Important Formulae – IE2110 (formerly known as IE2010 or EE2010/IM2004)

Even and Odd Functions

$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Energy and Power Formulae

$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$P_x = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x[n]|^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

Periodic signals, $P_x = \frac{1}{K_0} \sum_{n=k}^{k+K_0-1} |x[n]|^2 = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$

Properties of Impulse Function

$\frac{d}{dt}[u(t)] = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} = \delta(t); \quad \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$

$\int_{-\infty}^{\infty} \delta(t) dt = 1$

$x(t) \times \delta(t - T_0) = x(T_0) \times \delta(t - T_0)$

$x(t) * \delta(t - T_0) = x(t - T_0)$

Convolution

$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Step Response

$s[n] = u[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m]u[n-m] = \sum_{m=-\infty}^{n} h[m]$

$s(t) = u(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h(\tau)d\tau$

Autocorrelation

$R_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]x^*[n+m]$

$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)\, dt$

$R_{xx}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n]x^*[n+m]$

$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t+\tau)\, dt$

LTI System Properties

Memoryless: $h[n] = c\delta[n]$ or $h(t) = c\delta(t)$

Causal: $h[n] = 0$ for $n < 0$ or $h(t) = 0$ for $t < 0$

Stable: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ or $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

Periodicity of Sinusoidal Sums

If $T_1/T_2 \in \mathbb{Q}$ then $T_{\text{new}} = \text{LCM}(T_1, T_2)$

If $f_1/f_2 \in \mathbb{Q}$ then $f_{\text{new}} = \text{HCF}(f_1, f_2)$

If $\omega_1/\omega_2 \in \mathbb{Q}$ then $\omega_{\text{new}} = \text{HCF}(\omega_1, \omega_2)$

Trigonometry and Complex Numbers

$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$, where $r = |z|$, $\theta = \angle z$

$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right); \quad \sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\cos 2A = \cos^2 A - \sin^2 A$

$\int_{T_0} \sin(n2\pi f_0 t + \theta)\, dt = \int_{T_0} \cos(n2\pi f_0 t + \theta)\, dt = 0$

Fundamental frequency/period: $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

Fourier Series Forms

Trigonometric: $a_0 + \sum_{n=1}^{\infty}(a_n \cos n2\pi f_0 t + b_n \sin n2\pi f_0 t)$

Amplitude-Phase (AP): $a_0 + \sum_{n=1}^{\infty} A_n \cos(n2\pi f_0 t + \theta_n)$

Complex-Exponential (CE): $a_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{jn2\pi f_0 t}$

Conversion Formulae

$A_n = \sqrt{a_n^2 + b_n^2}; \quad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$

$c_n = \frac{1}{2}(a_n - jb_n) = \frac{A_n}{2} e^{j\theta_n}; \quad c_{-n} = c_n^*$

$a_0 = A_0 = c_0; \quad a_n = c_n + c_{-n}; \quad b_n = j(c_n - c_{-n})$

Fourier Series Coefficients

$a_0 = \frac{1}{T_0} \int_{T_0} x(t) \cdot 1\, dt; \quad c_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn2\pi f_0 t} dt$

$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n2\pi f_0 t\, dt; \quad b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n2\pi f_0 t\, dt$

Fourier Transform

$X(f) = \mathscr{F}[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$

$x(t) = \mathscr{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$

$\int_{-\infty}^{\infty} e^{jxa} dx = \int_{-\infty}^{\infty} e^{-jxa} dx = 2\pi\delta(a)$

Parseval's/Rayleigh's Theorem

$P_{\text{avg}} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

Sampling

$x[n] = A\cos(2\pi f_d n + \theta)$ where $f_d = f_a/f_s$

$x_s(t) = x(nT_s) \Leftrightarrow X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s).$

Amplitude Modulation

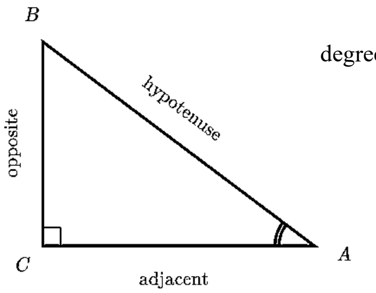
$x_{\text{AM}}(t) = A_c\left[1 + k_a m(t)\right] \cos(2\pi f_c t)$

