Intensity-based Registration
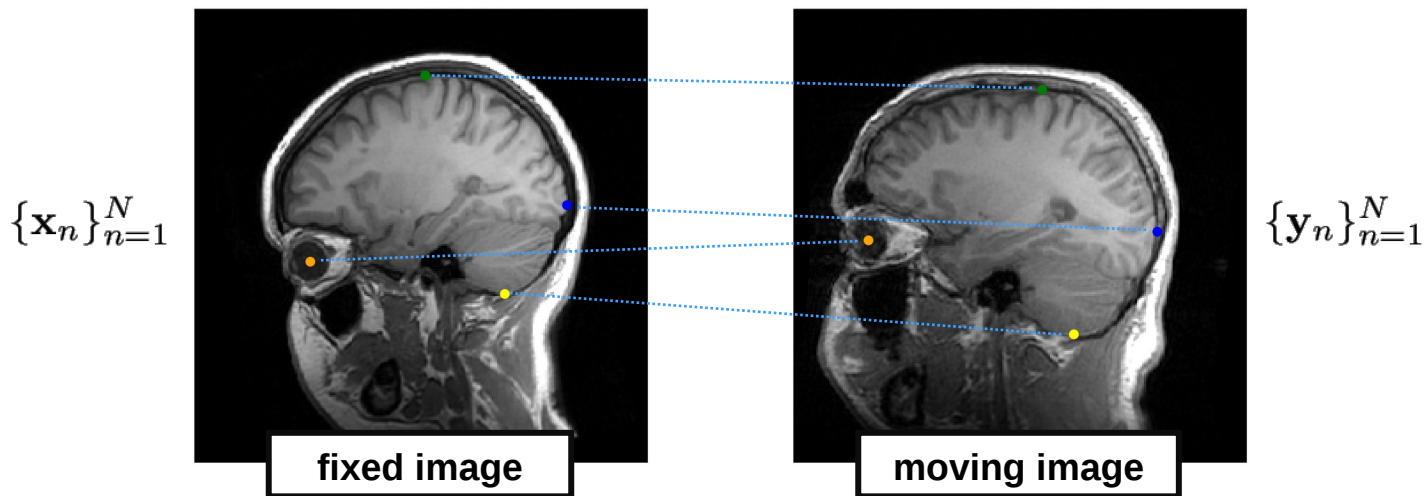
Medical Image Analysis

Koen Van Leemput

A"

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Aalto-universitetet
Aalto University

Recall landmark-based registration

✔ Manually annotate N corresponding points in two images:

$\{\mathbf{x}_n\}_{n=1}^N$ fixed image   moving image $\{\mathbf{y}_n\}_{n=1}^N$

✔ Register the images by minimizing the distance between matching point pairs:

$$E(\mathbf{w}) = \sum_{n=1}^N \|\mathbf{y}_n - \mathbf{y}(\mathbf{x}_n, \mathbf{w})\|^2$$

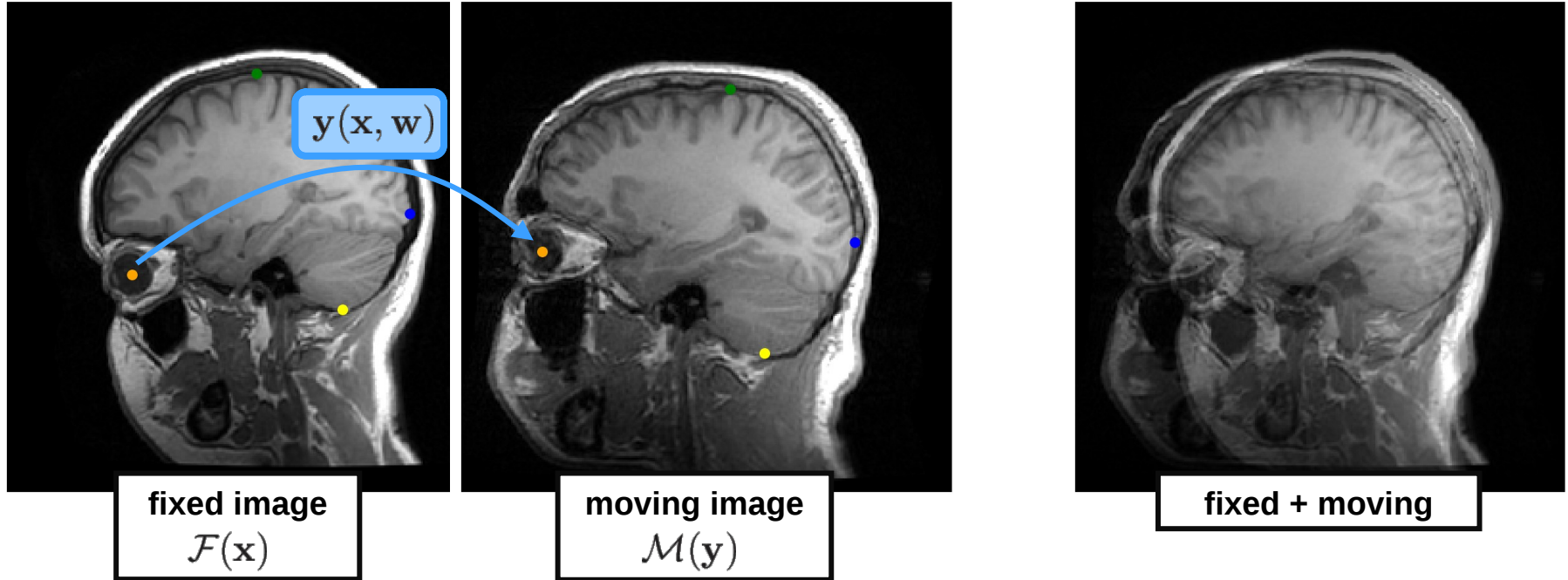**Spatial transformation model**

Spatial transformation models

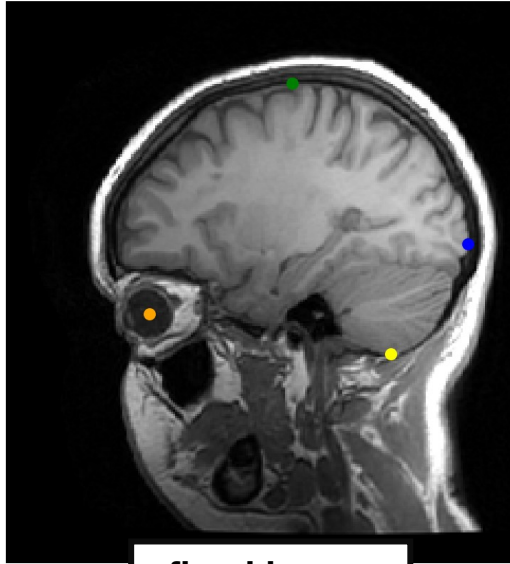**Spatial transformation** $\mathbf{y}(\mathbf{x}, \mathbf{w})$:

✔ maps world positions $\mathbf{x}$ in the fixed image to world positions $\mathbf{y}$ in the moving image
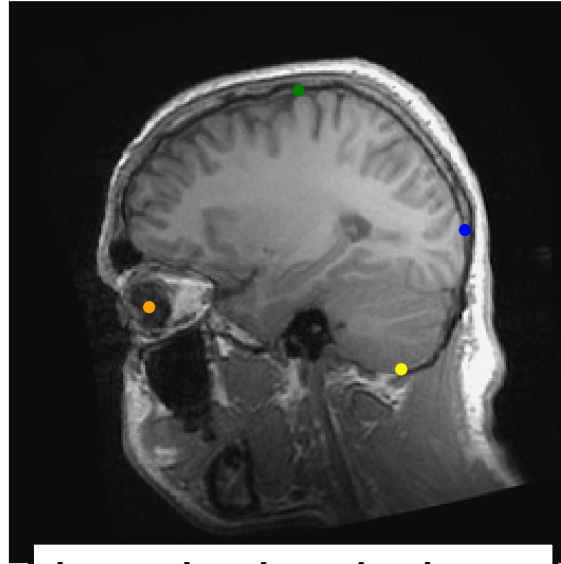
✔ controlled by parameters $\mathbf{w}$

affine

rigid

nonlinear

Landmark-based registration

fixed image
$\mathcal{F}(\mathbf{x})$

$\mathbf{y}(\mathbf{x}, \mathbf{w})$

moving image
$\mathcal{M}(\mathbf{y})$

fixed + moving

Before registration

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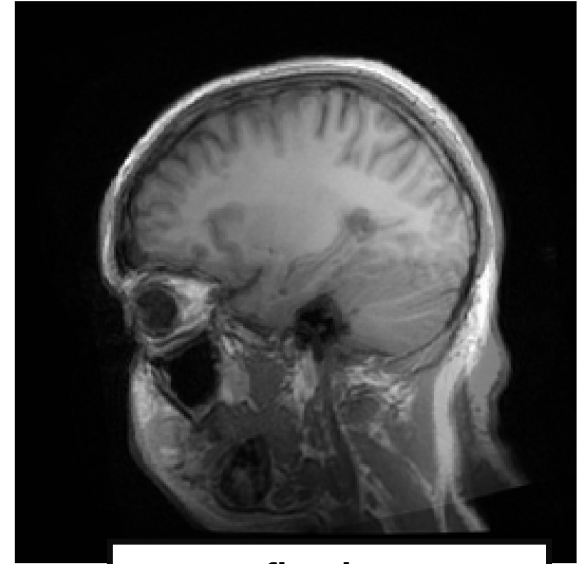Aalto University

