Qualitative Spatial Reasoning over Line-Region Relations

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Knowledge Representation
Seminar Presentation

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Motivation

9-Intersection

Snapshot Model

Smooth-Transition Model

Evaluation

Summary

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Smooth-Transition Model

Example

A pair of line-region relations that are conceptual neighbors (one can be obtained from the other via a "smooth transition")

Counterexample

A pair of line-region relations that **not** are conceptual neighbors

Formalization

A smooth transition occurs by moving around the line's

1. **boundary nodes**

   Q: Do they intersect with the same region part?
   Transition Rule 1 **if** Yes
   Transition Rule 2 **if** No

2. **interior**

   Transition Rule 3 **to** extend the intersection area *and*
   Transition Rule 4 **to** reduce it

What this means for the 9-intersection:
An entry or its adjacent entries gets changed from $\varnothing$ to $\neg\varnothing$ or v.v.

Formalization Extent of a Line Part

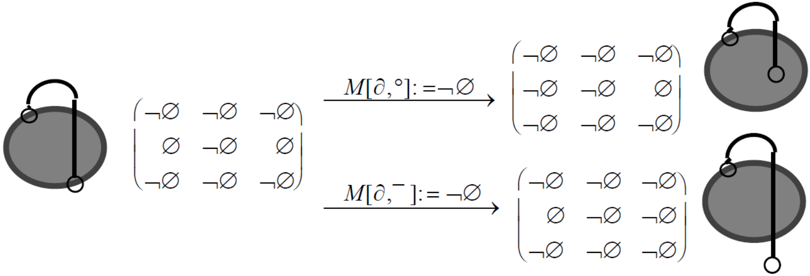
Definition (Extent of a line part i)

- Denoted by $\#M[i, \_]$
- Count of intersections betw. line part i and the region parts
- $\#M[i, \_]$ in the interval $[0 \dots 3]$

Transition Rule 1

If the line's two boundaries intersect with the **same** region part, then extend the intersection to either of the adjacent region parts:

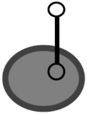
$$\#M[\delta, \_] = 1 \Rightarrow \forall i(M[\delta, i] = \neg\varnothing) : M_N[\delta, \text{adjacent}(i)] := \neg\varnothing$$



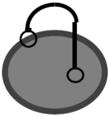
Transition Rule 2

If the line's two boundaries intersect with two **different** region parts then move either intersection to the adjacent region part:

$$\#M[\delta, \_] = 2 \Rightarrow \forall i(M[\delta, i] = \neg\varnothing) :$$
$$M_N[\delta, i] := \varnothing \textbf{ and } M_N[\delta, \mathrm{adjacent}(i)] := \neg\varnothing$$



$$\begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow[M[\partial,\partial]:=\neg\varnothing]{M[\partial,^{-}]:=\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



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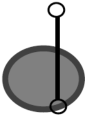
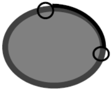


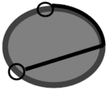
Transition Rule 3

Extend the line's interior-intersection to either of the adjacent region parts:

$$\forall i(M[^\circ, i] = \neg\varnothing) : M_N[^\circ, \mathrm{adjacent}(i)] := \neg\varnothing$$



$$\begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,^\circ]:=\neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



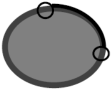
$$\begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,^-]:=\neg\varnothing} \begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$

Transition Rule 4

Reduce the line's interior intersection on either of the adjacent region parts.

$$\#M[^\circ,\_] = 2 \Rightarrow \forall i(M[^\circ,i] = \neg\varnothing) : M_N[^\circ,i] := \varnothing$$
$$\#M[^\circ,\_] = 3 \Rightarrow \forall i(i \neq \delta) : M_N[^\circ,i] := \varnothing$$

$$\begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,\partial]:=\varnothing} \begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



$$\begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,^-]:=\neg\varnothing} \begin{pmatrix} \varnothing & \varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$

Additional Consistency Constraints

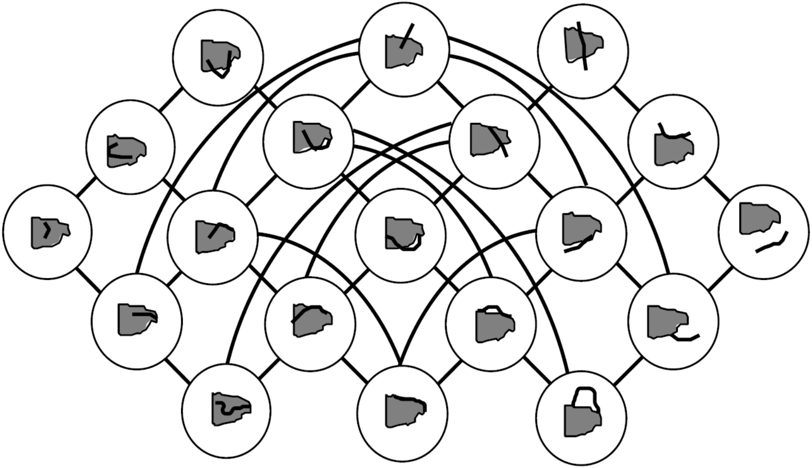
1. If the line's interior intersects with the region's interior *and* exterior, then the line's interior must also intersect with the region's boundary.

$$M[^\circ, ^\circ] = \neg\varnothing \textbf{ and } M[^\circ, ^-] = \neg\varnothing \Rightarrow M[^\circ, \delta] := \neg\varnothing$$

2. If the line's boundary intersects with the region's interior (exterior) then the line's interior must intersect with the region's interior (exterior) as well.

