Qualitative Spatial Reasoning over Line-Region Relations

Leena and Sibel

Knowledge Representation
Seminar Presentation

Agenda

Motivation

9-Intersection

Snapshot Model

Smooth-Transition Model

Evaluation

Summary

Motivation

- Modeling spatial relations
- How do humans conceptualize spatial relations?
- Strong correlation between Perceptual space and Language Space
- Understanding how language structures space

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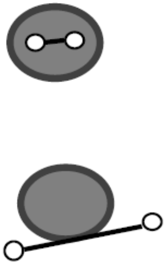
9-Intersection

Goal

A computational model to describe conceptual neighborhoods and enable the definition of a similarity metric for line region relations.

Conceptual Similarity



Formal Definitions

Line

A sequence of 1...n connected cells between two geometrically independent nodes such that they neither cross each other nor form cycles.

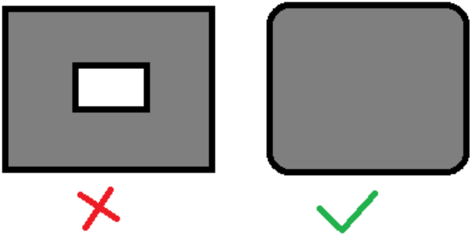
- Interior, Boundary, Exterior

Formal Definitions(contd.,)

Region

A region is defined as a connected, homogeneously 2-dimensional 2-cell. Its boundary forms a Jordan curve separating the region's exterior from its interior.

- Interior, Boundary, Exterior



Adjacency

Topological adjacency

- Adjacent(Interior A^0) $= \partial A$
- Adjacent(Boundary ∂A) $= A^0 and A^-$
- Adjacent(Exterior A^-) $= \partial A$

9-Intersection(contd..,)

Topological adjacency

- 9 intersections between the different topological parts of a line and a region

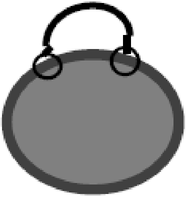
The 9-intersection Matrix(M)

$$\begin{pmatrix} L^0 \cap R^0 & L^0 \cap \partial R & L^0 \cap R^- \\ \partial L \cap R^0 & \partial L \cap \partial R & \partial L \cap R^- \\ L^- \cap R^0 & L^- \cap \partial R & L^- \cap R^- \end{pmatrix}$$

9-Intersection(contd..,)

- Binary assignment to intersections($\emptyset, -\emptyset$)
- **512** possible instances of M
- **19** of 512 instances can actually be realized.

Example



$$\begin{pmatrix} L^0 \cap R^0 & L^0 \cap \partial R & L^0 \cap R^- \\ \partial L \cap R^0 & \partial L \cap \partial R & \partial L \cap R^- \\ L^- \cap R^0 & L^- \cap \partial R & L^- \cap R^- \end{pmatrix}$$

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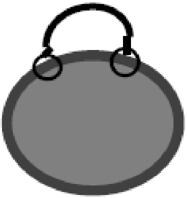

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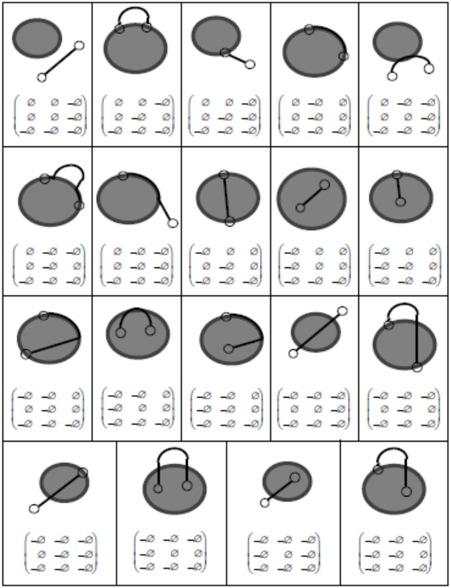
Compute the values of the matrix...

All 19 instances of M



Agenda

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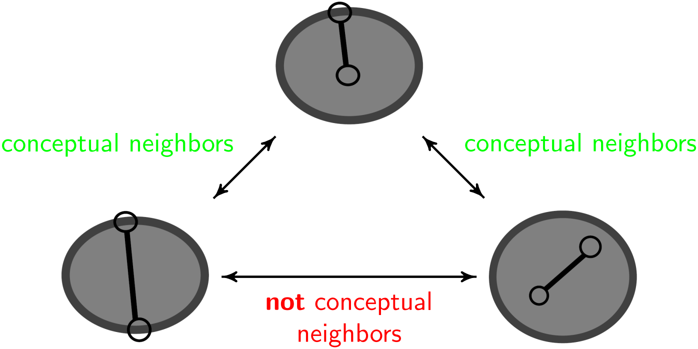
Smooth-Transition Model

Smooth Transition

An infinitesimally small deformation that changes the topological relation between the line and the region

Examples and Counterexamples



Formalization

A smooth transition occurs by moving around the line's

1. **boundary nodes**

   Q: Do they intersect with the same region part?
   Transition Rule 1 **if** Yes
   Transition Rule 2 **if** No

2. **interior**

   Transition Rule 3 **to** extend the intersection area *and*
   Transition Rule 4 **to** reduce it

What this means for the 9-intersection:
An entry or its adjacent entries gets changed from $\varnothing$ to $\neg\varnothing$ or v.v.