Landmark-based registration

fixed image
$\mathcal{F}(\mathbf{x})$

interpolated moving image
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$
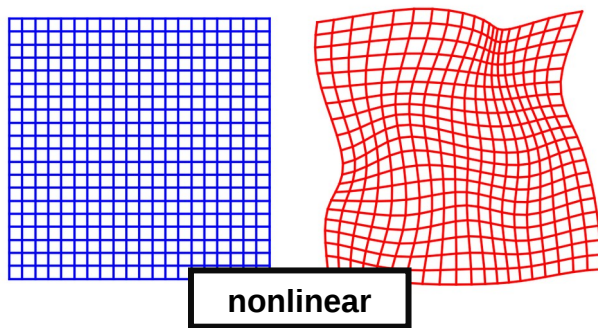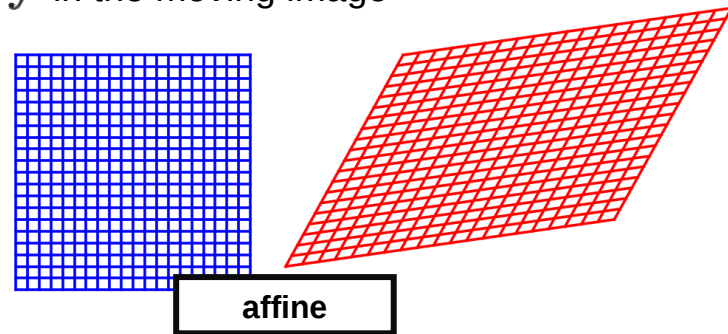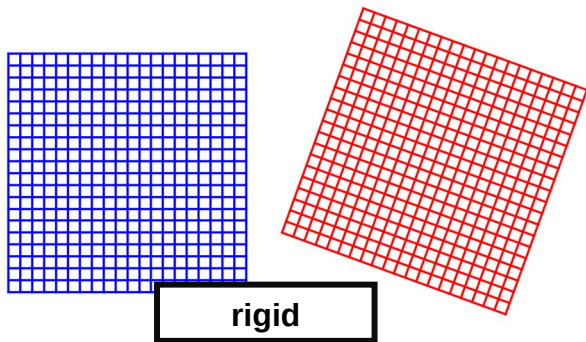
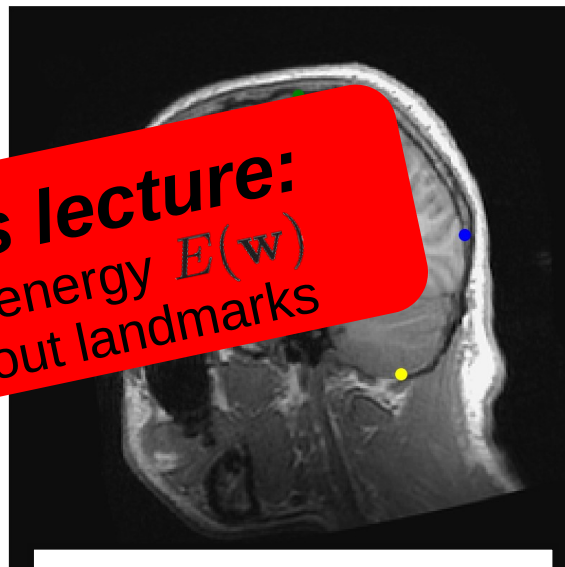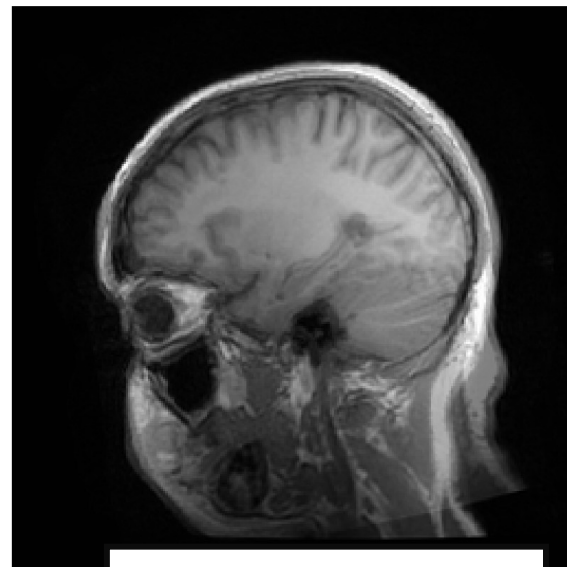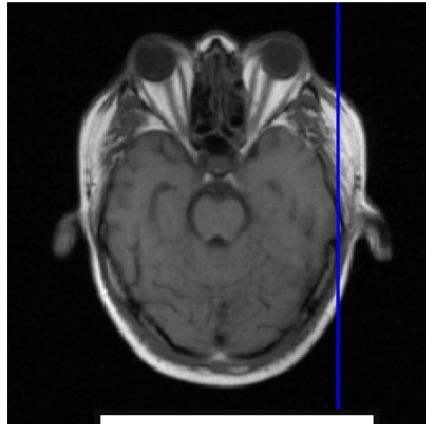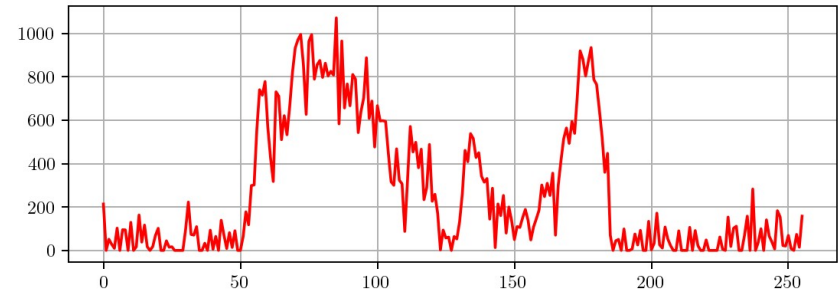fixed +
interpolated moving

After registration

Landmark-based registration

fixed image
$\mathcal{F}(\mathbf{x})$

interpolated moving image
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

fixed +
interpolated moving

This lecture:
define energy $E(\mathbf{w})$
without landmarks
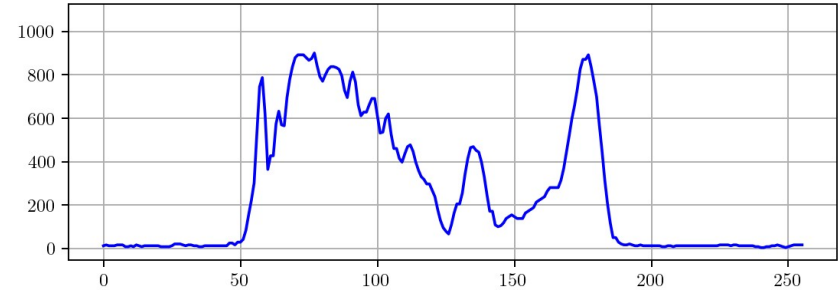
After registration

Intra-modal registration

Images have similar intensity characteristics

$\mathcal{F}(\mathbf{x})$

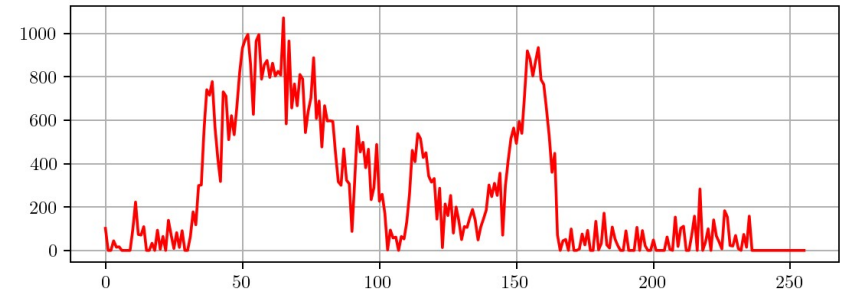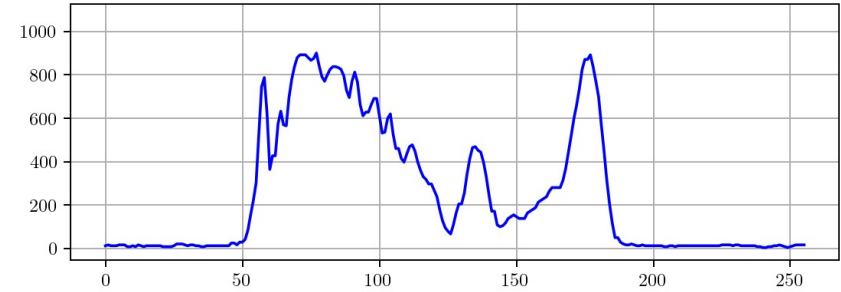$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

