Results

Model Designation	Model Description	Estimate	Std. Error	Test Statistic	p-value
LM	Linear Model	4.918e-2	1.455e-2	3.381	7.47e-4
LM-FE	Linear Model with Fixed-Effect Intercept	4.833e-2	1.381e-2	3.500	4.84e-4
LMM-RI	Linear Mixed Model with Random Intercept	4.920e-2	1.374e-2	3.579	3.6e-4
LMM-RS	Linear Mixed Model with Random Slope	5.938e-2	3.538e-2	1.678	1.19e-1
GEE	Generalized Estimating Equations	4.918e-2	1.455e-2	3.381**	7.47e-4

Table X!X. $MALAT1 \sim CD19$ **model estimates:** Fixed effect slope estimate, standard error, test statistic, and p-value for each model for the relationship between the predictor $CD19$ and the outcome $MALAT1$. ** Approximate Wald-Z distribution.

Model Designation	Model Description	Estimate	Std. Error	Test Statistic	p-value
LM	Linear Model	7.884e-1	4.92e-2	4.002	<2e-16
LM-FE	Linear Model with Fixed-Effect Intercept	1.31e-1	3.42e-2	3.818	1.42e-4
LMM-RI	Linear Mixed Model with Random Intercept	1.35e-1	3.42e-2	3.95	8.4e-5
LMM-RS	Linear Mixed Model with Random Slope	1.705e-1	7.29e-2	2.34	6.7e-2
GEE	Generalized Estimating Equations	7.884e-1	4.92e-2	4.002**	< 2e-16

Table X!X. $FBLN1 \sim CD34$ **model estimates:** Fixed effect slope estimate, standard error, test statistic, and p-value for each model for the relationship between the predictor $CD34$ and the outcome $FBLN1$. ** Approximate Wald-Z distribution.

Displayed in **table X!X - table X!X** below are percent change in: parameter estimate, standard error, and test statistic for the $MALAT1 \sim CD19$ variable pairing (**tables X!X-X!X**) and the $FBLN1 \sim CD34$ variable paring (**tables X!X-X!X**). Where the percent change in estimate is defined as:

$$\text{Percent Change } [A]_{ij} = \left(\frac{A_j - A_i}{A_i}\right) * 100$$

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	-1.7283	0.0407	20.7401	0.0000
LM-FE	1.7587	0	1.8001	22.8636	1.7587
LMM-RI	-0.0407	-1.7683	0	20.6911	-0.0407
LMM-RS	-17.1775	-18.6090	-17.1438	0	-17.1775
GEE	0.0000	-1.7283	0.0407	20.7401	0

Table X!X: Main effect slope percent change matrix, $MALAT1 \sim CD19$ variable pairing

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	-5.0859	-5.5670	143.1615	0.0000
LM-FE	5.3584	0	-0.5069	156.1912	5.3584
LMM-RI	5.8952	0.5095	0	157.4964	5.8952
LMM-RS	-58.8751	-60.9666	-61.1645	0	-58.8751
GEE	0.0000	-5.0859	-5.5670	143.1615	0

Table X!X: Main effect slope standard error percent change matrix, $MALAT1 \sim CD19$ variable pairing

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	3.5197	5.8563	-50.3697	0.0000
LM-FE	-3.4000	0	2.2571	-52.0571	-3.4000
LMM-RI	-5.5323	-2.2073	0	-53.1154	-5.5323
LM-RS	101.4899	108.5816	113.2896	0	101.4899
GEE	0.0000	3.5197	5.8563	-50.3697	0

Table X!X: Main effect slope test statistic percent change matrix, $MALAT1 \sim CD19$ variable pairing

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	-83.3841	-82.8767	-78.3739	0.0000
LM-FE	501.8321	0	3.0534	30.1527	501.8321
LM-RI	484.0000	-2.9630	0	26.2963	484.0000
LM-RS	362.4047	-23.1672	-20.8211	0	362.4047
GEE	0.0000	-83.3841	-82.8767	-78.3739	0

Table X!X: Main effect slope percent change matrix, $FBLN1 \sim CD34$ variable pairing

