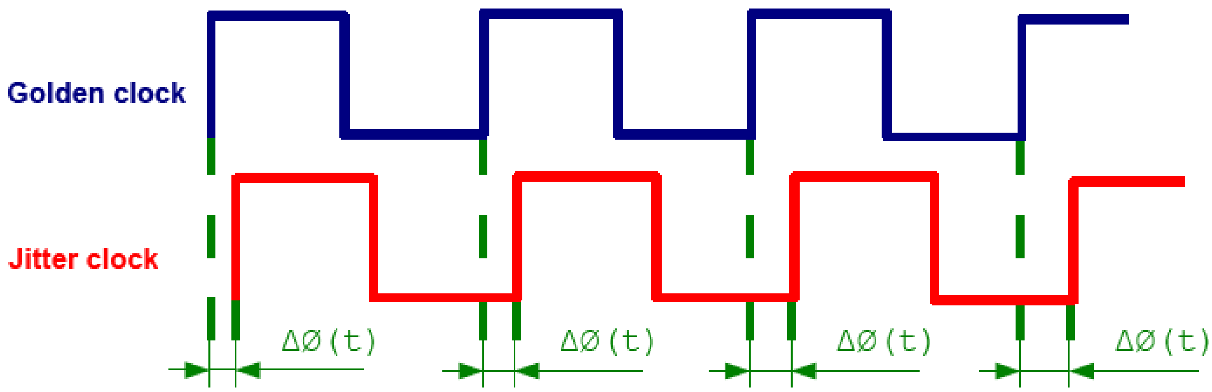
Outline

- Jitter definition - short vs long term(TIE)
- Jitter category - RJ, DJ and details
- Review of statistics - PDF, CDF, erf, erfc
- BERT vs Scope
- Dual Dirac Model - DJ_{pp} vs $DJ_{\delta\delta}$
- Decompose method - Spectrum vs Tail Fit
- Scope Jitter setting

TIE



- Deviation of the digital timing event from it is ideal position
- A specified number of observations.

Jitter definition

- Period Jitter (J_{PER})
  - Time difference between measured period and ideal period
- Cycle to Cycle Jitter (J_{CC})
  - Time difference between two adjacent clock periods
  - Important for budgeting on-chip digital circuits cycle time
- Accumulated Jitter (J_{AC})
  - Time difference between measured clock and ideal trigger clock
  - Jitter measurement most relative to high-speed link systems

Jitte Analysis Method

- Time domain
- Frequency domain

- Statistics domain
- Decompose to RJ and DJ

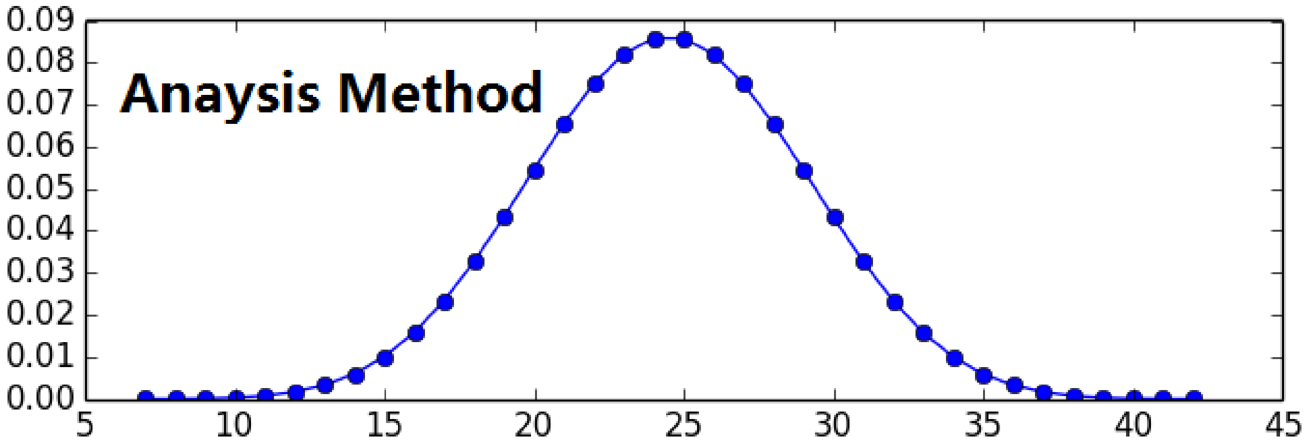
Monte Carlo

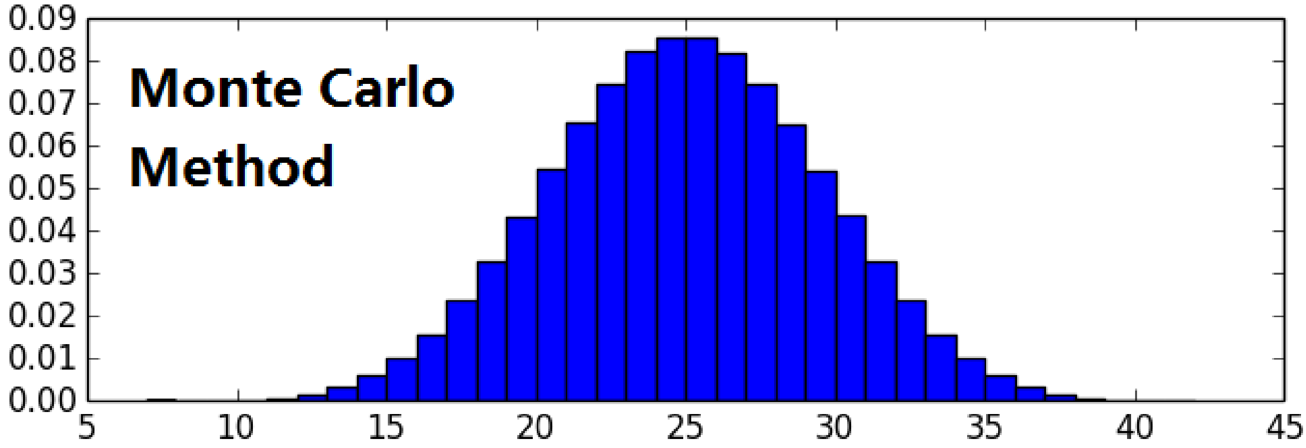
- Monte Carlo Simulation is a way of studying probability distributions with sampling
- Define a domain of possible inputs.
- Generate inputs randomly from a probability distribution over the domain.
- Perform a deterministic computation on the inputs.
- Aggregate the results.

Dice gambling

- 7 dices , 6 faces.
- summarize the 7 dices points
- what is the PDF?

