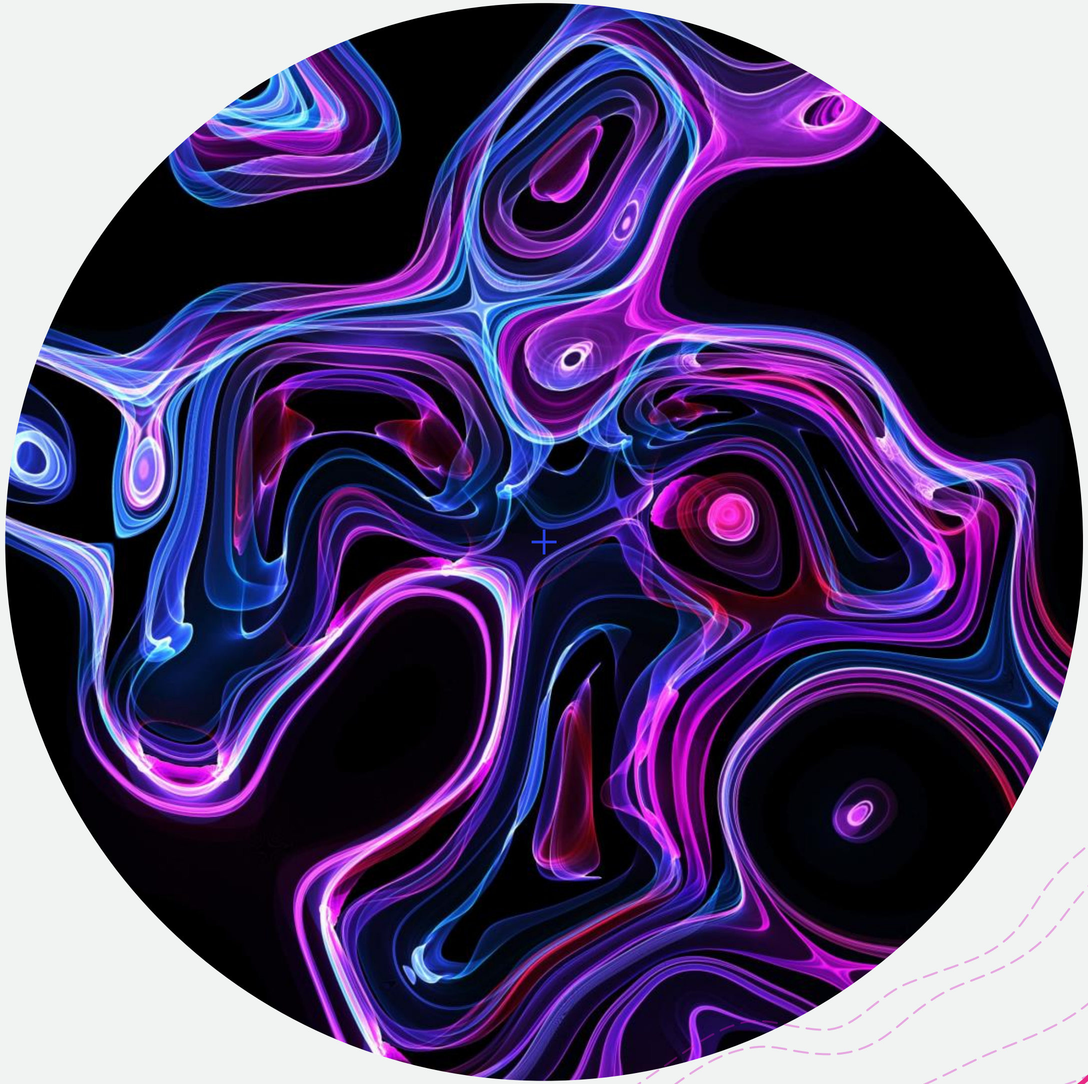
IE2110 Signals & Systems

Revision Session: *17 April 2023*



Brief Introduction

- Signals and Systems Peer Tutor for **5** semesters!
- Final year, final semester