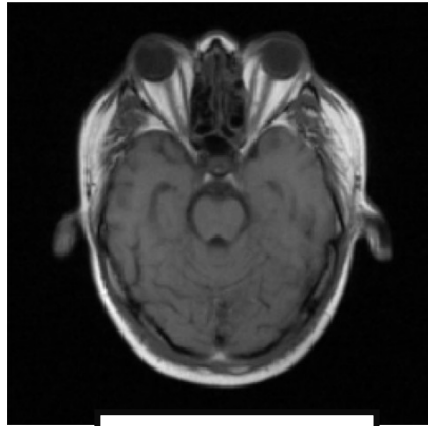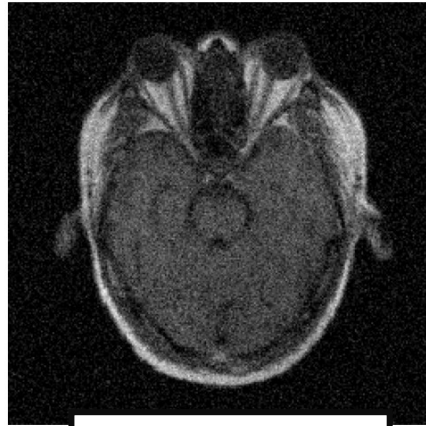**Images have similar intensity characteristics**

$\mathcal{F}(\mathbf{x})$
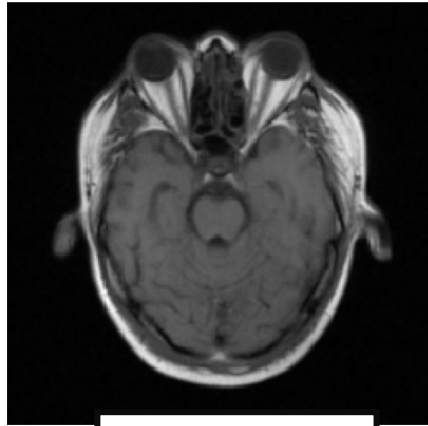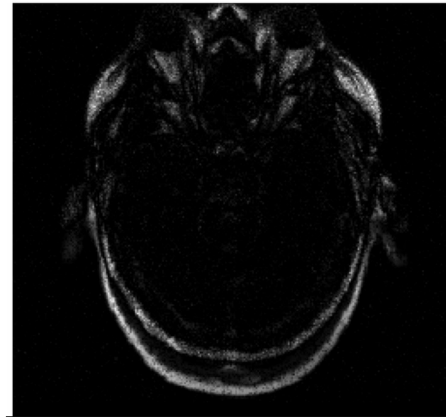
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

Images have similar intensity characteristics

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$
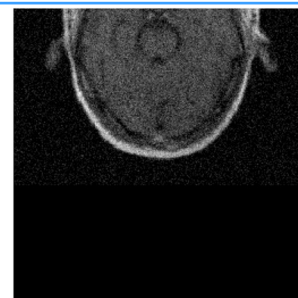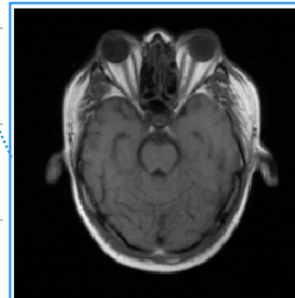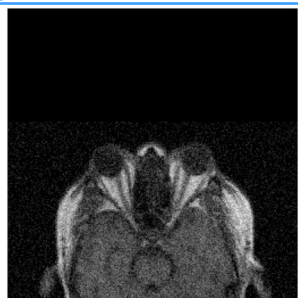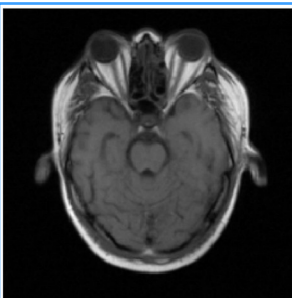
$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$

$$E(\mathbf{w}) = \sum_{n=1}^{N} [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2$$

sum over all voxels

Intra-modal registration

Images have similar intensity characteristics
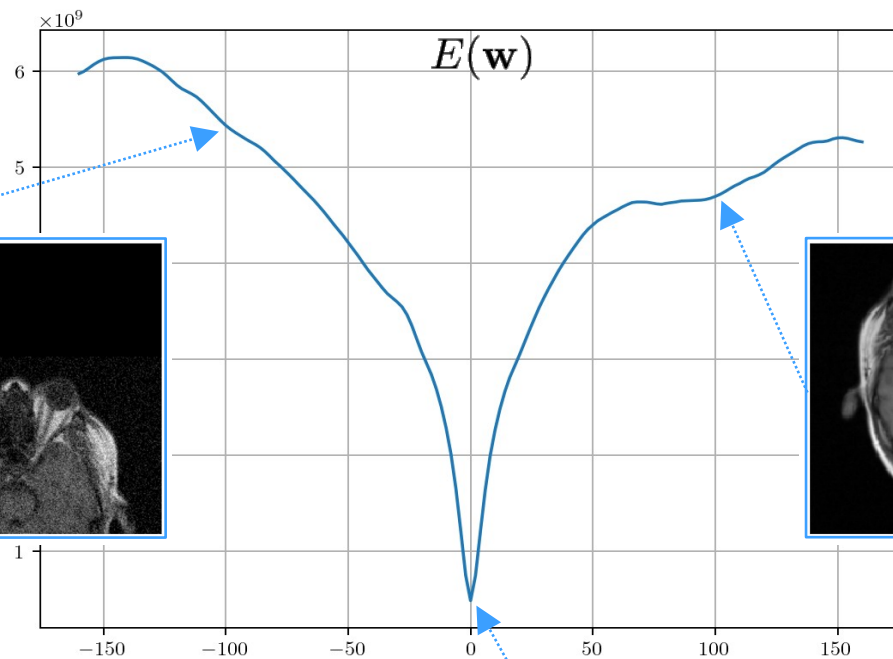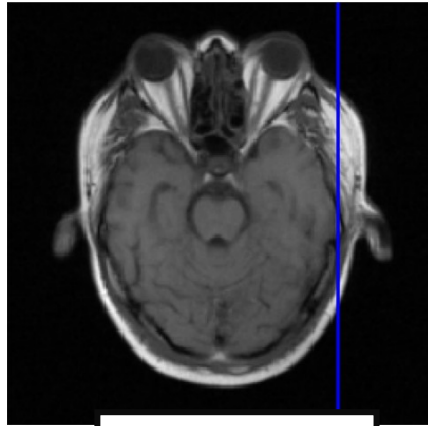

$$\mathcal{F}(\mathbf{x})$$

$$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$$

$$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$$

$$E(\mathbf{w}) = \sum_{n=1}^{N} [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2$$
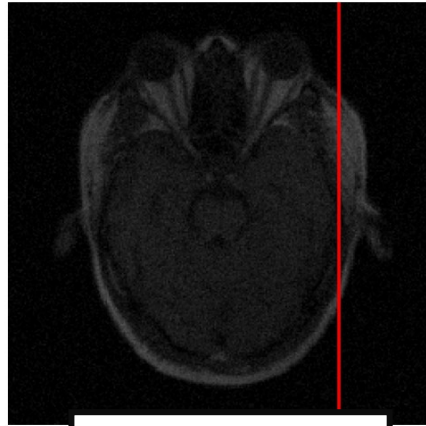
sum over all voxels
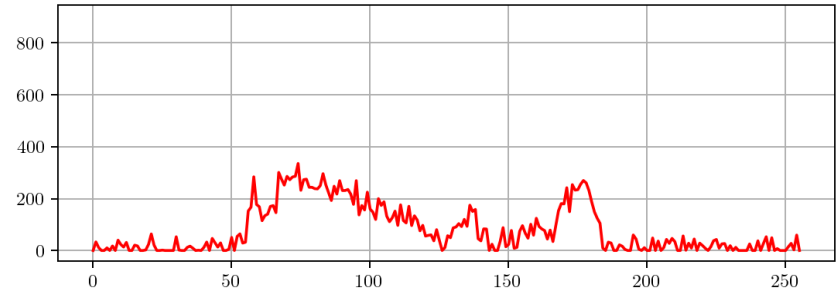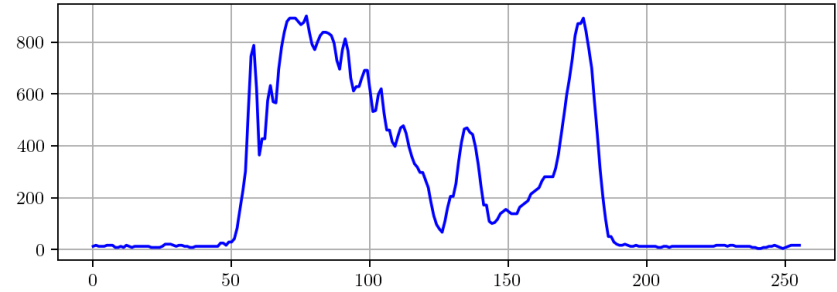
$E(\mathbf{w})$