Dice PDF





Code

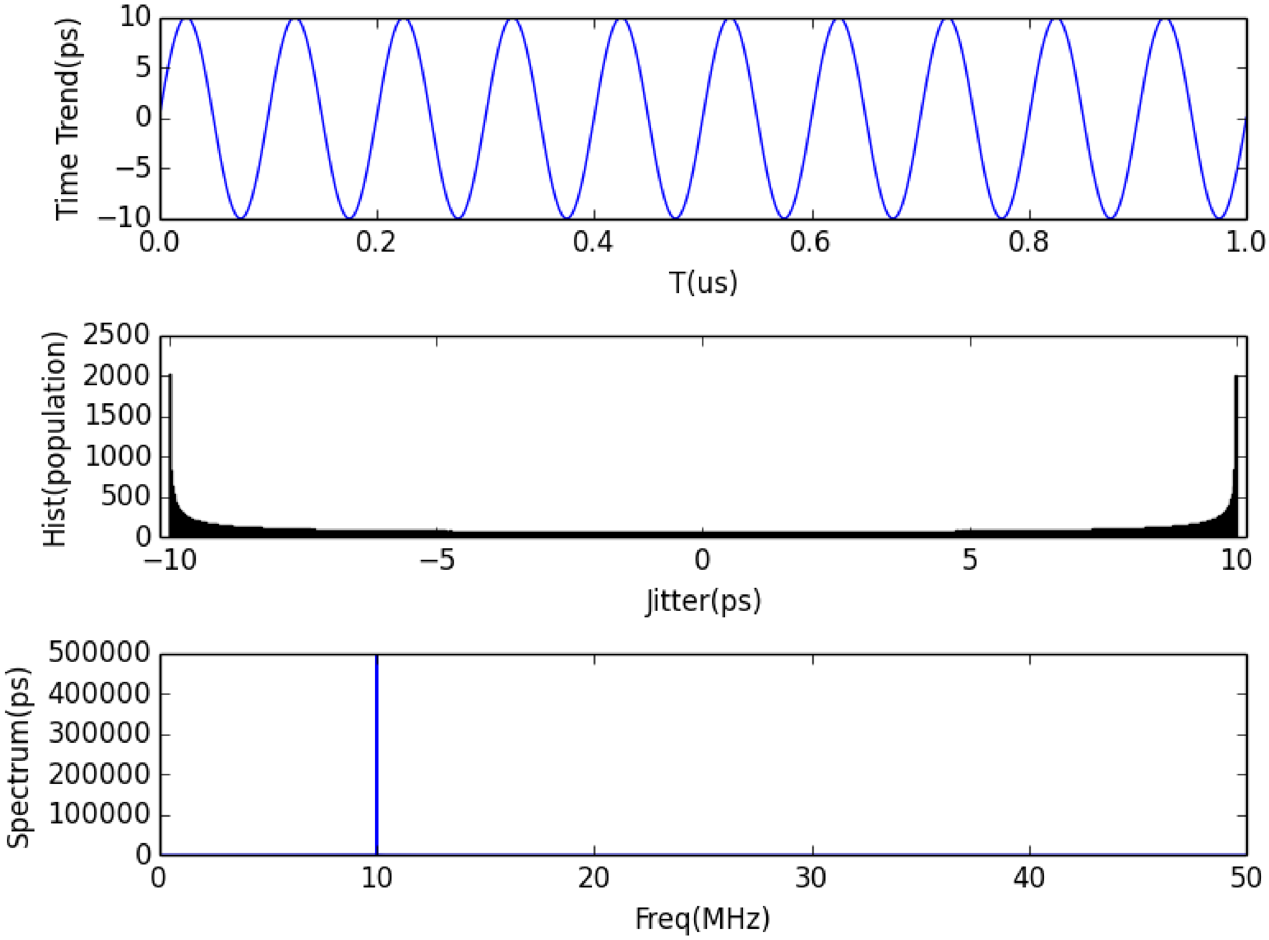
```
import numpy as np
from numpy.random import random_integers
import matplotlib.pyplot as plt

sample_len = 1000000
res = np.zeros(sample_len)
for i in range(sample_len):
    res[i] = np.sum(random_integers(1,6,7))
plt.hist(res,bins=35,normed=True)
plt.show()
```

Histogram(Simulation)

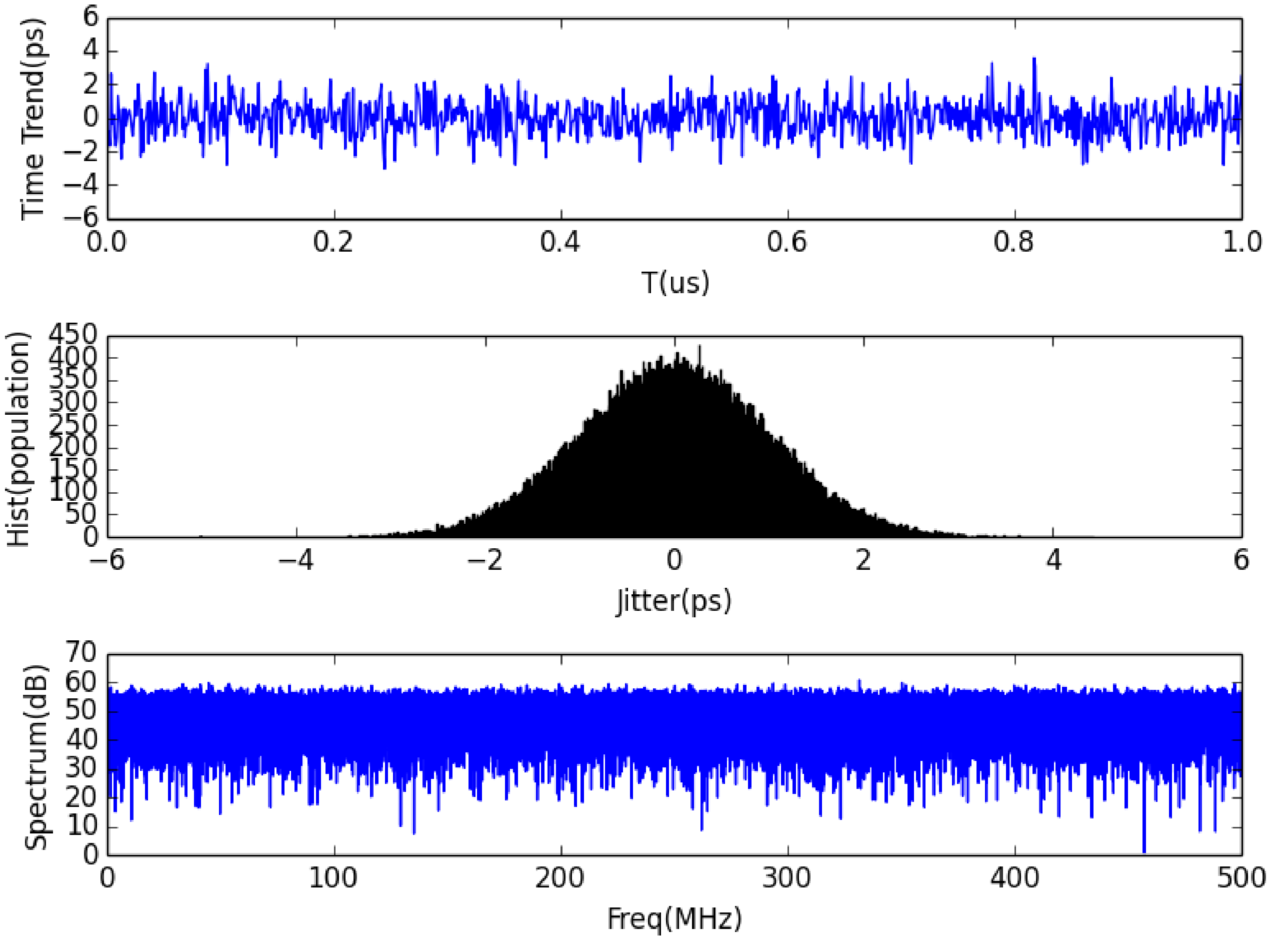
```python
import numpy as np
from numpy.fft import rfft, rfftfreq
from numpy.random import normal
import matplotlib.pyplot as plt

PJ_freq = 10e6 # 10MHz
PJ_amp = 10 # 10ps
RJ_rms = 1 # 1ps
sample_rate = 1e9 #1Gbps
sample_interval = 1./sample_rate
N_cycle =1000
pts = sample_rate/PJ_freq*N_cycle


t = sample_interval*np.arange(pts)*1E6

# sin jitter
plt.figure()
tie_sin = PJ_amp*np.sin(np.linspace(0,2*np.pi*N_cycle,sample_rate/PJ_freq*N_cycle))
plt.subplot(311)
plt.plot(t,tie_sin)
plt.xlabel("T(us)")
plt.ylabel("Time Trend(ps)")
plt.xlim([0,1]) # only plot 1us
plt.subplot(312)
plt.hist(tie_sin,bins=1000,normed=False)
plt.xlabel("Jitter(ps)")
plt.ylabel("Hist(population)")
plt.xlim([-10.2,10.2])
plt.subplot(313)
plt.plot(rfftfreq(len(tie_sin),1/sample_rate)/1e6,np.abs(rfft(tie_sin)))
plt.xlabel("Freq(MHz)")
plt.ylabel("Spectrum(ps)")
plt.xlim([0,50])
plt.tight_layout()

### random jitter
plt.figure()
tie_normal = normal(loc=0,scale=RJ_rms,size=pts)
plt.subplot(311)
plt.plot(t,tie_normal)
plt.xlabel("T(us)")
plt.ylabel("Time Trend(ps)")
plt.xlim([0,1]) # only plot 1us
plt.subplot(312)
plt.hist(tie_normal,bins=1000,normed=False)
plt.xlabel("Jitter(ps)")
plt.ylabel("Hist(population)")
plt.subplot(313)
plt.plot(rfftfreq(len(tie_normal),1/sample_rate)/1e6,20*np.log10(np.abs(rfft(tie_normal))))
plt.xlabel("Freq(MHz)")
plt.ylabel("Spectrum(dB)")

plt.tight_layout()
```

```
#Combined jitter
plt.figure()
tie_combine = tie_sin+tie_normal
plt.subplot(311)
plt.plot(t,tie_combine)
plt.xlabel("T(us)")
plt.ylabel("Time Trend(ps)")
plt.xlim([0,1]) # only plot 1us
plt.subplot(312)
plt.hist(tie_combine,bins=1000,normed=False)
plt.xlabel("Jitter(ps)")
plt.ylabel("Hist(population)")
plt.subplot(313)
plt.plot(rfftfreq(len(tie_combine),1/sample_rate)/1e6,20*np.log10(np.abs(rfft(tie_combine))))
plt.xlabel("Freq(MHz)")
plt.ylabel("Spectrum(dB)")
plt.xlim([0,50])
plt.tight_layout()


plt.show()
```