EEE Club PYP Solutions | https://ntueeeclub.github.io/

Agenda (2 hours)

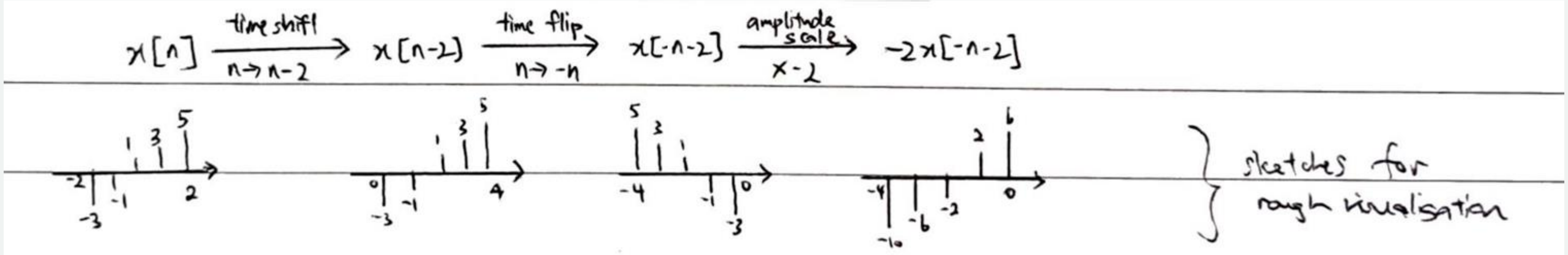
- Brief Course Overview
- Exam Tips
- Cheat Sheet
- Q&A (most of the time)

Brief Course Overview

Part 1: Time Domain

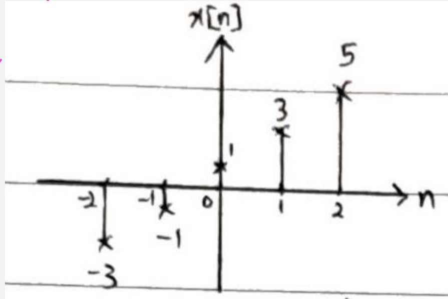
- Basic Signals
  - Delta, Step, Signum, Negative Exponential, Triangle
  - Rectangle (two forms), $n \cdot u[n]$
  - Delta properties
- Operations on Signals
  - Amplitude (x) axis: multiply, shift
  - Time (t) axis: flip, shift, stretch

- Example: Draw $-2x[-n-2]$.



Part 1: Time Domain

- Energy-Type *vs* Power-Type Signal
  - Energy – finite signals
  - Power – period, infinite signals
  - Formula important
- Even *vs* Odd Signal
  - Formula important
  - Know how to draw

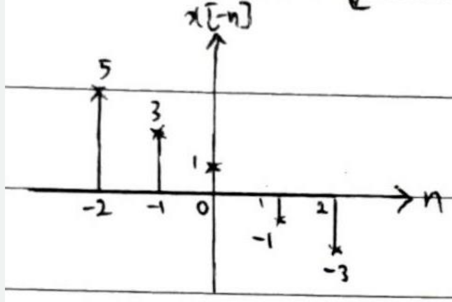


$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= (-3)^2 + (-1)^2 + 1^2 + 3^2 + 5^2$$

$$= 45 \quad \#$$

+ or −

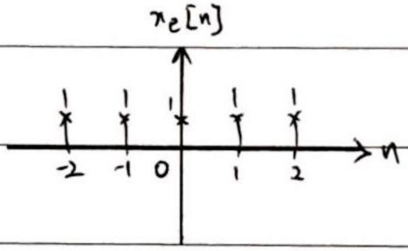


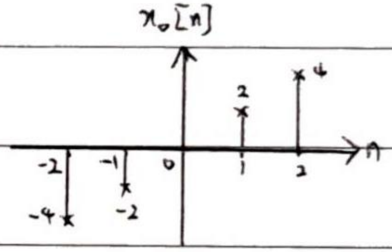
$$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right) \qquad E_{x_e} = 5$$

