Modeling COVID-19 Outbreaks with the SIR Framework

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Abstract

This project explores the effectiveness of the classical Susceptible-Infected-Recovered (SIR) model in capturing and predicting the spread of the COVID-19 virus. While the SIR model is mathematically simpler and widely used in epidemiology, its underlying assumptions (such as homogeneous mixing and fixed parameters) raise questions about its applicability to real-world, stochastic disease dynamics. We begin by deriving the SIR model and simplifying it through dimensional analysis, introducing the basic reproduction number R_0 to understand its role in outbreak behavior. The system of differential equations is then solved numerically using Python to simulate disease trajectories under different parameters. To reflect real-world interventions, we extend the model by incorporating a small-scale vaccination term and apply regular perturbation methods to quantify its effects. Finally, using a publicly available COVID-19 cases across the United States from John Hopkins University, we perform parameter estimation by fitting the model to real infection trends, evaluating its predictive capability. By combining theoretical modeling and empirical data fitting, this study assesses the SIR model's strengths and limitations, while exploring how even low-rate interventions can shift epidemic outcomes.

Introduction and Problem Statement

The COVID-19 pandemic has been one of the most disruptive global health crises in modern history, affecting millions of lives and fundamentally reshaping education and cultural norms. Its rapid transmission, primarily through close human contact, and the typical recovery time of one to four weeks led to a wide range of policy interventions such as social distancing, mask mandates, quarantines, and large-scale vaccination efforts. In response to this unprecedented challenge, there has been renewed interest in mathematical modeling of infectious diseases to control, predict, and prevent large-scale spreads [1]. One of the most fundamental models in epidemiology is the Susceptible-Infected-Recovered (SIR) model, introduced by Kermack and McKendrick, which uses compartmental dynamics to capture the progression of an epidemic [5]. Despite its simplicity, the SIR model still remains as a valuable framework for modeling disease transmission and serves as a basis for more complex extensions. However, given the inherently stochastic nature of real-world outbreaks, it is worth questioning how well the SIR model reflect reality. Thus, our goal is to explore the dynamics of the classical SIR model, investigate how it changes under intervention strategies such as small-scale vaccination, and evaluate its ability to capture and predict the spread of COVID-19 using real-world data. Specifically, we aim to address the following question: *To what extent can the classical SIR model accurately describe the real-world spread of COVID-19, and how do small-scale interventions affect its predictions?*

Literature Review

There exist plenty of literature that defines, derives and solves the SIR, which is represented as a system of ordinary differential equations (ODE) given by [6]:

$$\begin{aligned}\frac{dS(t)}{dt} &= -\beta S(t)I(t),\\ \frac{dI(t)}{dt} &= \beta S(t)I(t) - \gamma I(t),\\ \frac{dR(t)}{dt} &= \gamma I(t).\end{aligned}$$

Where β is the rate of contact between the susceptible and the infected (transmission rate) and γ is the recovery rate. Due to its simplicity, such as a constant population size assumption [9], studies address limitations of this model, showing that small deviations from these model assumptions can lead to errors in forecasting epidemic durations [7]. Thus, many studies extend this SIR framework in order to improve its applicability to real-world outbreaks like COVID-19, such as including global dynamics of infection, which has shown to achieve higher predictive accuracies [1].

Proposed Mathematical Methods

To simplify our analysis, we will primarily focus on the classic SIR model. We propose the following methodology:

1. **Model Derivation**: We will begin with deriving the SIR model ODEs, discuss its application to epidemiology, and address the underlying assumptions.
2. **Dimensional Analysis**: To better understand the system's governing parameters and simplify the analysis, we perform dimensional analysis and nondimensionalize the model. This introduces the basic reproduction number $R_0 = \frac{\beta S_0}{\gamma}$, discussing how this gives insight on model behavior.
3. **Numerical Integration**: Leveraging Python packages, we will solve the system of ODEs and simulate disease spread over time for various parameter values.
4. **Vaccination Extension and Perturbation Methods**: We will briefly extend the SIR model by introducing a small vaccination term that removes a fraction ϵ of the susceptible population. We then apply regular perturbation theory to analyze how this low-rate intervention alters disease dynamics, yielding approximate solutions that could quantify the effect of vaccination.
5. **Data Fitting and Parameter Estimation**: Using real-world COVID-19 case data adopted from the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University [3], we will estimate the model's parameters (β and γ) by fitting the model that minimizes the mean squared error (MSE) between observed and simulated infection curves.

Anticipated Challenges (with mitigation proposals)

As briefly mentioned earlier, real-world COVID-19 data may be noisy, incomplete, or inconsistently reported across regions and time. In addition, having quality data with controlled interventions (such as small-scale vaccinations) are difficult and unfeasible to find in a large-scale. To best mitigate this, we will (1) focus on the time intervals with consistent reporting in the data, and (2) perform data cleaning pipelines to ensure quality of the data.
Another potential challenge could arise in parameter estimation. Since different combinations of parameters could potentially produce similar model outputs, estimating accurate β and γ can be a challenge. We aim to address this by using sensitivity analysis to better understand parameter influence.

Timeline for Completion

Week 6: Introduction, Tasks 1-3.
Week 7: Tasks 3, 4.
Week 8: Task 5, Conclusions, Future directions.
Week 9: Prepare presentation.
Week 10: Complete final report.

References

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