One More Thing...

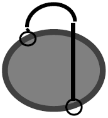
Definition (Extent of a line part i)

- Denoted by $\#M[i, \_]$
- Count of intersections betw. line part i and the region parts
- $\#M[i, \_]$ in the interval $[0 \ldots 3]$

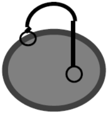
Transition Rule 1

If the line's two boundaries intersect with the **same** region part, then extend the intersection to either of the adjacent region parts:

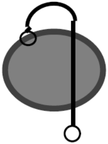
$$\#M[\delta, \_] = 1 \Longrightarrow \forall i (M[\delta, i] = \neg\varnothing) : M_N[\delta, \text{adjacent}(i)] := \neg\varnothing$$



$$\begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\partial, ^\circ] := \neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



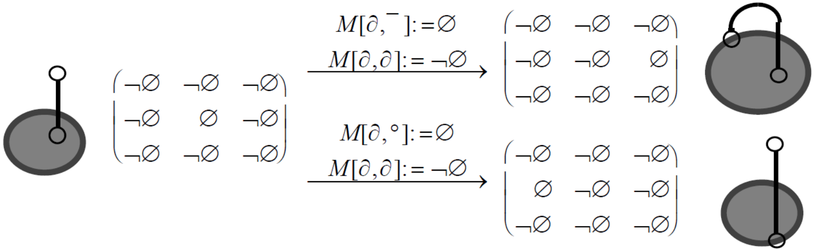
$$\begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\partial, ^-] := \neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \neg\varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



Transition Rule 2

If the line's two boundaries intersect with two **different** region parts then move either intersection to the adjacent region part:

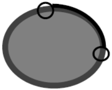
$$\#M[\delta, \_] = 2 \Longrightarrow \forall i(M[\delta, i] = \neg\varnothing) :$$
$$M_N[\delta, i] := \varnothing \wedge M_N[\delta, \mathrm{adjacent}(i)] := \neg\varnothing$$



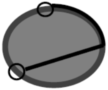
Transition Rule 3

Extend the line's interior-intersection to either of the adjacent region parts:

$$\forall i (M[^\circ, i] = \neg\varnothing) : M_N[^\circ, \text{adjacent}(i)] := \neg\varnothing$$



$$\begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,^\circ]:=\neg\varnothing} \begin{pmatrix} \neg\varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



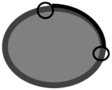
$$\begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[^\circ,^-]:=\neg\varnothing} \begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$

Transition Rule 4

Reduce the line's interior intersection on either of the adjacent region parts.

$$\#M[\circ,\_] = 2 \Longrightarrow \forall i(M[\circ,i] = \neg\varnothing) : M_N[\circ,i] := \varnothing$$
$$\#M[\circ,\_] = 3 \Longrightarrow \forall i(i \neq \delta) : M_N[\circ,i] := \varnothing$$

$$\begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\circ,\partial]:=\varnothing} \begin{pmatrix} \varnothing & \neg\varnothing & \varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$



$$\begin{pmatrix} \varnothing & \neg\varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix} \xrightarrow{M[\circ,^-]:=\neg\varnothing} \begin{pmatrix} \varnothing & \varnothing & \neg\varnothing \\ \varnothing & \neg\varnothing & \varnothing \\ \neg\varnothing & \neg\varnothing & \neg\varnothing \end{pmatrix}$$

Additional Consistency Constraints

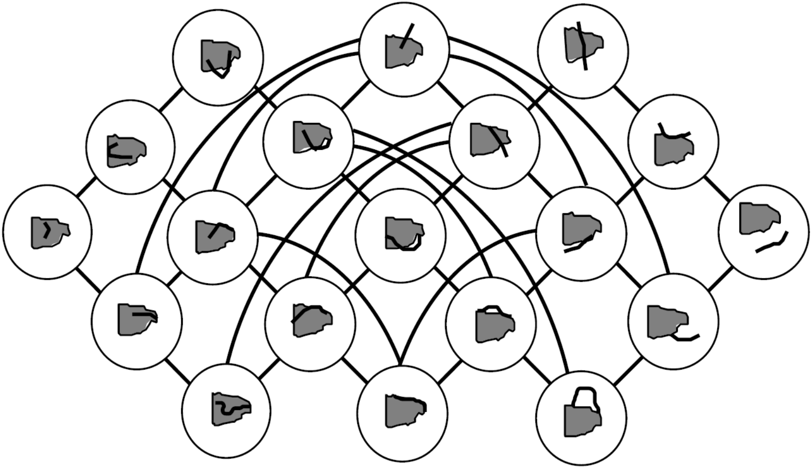
1. If the line's interior intersects with the region's interior *and* exterior, then the line's interior must also intersect with the region's boundary.

$$M[^\circ, ^\circ] = \neg\varnothing \wedge M[^\circ, ^-] = \neg\varnothing \Longrightarrow M[^\circ, \delta] := \neg\varnothing$$

2. If the line's boundary intersects with the region's interior (exterior) then the line's interior must intersect with the region's interior (exterior) as well.

