Final Project Outline

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Contents

1 Introduction

1.1 Background & Context necessary for project

- The Patient Health Questionnaire-Nine (PHQ9)
  - Self-administered module
  - Can be administed by medical staff
  - Nine questions, answers correspond to the frequency with which respondents feel certain feelings
  - Used for screening, monitoring, and grading severity of depressive symptoms related to nine criteria outlined by the Diagnostic and Statistical Manual of Mental Health Disorders (DSM-IV) [1]
  - Developed by Dr. Robert J. Spitzer, Dr. Janet B.W. Williams, Dr. Kurt Kroenke in 1999 with a grant from Pfizer [2]
  - Response sets are classified into a discrete set of categories corresponding to depression symptom magnitude
  - Three categories corresponding to: “Not clinically Depressed”, “Sub-threshold Depression”, and “Major Depression” will be used for this analysis

1.2 Data

1.2.1 Data Origin:

- Federally Qualified Health Research Center in Montana [3]
- Collected over six month timeframe
- Tablet-administered PHQ9s and Quick Diagnostic Panels (QDPs)

1.2.2 Data Properties:

- Technically a Randomized Control Design based upon assignment of QDPs, but observational based upon PHQ9 data
- 2495 observations
- 286 variables: PHQ9 data, QDP variables, Control variables, Demographic variables, Record-keeping (time, date, etc) variables
- De-identified, and left without validation outcome measurement
- No theoretically “correct” classification
- Each observation contains integer-value responses (0-3) for the nine PHQ9 questions
- No missing data
- Original test was administered sequentially, with all questions being asked
- Some respondents had taken the QDP first

1.3 Classification Methods

1.3.1 Traditional Classification

- Traditional classification class representations:
  - $\mathbf{C}^{TR} = \left(C_1^{TR}, C_2^{TR}, C_3^{TR}\right)$
- Assign numerical quantities to each response (0-3) for questions (1-9).
  - An answer set corresponding to observation $i = 1, \ldots, N$ is represented as
  - $\mathbf{A}_i = \{a_q\}_{q=1}^{9}$ where $i = 1, \ldots, N$ and $a_q \in \{0, 1, 2, 3\}$ for $q = 1, \ldots, 9$
- Calculate sum of assigned numerical quantities (0-27)
  - The sum of an answer set provided in observation $i = 1, \ldots, N$ is represented by:
  - $S_i = \sum_{q=1}^{9} a_q$
- Use sum to classify observation
- Depression classes are distinguished by "threshold values"
  - let $\alpha_1,\ \alpha_2\ \in \{0, \ldots, 27\}$ with $\alpha_1 \leq \alpha_2$ be threshold values
  - We can define the Traditional class representations C_c^{TR} for $c = 1, 2, 3$ as Level sets according to:
    - $C_1^{TR} = \left\{i \;\middle|\; S_i < \alpha_1\right\}$
    - $C_2^{TR} = \left\{i \;\middle|\; \alpha_1 \leq S_i < \alpha_2\right\}$
    - $C_3^{TR} = \left\{i \;\middle|\; \alpha_2 \leq S_i\right\}$
- for this analysis, threshold values are: $\alpha_1 = 7$ and $\alpha_2 = 10$
- 88% accurate [4]
- 12% FP/FN causes issues with Clinical Fatigue, patient concern, and healthcare system burdens
- Outcomes provide limited information
- Need to take the entire PHQ9 in order to achieve results

1.3.2 Probabilistic Classification

- Possible benefits:
  - Reduced test-taking requirements (early convergence)
  - Increased relatability with other Mental Health disorders, and better result integration in general
  - Not dependent on numerical assignments: integer differences of outcomes less influential.
  - Training-sample accuracy increase possibility
- The algorithm:
  - Probabilistic classification class representation
    - $\mathbf{C}^{PR} = (C_1^{PR}, C_2^{PR}, C_3^{PR})$
  - Let the question number be represented by: $Q = q \in \{1, 2, \ldots, 9\}$
  - Let the provided answer to a particular question be representy by: $A = a \in \{0, 1, 2, 3\}$

- Given training set, a set of weights is calculated using the formula:

$$W_{AQ}^{C^{PR}} = \frac{P\left(Q = q \;\middle|\; C^{TR} = c\right)}{P\left(Q = q\right)} \tag{1.3.2 - 1}$$