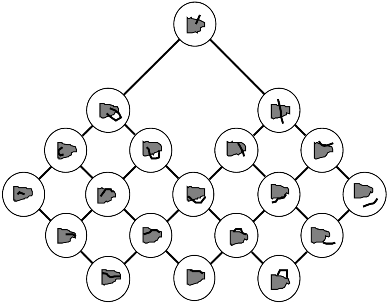
$$M[\delta, ^\circ] = \neg\varnothing \Rightarrow M[^\circ, ^\circ] := \neg\varnothing$$
$$M[\delta, ^-] = \neg\varnothing \Rightarrow M[^\circ, ^-] := \neg\varnothing$$

Resulting Neighborhood Graph

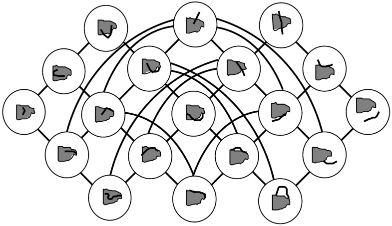


Comparison

Snapshot Model



Smooth-Transition Model



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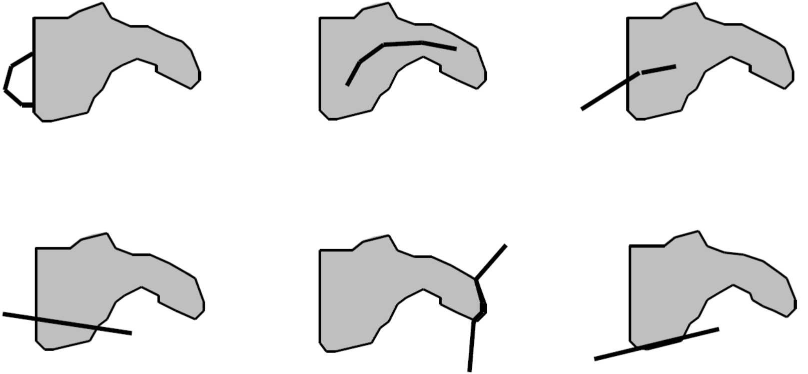
Setup

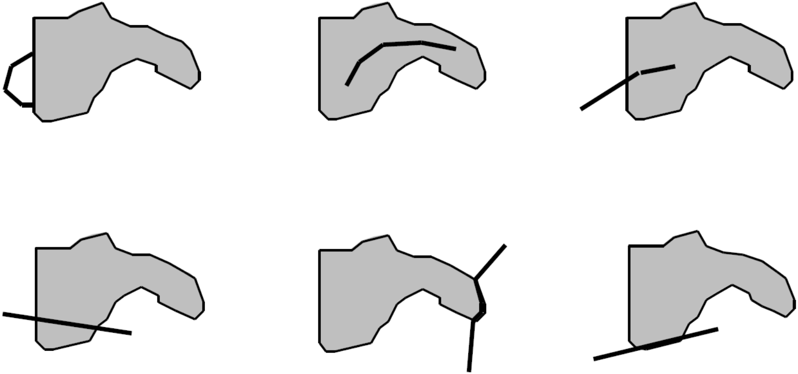
- 38 diagrams each of which showed a line and a region, said to be a road and a park respectively
- 2 geometrically distinct placements of the road corresponding to each of the 19 topologically distinct relations
- the parks were all the same size and shape
- some examples of the stimuli used in this research. The right and middle examples in the lower row are topologically identical but geometrically-distinct.

Setup (cont.)



Here are six of the diagrams that were used in this research, which are obviously all geometrically distinct. Find the pair that is topologically identical.

Setup (cont.)



Answer: The right and middle examples in the lower row are topologically identical but geometrically distinct.

Task

- group spatial relations between line and region, road and park (parks were all the same size and shape)
- arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group

Goal

Goal

- analyse how the subjects formed groups of similar relations
- check similarity with presented conceptual neighborhood models

Results

- The pairs that were neighbors by both snapshot and smooth-transition models were grouped from 0 to 78 times, with a mean of 33.6.
- Those pairs that were neighbors for smooth transitions-but not snapshots- were grouped between 0 and 66 times, with a mean of 17.3 (15.4 per cent).
- The two pairs that were snapshot neighbors-but not smooth transition neighbors- were grouped 10 and 16 times (mean = 14; 11.6 per cent).
- Perhaps most significant, however, is the fact that the 131 pairs that were neighbors by neither the snapshot model nor the smooth transitions were grouped an average of only 6.0 times by the subject (5.3 per cent of the maximum).
- Sixty pairs were never grouped by any of the 28 subjects nor any of the four possible stimulus pairs. The most frequently-grouped pair in this category was 54 times (48 per cent), but only 20 stimulus pairs with neither smooth transitions nor minimum snapshot difference were grouped 12

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Summary

Two Conceptual Neighborhood Models

1. Snapshot Model
2. Smooth-Transition Model

Finding: *Almost* identical Conceptual-Neighborhood Graphs

Human-Subject Experiment

Findings:

- confirmed that the conceptual neighborhoods identified by the two models correspond largely to the way humans conceptualize similarity about spatial relations
- **Finding:** groupings the subjects made indicate that the smooth-transition model captures more important aspects of the similarity of topological line-region relations than the snapshot model
- the majority of conceptual neighbors is the same in both

References

- Egenhofer, Max J., and David M. Mark. "Modelling conceptual neighbourhoods of topological line-region relations." *International journal of geographical information systems 9*, no. 5 (1995): 555-565.
- Mark, David M., and Max J. Egenhofer. "Modeling spatial relations between lines and regions: combining formal mathematical models and human subjects testing." *Cartography and geographic information systems 21*, no. 4 (1994): 195-212.