

# **Qualitative Spatial Reasoning over Line-Region Relations**

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Knowledge Representation  
Seminar Presentation

# Agenda

## Motivation

## 9-Intersection

## Snapshot Model

## Smooth-Transition Model

## Evaluation

## Summary

# Motivation

- ▶ Modeling spatial relations
- ▶ How do humans conceptualize spatial relations?
- ▶ Strong correlation between Perceptual space and Language Space
- ▶ Understanding how language structures space

# Agenda

Motivation

**9-Intersection**

Snapshot Model

Smooth-Transition Model

Evaluation

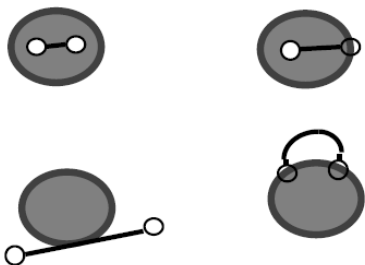
Summary

## 9-Intersection

### Goal

A computational model to describe conceptual neighborhoods and enable the definition of a similarity metric for line region relations.

### Conceptual Similarity



# Formal Definitions

## Line

A sequence of  $1 \dots n$  connected cells between two geometrically independent nodes such that they neither cross each other nor form cycles.

- Interior, Boundary, Exterior

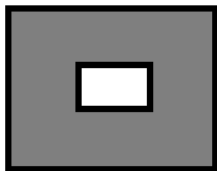


# Formal Definitions(contd.,)

## Region

A region is defined as a connected, homogeneously 2-dimensional 2-cell. Its boundary forms a Jordan curve separating the region's exterior from its interior.

- Interior, Boundary, Exterior



# Adjacency

## Topological adjacency

- ▶  $\text{Adjacent}(\text{Interior } A^0) = \partial A$
- ▶  $\text{Adjacent}(\text{Boundary } \partial A) = A^0 \text{ and } A^-$
- ▶  $\text{Adjacent}(\text{Exterior } A^-) = \partial A$



## 9-Intersection(contd..)

### Topological adjacency

- 9 intersections between the different topological parts of a line and a region

### The 9-intersection Matrix(M)

$$\begin{pmatrix} L^0 \cap R^0 & L^0 \cap \partial R & L^0 \cap R^- \\ \partial L \cap R^0 & \partial L \cap \partial R & \partial L \cap R^- \\ L^- \cap R^0 & L^- \cap \partial R & L^- \cap R^- \end{pmatrix}$$

## 9-Intersection(contd..)

- ▶ Binary assignment to intersections( $\emptyset, -\emptyset$ )
- ▶ **512** possible instances of M
- ▶ **19** of 512 instances can actually be realized.

### Example



$$\begin{pmatrix} L^0 \cap R^0 & L^0 \cap \partial R & L^0 \cap R^- \\ \partial L \cap R^0 & \partial L \cap \partial R & \partial L \cap R^- \\ L^- \cap R^0 & L^- \cap \partial R & L^- \cap R^- \end{pmatrix}$$

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Compute the values of the matrix...



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# Snapshot Model

- ▶ A model of conceptual neighborhood among topological relations between a line and a region.

## Characteristics

- ▶ No prior knowledge of the potential transformations that could lead from one configuration to the other.
- ▶ Comparison on the basis of a pre-defined distance metric

## Differences of Intersections( $\emptyset = 0$ , $\neg\emptyset = 1$ )

$$\begin{array}{rclcl} \emptyset & - & \emptyset & = & 0 \\ \neg\emptyset & - & \neg\emptyset & = & 0 \\ \emptyset & - & \neg\emptyset & = & -1 \\ \neg\emptyset & - & \emptyset & = & 1 \end{array}$$

# Defining topological neighbors

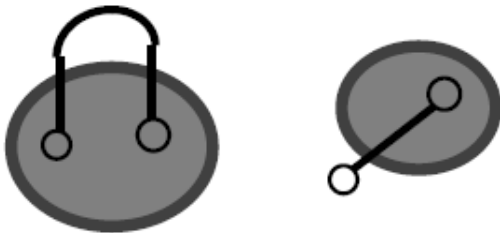
Distance between any two relationships  $r_A, r_B$  is given by:

$$T_{r_A, r_B} = \sum_{i=0}^{\bar{\quad}} \sum_{j=0}^{\bar{\quad}} |M_A[i, j] - M_B[i, j]| \quad (1)$$

- ▶ It is the count of differences of empty/non empty entries of corresponding elements in the 9 intersections.
- ▶ Shortest Nonzero distance between relations is 1
- ▶ Spatial relations with the shortest non zero distance are considered topological neighbors.