$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right) \qquad E_{x_o} = 40$$

E_x energy is conserved!

$$x[n] = x_e[n] + x_o[n] \quad \#$$





Part 1: Time Domain

- Properties of Systems, $x[n] \rightarrow y[n]$
  - Stability, Memory, Causality, Linearity, Time Invariant
  - For Linearity, Time Invariant – Just follow their format
- LTI System Properties
  - $h(t)$ impulse response describes the whole system
  - Analyse $h(t)$ in terms of stability, memory, causality
  - Parallel and cascade systems

Linearity :

$$x_1(t) \rightarrow \boxed{H} \rightarrow y_1(t) = x_1(5t)$$

$$x_2(t) \rightarrow \boxed{H} \rightarrow y_2(t) = x_2(5t)$$

linear ✓ #

$$\alpha x_1(t) + \beta x_2(t) \rightarrow \boxed{H} \rightarrow y(t) = \alpha x_1(5t) + \beta x_2(5t) \equiv \alpha y_1(t) + \beta y_2(t)$$

Time invariant :

$$\left.\begin{aligned} &x(t) \rightarrow \boxed{H} \rightarrow y(t) = x(5t) \\ &x(t-T) \rightarrow \boxed{H} \rightarrow x(5(t-T)) \neq x(5t-T) = y(t-T) \end{aligned}\right\}$$

non-time invariant

#

Part 1: Time Domain

- Convolution
  - Has delta → Use delta properties
  - Graphical method?
    - 1) Identify easier signal to be used as $h(t)$
    - 2) Flip and shift $h(t)$: negate and add '$+t$' to each value
    - 3) Identify important time values [CT], Shift one by one [DT]
    - 4) Calculate values at time values by multiply THEN sum
  - In doubt? Use formula
  - Step response, start from definition first.
    - Integral variant formula hardly used

Part 1: Time Domain

- Correlation
  - Graphical method?
    - 1) Identify easier signal to be used as $h(t)$
    - 2)** $y(t)$: add '$-\tau$' to each value
    - 3) Identify important time values [CT], Shift one by one [DT]
    - 4) Calculate values at time values by multiply THEN sum
  - Auto-correlation properties
  - Identify correct energy/power formula to be used.

Q & A?

Part 2: Frequency Domain

- Fourier Series
  - Test for periodicity: Suggested to used HCF for f_0
  - GET RID of weird looking periodic functions, e.g. $\sin^2 2t$.
  - Rewrite in the form $\cos(2\pi \cdot nf_0 t)$ or sin, so that you can identify the harmonic easily
  - 3 forms: Trigonometric, Amplitude-Phase, Complex Exponential
  - Lots of formulas in this chapter ☹ → Cheat sheet
  - FS coefficients and properties

Q2. Consider a periodic signal $x(t) = \frac{\sqrt{2}}{2}\cos\left(\frac{13}{6}t\right) - \frac{\sqrt{2}}{2}\sin\left(\frac{13}{6}t\right) + 6\sin(3t)$, t is in seconds.

(a) What is the fundamental frequency of $x(t)$ in rads/sec?

(b) Express the signal $x(t)$ in *amplitude-phase* Fourier series form and sketch the one-sided plot of the amplitude and phase spectra of $x(t)$.

(c) Express the signal $x(t)$ in *complex-exponential* Fourier series form and sketch the two-sided plot of the magnitude and phase spectra of $x(t)$.