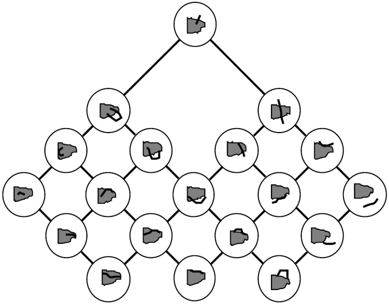
$$M[\delta, ^\circ] = \neg\varnothing \Longrightarrow M[^\circ, ^\circ] := \neg\varnothing$$
$$M[\delta, ^-] = \neg\varnothing \Longrightarrow M[^\circ, ^-] := \neg\varnothing$$

Resulting Neighborhood Graph

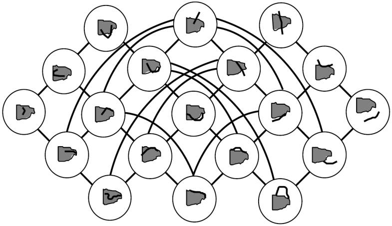


Comparison

Snapshot Model



Smooth-Transition Model



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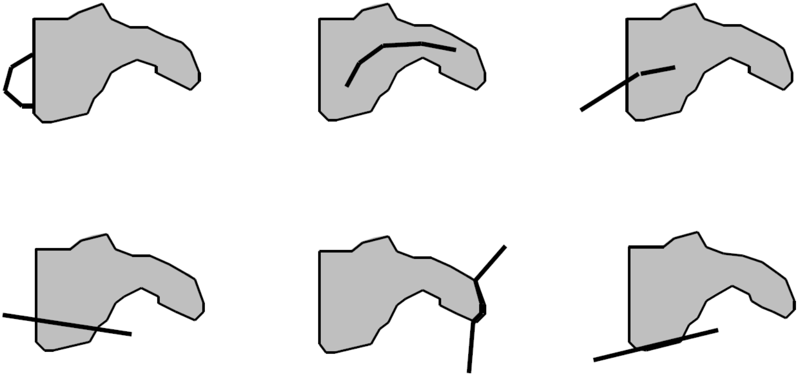
Setup

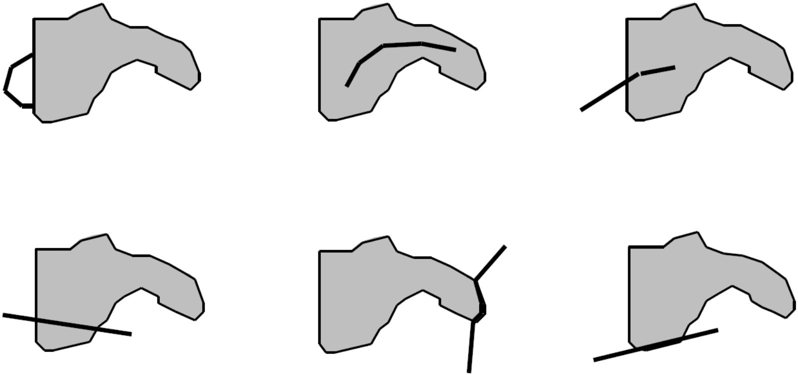
- **2** geometrically distinct placements of the line for each of the **19** topologically distinct relations
- a total of **38** diagrams each showing a line and a region
- line $\rightarrow$ road, region $\rightarrow$ park
- parks in all diagrams same size and shape
- 28 participants

Setup (cont.)



Q: Find the pair that is topologically identical from among all geometrically distinct diagrams.

Setup (cont.)



Q: Find the pair that is topologically identical from among all geometrically distinct diagrams.

A: The right and middle examples in the lower row.

Task

- arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group

Goal

Goal

- analyse how the subjects formed groups of similar relations
- check similarity with presented conceptual neighborhood models

Results

Each spatial relation could be grouped as many as 112 times (4 pairs times 28 subjects) with each other relation.

Number of times conceptual neighbors are grouped:

	...[1]	*min*	*max*	*mean*	*%*[2]
snapshot model only	2	10	16	13.0	11.6
smooth-transition model only	12	0	66	17.3	15.4
both models	26	0	78	33.6	29.5
neither model	131	-	-	6.0	5.3

[1] Number of relations that are conceptual neighbors
[2] percentage = mean / 112

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Two Conceptual Neighborhood Models:

1. Snapshot Model
2. Smooth-Transition Model

Finding: *Almost* identical Conceptual-Neighborhood Graphs

Findings from the Human-Subject Experiment:

- models correspond largely to the way humans conceptualize similarity about line-region relations
- smooth-transition model captures more aspects of the similarity of topological line-region relations than the snapshot model

References

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