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	-30.4878	-30.4878	48.1707	0.0000
LM-FE	43.8596	0	0.0000	113.1579	43.8596
LM-RI	43.8596	0.0000	0	113.1579	43.8596
LM-RS	-32.5103	-53.0864	-53.0864	0	-32.5103
GEE	0.0000	-30.4878	-30.4878	48.1707	0

Table X!X: Main effect slope standard error percent change matrix, $FBLN1 \sim CD34$ variable pairing

Model	LM	LM-FE	LMM-RI	LMM-RS	GEE
LM	0	-4.5977	-1.2994	-41.5292	0.0000
LM-FE	4.8193	0	3.4573	-38.7114	4.8193
LM-RI	1.3165	-3.3418	0	-40.7595	1.3165
LM-RS	71.0256	63.1624	68.8034	0	71.0256
GEE	0.0000	-4.5977	-1.2994	-41.5292	0

Table X!X: Main effect slope test statistic percent change matrix, $FBLN1 \sim CD34$ variable pairing

Parameter Value Comparisons

A comparison of main effect slope coefficient, standard error and test statistic across modeling approaches within variable pairings indicates that estimates produced by the LM and GEE methods are numerically indistinguishable at resolutions greater than 10^{-4}. The LM-FE and LMM-RI method estimates are also similar since estimates for each parameter type (coefficient, standard error and test statistic) exhibit magnitude and directional similarities in both variable pairings.

The LMM-RS estimates for the fixed effect slope parameter standard error is the highest when compared to the corresponding estimates within variable pairing as generated by other modeling methods. In contrast, the standard error of the fixed effect slope parameter is lowest for the LMM-RI model within variable pairings. The LM-FE model has a lower fixed effect slope standard error than either the LM or the GEE model within both variable combinations.

The differences in test statistics of the fixed effect slope parameter for each modeling method within each variable pairing are analogous to the differences in slope coefficients previously noted. In particular, test statistics have similar values between the LM and GEE models as well as between the LM-FE and LMM-RI models. Test statistics calculated for the LMM-RS model have the most irregular values, and also result in calculated p-values that have decreased significance in up to three orders of magnitude of percent change.

Nested Model Comparisons

Variable Pair	Model	Resid DF	RSS	DF	Sum of Squares	F-stat	P(>F)
MALAT1-CD19	LM	1108	1167.76				
	LM-FE	1094	935.89	14	231.87	19.36	6.4776e-44
FBLN1-CD34	LM	1108	650.51				
	LM-FE	1094	214.92	14	435.59	158.38	2.8058e-251

Table X!X: ANOVA nested model comparison table for testing the inclusion of the subject-specific fixed-effect intercept

Table X!X above is a nested model comparison, the result of which is an F-test statistic indicating that there is very strong evidence to support the inclusion of the subject-specific fixed-effect intercept into the LM model.

Variable Pair	Model	df	AIC	logLik	L.Ratio	p-value
MALAT1-CD19	LM	3	3224.097	-1609.048		
	LMM-RI	4	3032.024	-1512.012	194.0722	4.1068e-44
FBLN1-CD34	LM	3	2572.807	-1283.403		
	LMM-RI	4	1438.086	-715.043	1136.72	3.4517e-249

Table X!X: ANOVA nested model comparison table for testing the inclusion of the subject-specific random effect intercept

Table X!X above is a nested model comparison, the result of which is a likelihood ratio statistic indicating that there is very strong evidence to support the inclusion of the subject-specific random effect intercept into the LM model.

Variable Pair	Model	df	AIC	logLik	L.Ratio	p-value
MALAT1-CD19	LMM-RI	4	3032.024	-1512.012		
	LMM-RS	6	2993.820	-1490.910	42.20503	6.8437e-10
FBLN1-CD34	LMM-RI	4	1438.086	-715.043		
	LMM-RS	6	1438.068	-713.034	4.018095	0.1341

Table X!X: ANOVA nested model comparison table for testing the inclusion of the subject-specific random effect slope

Table X!X above is a nested model comparison, the result of which is a likelihood ratio statistic idicating that there is very strong evidence to support the inclusion of the subject-specific random effect slope into the LMM-RI model for the $MALAT1 \sim CD19$ variable pairing. However, there is insufficient evidence to support the inclusion of the subject-specific random effect slope into the LMM-RI model for the $FBLN1 \sim CD34$ variable pairing.