Sinusoid

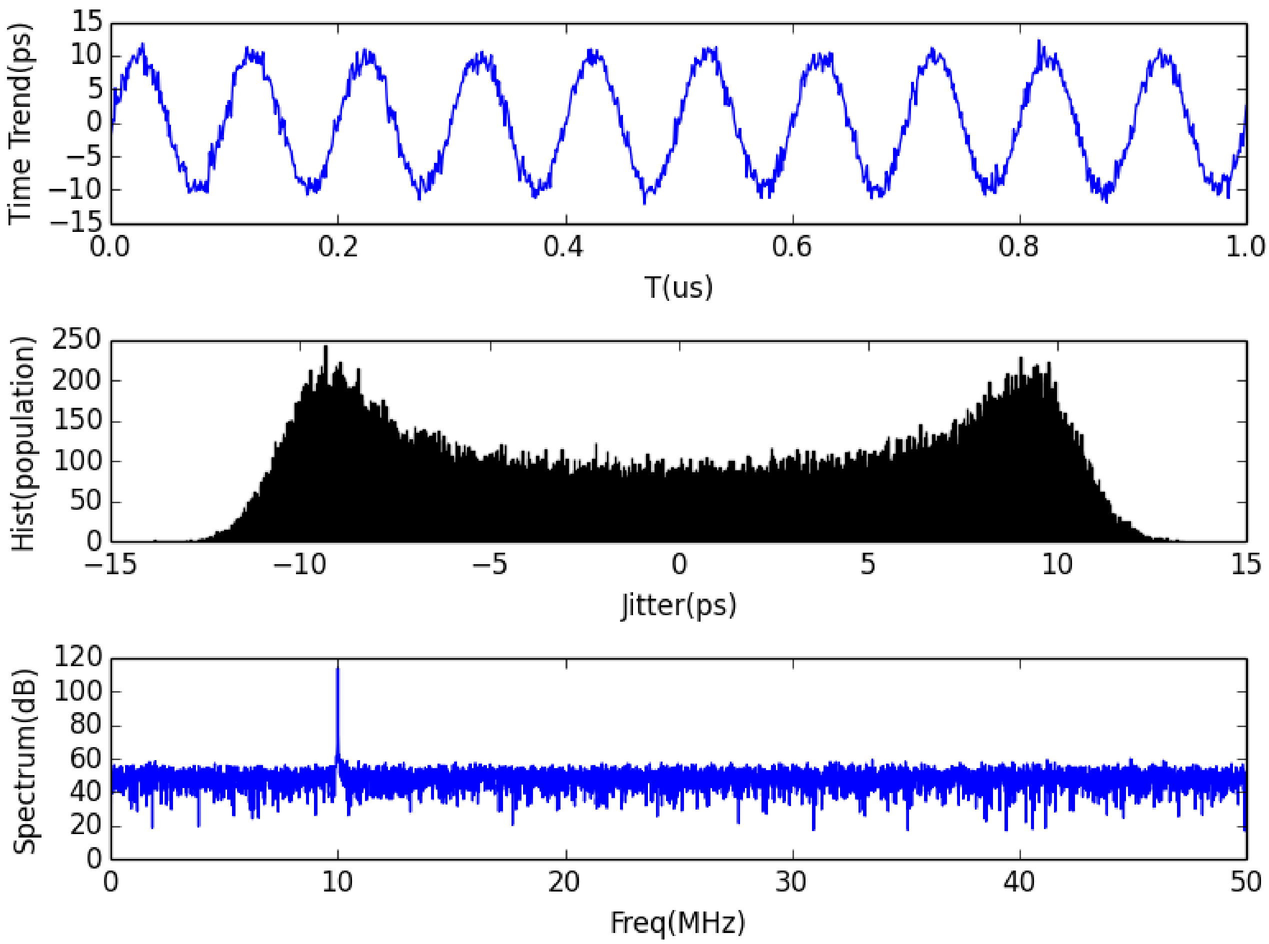


Random(Gaussian)

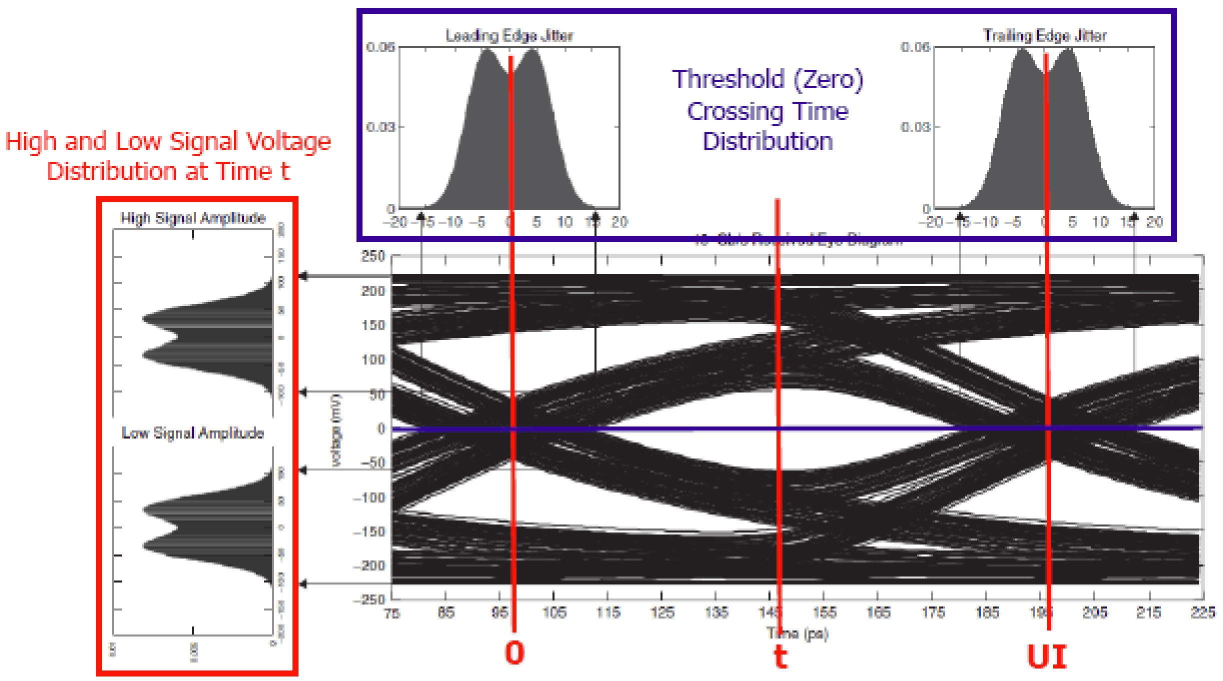


Time Trend(ps)
6
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0
T(us)
Hist(population)
450
400
350
300
250
200
150
100
50
0
−6
−4
−2
0
2
4
6
Jitter(ps)
Spectrum(dB)
70
60
50
40
30
20
10
0
0
100
200
300
400
500
Freq(MHz)

