Qualitative Spatial Reasoning over Line-Region Relations

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Knowledge Representation
Seminar Presentation

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Smooth-Transition Model

Smooth Transitions

Inhalt...

Conceptual Neighborhood

A line-region relation is topologically different from another one by an infinitesimally small deformation of its geometry.

Possible Changes

A total of four rules:

- Moving around a line's boundary nodes

Rule 1 Line's two boundary nodes intersect with same region part
Rule 2 Line's two boundary nodes intersect with different region part

- Moving around a line's interior

Rule 3 Extend line's interior-intersection partially
Rule 4 Reduce line's interior-intersection partially

In terms of 9-Intersection, a smooth transition means that an intersection or its adjacent intersection gets changed from empty to non-empty, or reverse.

Extent of a Line Part

Extent of a part i: Denoted by $\#M[i, \_]$; number of non-empty intersections between i and th three parts of the second object. Define extent of a part i Draw 9-intersection model on the board for reference

- The extent of a line's interior with respect to a region is in the interval of 1 to 3, the extent of the lines boundary is either 1 (if both nodes are located in the same region part) or 2 (if the nodes are located in different parts of the region), and the extent of a line's interior is always 3.

Moving the Line’s Boundaries

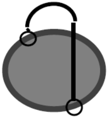
Rule 1

If the line’s two boundaries intersect with the same region part, then extend the intersection to either of the adjacent region parts.

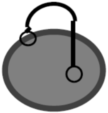
Formalization

$$\#M[\delta, \_] = 1 \Rightarrow \forall i(M[\delta, i] = \neg\varnothing) : M_N[\delta, \mathsf{adjacent}(i)] := \neg\varnothing$$

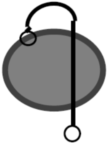
Example



$$\begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\partial,^\circ]:=\neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



$$\xrightarrow{M[\partial,^-]:=\neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



Moving the Line's Boundaries

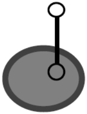
Rule 2

If the line's two boundaries intersect with two different region parts then move either intersection to the adjacent region part.

Formalization

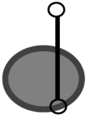
$$\#M[\delta, \_] = 2 \Rightarrow \forall i(M[\delta, i] = \neg\varnothing) : M_N[\delta, i] := \varnothing \textbf{ and } M_N[\delta, \mathrm{adjacent}(i)] := \neg\varnothing$$

Example



$$\begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow[M[\partial,\partial]:=\neg\varnothing]{M[\partial,^{-}]:=\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$

$$\xrightarrow[M[\partial,\partial]:=\neg\varnothing]{M[\partial,^{\circ}]:=\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



Moving the Line's Interior

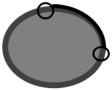
Rule 1

Extend the line's interior-intersection to either of the adjacent region parts.

Formalization

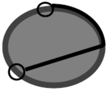
$$\forall i (M[^\circ, i] = \neg\emptyset) : M_N[^\circ, \text{adjacent}(i)] := \neg\emptyset$$

Example



$$\begin{pmatrix} \emptyset & \neg\emptyset & \emptyset \\ \emptyset & \neg\emptyset & \emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{pmatrix}$$

$$\xrightarrow{M[^\circ,^\circ]:=\neg\emptyset} \begin{pmatrix} \neg\emptyset & \neg\emptyset & \emptyset \\ \emptyset & \neg\emptyset & \emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{pmatrix}$$



$$\xrightarrow{M[^\circ,^-]:=\neg\emptyset} \begin{pmatrix} \emptyset & \neg\emptyset & \neg\emptyset \\ \emptyset & \neg\emptyset & \emptyset \\ \neg\emptyset & \neg\emptyset & \neg\emptyset \end{pmatrix}$$

Moving the Line's Interior

Rule 2

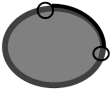
Reduce the line's interior intersection on either of the adjacent region parts.