- Suppose that $\mathbf{A}_i = \{a_q\}_{q=1}^{9}$ represents a set of answers to the PHQ9, contained in observation i.
- For a provided confidence threshold value $\gamma \in (0, 1)$, the probabilistic scoring sequence is given recursively by:

$$\left(P_0^{(1)}, P_0^{(2)}, P_0^{(3)}\right) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$\left(P_{q+1}^{(1)}, P_{q+1}^{(2)}, P_{q+1}^{(3)}\right) = \frac{\left(P_q^{(1)} W_{a_q}^{(1)}, P_q^{(2)} W_{a_q}^{(2)}, P_q^{(3)} W_{a_q}^{(3)}\right)}{\sum_{j=1}^{3} P_q^j W_{a_q q}^{(j)}} \tag{1.3.2 - 2}$$

- If we define:

  $$j_q^* = \underset{j}{max}\ P_q^{(j)}$$

  and

  $$q^* = \underset{q}{min}\ \left\{q : P_q^* > \gamma\right\}$$

  then (provided that q^* exists), the probabilistic scoring classification is:

  $$j_{q^*}^* = \underset{j}{max}\ P_{q^*}^j$$

  more specifically, it is:

  $$C_{j_{q^*}^*}^{PR}$$

1.4 Consultation goals and deliverables

1.4.1 Client's Specified Goals:

> "...mathematically prove that probabilistic scoring is more accurate than conventional scoring"

> "...mathematically prove that probabilistic scoring derived from a conventional scored validation dataset is essentially as accurate as using the original validation dataset and therefore still more accurate than conventional scoring"

1.4.2 Analysis Goals:

- Compare Probabilistic Scoring accuracy to Conventional Scoring accuracy measured against simulated responses generated using information in PHQ9 data.
- Determine how accuracy comparisons vary as a function of training sample size.

1.4.3 Deliverables:

- Presentable results. Evidence of outcomes that can be shown to potential clients with the purpose of demonstrating the practical advantages of Probabilistic Scoring.
- Relationship Diagrams. Visualizations that can be utilized to show informational gain when Probabilistic Scoring is implemented. Particular interest in relating outcomes to specific question answers.

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2 Model and Methods

2.1 Quantifying Accuracy

- Accuracy as a function of model value and validation outcome

$$\text{Accuracy} = f(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{N}\sum_{i=1}^{N} I\left(y_i = \hat{y}_i\right)$$

  - where $\mathbf{y} = (y_1, \ldots, y_N)$ is the validation outcome
  - $\hat{\mathbf{y}} = (\hat{y}_1, \ldots, \hat{y}_N)$ is the model value
  - $I\left(y_i = \hat{y}_i\right) = \begin{cases} 0 & \text{if} \quad y_i \neq \hat{y}_i \\ 1 & \text{if} \quad y_i = \hat{y}_i \end{cases}$
- Accuracy as a function of model value and *simmulated* validation outcome

$$\tilde{\text{Accuracy}} = f(\tilde{\mathbf{y}}, \hat{\mathbf{y}}) = \frac{1}{N}\sum_{i=1}^{N} I\left(\tilde{y}_i = \hat{y}_i\right)$$

  - where $\tilde{\mathbf{y}} = (\tilde{y}_1, \ldots, \tilde{y}_N)$ is the simmulated validation outcome
- Simmulated accuracy calculation can be performed for Traditional and Probabilistic Classifications, and then compared:

$$\tilde{Accuracy}_{TR} \sim \tilde{Accuracy}_{PR}$$

$$f\left(\tilde{\mathbf{y}}, \hat{\mathbf{y}}^{TR}\right) \sim f\left(\tilde{\mathbf{y}}, \hat{\mathbf{y}}^{PR}\right) \tag{2.1-1}$$

- This analysis will focus on estimating various values of $\tilde{accuracy}$ using multiple methods of generating values of $\tilde{\mathbf{y}}$

2.2 Cross Validation Processing:

- Estimating $\tilde{Accuracy}$ using Cross Validation allows us to obtain an average-stabilized estimate of both Pscore and traditional classifications simmultanneuously, and to establish the relationship between training sample size and accuracy