Intra-modal registration

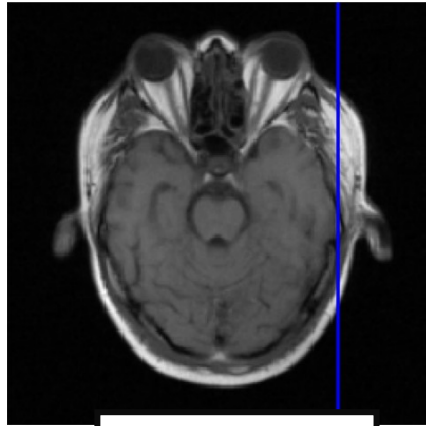**Same but images are scaled differently**

$\mathcal{F}(\mathbf{x})$

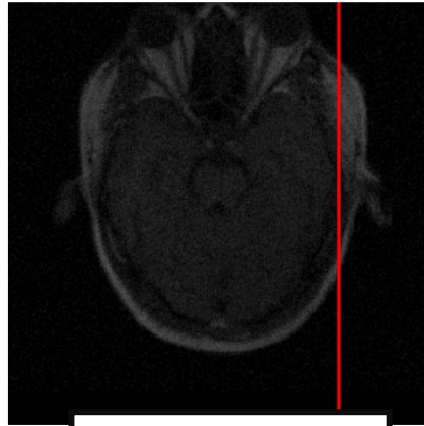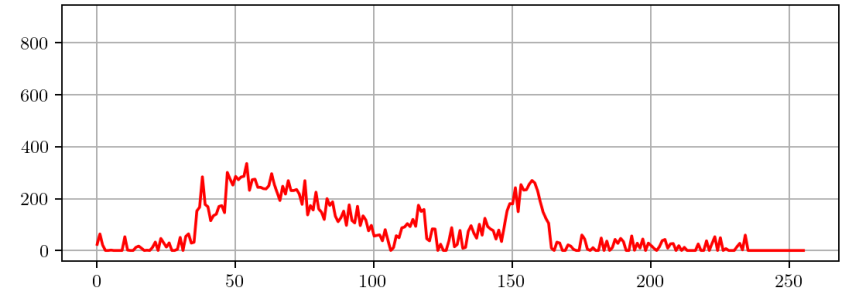$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Task: what's a good energy function $E(\mathbf{w})$?
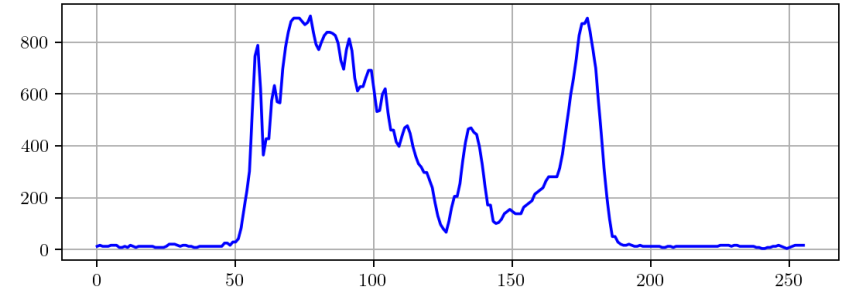
Intra-modal registration

Same but images are scaled differently
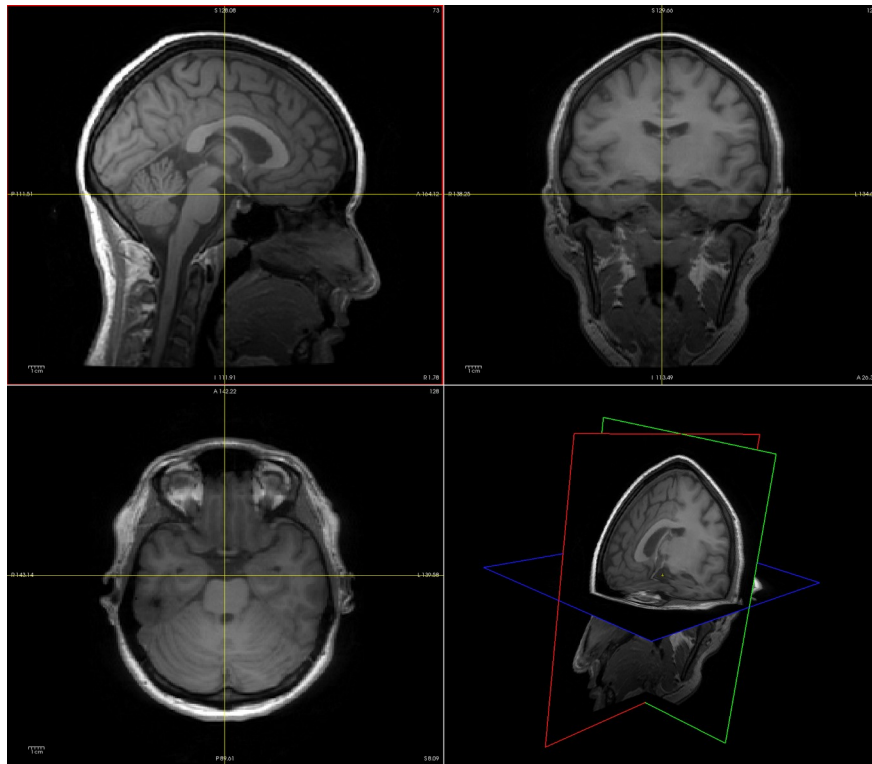

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

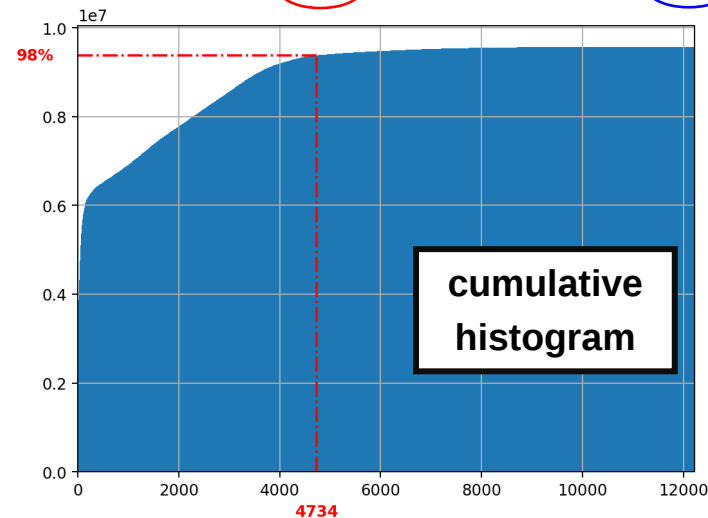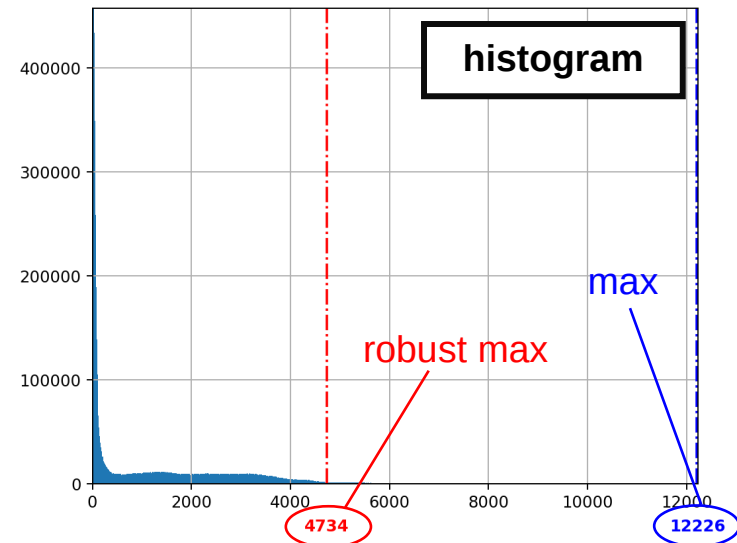**Task:** what's a good energy function $E(\mathbf{w})$?

"Maximum" intensity

Inter-modal registration

Images have different intensity characteristics

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Task: what's a good energy function $E(\mathbf{w})$?