$\mu = \max(|k_a m(t)|) = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$

$P_c = \frac{A_c^2}{2}; \quad P_s = \frac{\mu^2 A_c^2}{4} \Rightarrow \therefore \eta = \frac{P_s}{P_s + P_c} = \frac{\mu^2}{2 + \mu^2}$

Appendix: High School Math Recap

Trigonometry



$$\text{degree} \underset{\times 180°/\pi}{\overset{\times \pi/180°}{\rightleftharpoons}} \text{radian}$$

$$\sin A = \frac{\text{Opp.}}{\text{Hyp.}}$$
$$\cos A = \frac{\text{Adj.}}{\text{Hyp.}}$$
$$\tan A = \frac{\text{Opp.}}{\text{Adj.}}$$

Even and Odd:
$\cos(-A) = \cos A$; $\sin(-A) = -\sin A$

Complementary Angles, Phase Shift:
$\sin A = \cos(90° - A)$ [complementary angles]
$\quad = \cos(A - 90°)$ [since $\cos\alpha = \cos(-\alpha)$]
$\tan A = \dfrac{1}{\tan(90° - A)}$; $\sin A = \sin(180° - A)$

Common Angles:

A	0°	30°	45°	60°	90°
$\sin A$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\cos A$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{0}}{2} = 0$
$\tan A$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–

You can also use 30°-60°-90° and 45°-45°-90° triangles.

Pythagorean Identities:
$\sin^2 A + \cos^2 A = 1$
$\tan^2 A + 1 = \sec^2 A$ $[\div \cos^2 A]$
$1 + \cot^2 A = \csc^2 A$ $[\div \sin^2 A]$

For $n \in \mathbb{Z}$:
$\sin n\pi = 0$; $\cos n\pi = e^{jn\pi} = (-1)^n$
$\sin 2n\pi = 0$; $\cos 2n\pi = e^{j2n\pi} = 1$

Law of Sines and Cosines
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$; $a^2 = b^2 + c^2 - 2ab\cos A$

Calculus

$\frac{d}{dt}(\sin\alpha t) = \alpha\cos\alpha t$; $\int \sin\alpha t\, dt = -\frac{1}{\alpha}\cos\alpha t + C$
$\frac{d}{dt}(\cos\alpha t) = -\alpha\sin\alpha t$; $\int \cos\alpha t\, dt = \frac{1}{\alpha}\sin\alpha t + C$
$\frac{d}{dt}\left(e^{\alpha t}\right) = \alpha\, e^{\alpha t}$; $\int e^{\alpha t} dt = \frac{1}{\alpha}e^{\alpha t} + C$

Integration by parts:
$\int uv'\, dt = uv - \int vu'\, dt$
Determine u using <u>L</u>og <u>A</u>lg <u>T</u>rigo <u>E</u>xp (LATE)

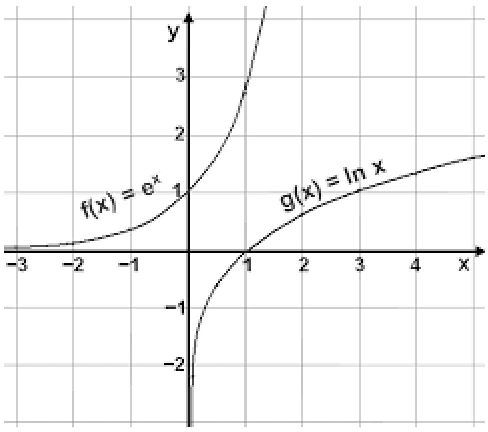
Common Algebraic Identities

$(a+b)(a-b) = a^2 - b^2$
$(a \pm b)^2 = a^2 \pm 2ab + b^2$
$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

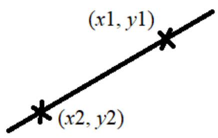
Complex Numbers

If $z = x + jy$, then:
$z = x + jy = re^{j\theta} = r(\cos\theta + j\sin\theta)$
$z^* = x - jy$; $zz^* = |z|^2$
$r = |z| = \sqrt{x^2 + y^2}$; $\theta = \angle z = \tan^{-1}\left(\frac{y}{x}\right)$

Exponential and Logarithmic Graphs



Coordinate Geometry



$m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of straight line:
$y = mx + c$ (slope-intercept form)

For equation using two methods:
1) 2 points
2) 1 gradient 1 point

Forming an equation:
1) Find m if not given.
2) $y = mx + c$; Sub (x_i, y_i) and find c.
3) $m = \dfrac{y - y_i}{x - x_i} \Rightarrow y = m(x - x_i) + y_i$