- For a fixed value of $K \in \{2, \ldots, N-1\}$ we partition the full data set into K equal subsets, each of which has N_K observations sampled randomly without replacement from the original data, where:
$$N_K = \left\lfloor \frac{N}{K} \right\rfloor$$
- We now have K distinct data sets that have been subset from the original data, that we will combine into test-train pairs
- There are K different ways in which $K-1$ data sets can be chosen from K sets when order does not matter, for each of these K combinations we create a train-test combination by combining the $K-1$ selected sets to form the training set, and the remaing set to form the test set.
- We will denote these train/test pairs
$$(TR, TE)_j^k = \left(TR_j^k, TE_j^k\right)$$
for $j = 1, \ldots, K$
- From the definition of N_K, and the fact that each training set is a union of $(K-1)$ sets of length N_K, we can establish a relationship between training set length ($|TR_j^k|$-the number of observations in the jth training set) and the value of K from which the original test-train partitions were originally formed:
$$|TR_j^k| = (K-1)N_K = (K-1)\left\lfloor \frac{N}{K} \right\rfloor$$
- Similarly, we may define the length of the test set:
$$|TE_j^k| = N_K = \left\lfloor \frac{N}{K} \right\rfloor$$
- noting that:
$$\left\lfloor \frac{N}{K} \right\rfloor \leq \frac{N}{K}$$
We have
$$\begin{aligned} |TR_j^K| + |TE_j^K| &= (k-1)\left\lfloor \frac{N}{K} \right\rfloor + \left\lfloor \frac{N}{K} \right\rfloor \\ &= K\left\lfloor \frac{N}{K} \right\rfloor \\ &\leq K\frac{N}{K} = N \end{aligned}$$
Implying that the some observations may be left out of the CV process.
- We are interested in establishing a relationship between $|TR_j^k|$ and $\tilde{accuracy}$, idealistically represented by:
$$acc\tilde{u}racy\left(|TE_j^k|\right) = acc\tilde{u}racy\left(\left\lfloor \frac{N}{K} \right\rfloor\right)$$
This formula establishes a connection between the value of K and $acc\tilde{u}racy$. Demonstrating that by calculating the value of $acc\tilde{u}racy$ at varying the value of K, we may establish the relationship between $acc\tilde{u}racy$ and $|TR_j^k|$ (i.e. Training Sample Length).

- We can calculate the calculate the classification accuracy for each train-test pairing within a specific value of K.

$$\tilde{accuracy}\left(TR, TE\right)_j^k = \frac{1}{|TE_j^k|}\sum_{i=1}^{|TE_j^k|} I\left(\tilde{y}_i = \hat{y}_i\right)$$

- and we will use the average of these values to represent the accuracy at a particular K-value

$$\begin{aligned}\tilde{accuracy}\left(TR, TE\right)^k & = \tilde{accuracy}_m\left(TR, TE\right)^k \\ & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tilde{accuracy}\left(TR, TE\right)_j^k\right\} \\ & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tfrac{1}{|TE_j^k|}\sum_{i=1}^{|TE_j^k|} I\left(\tilde{y}_i = \hat{y}_i\right)\right\}\end{aligned}$$