Inter-modal registration

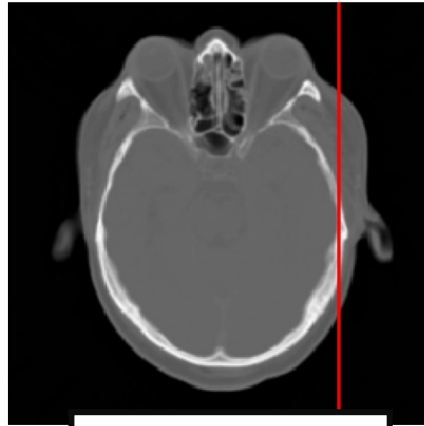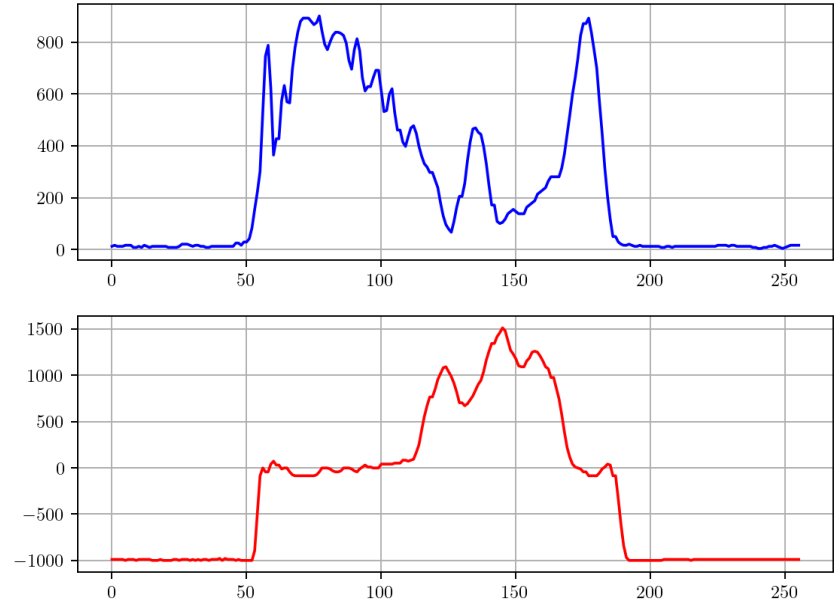**Images have different intensity characteristics**

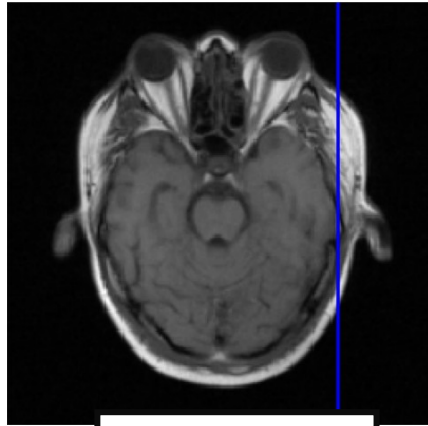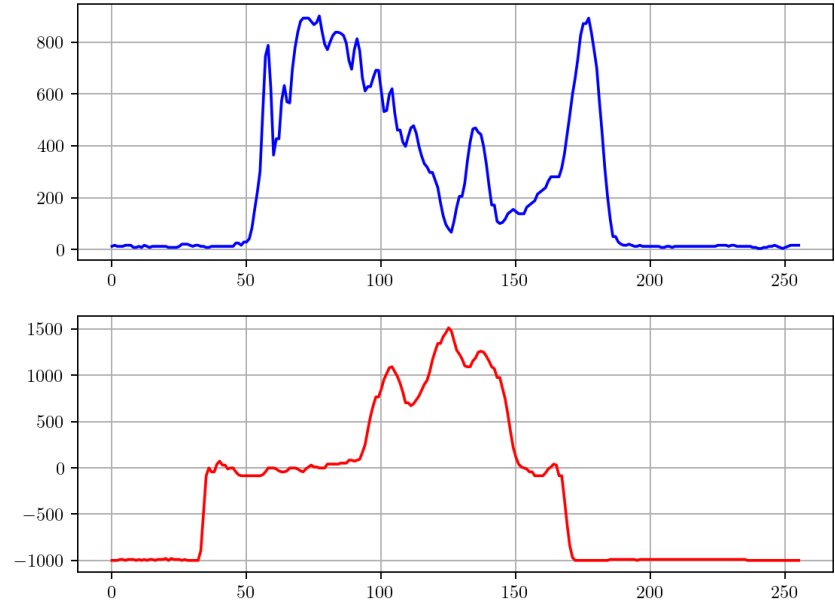

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Task: what's a good energy function $E(\mathbf{w})$?

Inter-modal registration

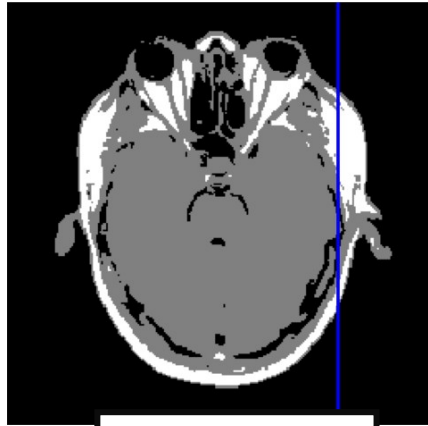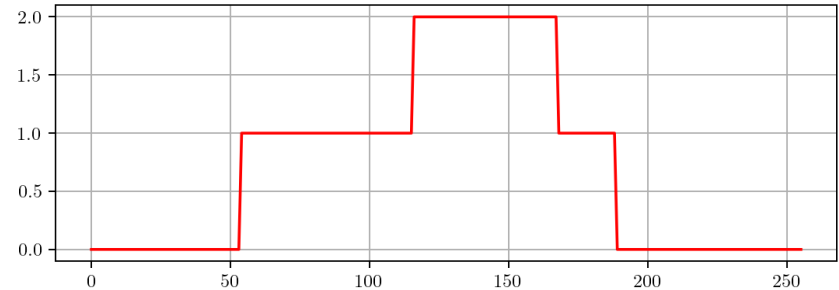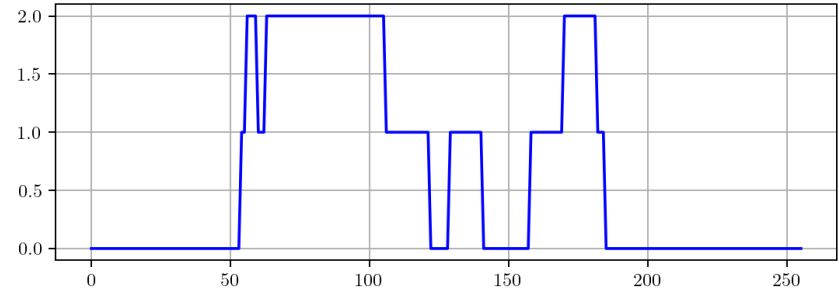**Images have different intensity characteristics**

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Easier task: what's a good energy function $E(\mathbf{w})$ now?
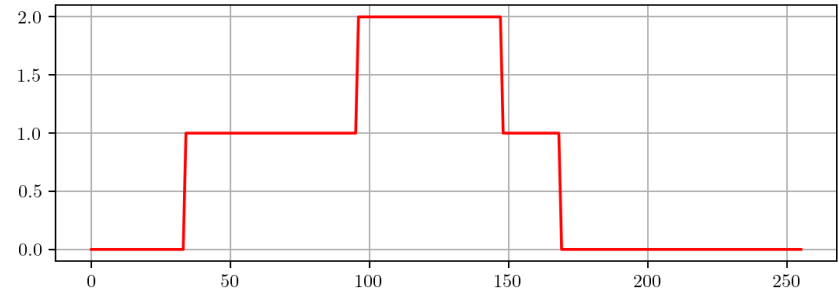
Inter-modal registration

Images have different intensity characteristics

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

Easier task: what's a good energy function $E(\mathbf{w})$ now?

Inter-modal registration

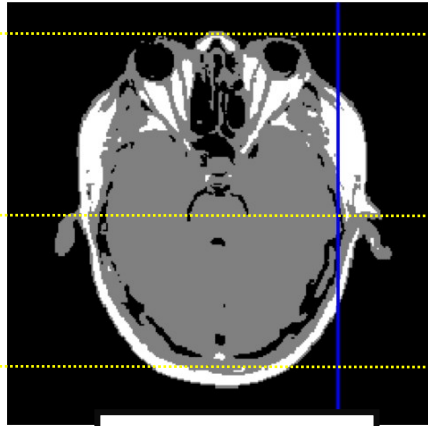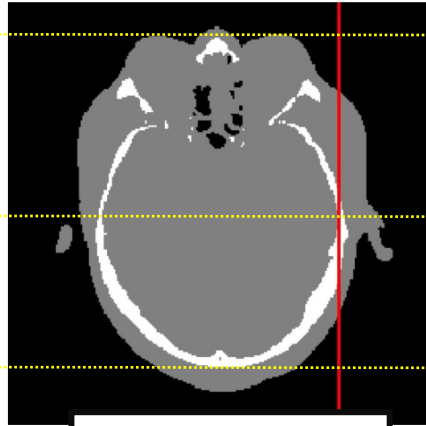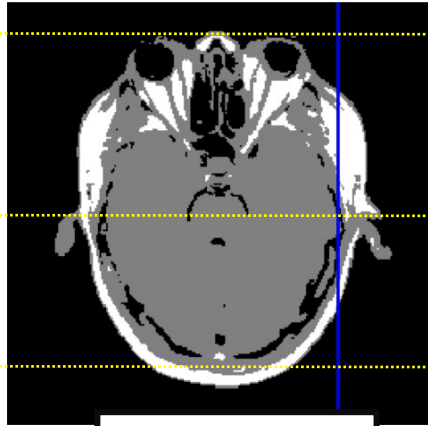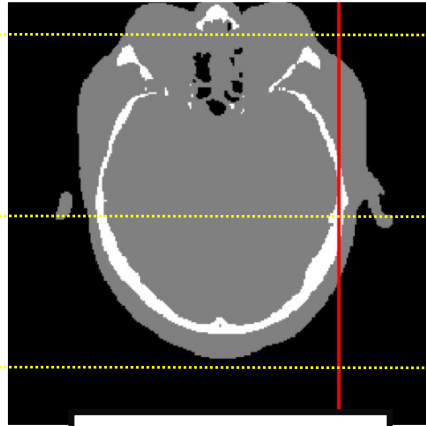**Images have different intensity characteristics**