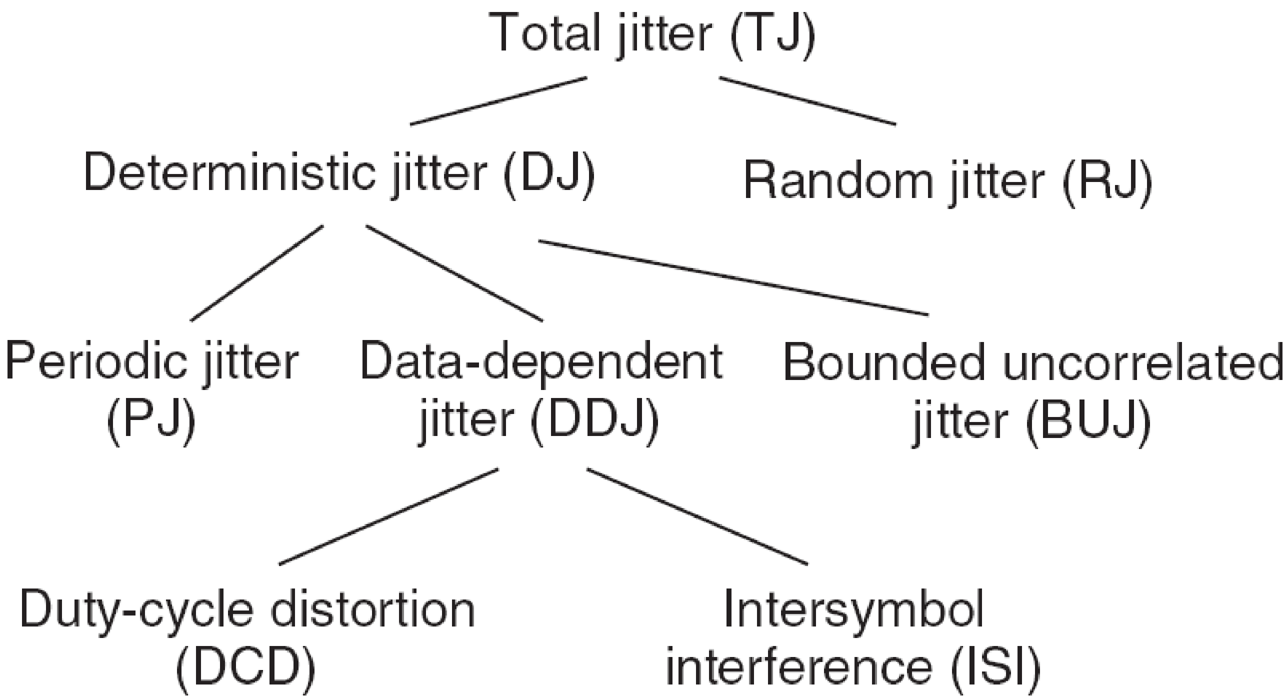
Combined



Jitter Histogram



Jitter category



Bounded vs Unbounded

- RJ is unbounded - long tail of Gaussian distribution
- Dj are bounded - must within a fixed range
- This is a significant differences, that's the reason why tail fit works

Uncorrelated vs Dependent

- Uncorrelated or dependent is relative to data stream
- Uncorrelated jitter need "**convolve**" , dependent jitter need "**add**".
- RJ and DJ is uncorrelated
- DCD,ISI is data dependent
- PJ is also uncorrelated to data, but still "add"
- Crosstalk is BUJ(Bounded uncorrelated jitter)

Review of statistics

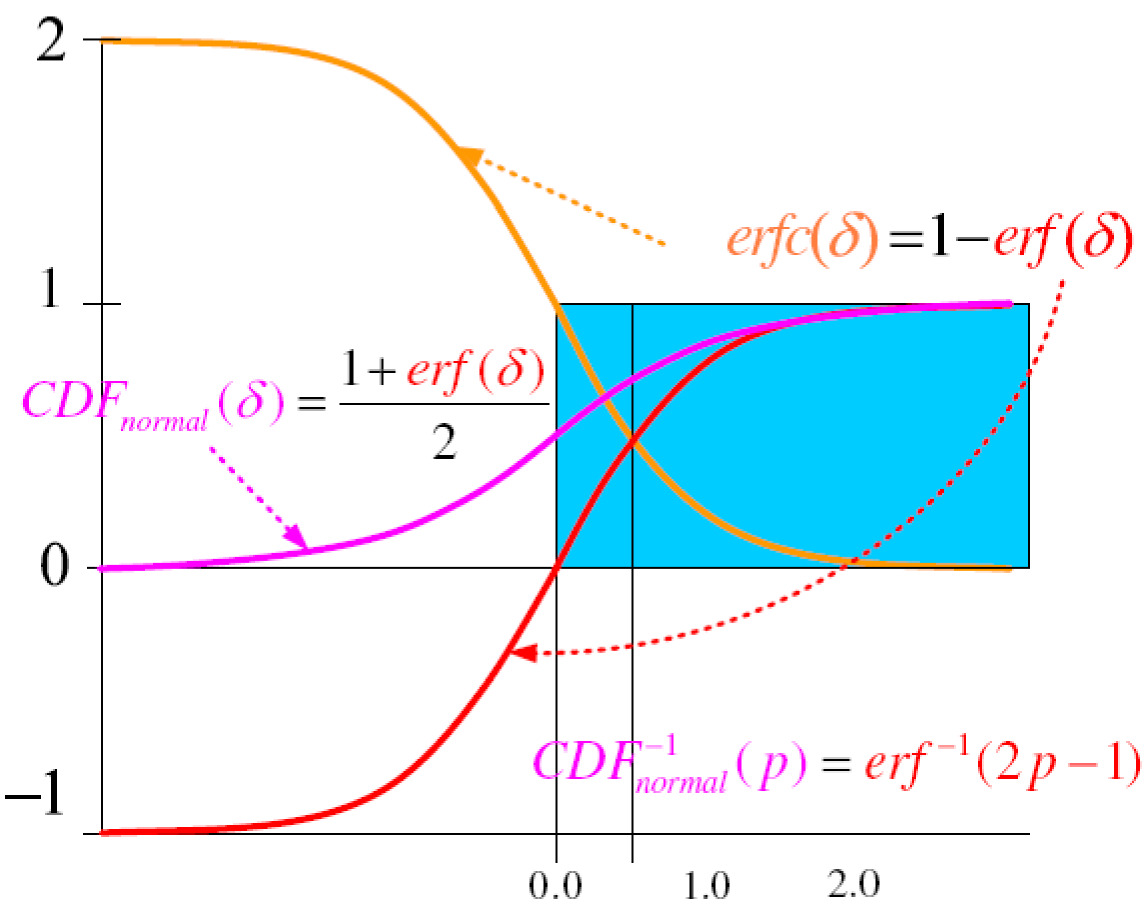
- PDF - The Probability density function, sometimes written $\rho(x)$.Histograms are closest in terms of a measurable probability density, and are said to approximate the PDF when normalized.
- CDF - The Cumulative Distribution Function is $CDF(x) = \int_{-\infty}^{x} PDF(u)du$

- PDF of Gaussion. $\rho(x) = \frac{w}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Statistics cont'd

- Error Function. $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$. It is a basic a mathematical function closely related to the cumulative distribution function for a Gaussian. $1 - 2CDF_{Gaussian}(x) = erf(x)$
- Complimentary Error Function. $erfc(x) \equiv 1 - erf(x)$ or $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$
- Inverse Error Function, $erf^{-1}(x)$, This function has no closed analytic form but can be obtained from numerical methods.