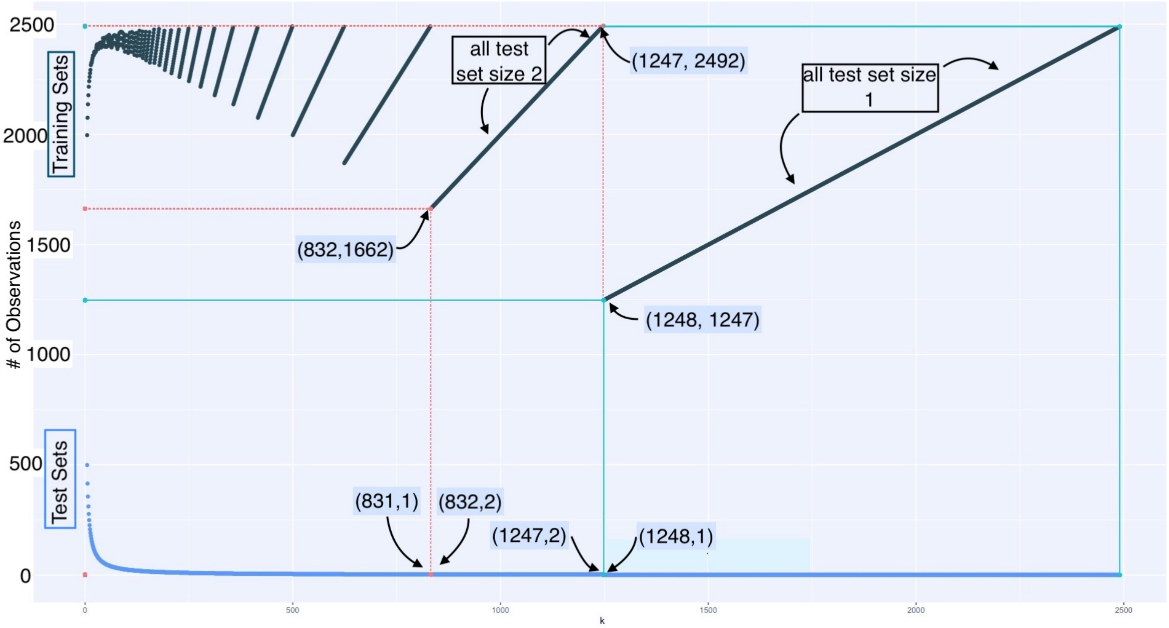
- This process can be applied to Traditional and Probabilistic Scoring methods:

$$\begin{aligned}\tilde{accuracy}_m\left(TR, TE\right)^k & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tilde{accuracy}_m\left(TR, TE\right)_j^k\right\} \\ & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tfrac{1}{|TE_j^k|}\sum_{i=1}^{|TE_j^k|} I\left(\tilde{y}_i = \hat{y}_i^m\right)\right\}\end{aligned}$$

  where $m \in \{Pscore,\ Tscore\}$, $\hat{y}^{Pscore}$ is a Probabilistic Scoring generated outcome, and $\hat{y}^{Tscore}$ is a Traditional Scoring generate outcome
- It will also be applied to a multitude of different simulated validation outcomes

$$\begin{aligned}\tilde{accuracy}_m^h\left(TR, TE\right)^k & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tilde{accuracy}_m^h\left(TR, TE\right)_j^k\right\} \\ & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tfrac{1}{|TE_j^k|}\sum_{i=1}^{|TE_j^k|} I\left(\tilde{y}_i^h = \hat{y}_i^m\right)\right\}\end{aligned}$$

  where $h \in \{1, 2, 3, 4, 5\}$
- Since notation is obviously extremely cumbersome, indices displayed will be limited only to those relevant.
- **Figure 1**



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- **Figure 1 Caption:** Training and Test set observation counts vs K
- We see that the possible training set observation counts using the CV method outlined above on the data provided range between 1247 and 2492 observations.
- The figure also shows that for K-values greater than 1248, test set sizes are confined to one.
- Training set sizes were selected for analysis by sampling 50 or more different values of K from k=5 to k=2490.
- Plots of comparing accuracies in the form of (2.1-1) are shown for each value of K selected as the value of training set length corresponding to K increases
- It was assumed that subsets taken from the data which were sampled randomly without replacement are at least partially representative of the population as a whole.
- It was assumed (incorrectly) for this analysis that test-set accuracy evaluations are comparable at all values of K.

2.3 Optimal Convergence Criterion

- The Pscore algorithm requires the specification of a confidence level for termination.
- For the purposes of this analysis, a confidence level of 0.75 was chosen.
- This value optimized probabilistic scoring classification accuracy of the first simulated validation outcome (see below) when the Pscoring weights were trained on the full data set.