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram

f

m

intensity in
fixed image

intensity in
moving image

Inter-modal registration

Images have different intensity characteristics
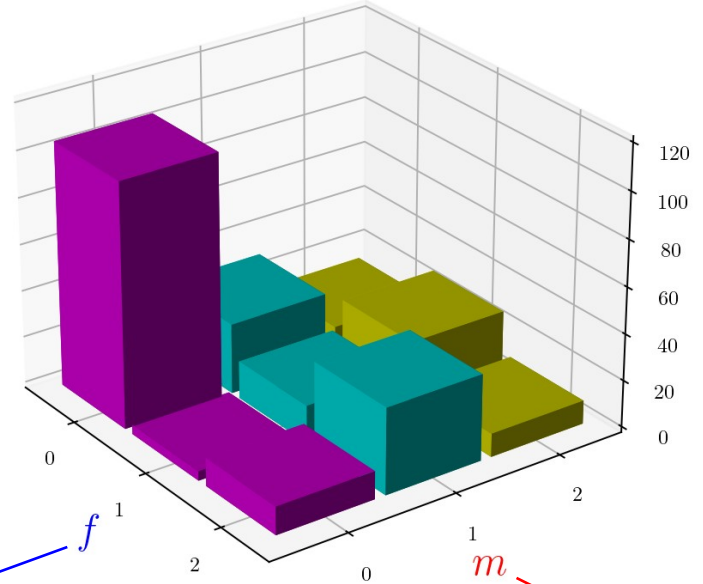

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$
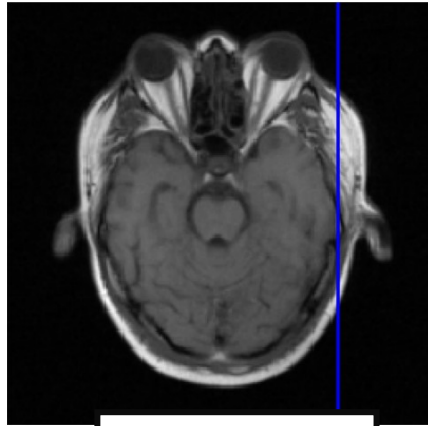
joint histogram
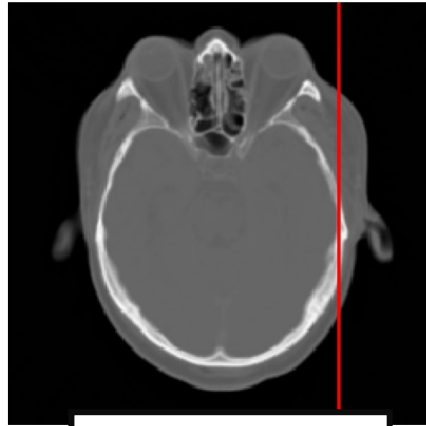
f

m

intensity in
fixed image

intensity in
moving image

Inter-modal registration

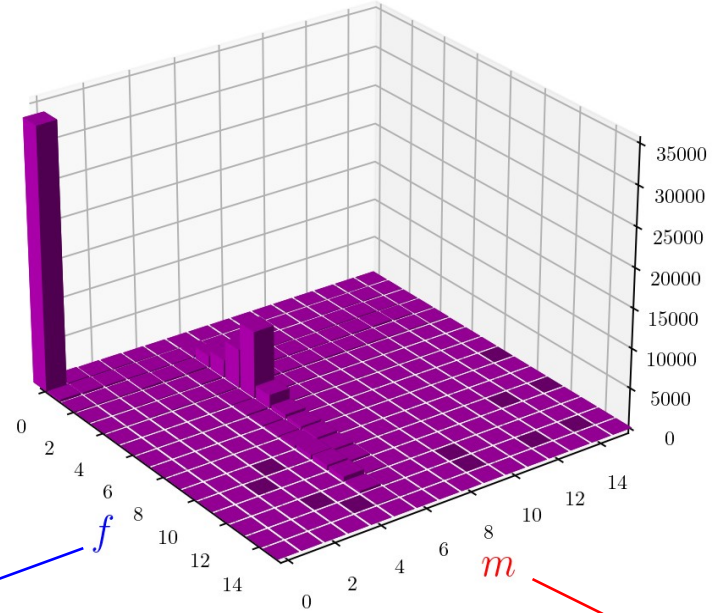**Images have different intensity characteristics**

$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram

f

intensity in
fixed image

m

intensity in
moving image

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Aalto University

Inter-modal registration

Images have different intensity characteristics

$\mathcal{F}(\mathbf{x})$

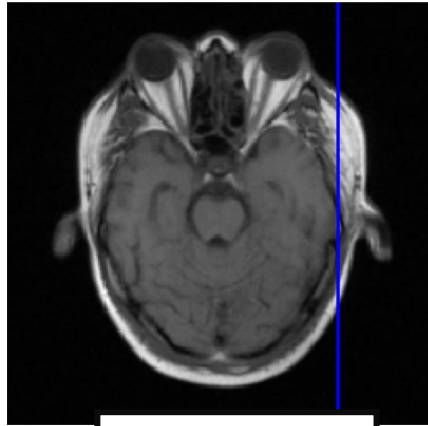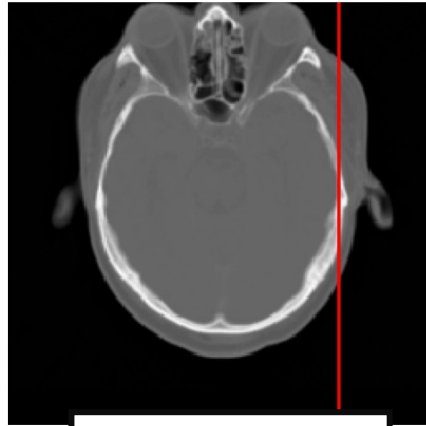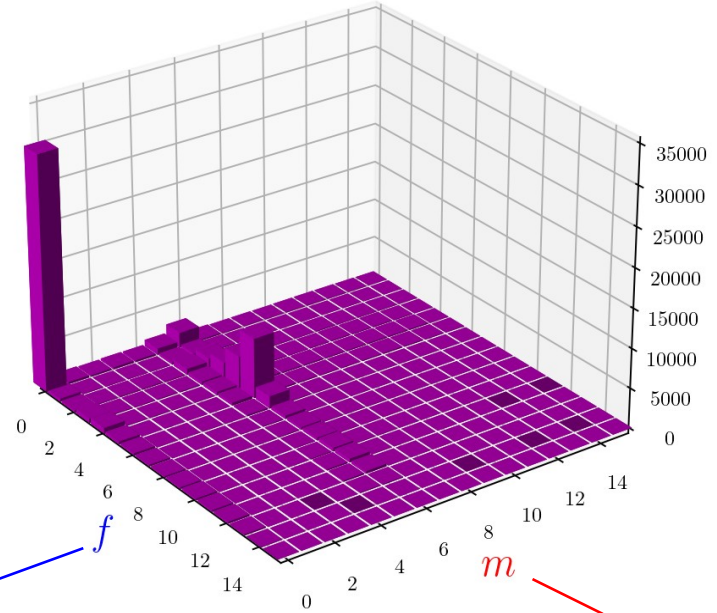$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram

f

intensity in
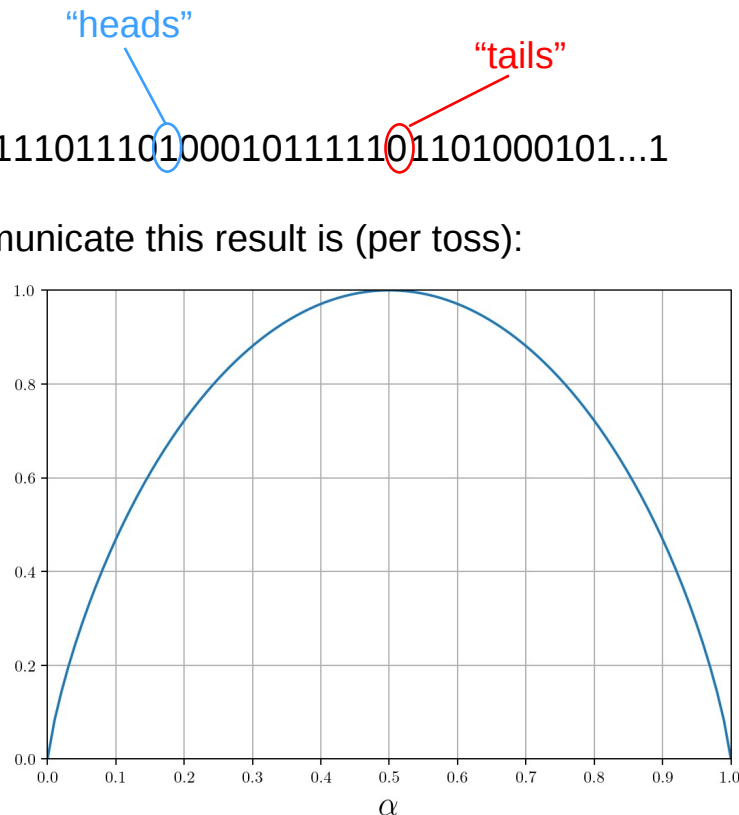fixed image

m

intensity in
moving image

A bit of information theory...