ERF relations

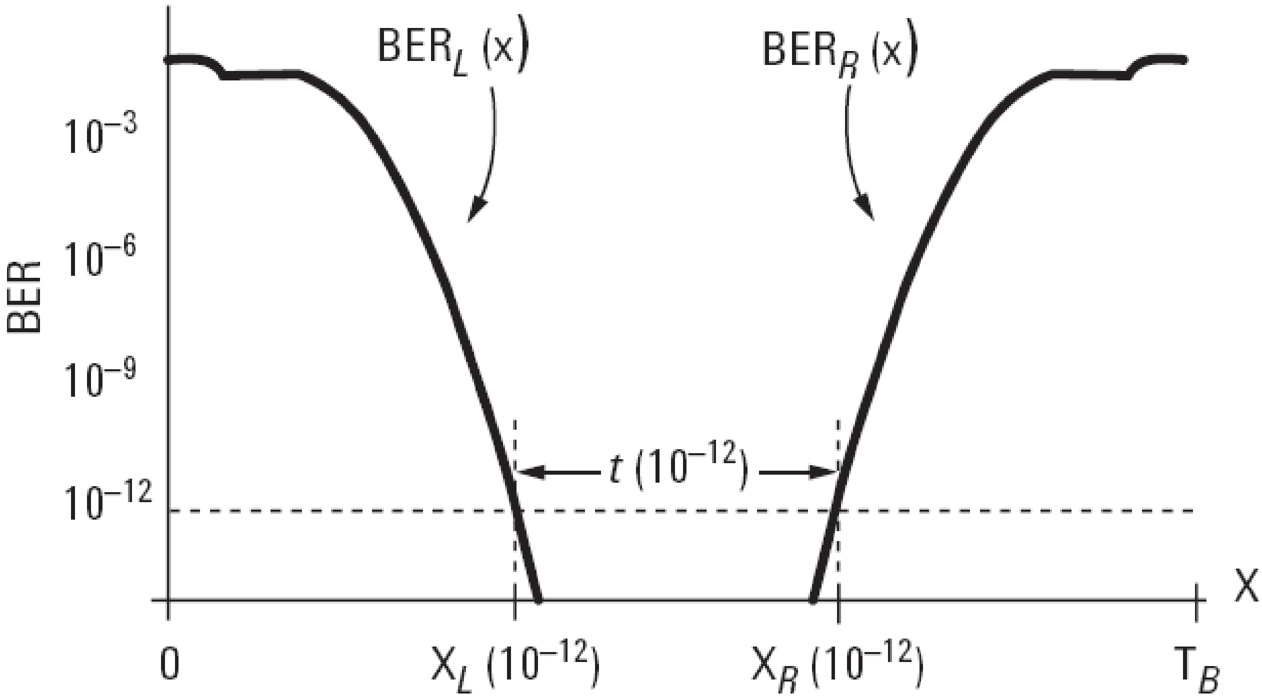


BERT vs Scope

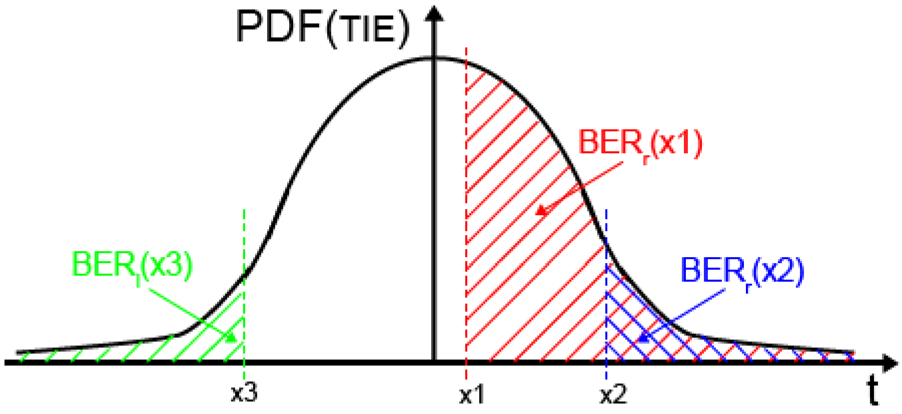
- BERT
  - Shift the sampling point across an entire bit time
  - Base on number of errors found
  - Accuratly measures the total jitter
- Scope

- Get a limited number of samples
- Good at determining the jitter components
- Need post-process to estimate RJ

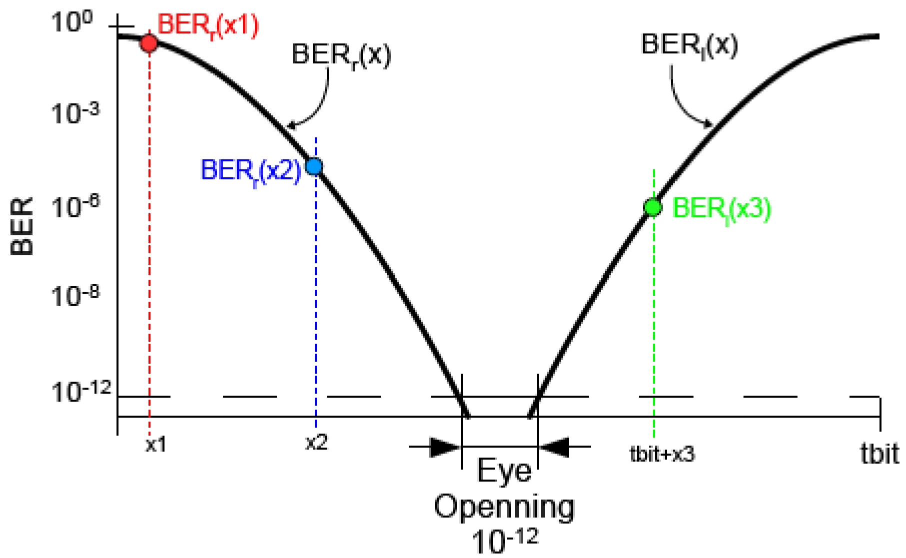
Bathtub



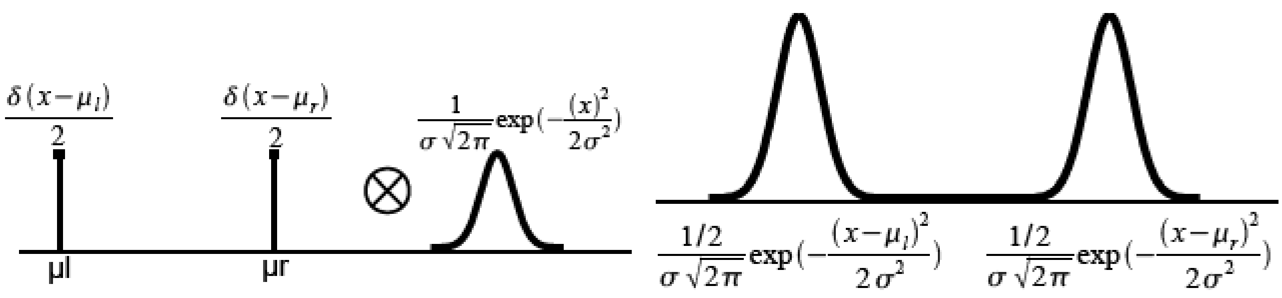
BER vs Jitter



(b) PDF of TIE



Dual Dirac Model



(a) PDF of each component

(b) $TJ_{(\delta\delta)}$ PDF

- An approximate method
- Quickly estimating of total jitter
- Not reflect the true jitter

Assumption

1. Jitter can be separated into two categories, random jitter (RJ) and deterministic jitter (DJ).

2. RJ follows a Gaussian distribution and can be fully described in terms of a single relevant parameter, the rms(σ) value of the RJ distribution.
3. DJ follows a finite, bounded distribution.
4. DJ follows a distribution formed by two Dirac-delta functions.
5. Jitter is a stationary phenomenon. Give the same result regardless of when that time interval is initiated.