2.4 Simulated Validation Outcomes

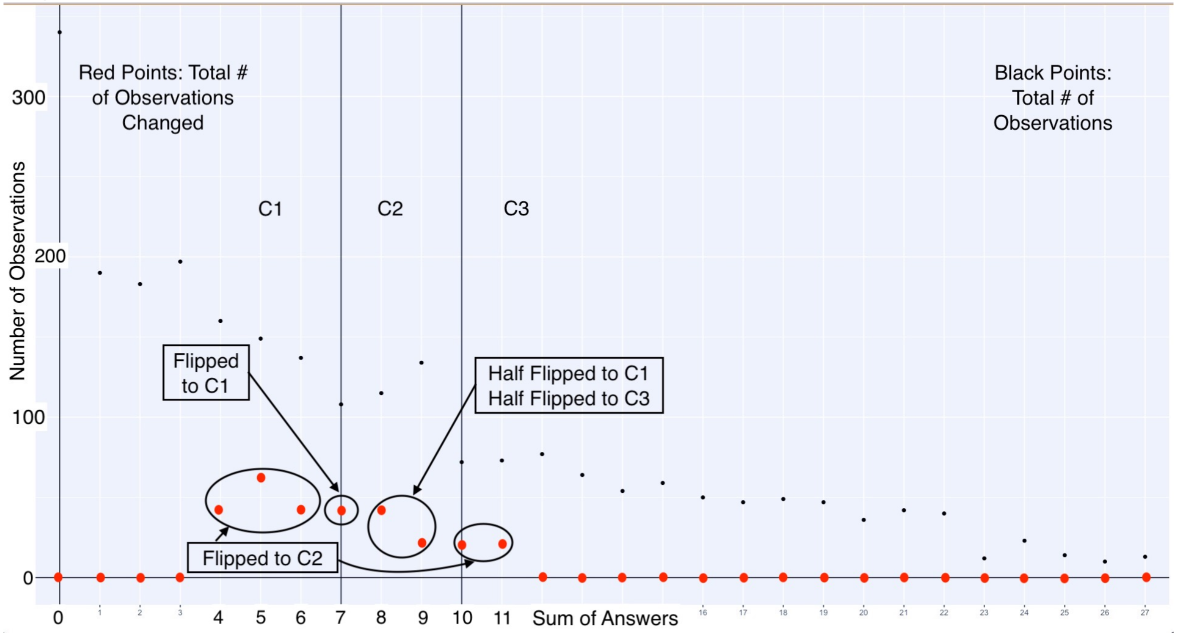
2.4.1 Inferential Simulations

- Change traditional classification based upon the following assumptions:
  - Depression Classifications are a hierarchy, so outcomes for group 1 are lower than group 2,…etc
  - The 12 percent misclassification is due to the traditional scoring algorithm misclassifying observations into a proximal class. Group 1 into group 2, group 3 into group 2 or group 2 into either group 1 or 2
  - Within each class there is a spectrum of conditions, and transitions between classes are essentially continuous.
- Based on the number of observations with the same traditional sum-score.
  - Let $\mathbb{S}_T^{PR} = \left\{ i \,\middle|\, S_i = T \right\}$ for $i = 1, \ldots, N$ and $T = 0, 1, \ldots, 27$ where $S_i = \sum_{q=1}^{9} a_q$
  - Then the number of observations with the same traditional sum-score (S_i) is the value: $|\mathbb{S}_T^{PR}|$
- Based upon distance from a threshold value. Meaning that observations which are closer to a threshold value are more likely to be mis-classified (switched)
  - If we suppose that $\alpha_1 \leq \alpha_2 \in \{0, 1, \ldots, 27\}$ are threshold values. The distance to the nearest threshold value of an observation i that is classified into traditional

sum value S_i is:

$$D_i = \min_{k=1,2} \left\{ \left| \alpha_k - S_i \right| \right\}$$

- The final probability of a traditional classification being altered in its representation in the simulated outcome data set then follows:
  - $P\left(C_i^{TR} \neq \tilde{C}_i^1\right) \propto \frac{|\mathbb{S}_T^{PR}|}{D_i}$
  - Proportionality constants were chosen so that the total difference between Traditional Scoring classification and simmulated outcome was 12%
- **Figure 2:**