Imagine that a coin is "rigged":

✔ lands on heads with probability $0 \leq \alpha \leq 1$

"heads"

"tails"

✔ I toss it _many_ times, and the result is 11010001011111011101000101111101101000101...1

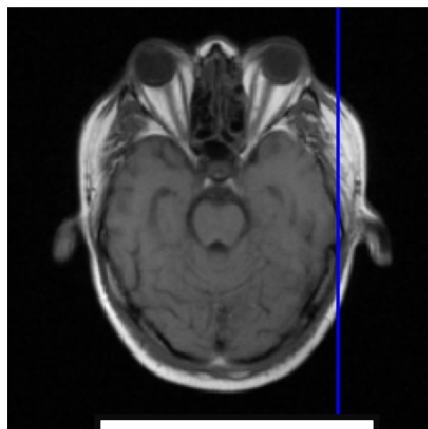✔ The minimum number of bits required to store/communicate this result is (per toss):

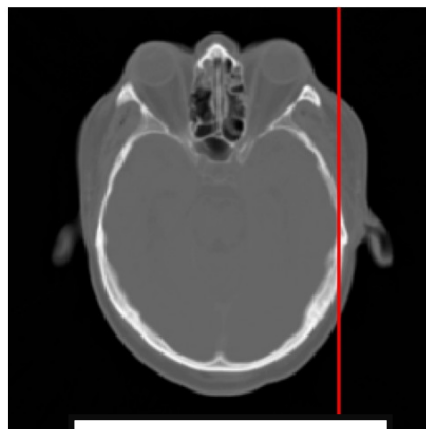$$-\alpha \log_2(\alpha) - (1 - \alpha) \log_2(1 - \alpha)$$

"entropy"

Inter-modal registration

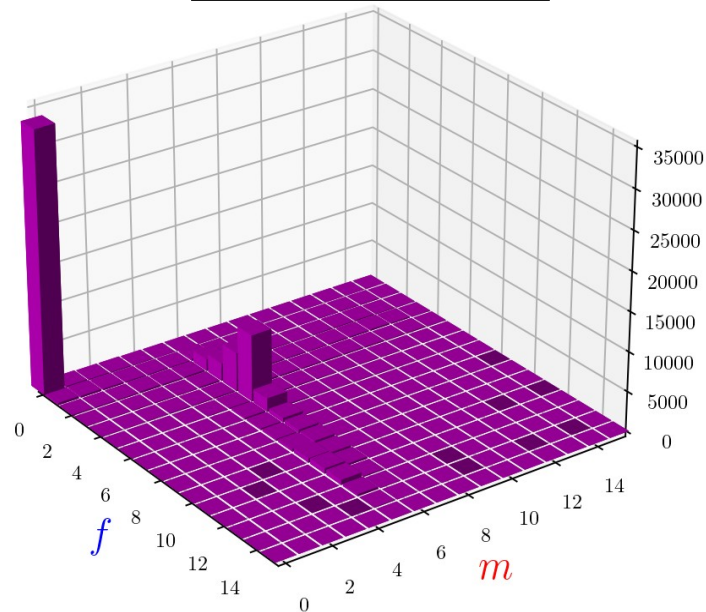**Images have different intensity characteristics**

$\mathcal{F}(\mathbf{x})$

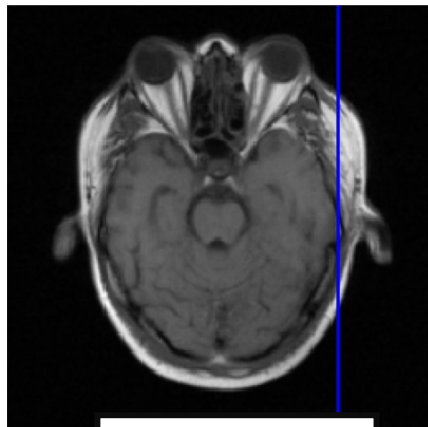$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram

$$E(\mathbf{w}) = H_{F,M} \text{ where } H_{F,M} = -\sum_{f=1}^{B}\sum_{m=1}^{B} p_{f,m} \log(p_{f,m})$$
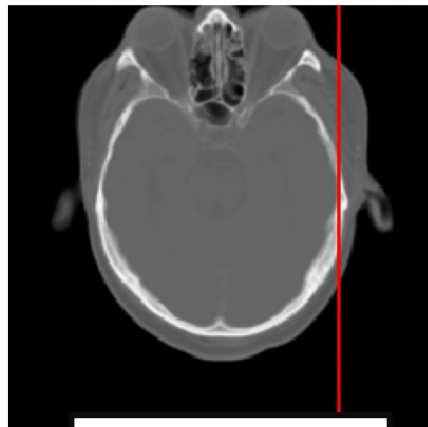
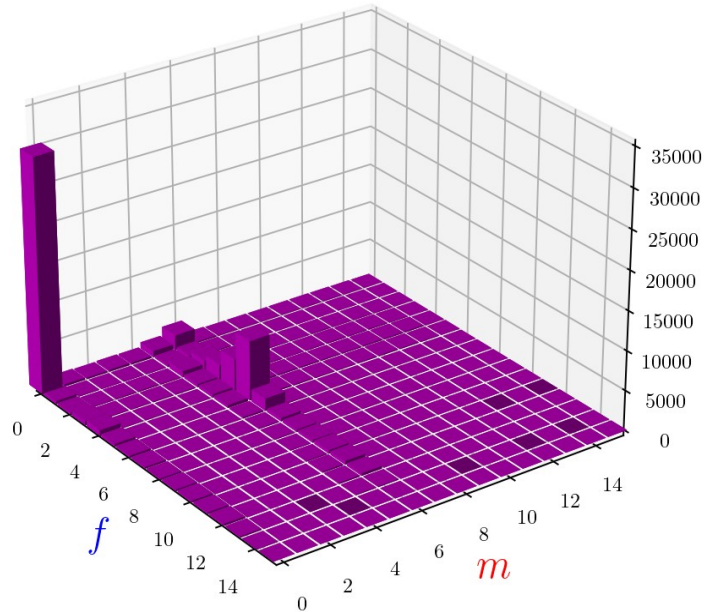normalized
histogram counts

Inter-modal registration

Images have different intensity characteristics
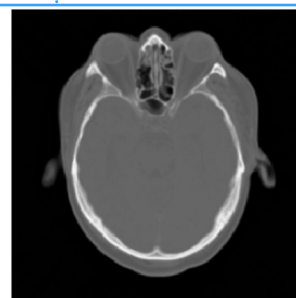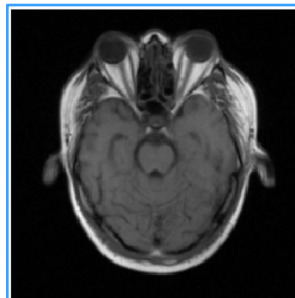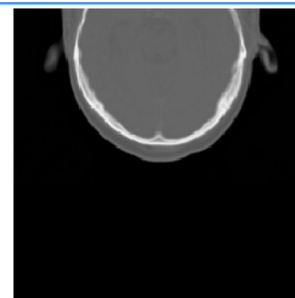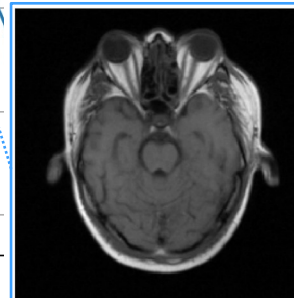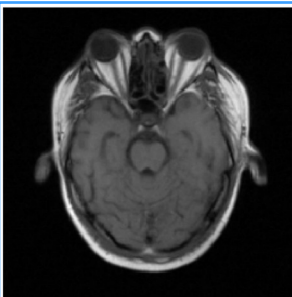