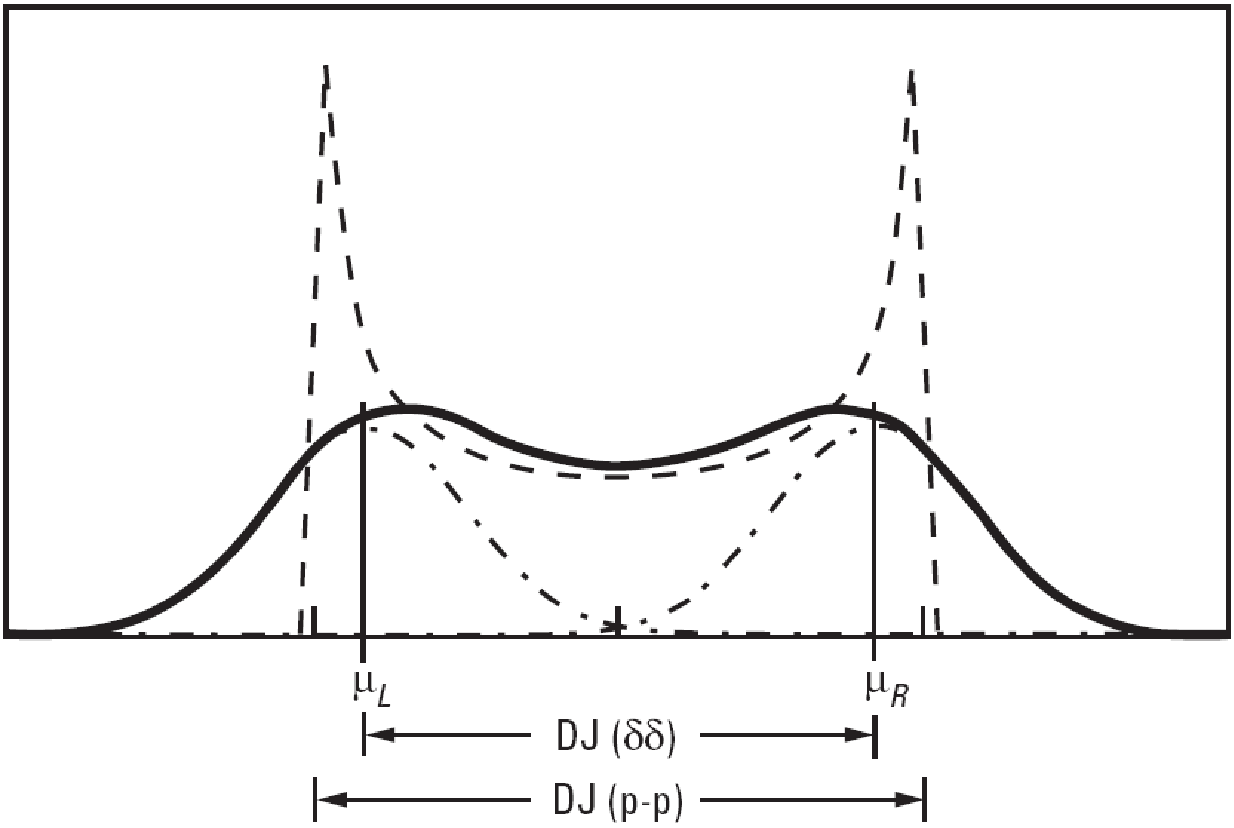
Math Expression

- PJ pdf is $A * \delta(x - \mu_l) + B * \delta(x - \mu_r)$
- RJ pdf is $\frac{1}{\sigma_{(\delta\delta)}\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2_{(\sigma\sigma)}}\right]$
- TJ combined by convolve, as it is uncorrelated $\frac{A}{\sigma_{(\delta\delta)}\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_l)^2}{2\sigma^2_{(\sigma\sigma)}}\right] + \frac{B}{\sigma_{(\delta\delta)}\sqrt{2\pi}} \exp\left[-\frac{(x-\mu_r)^2}{2\sigma^2_{(\sigma\sigma)}}\right]$
- RJ RMS combine: $\sigma_{Total} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots \sigma_n^2}$
- DJ pp combine: $DJ_{total}(\delta\delta) = DJ_1(\delta\delta) + DJ_2(\delta\delta) + \ldots + DJ_n(\delta\delta)$

DJ_{pp} vs $DJ_{\delta\delta}$

- Dual-Dirac DJ is a completely different quantity than the peak-to-peak DJ(very confused)
- To distinguish the two use the notation DJ_{pp} as the real jitter, $DJ_{\delta\delta}$ as dual-Dirac model DJ
- DJ_{pp} never follows the simple dual-Dirac distribution
- $DJ_{\delta\delta}$ is a model dependent quantity that must be derived under the assumption that DJ follows a distribution formed by two Dirac-delta. Generally, $DJ_{\delta\delta} < DJ_{pp}$

DJ_{pp} vs $DJ_{\delta\delta}$ cont'd



TJ vs BER

$TJ(BER) = 2 \cdot N(BER) \cdot \sigma_{total} + DJ_{total}$

$N = \sqrt{2} \cdot erfc^{-1}(2 \cdot BER)$

if BER if 1e-12, we could get the factor as 14.07 by the following code. Most of time we use a look-up table

```
from scipy.special import erfcinv
import numpy as np
BER = 1E-12
print 2*np.sqrt(2)*erfcinv(2*BER)
```

Jitter decompose

- Spectral Method
- Tail Fit method

Spectral Method

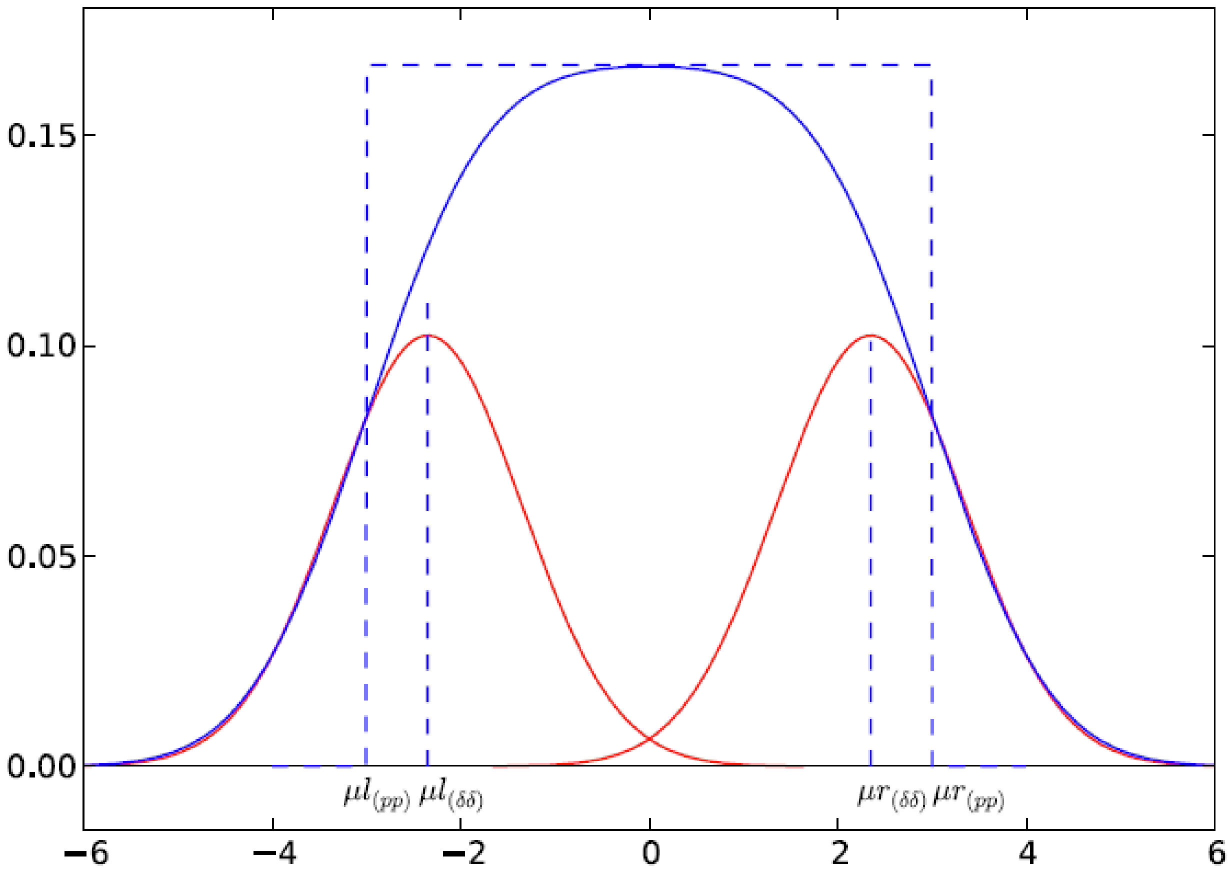
- FFT of TIE(t)