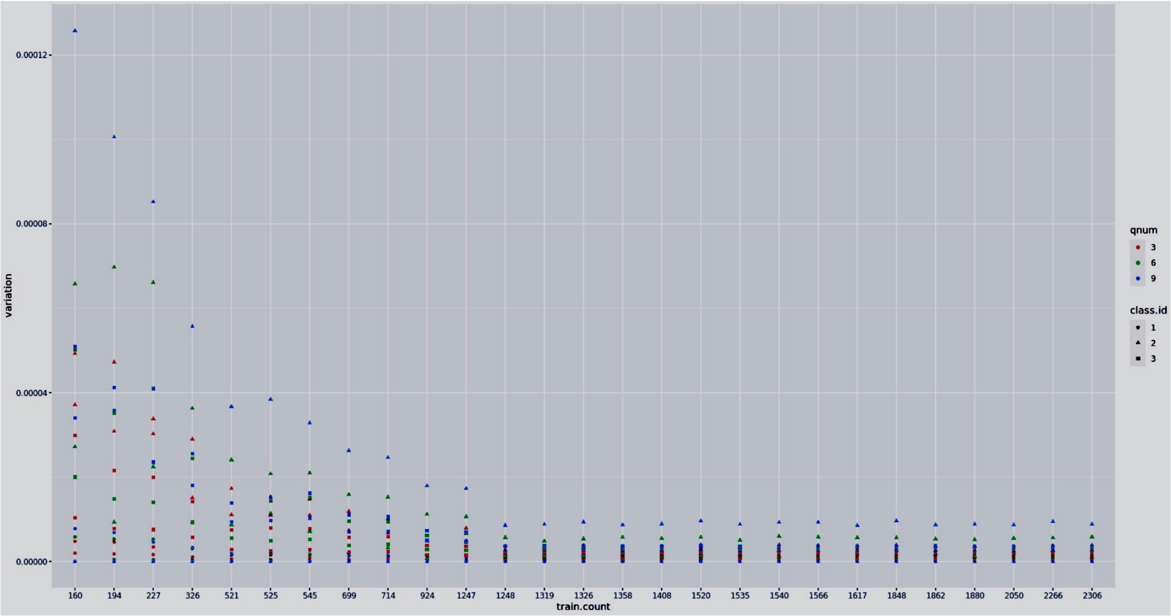
- **Figure 2 Caption:** Figure depicts observational distributions as classified by traditional methods, and how the first simulated validation outcome induced change
- Simmulated validation outcomes generated using this method depended only on information relevant to the observation for which the outcome was being simmulated
- Pairing of simmulated validation outcomes performed with full-data
- Accuracy comparisons of Probabilistic Scoring and Traditional Scoring classifications were obtained on 100 different values of K using the CV algorithm previously outlined.
- The simulated outcome generated using this method was implemented in the process to obtain the optimal convergence confidence level in section 2.3

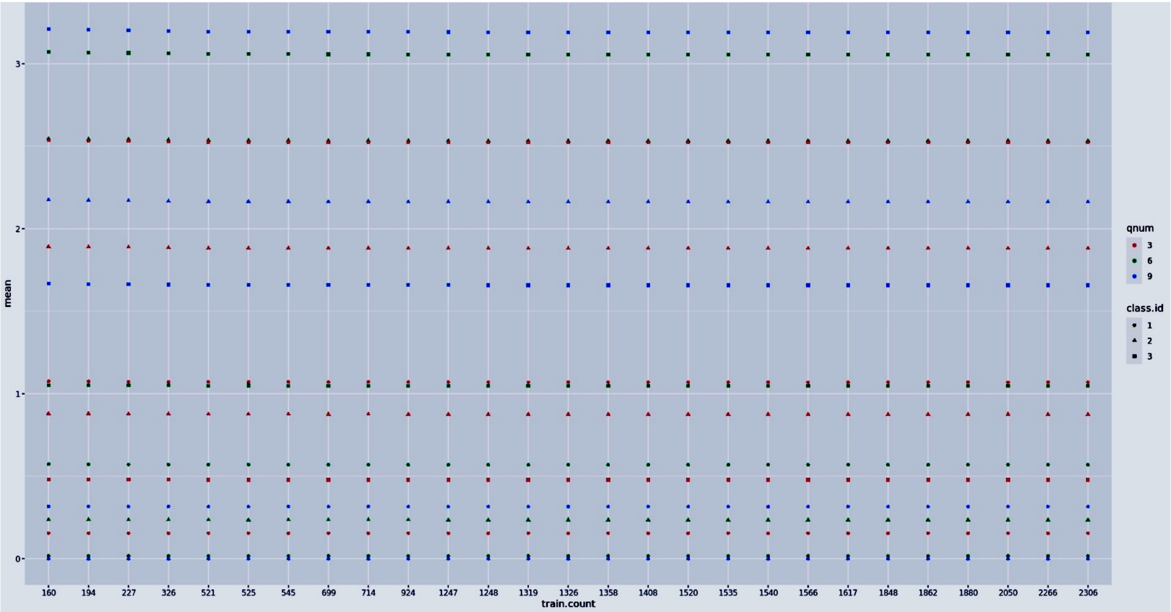
2.4.2 Probabilistic Algorithm Simmulations