(35 marks)

2 a) $\omega_0 = \text{HCF}\left(\frac{13}{6}, 3\right)$

$= \text{HCF}\left(\frac{13}{6}, \frac{18}{6}\right)$

$= \frac{1}{6}$ rad s^{-1} #

Change $\omega_0 \rightarrow 2\pi f_0$

b) $x(t) = \frac{\sqrt{2}}{2}\cos(13\omega_0 t) - \frac{\sqrt{2}}{2}\sin(13\omega_0 t) + 6\sin(18\omega_0 t)$, where $\omega_0 = \frac{1}{6}$

$$A = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = 1$$

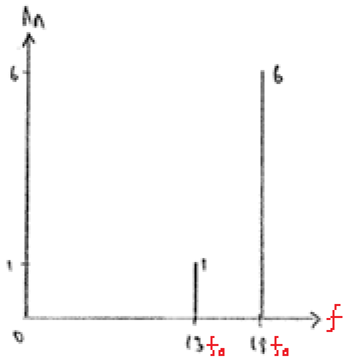
$$6\sin(18\omega_0 t)$$
$$= 6\cos(18\omega_0 t - 90^\circ)$$

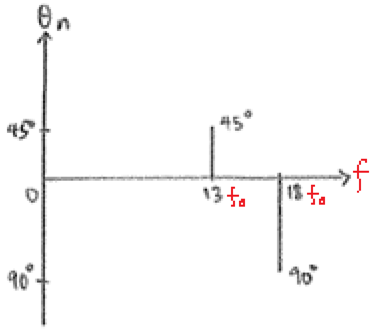
$$\theta = \tan^{-1}\left(\frac{-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}}\right) = 45^\circ$$

Change $\omega_0 \rightarrow 2\pi f_0$

$\therefore\ x(t) = \cos(13\omega_0 t + 45^\circ) + 6\cos(18\omega_0 t - 90^\circ)$, where $\omega_0 = \frac{1}{6}$ #

$A_{13} = 1$, $\theta_{13} = 45^\circ$, $A_{18} = 6$, $\theta_{18} = -90^\circ$



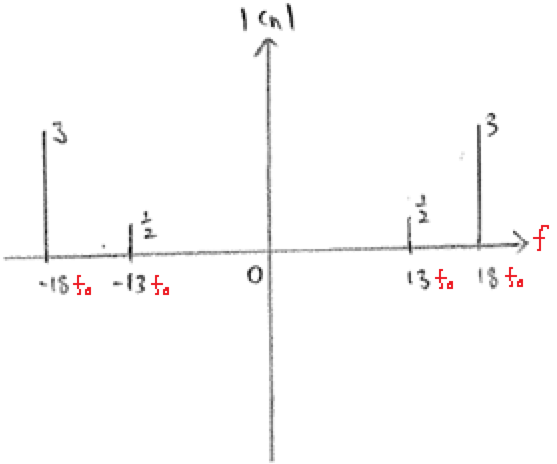


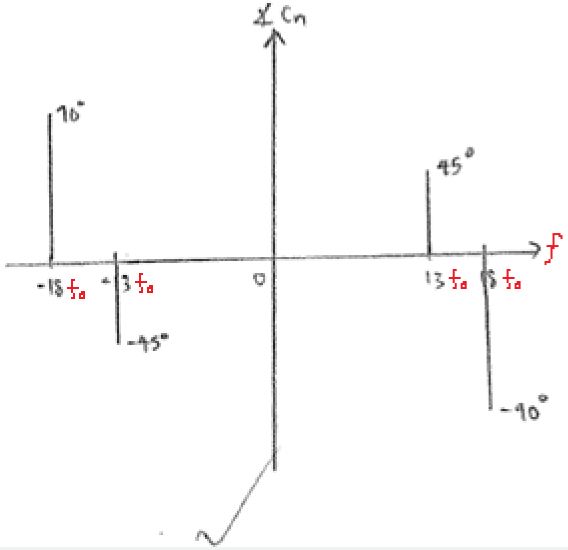
Change $\omega_0 \rightarrow 2\pi f_0$

c) $x(t) = \cos(13\omega_0 t + 45°) + 6\cos(18\omega_0 t - 90°)$

$$= \frac{1}{2}\left[e^{j(13\omega_0 t + 45°)} + e^{-j(13\omega_0 t + 45°)}\right] + 6\left(\frac{1}{2}\right)\left[e^{j(18\omega_0 t - 90°)} + e^{-j(18\omega_0 t - 90°)}\right]$$