## Example 1

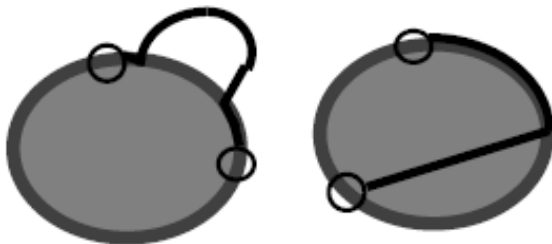
Are these relations neighbors according to the snapshot model?



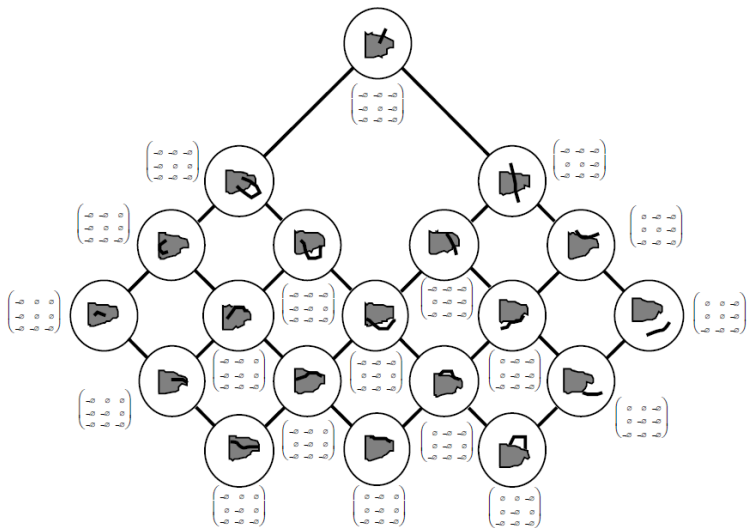


## Example 2

Another example... Are these relations topological neighbors ?



# Conceptual neighborhoods derived from the snapshot model



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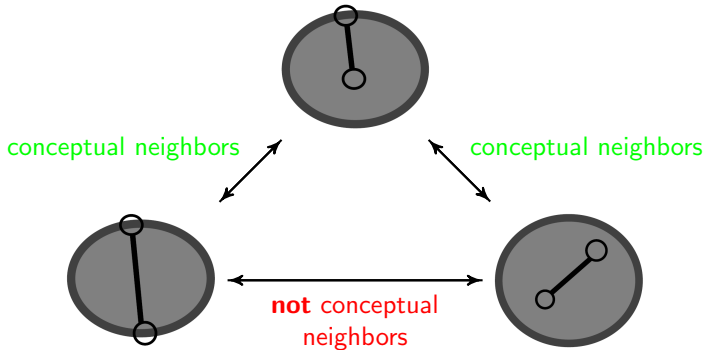
Summary

# Smooth-Transition Model

## Smooth Transition

An infinitesimally small deformation that changes the topological relation between the line and the region

## Examples and Counterexamples



# Formalization

A smooth transition occurs by moving around the line's

## 1. boundary nodes

**Q:** Do they intersect with the same region part?

Transition Rule 1 **if** Yes

Transition Rule 2 **if** No

## 2. interior

Transition Rule 3 **to** extend the intersection area *and*

Transition Rule 4 **to** reduce it

**What this means for the 9-intersection:**

An entry or its adjacent entries gets changed from  $\emptyset$  to  $\neg\emptyset$  or v.v.

# One More Thing...

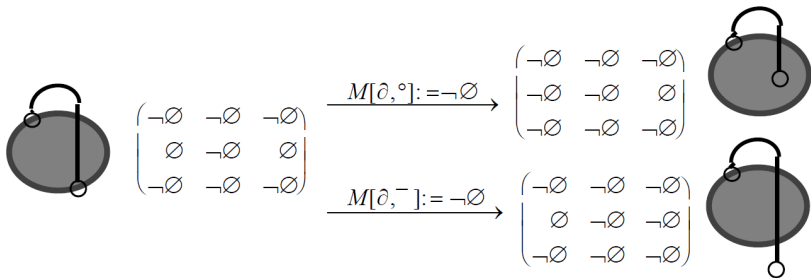
## Definition (Extent of a line part $i$ )

- ▶ Denoted by  $\#M[i, \_]$
- ▶ Count of intersections betw. line part  $i$  and the region parts
- ▶  $\#M[i, \_]$  in the interval  $[0 \dots 3]$

# Transition Rule 1

If the line's two boundaries intersect with the **same** region part, then extend the intersection to either of the adjacent region parts:

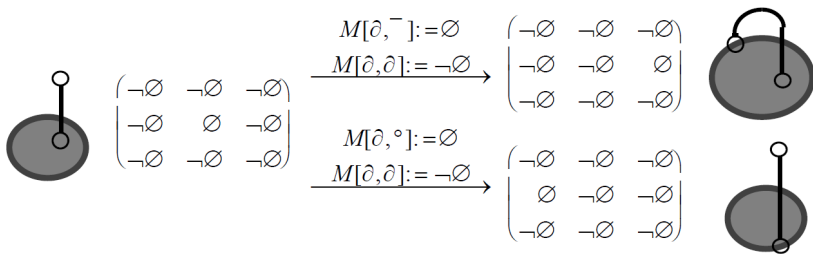
$$\#M[\delta, \_]=1 \Rightarrow \forall i(M[\delta, i] = \neg\emptyset) : M_N[\delta, \text{adjacent}(i)] := \neg\emptyset$$



## Transition Rule 2

If the line's two boundaries intersect with two **different** region parts then move either intersection to the adjacent region part:

$$\begin{aligned} \#M[\delta, \_ ] = 2 &\implies \forall i (M[\delta, i] = \neg\emptyset) : \\ M_N[\delta, i] &:= \emptyset \wedge M_N[\delta, \text{adjacent}(i)] := \neg\emptyset \end{aligned}$$

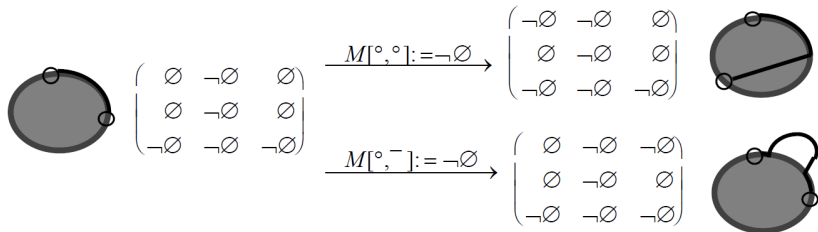




# Transition Rule 3

**Extend** the line's interior-intersection to either of the adjacent region parts:

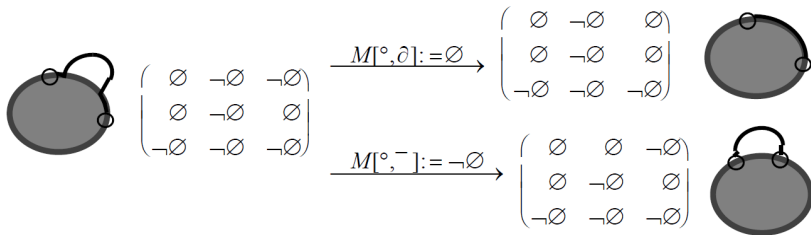
$$\forall i (M[^\circ, i] = \neg\emptyset) : M_N[^\circ, \text{adjacent}(i)] := \neg\emptyset$$



## Transition Rule 4

**Reduce** the line's interior intersection on either of the adjacent region parts.

$$\begin{aligned} \#M[^\circ, -] = 2 &\implies \forall i (M[^\circ, i] = \neg\emptyset) : M_N[^\circ, i] := \emptyset \\ \#M[^\circ, -] = 3 &\implies \forall i (i \neq \delta) : M_N[^\circ, i] := \emptyset \end{aligned}$$



## Additional Consistency Constraints

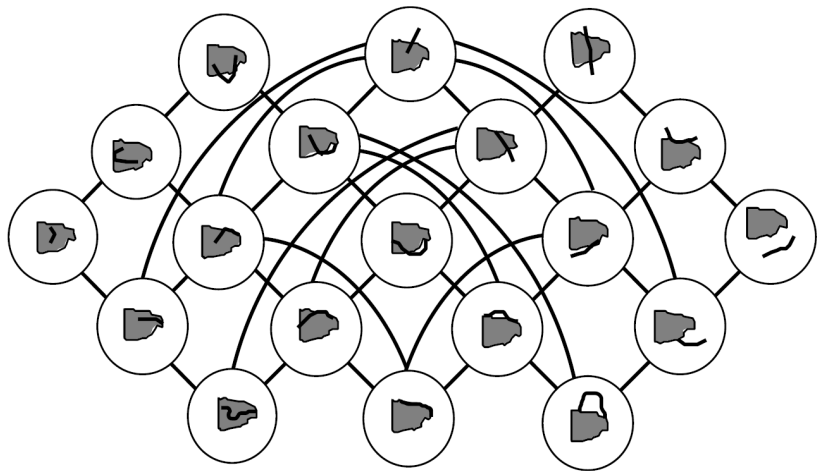
1. If the line's interior intersects with the region's interior *and* exterior, then the line's interior must also intersect with the region's boundary.

$$M[^\circ, ^\circ] = \neg\emptyset \wedge M[^\circ, ^-] = \neg\emptyset \implies M[^\circ, \delta] := \neg\emptyset$$

2. If the line's boundary intersects with the region's interior (exterior) then the line's interior must intersect with the region's interior (exterior) as well.

