020 (FFTDX) comprising of years 2012 through 2017.

The two graphs below map the trend of the data sets’ daily arithmetic returns over the span of the 6 years in percentages. Within the graphs, we observed that the overall trend in both funds have been increasing steadily with the exceptions of slight dips between the period 2002 to 2003 and the period between 2016 and 2017.

A close up of a map

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As an analysis on the graphs of the daily return of both the 2010 and 2020 funds, a very important property of return to note is that both graphs have fat tails. Upon further calculations on both data set’s kurtosis, the 2010 fund has a kurtosis of 4.401, whereas the 2020 fund has a kurtosis of 6.184. These results signify that the series of 2020 returns has a fatter tail than the 2010 returns, and therefore is more likely to experience occasional extreme positive or negative returns.

**Methodology**

In the implementation of our methodology and in consideration of the various risk measurements available for use in this project, we are guided by the fact that daily returns may have persistence of time dependency, that is, they may be autocorrelated. At the same time, we also recognize that the returns of the stock market usually appear to be fat tailed. As a result, independent normality assumption may not be appropriate to this data.

It is our view that historical simulation would be a good way to deal with autocorrelation and fat tail problems. However, this approach would be slow to react to changing market environments, which is a key criticism of the traditional risk management approach espoused by Target Date Funds. To evaluate different risk management performance measurements before and after the crisis, historical simulation may be weak at capturing differences in market conditions in different time periods.

To make our risk measurements robust, we used several different methodologies to estimate the VaR for the 2010 Fund, FFFCX, and the 2020 Fund, FFFDX.

1. **GARCH Model:**

Daily volatility of portfolios may vary over time. Our approach is to introduce the GARCH model to update volatility. Daily standard deviation is updated daily based on the characteristics of the portfolio on the previous day. Therefore, we willcalculate VaR each day and compare it to the actual return.

1. **ARMA Process:**

Portfolio returns may have autocorrelation. Introduce the ARMA process of stock returns. Expected value of daily return will be estimated based on the characteristics of the portfolio on the previous day. Calculate VaR daily and compare it to actual return.

1. **Generalized Pareto Distribution:**

There is a common thrust that any tailed distribution should follow a Generalized Pareto Distribution (GPD). Choose a threshold and distribute the tail of returns by GPD and use GPD to estimate VaR.

We will estimate the VaRs using the three methodologies described above and compare the results with historical simulation and normal distribution maximum likelihood estimation. We can then derive the risk measurements using the different approaches and compare the results to see which approaches would be more appropriate for Target-Date Funds.

We will also compare the VaRs across the funds to evaluate the difference of risk management performance before and after the Financial crisis of 2008.

**Results**

1. **Historical Approach**

Historical Approach is one of the most commonly used approaches to estimating VaRs. Despite its lack of ability finding the shape and dealing with market conditions, the historical approach is simple and straightforward. It can also give us a good baseline for VaR estimations. The result is expressed by the plots below:

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1. **Normal Assumption Approach**

Another very commonly used approach for VaR estimation is the normal approach. That is, assuming daily returns follow a normal distribution according to the central limit theorem. The shortage for this approach is that it lacks ability to capture the fat tail or thin tail properties. Here, data is fat tailed. Similarly, we use a plot to express the results:

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1. **Garch Approach**

A good way to capture the difference in daily volatility is to use the Garch model. Garch model assumes a constant mean return each day but allows a changing standard error.

For simplicity, we use the Garch (1,1) model here, which allows volatility to update daily. Below are the simulated returns against real returns and it can be inferred that Garch(1,1) captures the shape of daily returns very well.

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Then, we can use daily standard errors to estimate daily VaRs:

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1. **Arima Approach**

A similar idea is to capture the difference in expected return every day. To capture this difference, a straightforward way is to apply the Arima model.

However, the data does not exist as an obvious autocorrelation. By plotting the ACF plot, we can see that the autocorrelation of daily returns is close to zero for both funds:

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The ACF plot suggests that it is not a good idea to apply the ARIMA model. Even if we do try to apply AR(1) model on daily returns, the result is very poor. It can be show in the plot below:

A picture containing implement, pencil

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It seems that VaR is almost not changing over time. If we do want to capture the serial correlation of daily returns, a possible way is to use Kalman Filter, and this can be the future research topic.

1. **GPD approach**

There is a common trust that tailed distributions should follow a generalized Pareto Distribution (GPD). Compared to normal, it can capture both fat tailed and thin tailed cases.

In the data of both funds, we have 1509 daily returns. We choose the 300 lowest returns for both funds as the lower tail of both funds. Then we apply GPD distribution on both return sets. The results are:

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|  | Fund 2010 | Fund 2020 |
| Shape | 81.30610099869736 | 69.48475579646345 |
| Location | -0.24336102575795365 | -0.22957449718344247 |
| Scale | 0.2465278703274249 | 0.23216013442897443 |

Looking at the results carefully, we can see that the shape parameter is larger for the 2010 fund (81.3 vs 69.5), which matches the fact that fund 2010 has a lower kurtosis. Both the lower shape parameter and high kurtosis for 2020 fund suggest that 2020 fund has a fatter tail.

The result of VaR estimation is below:

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**Conclusion**

We compare in two different approaches:

1. VaR estimation compare: Mean VaR estimation for the two funds using different approach
2. Performance compare: Number of days that performance is worse than VaR estimation for the two funds using different approaches.

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| --- | --- | --- | --- | --- |
| VaR Estimation | 95% VaR 2010 | 99% VaR 2010 | 95% VaR 2020 | 99% VaR 2020 |
| Historical | 0.764 | 1.143 | 0.720 | 1.304 |
| Normal | 0.717 | 1.025 | 0.710 | 1.019 |
| Garch | 0.678 | 0.973 | 0.672 | 0.969 |
| Arima | 0.717 | 1.025 | 0.710 | 1.017 |
| GPD | 0.741 | 1.242 | 0.726 | 1.281 |

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| --- | --- | --- | --- | --- |
| Performance | 95% VaR 2010 | 99% VaR 2010 | 95% VaR 2020 | 99% VaR 2020 |
| Theoretical  /Historial | 75.45 | 15.09 | 75.45 | 15.09 |
| Normal | 90 | 25 | 78 | 31 |
| Garch | 86 | 27 | 86 | 31 |
| Arima | 92 | 26 | 77 | 29 |
| GPD | 82 | 8 | 75 | 18 |

One of the most obvious findings is that Garch gives a much lower mean daily VaR estimation while not doing worse than other approaches. By performance compare table, we can see Garch has a similar performance compared to normal and Arima approach. But it gives a much lower mean daily VaR estimation. This means that the Garch model is effective at capturing daily volatility change overtime and is able to provide a more accurate daily VaR estimation.

Another very straightforward finding is that GPD approach has the best performance. The reason is that only GPD can deal with the fat tail case. All other estimations will give the results that underestimate the VaR. The further step of this approach is that we can try to use weighted maximum likelihood estimation on the data. This will avoid a sharp cutoff and have more tolerance on the choice of threshold. This improved approach may help us get more stable estimations on extreme values at different levels.

Last but not least, all approaches give a higher 95% VaR estimation for the 2010 fund. It shows that the portfolio can control most of the risks better after the crisis. However, 99% VaRs are similar for both funds. This means that super extreme risks still cannot be better controlled.

1. United States Department of Labor, Employee Benefits Security Administration: Private Pension Plan Bulletin, Abstract of 2017 Form 5500 Annual Reports [↑](#footnote-ref-1)