$$= \underbrace{\frac{1}{2} e^{j(45°)}}_{c_{13} = \frac{1}{2} \angle 45°} e^{13j\omega_0 t} + \underbrace{\frac{1}{2} e^{j(-45°)}}_{c_{-13} = \frac{1}{2} \angle -45°} e^{-13j\omega_0 t} + \underbrace{3 e^{j(-90°)}}_{c_{18} = 3 \angle -90°} e^{18j\omega_0 t} + \underbrace{3 e^{j(90°)}}_{c_{-18} = 3 \angle 90°} e^{-18j\omega_0 t}$$





Part 2: Frequency Domain

- Fourier Transform
  - Be familiar with the transform pairs, and transform operations
  - Keep track of any transformation performed, working should be clear
  - Partial fractions may be needed
  - Memorise FT formula
- Filtering
  - Remember 4 types of ideal filters:
    low-pass, high-pass, bandpass, bandstop
  - $y(t) = x(t) * h(t) \quad \Leftrightarrow \quad Y(f) = X(f)H(F)$

E.g. $H(f) = \frac{4e^{-j6\pi f}}{4+4\pi^2 f^2}$, find $h(t)$.

$e^{-a\|t\|}$	$\frac{2a}{a^2+(2\pi f)^2}$	$a>0$
$x(t-t_0)$	$X(f)e^{-j2\pi f t_0}$	

$$H(f) = \frac{4}{4+4\pi^2 f^2}\cdot e^{-j6\pi f} = \frac{2(2)}{2^2+(2\pi f)^2}\cdot e^{-j2\pi f(3)} = \left.\frac{2(2)}{2^2+(2\pi f)^2}\right|_{t\to t-3}$$

$$\therefore h(t) = \left.e^{-2|t|}\right|_{t\to t-3} = e^{-2|t-3|}$$

E.g. $H(f) = \frac{6e^{-j6\pi f}}{4+4\pi^2 f^2}$, find $h(t)$.

$e^{-a\|t\|}$	$\frac{2a}{a^2+(2\pi f)^2}$	$a>0$
$x(t-t_0)$	$X(f)e^{-j2\pi f t_0}$	

$$H(f) = \frac{6}{4+4\pi^2 f^2}\cdot e^{-j6\pi f} = \frac{3}{2}\cdot\frac{2(2)}{2^2+(2\pi f)^2}\cdot e^{-j2\pi f(3)} = \frac{3}{2}\cdot\frac{2(2)}{2^2+(2\pi f)^2}\bigg|_{t\to t-3}$$

$$\therefore h(t) = \frac{3}{2}e^{-2|t|}\bigg|_{t\to t-3} = \frac{3}{2}e^{-2|t-3|}$$

Part 2: Frequency Domain

- Sampling
  - ➢Identify Nyquist rate → MAXIMUM not LCM/HCF
  - ➢Which case of sampling? Under, critical, over
  - ➢Convert time domain signal to discrete
  - ➢Make sure $|f_d| \leq 0.5$ by $\pm 2n\pi$, use even/odd properties
  - ➢For frequency analysis, draw original signal
    Then, duplicate the signal ≥ 3 times, and scale with f_s
  - ➢Familiarity with the following frequencies: f_s, f_N, f_m, f_a, f_d

$$x[n] = x(t)\big|_{t=n/f_s}$$

$2\pi \cdot \frac{f_a}{f_d} \cdot n$

$\downarrow$ format

$$= 10\cos\left(2\pi \cdot \frac{50}{420} \cdot n\right) - 5\sin\left(2\pi \cdot \frac{300}{420} \cdot n - 60^\circ\right)$$

$$= 10\cos\left(2\pi \frac{50}{420} \cdot n\right) - 5\sin\left(2\pi \cdot \frac{-120}{420} n - 60^\circ\right)$$

$$= 10\cos\left(2\pi \cdot \frac{50}{420} \cdot n\right) + 5\sin\left(2\pi \cdot \frac{120}{420} \cdot n + 60^\circ\right)$$

