# Qualitative Spatial Reasoning over Line-Region Relations

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Knowledge Representation Seminar Presentation

### Motivation

9-Intersection

**Snapshot Model** 

**Smooth-Transition Model** 

**Evaluation** 

Motivation

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## **Snapshot Model**

**Smooth-Transition Model** 

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**Snapshot Model** 

### **Smooth-Transition Model**

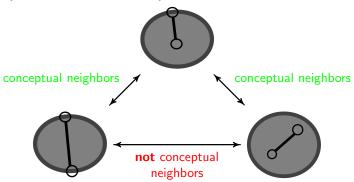
**Evaluation** 

## **Smooth-Transition Model**

#### **Smooth Transition**

An infinitesimally small deformation that changes the topological relation between the line and the region

## **Examples and Counterexamples**



## **Formalization**

### A smooth transition occurs by moving around the line's

### 1. boundary nodes

**Q:** Do they intersect with the same region part? Transition Rule 1 if Yes

Transition Rule 2 if No

#### 2. interior

Transition Rule 3 **to** extend the intersection area *and* Transition Rule 4 **to** reduce it

#### What this means for the 9-intersection:

An entry or its adjacent entries gets changed from  $\emptyset$  to  $\neg \emptyset$  or v.v.

## One More Thing...

## Definition (Extent of a line part i)

- ▶ Denoted by  $\#M[i, \_]$
- ► Count of intersections betw. line part *i* and the region parts
- ▶  $\#M[i, \_]$  in the interval [0...3]

If the line's two boundaries intersect with the **same** region part, then extend the intersection to either of the adjacent region parts:

$$\#M[\delta, \_] = 1 \Longrightarrow \forall i (M[\delta, i] = \neg \varnothing) : M_N[\delta, adjacent(i)] := \neg \varnothing$$

If the line's two boundaries intersect with two different region parts then move either intersection to the adjacent region part:

$$\#M[\delta, \_] = 2 \Longrightarrow \forall i (M[\delta, i] = \neg \varnothing) :$$
  
 $M_N[\delta, i] := \varnothing \land M_N[\delta, \operatorname{adjacent}(i)] := \neg \varnothing$ 

$$\begin{array}{c} M[\partial, \overline{\ \ }] := \varnothing & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \\ \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} & M[\partial, \partial] := \neg\varnothing \end{pmatrix} & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \\ \begin{pmatrix} \neg\varnothing & \varnothing & \neg\varnothing \end{pmatrix} & M[\partial, \circ] := \varnothing \\ M[\partial, \partial] := \neg\varnothing \end{pmatrix} & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \\ \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \\ \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \\ \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} & \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \end{pmatrix}$$





**Extend** the line's interior-intersection to either of the adjacent region parts:

$$\forall i(M[^{\circ}, i] = \neg \varnothing) : M_N[^{\circ}, adjacent(i)] := \neg \varnothing$$

**Reduce** the line's interior intersection on either of the adjacent region parts.

$$#M[^{\circ}, \_] = 2 \Longrightarrow \forall i (M[^{\circ}, i] = \neg \varnothing) : M_{N}[^{\circ}, i] := \varnothing$$
$$#M[^{\circ}, \_] = 3 \Longrightarrow \forall i (i \neq \delta) : M_{N}[^{\circ}, i] := \varnothing$$

$$\begin{pmatrix}
\varnothing & \neg \varnothing & \neg \varnothing \\
\varnothing & \neg \varnothing & \varnothing \\
\varnothing & \neg \varnothing & \varnothing \\
\neg \varnothing & \neg \varnothing & \neg \varnothing
\end{pmatrix}$$

$$\xrightarrow{M[°, ^{\circ}] := \varnothing}
\begin{pmatrix}
\varnothing & \neg \varnothing & \varnothing \\
\varnothing & \neg \varnothing & \varnothing \\
\neg \varnothing & \neg \varnothing & \neg \varnothing
\end{pmatrix}$$

$$\xrightarrow{M[°, ^{\circ}] := \neg \varnothing}
\begin{pmatrix}
\varnothing & \varnothing & \neg \varnothing \\
\varnothing & \neg \varnothing & \varnothing \\
\varnothing & \neg \varnothing & \varnothing \\
\neg \varnothing & \neg \varnothing & \neg \varnothing
\end{pmatrix}$$