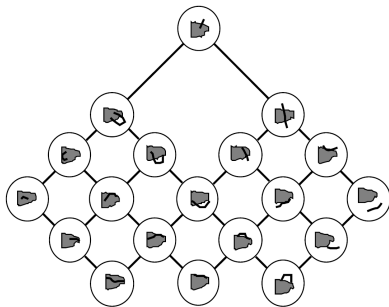
$$\begin{aligned} M[\delta, ^\circ] = \neg\emptyset &\implies M[^\circ, ^\circ] := \neg\emptyset \\ M[\delta, ^-] = \neg\emptyset &\implies M[^\circ, ^-] := \neg\emptyset \end{aligned}$$

## Resulting Neighborhood Graph

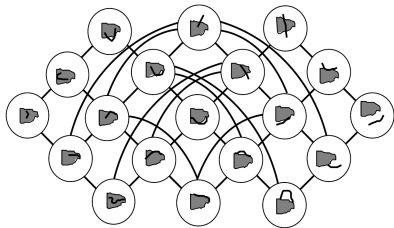


# Comparison

**Snapshot Model**



**Smooth-Transition Model**



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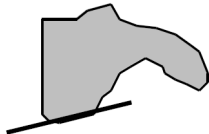
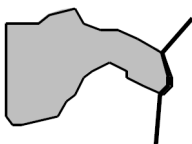
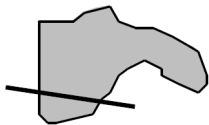
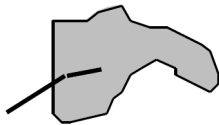
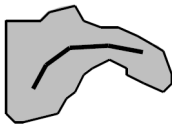
**Evaluation**

Summary

# Setup

- ▶ **2** geometrically distinct placements of the line for each of the **19** topologically distinct relations
- ▶ a total of **38** diagrams each showing a line and a region
- ▶ line → road, region → park
- ▶ parks in all diagrams same size and shape
- ▶ 28 participants

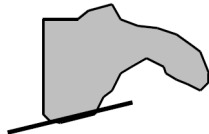
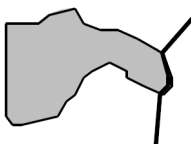
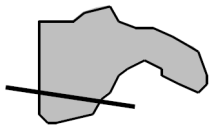
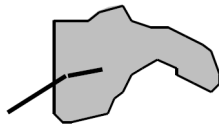
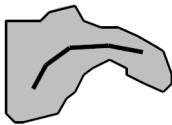
## Setup (cont.)



**Q:** Find the pair that is topologically identical from among all geometrically distinct diagrams.



## Setup (cont.)



**Q:** Find the pair that is topologically identical from among all geometrically distinct diagrams.

**A:** The right and middle examples in the lower row.

# Task

- ▶ arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group

# Goal

## Goal

- ▶ analyse how the subjects formed groups of similar relations
- ▶ check similarity with presented conceptual neighborhood models

# Results

Each spatial relation could be grouped as many as 112 times (4 pairs times 28 subjects) with each other relation.

## Number of times conceptual neighbors are grouped:

	... <sup>1</sup>	<i>min</i>	<i>max</i>	<i>mean</i>	% <sup>2</sup>
snapshot model only	2	10	16	13.0	11.6
smooth-transition model only	12	0	66	17.3	15.4
both models	26	0	78	33.6	29.5
neither model	131	-	-	6.0	5.3

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<sup>1</sup>Number of relations that are conceptual neighbors

<sup>2</sup>percentage = mean / 112

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## Two Conceptual Neighborhood Models:

1. Snapshot Model
2. Smooth-Transition Model

**Finding:** *Almost* identical Conceptual-Neighborhood Graphs

## **Findings** from the Human-Subject Experiment:

- ▶ models correspond largely to the way humans conceptualize similarity about line-region relations
- ▶ smooth-transition model captures more aspects of the similarity of topological line-region relations than the snapshot model

## References

- ▶ Egenhofer, Max J., and David M. Mark. "Modelling conceptual neighbourhoods of topological line-region relations." *International journal of geographical information systems* 9, no. 5 (1995): 555-565.
- ▶ Mark, David M., and Max J. Egenhofer. "Modeling spatial relations between lines and regions: combining formal mathematical models and human subjects testing." *Cartography and geographic information systems* 21, no. 4 (1994): 195-212.
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- ▶ Talmy, L. Herbert L. Pick Jr, L. P. A. (Ed.) How language structures space Springer, 1983
- ▶ Clark, H. H. Space, time, semantics and the child Cognitive development and acquisition of language, 1973