$|f_d| \leq 0.5$

minus 1

$f_d > 0$

factorise '-' value

$V(t) = 1 + \cos(400\pi t) + 3\sin(300\pi t)$

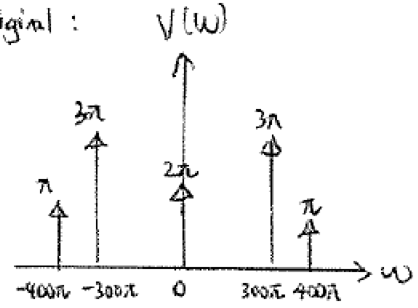
Change $\omega_0 \rightarrow 2\pi f_0$

$f_N = 2 f_{max} = 2 \max\{200, 150\} = 400\text{Hz}$ #

$\cos \omega_0 t \Leftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \rightarrow$ magnitude : π

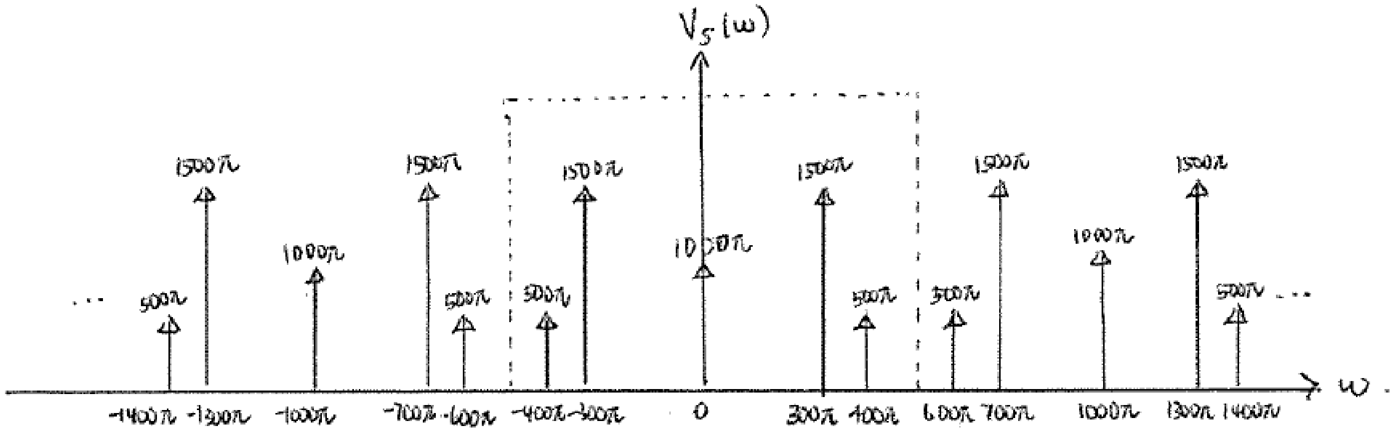
$\sin \omega_0 t \Leftrightarrow j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)] \rightarrow$ magnitude : π

original :



$f_s = 500\text{ Hz} > f_N = 400\text{ Hz} \Rightarrow$ oversampling (no aliasing!)

Amplitude scaled by $\frac{1}{T_s} = f_s = 500$

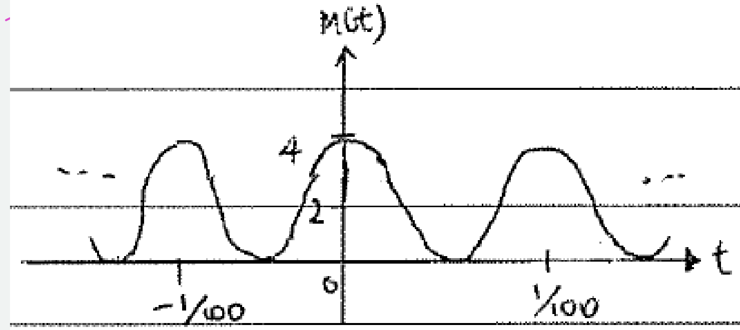


Q & A?