$\mathcal{F}(\mathbf{x})$

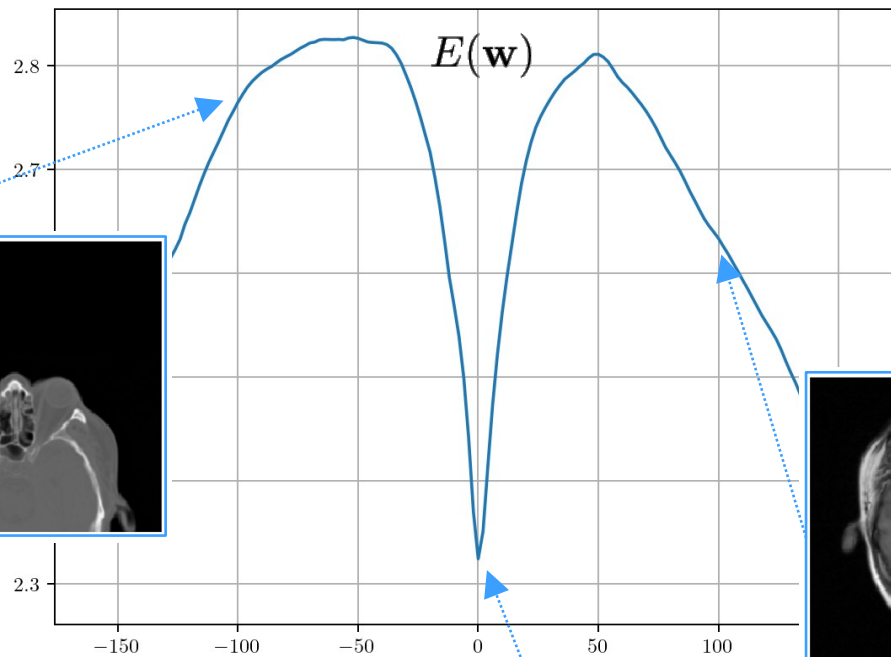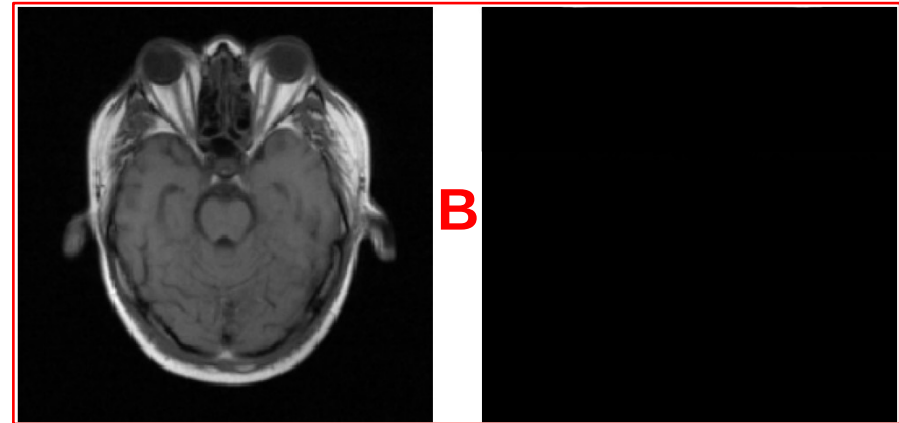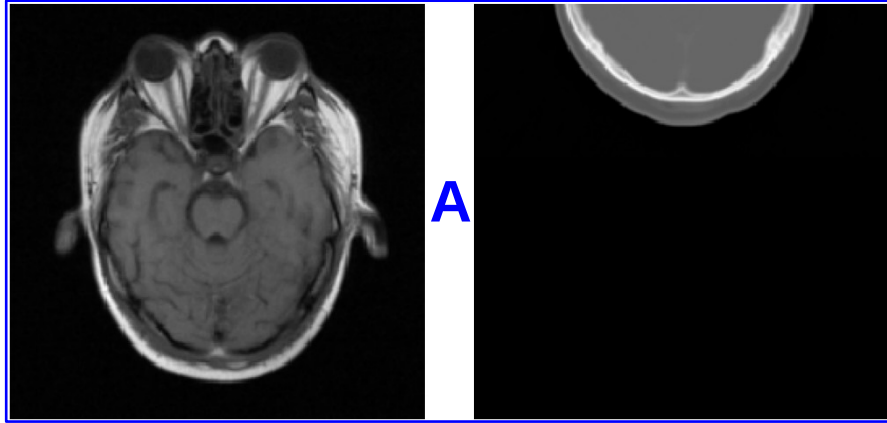$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

$$E(\mathbf{w}) = H_{F,M} \text{ where } H_{F,M} = -\sum_{f=1}^{B} \sum_{m=1}^{B} p_{f,m} \log(p_{f,m})$$

normalized histogram counts

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Aalto-universitetet
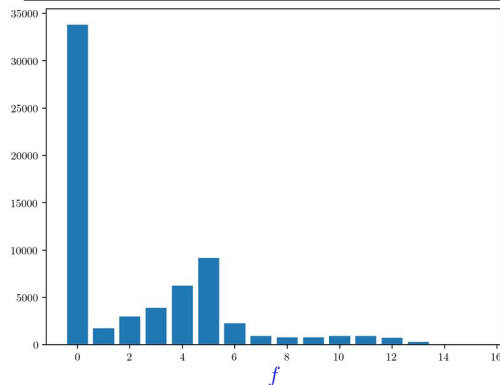Aalto University

$E(\mathbf{w})$

Diagnosing the problem

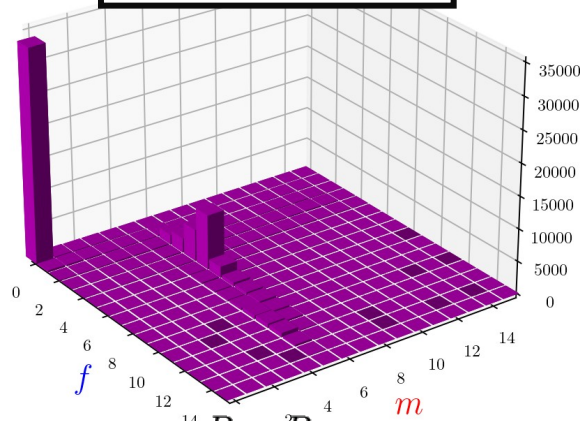**Question:** which image pair takes more bits to encode?
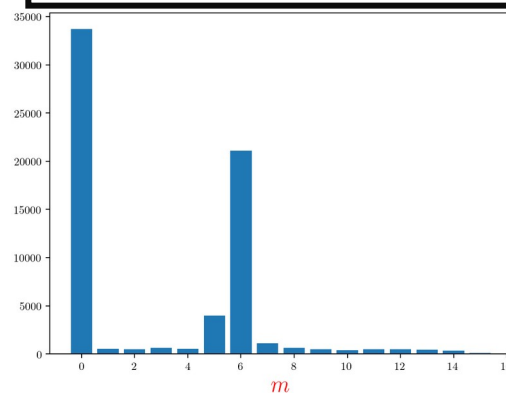
Solution

histogram fixed image

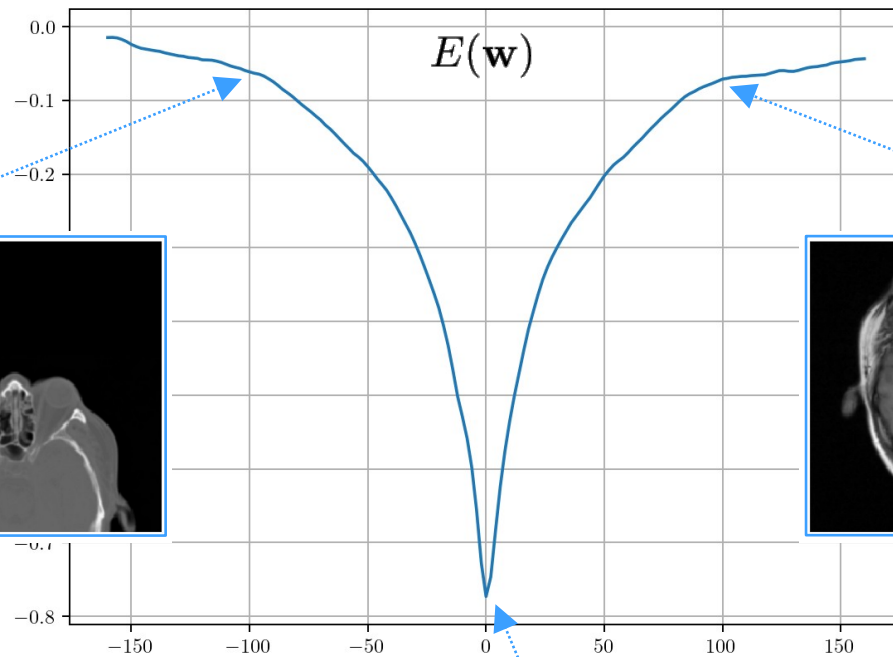$$H_F = -\sum_{f=1}^{B} p_f \log(p_f)$$

joint histogram

$$H_{F,M} = -\sum_{f=1}^{B} \sum_{m=1}^{B} p_{f,m} \log(p_{f,m})$$

histogram moving image

$$H_M = -\sum_{m=1}^{B} p_m \log(p_m)$$

$$E(\mathbf{w}) = H_{F,M} - H_F - H_M \quad \text{(negative "mutual information")}$$

$E(\mathbf{w})$

Numerical optimization

Find transformation parameters $\mathbf{w}$ **that minimize** $E(\mathbf{w})$