- spectral “peaks” are manifestations of deterministic jitter, and thus provide a measure of DJ
- The remaining spectrum, or “noise floor” accounts for all of RJ
- Remove the "peaks", and IFFT the "noise floor" get RJ
- Deconvolve and get DJ

Spectral problem

- BUJ like crosstalk is will impact noise floor
- Non-Stationary Periodic aggressors (like Spread Spectrum) manifest broadly in the jitter spectrum and cannot always be identified as “peaks”
- In effect, any wide-band aggressor which contributes bounded timing fluctuations will be indistinguishable from the noise floor, and consequently “counted” as RJ.
- Then it is easy to overestimations of Tj

Tail fit method



Tail fit difficulties

- Exponential waveform fit. - Q-scale

- Find the region of pure Gaussian PDF.   - Re-normalized Q-scale
- Need enough samples to get more long tail data.

Q-scale

- A smart method transform the exponential waveform to linear
- Q-scale is a simple coordinates transformation. Similar method like Cartesian coordinates to Polar coordinates, time domain to frequency domain. Different view point of the same physical object.
- Replace x with $Q = \frac{x-\mu}{\sigma}$ in BER expression, we get Q scale.

Math deduction

At the region far from DJ(long tail), the distribution is Gaussion, it also have left and right part. Just take left side as example.

$$BER_l(x) = K_l \frac{1}{\sigma_{l\delta\delta}\sqrt{2\pi}} \oint_x^\infty \exp\left[-\frac{(x' - \mu_{l\delta\delta})}{2\delta_{l\sigma\sigma}}\right] dx'$$

Let $Q = \frac{x-\mu_{l\delta\delta}}{\sigma_{l\delta\delta}}$, we get

$$BER_l(Q) = K_l \frac{1}{\sigma_{l\delta\delta}\sqrt{2\pi}} \oint_Q^\infty \exp\left[-\left(\frac{Q'}{\sqrt{2}}\right)\right] dQ'$$

Math cont'd 1

Remember that complementary error function is given by

$$erfc(x) = \frac{2}{\sigma\sqrt{2\pi}} \oint_x^\infty \exp(-u^2) du$$

then we could rewrite $BER_l(Q)$ as

$$BER_l(Q) = K_l erfc(\frac{Q}{\sqrt{2}}) = K_l(1 - erf(\frac{Q}{\sqrt{2}}))$$

inverse the function, $A_l = \frac{1}{K_l}$we get

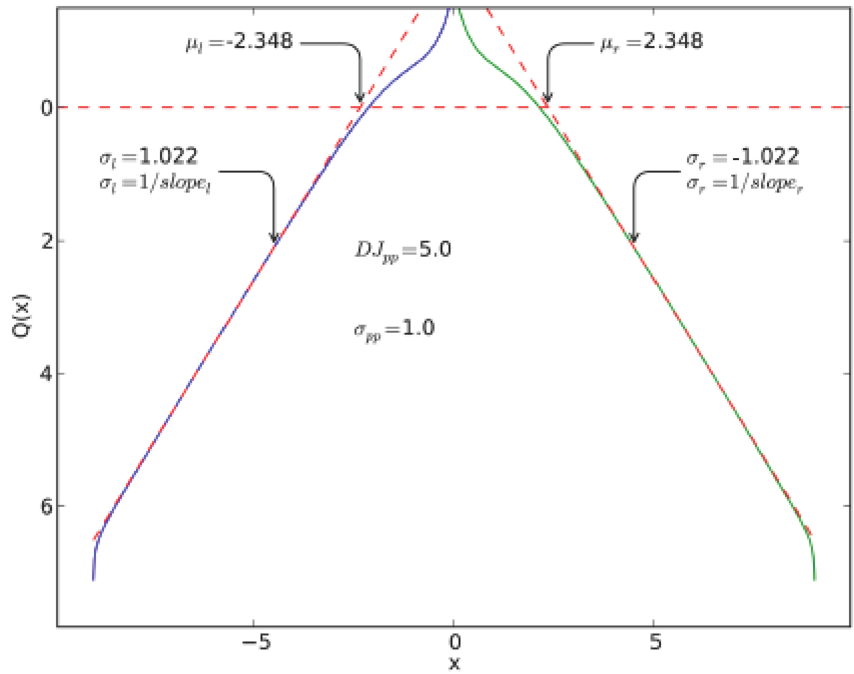
$$Q_l(x) = \sqrt{2} erf^{-1}\left[1 - BER_l \cdot A_l\right]$$

Math cont'd 2

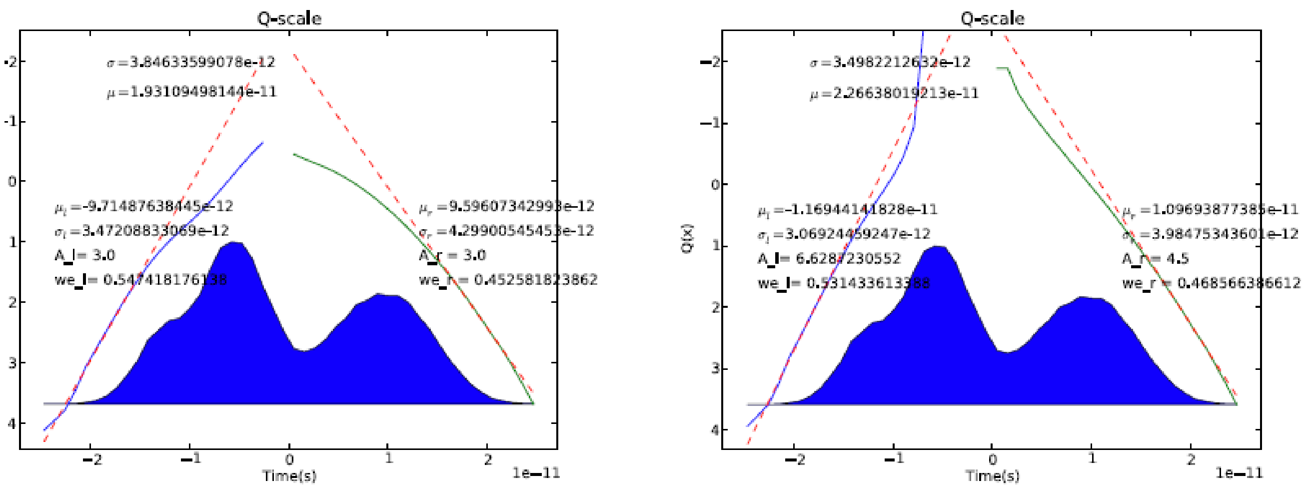
compare the 2 equation, the target is fit the 2.