Part 3: Application

- Modulation
  - Mainly amplitude modulation covered
  - Which case of modulation? Under, over
  - You can tell by analysing the range of the amplitude of $x_{AM}(t)$
  - Drawings: make sure your diagram is labelled clearly
  - Some formulas you have to memorise, few marks allocated
  - Bandwidth analysis – only positive freqs are important

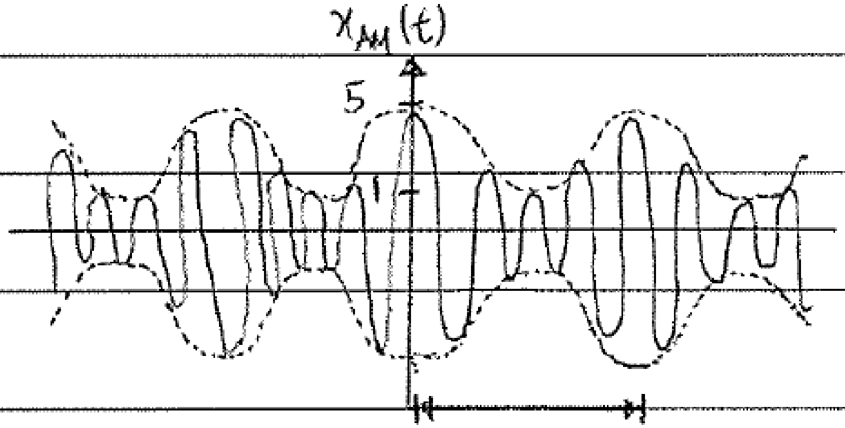
$$m(t) = 4\cos^2(100\pi t) = 4 \cdot \frac{1}{2}[1+\cos(200\pi t)] = 2\cos(200\pi t) + 2 \qquad T_0 = \frac{2\pi}{200\pi} = \frac{1}{100}\ s$$



$m(t) : [0, 4]$

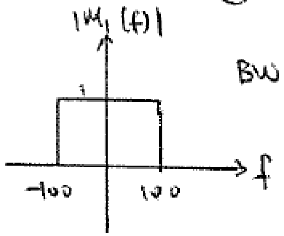
$1+m(t) : [1, 5]$

Find range of $(1 + k_a m(t))$ to determine which case of modulation.

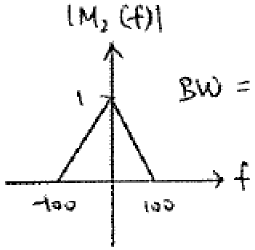


(from $\frac{1000}{200} = 5$) 5 cycles per period

2) b) i) Bandwidth : only +ve frequencies !