- Motivating Principle: The Pscoring algorithm incorperates more information contained in the data than anything else, therefore it should generate the most appropriate estimates
- Motivating Principle: The Pscoring algorithm can estimate multiple amounts of inherently different estimates for the same observations based upon different training sets.

- The process:
  - Separate data into K-CV sets, and combine into train-test pairings as before in the CV analysis
  $$(TR, TE)_j^K$$
  where $j = 1, \dots, K$
  - For each training set, calculate the corresponding set of Pscore weights
  $$TR_j^K \ \rightarrow \text{equation (1.3.2 - 1)} \ \rightarrow \ W_{AQj}^C$$
    - K train-test pairs means there will be K distinct estimates of the training weights
  - Calculate a representative weight set for the value of K by averaging over the K weight values
  $$W_{AQk}^C = \frac{1}{K}\sum_{j=1}^{K} W_{AQj}^C$$
  - For each of the K test sets use the calculated, representative weights to Probabilistically Classify a corresponding set of *simulated validation outcomes*
  $$\left(W_{AQk}^C, TE_j^k\right) \ \rightarrow \ \text{equation (1.3.2 - 2)} \ \rightarrow \ \tilde{y}_j^2 k$$
  - The outcomes generated using this method depend on information obtained from a training set used to generate Pscore weights which were used in its classifications.

  - Each simulated outcome will therefore be paired with data that was simulated using the same set of information. (i.e. categorized using the same training data).
  - The values of the simmulated outcome specific to training sample within each K value are then substituted into the CV algorithm for $\tilde{y}^h$
  $$\begin{aligned} \tilde{accuracy}_m^h \left(TR, TE\right)^k \quad & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tilde{accuracy}_m^h \left(TR, TE\right)_j^k\right)\right\} \\ & = \tfrac{1}{K}\textstyle\sum_{j=1}^{K}\left\{\tfrac{1}{|TE_j^k|}\textstyle\sum_{i=1}^{|TE_j^k|}\left\{I\left(\tilde{y}_i^h = \hat{y}_i^m\right)\right\}\right\} \end{aligned}$$
- The method makes the assumption that estimates of average weights are stable enough to produce reasonably consistent estimates
- **Figure 3:**

- 

- **Figure 3 Caption:** Variance of average weight values for several question number/class ID combinations plotted against training data count length
- **Figure 4:**
- 

- **Figure 4 Caption:** Mean value of the average weight estimates for several question number/class ID combinations plotted against training data count length
- Stability of the average estimate value, as training sample increases, in combination with a dramatic decrease in variance as training sample size increases are indications that the average weight values are capable of producing stable outcome estimates

2.4.3 Unsupervised Learning Classifications

- Using unsupervised learning techniques to gather trends in

2.4.3.1 Kmeans

Assumptions of the method, including experimental design and target population

Were any pre-filtering or pre-processing steps applied to the data?

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3 Analysis and Results

- **Results of analysis conducted**
- **Interpretations and "walk-throughs" of results & findings**

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4 Discussion

- **Discussion of Results (separate from Results section)**
- **Have you addressed your client's goals?**
- **Are there any caveats to your analysis?**
- **What are the next steps from such an analysis?**
- **Recommended resources**

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5 Appendix

- **Necessary code or solftware instructions to carry our your analysis.**

- **Include any resources you have referred to or found helpful**
- **If appropriate, include other deliverables**

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6 References

1. Kroenke K, Spitzer RL (2002) The phq-9: A new depression diagnostic and severity measure. *Psychiatric annals* 32: 509–515.

2. Kroenke K, Spitzer R (2010) Instruction manual: Instructions for patient health questionnaire (phq) and gad-7 measures.

3. Malik A More effective and cost effective use of the phq-9.

4. Kroenke K Spitzer, rl & williams, jb (2001). The phq-9. *Journal of General Internal Medicine* 16: 606–613.