Formalization

$$\#M[\circ, \_] = 2 \Rightarrow \forall i (M[\circ, i] = \neg\varnothing) : M_N[\circ, i] := \varnothing$$
$$\#M[\circ, \_] = 3 \Rightarrow \forall i (i \neq \delta) : M_N[\circ, i] := \varnothing$$

Example

$$\begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\circ,\partial] := \varnothing} \begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



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Consistency Constraints

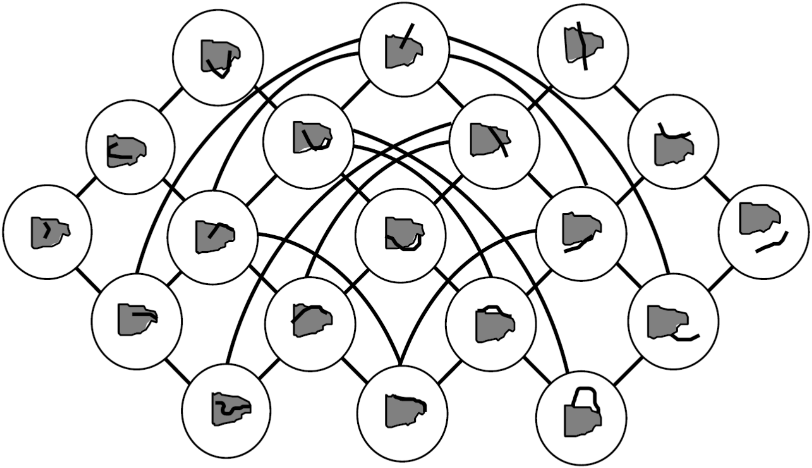
1. If the line's interior intersects with the region's interior *and* exterior, then the line's interior must also intersect with the region's boundary.

$$M[^\circ, ^\circ] = \neg\varnothing \text{and} M[^\circ, ^-] = \neg\varnothing \Rightarrow M[^\circ, \delta] := \neg\varnothing$$

2. If the line's boundary intersects with the region's interior (exterior) then the line's interior must intersect with the region's interior (exterior) as well.

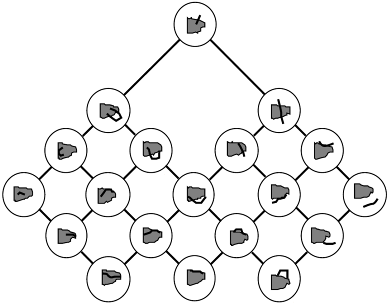
$$M[\delta, ^\circ] = \neg\varnothing M[^\circ, ^\circ] := \neg\varnothing$$
$$M[\delta, ^-] = \neg\varnothing M[^\circ, ^-] := \neg\varnothing$$

Resulting Neighborhood Graph

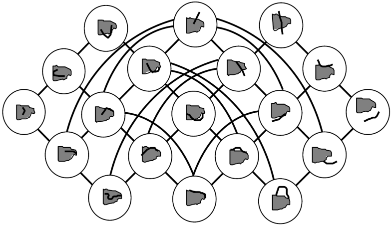


Juxtaposition of Neighborhood Graphs

Snapshot Model



Smooth-Transition Model



Juxtaposition of Neighborhood Graphs

- 19 line-region relations $\rightarrow$ 171 distinct pairs of relations that can possibly be conceptual neighbors
- 26 under snapshot model and the smooth-transition model
- 2 under snapshot model
- 12 under smooth-transition model
- 131 pairs under neither model

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Experiment

- GOAL: Within the context of different models for conceptual neighbors, it is particularly enlightening to analyse how the subjects formed groups of similar relations.
- group spatial relations between line and region, road and park (parks were all the same size and shape)
- 28 subjects performing tasks
- 38 diagrams 2 geometrically distinct placements of the road corresponding to each of the 19 topologically distinct relations TODOExample from Mark1994
- each spatial relation could be grouped as many as 112 times (4 pairs times 28 subjects) with each other relation

Participants

Results

- The pairs that were neighbors by both snapshot and smooth-transition models were grouped from 0 to 78 times, with a mean of 33.6.
- Those pairs that were neighbors for smooth transitions-but not snapshots- were grouped between 0 and 66 times, with a mean of 17.3 (15.4 per cent).
- The two pairs that were snapshot neighbors-but not smooth transition neighbors- were grouped 10 and 16 times (mean = 14; 11.6 per cent).
- Perhaps most significant, however, is the fact that the 131 pairs that were neighbors by neither the snapshot model nor the smooth transitions were grouped an average of only 6.0 times by the subject (5.3 per cent of the maximum).
- Sixty pairs were never grouped by any of the 28 subjects nor any of the four possible stimulus pairs. The most frequently-grouped pair in this category was 54 times (48 per cent), but only 20 stimulus pairs with neither smooth

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References