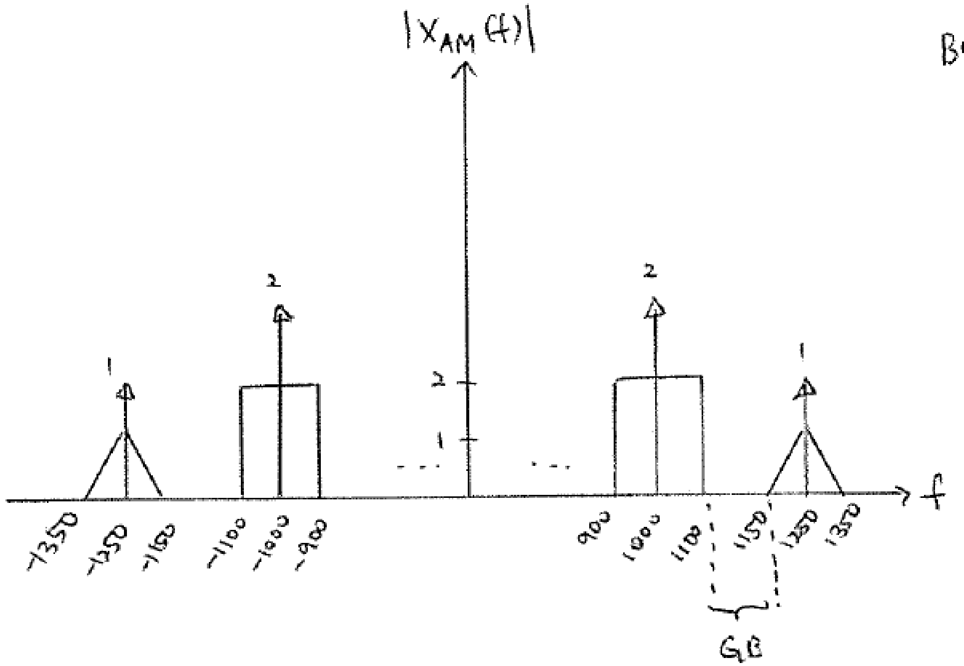
BW = 100 Hz



BW = 100 Hz

ii) Amplitudes : $\frac{A_c}{2}$ for carrier ; $\frac{k_a A_c}{2}$ for M

$\{ A_{c_1} = 4,\ k_{a_1} = 1,\ A_{c_2} = 2,\ k_{a_2} = 1 \}$



$$BW = (1350 - 1150) + (1100 - 900)$$
$$= 200 + 200$$
$$= 400\,Hz$$

Q & A?

Exam Tips

Disclaimer: The next section is only personal advice, please use your own discretion whether to follow or not.

Commencing Notes

- Part 2 has changed (Prof Ma → Prof Er)
- Maybe new format for PYP?

- 2 hours (divide) 4 questions = 30 mins each
  - Try to finish ‘easy’ questions within 25 mins

Question 1 and 2

- Prof Teh’s content
- Do convolution/operations on signals last as it takes up some time
- Make sure to draw out graphs (sketch). It helps with visualisation!
- Try to score full marks for modulation.

Question 3

[REPEATED FROM EARLIER]

- Fourier Series – Prof Er's content
  - Test for periodicity: Suggested to used HCF for f
  - GET RID of weird looking periodic functions, e.g. $\sin^2 2t$.
  - Rewrite in the form $\cos(n \cdot 2\pi f_0 \cdot t)$ or sin, so that you can identify the harmonic easily
  - 3 forms: Trigonometric, Amplitude-Phase, Complex Exponential
  - Lots of formulas in this chapter ☹ → Cheat sheet
  - FS coefficients and properties

Question 4

- Prof Er’s content
- Most unpredictable question ☹
- Many forms so your fundamentals must be good
- Try to score full marks for sampling.

* Tips to Score : ~PL :)

- Understand the module on a very fundamental level.
- Have a cheat sheet for formulae to memorise, eg. energy, fourier transform
- Start by sketching any basic graphs with pencil and split your transformations one-by-one : perform on the graph. Eg $\sum_{n=-1}^{1} x(t-3n) = x(t+3) + x(t) + x(t-3)$. Expand out first and realise that the transformation makes three copies of x, shift one to the left, right and no change.

Q & A?

Cheat Sheet

IE2110 Cheat Sheet Request Form

Important Formulae – IE2110 (formerly known as IE2010 or EE2010/IM2004)

Even and Odd Functions

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Energy and Power Formulae

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x[n]|^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Periodic signals, $P_x = \frac{1}{K_0} \sum_{n=k}^{k+K_0-1} |x[n]|^2 = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt$

Properties of Impulse Function

Trigonometry and Complex Numbers

$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$, where $r = |z|$, $\theta = \angle z$

$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$; $\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\int_{T_0} \sin(n2\pi f_0 t + \theta)\, dt = \int_{T_0} \cos(n2\pi f_0 t + \theta)\, dt = 0$$

Fundamental frequency/period: $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

Fourier Series Forms

Trigonometric: $a_0 + \sum_{n=1}^{\infty}(a_n \cos n2\pi f_0 t + b_n \sin n2\pi f_0 t)$

Q & A?

Thank you for coming!