## **Additional Consistency Constraints**

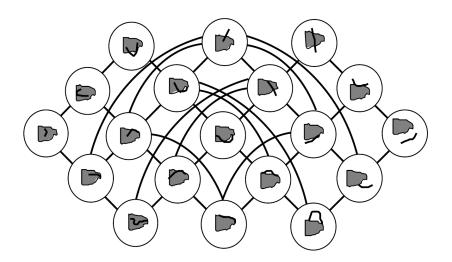
 If the line's interior intersects with the region's interior and exterior, then the line's interior must also intersect with the region's boundary.

$$M[^{\circ}, ^{\circ}] = \neg \varnothing \wedge M[^{\circ}, ^{-}] = \neg \varnothing \Longrightarrow M[^{\circ}, \delta] := \neg \varnothing$$

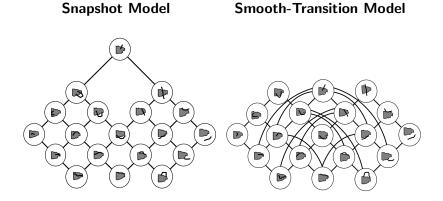
2. If the line's boundary intersects with the region's interior (exterior) then the line's interior must intersect with the region's interior (exterior) as well.

$$M[\delta,^{\circ}] = \neg \varnothing \Longrightarrow M[^{\circ},^{\circ}] := \neg \varnothing$$
$$M[\delta,^{-}] = \neg \varnothing \Longrightarrow M[^{\circ},^{-}] := \neg \varnothing$$

# Resulting Neighborhood Graph



## Comparison



Motivation

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**Snapshot Model** 

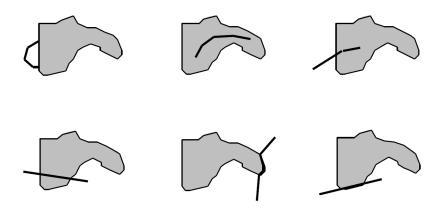
**Smooth-Transition Model** 

### **Evaluation**

## Setup

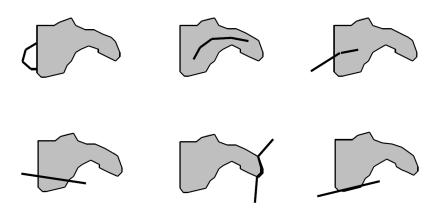
- 2 geometrically distinct placements of the line for each of the
   19 topologically distinct relations
- ► a total of 38 diagrams each showing a line and a region
- ▶ line  $\rightarrow$  road, region  $\rightarrow$  park
- ▶ parks in all diagrams same size and shape
- ► 28 participants

# Setup (cont.)



**Q:** Find the pair that is topologically identical from among all geometrically distinct diagrams.

# Setup (cont.)



Q: Find the pair that is topologically identical from among all geometrically distinct diagrams.

A: The right and middle examples in the lower row.

### **Task**

► arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group

## Goal

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- ► analyse how the subjects formed groups of similar relations
- check similarity with presented conceptual neighborhood models

## Results

Each spatial relation could be grouped as many as 112 times (4 pairs times 28 subjects) with each other relation.

## Number of times conceptual neighbors are grouped:

	1	min	max	mean	% <sup>2</sup>
snapshot model only	2	10	16	13.0	11.6
smooth-transition model only	12	0	66	17.3	15.4
both models	26	0	78	33.6	29.5
neither model	131	-	-	6.0	5.3

<sup>&</sup>lt;sup>1</sup>Number of relations that are conceptual neighbors

<sup>&</sup>lt;sup>2</sup>percentage = mean / 112

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**Evaluation** 

## Summary

## Two Conceptual Neighborhood Models:

- 1. Snapshot Model
- 2. Smooth-Transition Model

Finding: Almost identical Conceptual-Neighborhood Graphs

### Findings from the Human-Subject Experiment:

- models correspond largely to the way humans conceptualize similarity about line-region relations
- smooth-transition model captures more aspects of the similarity of topological line-region relations than the snapshot model

### References

- Egenhofer, Max J., and David M. Mark. "Modelling conceptual neighbourhoods of topological line-region relations." *International journal of geographical information systems 9*, no. 5 (1995): 555-565.
- Mark, David M., and Max J. Egenhofer. "Modeling spatial relations between lines and regions: combining formal mathematical models and human subjects testing." Cartography and geographic information systems 21, no. 4 (1994): 195-212.