$$Q_l(x) = \sqrt{2} erf^{-1}\left[1 - BER_l(x) \cdot A_l\right]$$

$$Q_l(x) = \frac{x - \mu_{l\delta\delta}}{\sigma_{l\delta\delta}}$$



Q-scale fit

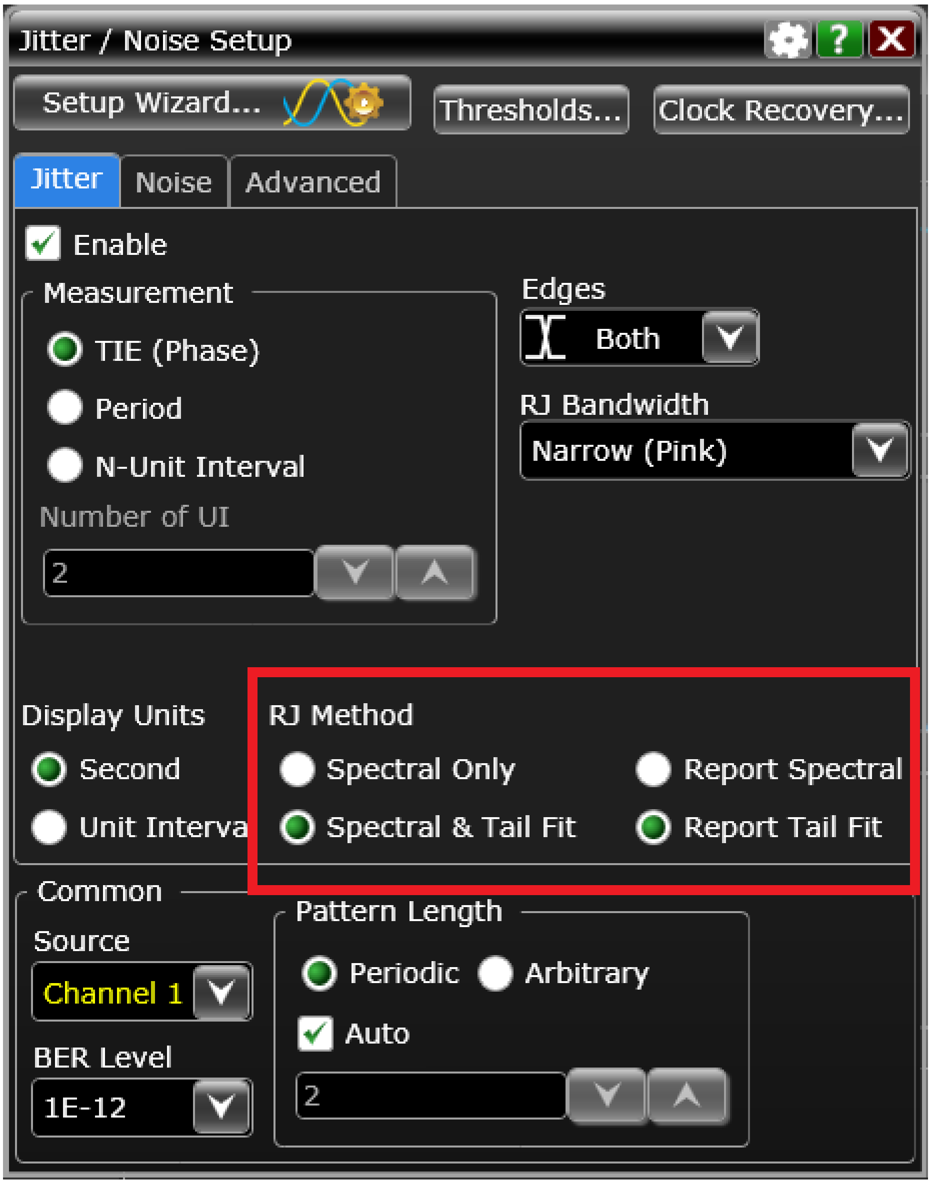


(a) Non-improved linear approximation (b) Improved linear approximation

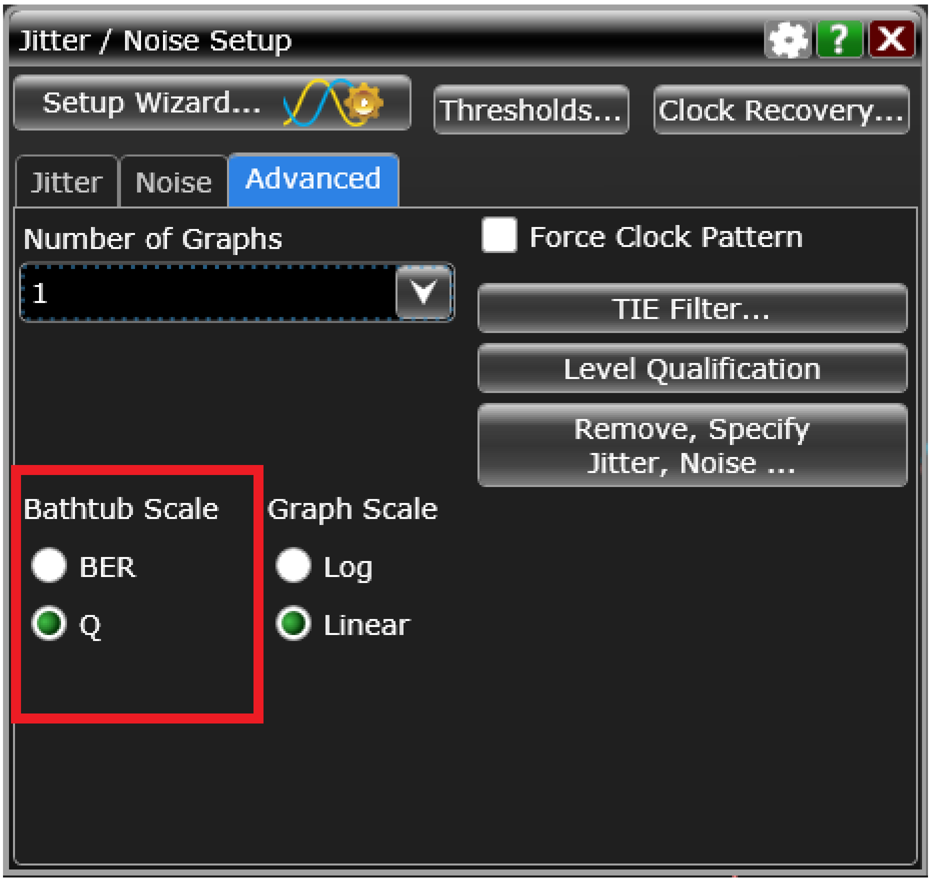
Pros of tail fit

- The noise like or flat spectrum jitter source like crosstalk and SSC could be remove from RJ
- If a system will too much crosstalk, it is better use tail fit method, otherwise spectrum method could be used.

Scope setting



Q-Scale option



Split threshold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File Control Setup Display Trigger Measure Math Analyze Utilities Demos Help
KEYSIGHT TECHNOLOGIES
Run Stop Single
80.0 GSa/s
4.11 Mpts
33 GHz
10 mV
Waveform Window 1
Jitter/Noise
Type All
Graphs 1
RJ,PJ Threshold (Periodic)
RJ+PJ Aliased Spectrum
RJ Aliased Spectrum
Separation Threshold
RJ Baseline
Power Spectral Density (Log)
0.0 Hz
1.250 MHz
2.490 MHz
3.740 MHz
4.980 MHz
Results (Measure All Edges)

BER scale

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Q scale

Jitter Result

Spectrum

Results (Measure All Edges)	
Source	Channel 1
RJ Method	Spectral
Data Rate	3.0000005 GHz
Pattern Length	---------------
TJ(1E-12)	74.31 ps
RJrms,narrow	2.02 ps
DJδδ	45.51 ps
Transitions	308.492 k
PJrms	13.11 ps
PJδδ	45.24 ps
DDJpp	990 fs
DCD	990 fs
ISIpp	0.0 s
DDPWS	990 fs
Clock Recovery	Constant Freq
Edge Direction	Both
Measurement	TIE (Phase)
ABUJrms	--------------

Tail Fit

Results (Measure All Edges)	
Source	Channel 1
RJ Method	Tail Fit
Data Rate	3.0000005 GHz
Pattern Length	Clock
TJ(1E-12)	87.08 ps
RJrms	3.87 ps
DJδδ	31.87 ps
Transitions	308.492 k
PJrms	13.11 ps
BUJδδ	32.27 ps
DDJpp	990 fs
DCD	990 fs
ISIpp	0.0 s
DDPWS	990 fs
Clock Recovery	Constant Freq
Edge Direction	Both
Measurement	TIE (Phase)
ABUJrms	0.0 s

Summary

- Dual-Dirac model is for estimate RJ, aks. extrapolation.
- RJ is the easy split than DJ, as its PDF feature.
- Both spectrum and tail fit have pros and cons