If $\sigma \geq 0$, W_α^* is an exponentially tilted stable random variable so that there exist exact samplers.

Jim Pitman, 5.3 said

if Levy density has the form $\rho_\alpha(x) = \frac{\alpha x^{-\alpha-1}}{\Gamma(1-\alpha)} \quad (x > 0)$, then total length $T =: S_\alpha$ follows $\operatorname{stable}(\alpha)$ with pdf $f(t)$

Furthermore, from Levy khintchine formula, we can get a Laplace transform as

$$\mathbb{E}\left\{\mathrm{e}^{-tT}\right\} = \exp\left\{\int_{(0,\infty]} \left(1 - \mathrm{e}^{-tx}\right) \rho_\alpha(dx)\right\} = \exp(-t^\alpha)$$

Jim Pitman, 4.2 said

If we set $\rho_{\alpha,b}(x) = \rho_\alpha(x)e^{-bx}$ as a Levy density, then $T =: S_{\alpha,b}$'s pdf $f^{(b)}(t) = f(t)e^{\psi(b)-bt}$ where $\psi(b) = \int_0^\infty \left(1 - e^{-bx}\right)\rho_\alpha(x)dx = \exp(-b^\alpha)$

Therefore, using a simple trick, we can get a Laplace transform as

$$\mathbb{E}\left\{e^{-tS_{\alpha,\lambda}}\right\} = \mathbb{E}_\alpha\left\{e^{-\lambda^\alpha-(t+\lambda)S_\alpha}\right\} = e^{-\lambda^\alpha-(t+\lambda)^\alpha}$$

Caron, 5.3 considers Levy measure

$$\begin{aligned}\rho(dw)\lambda([0,\alpha]) &= \frac{\alpha}{\Gamma(1-\sigma)} w^{-1-\sigma}\exp(-\tau w)dw \\ &= \frac{\alpha}{\sigma} \times \frac{\sigma}{\Gamma(1-\sigma)} w^{-1-\sigma}\exp(-\tau w)dw \\ &= \frac{\alpha}{\sigma}\rho_{\sigma,\tau}(dw)\end{aligned}$$

Then, from Levy khintchine formula,

$$\begin{aligned}\mathbb{E}\left\{\mathrm{e}^{-tW_\alpha^*}\right\} &= \exp\left\{\int_{(0,\infty]} \left(1 - \mathrm{e}^{-tx}\right)\rho(dx)\lambda([0,\alpha])\right\} \\ &= \exp\left\{\frac{\alpha}{\sigma} \times \int_{(0,\infty]} \left(1 - \mathrm{e}^{-tx}\right)\rho_{\sigma,\tau}(dx)\right\} \\ &= \exp\left\{\frac{\alpha}{\sigma} \times \left(-\tau^\sigma - (t+\tau)^\sigma\right)\right\} \\ &=: \exp\{M \times (-\tau^\sigma - (t+\tau)^\sigma)\} \\ &= \exp\left\{\left(-\left(M^{\frac{1}{\sigma}}\tau\right)^\sigma - \left(M^{\frac{1}{\sigma}}t + M^{\frac{1}{\sigma}}\tau\right)^\sigma\right)\right\} \\ &= \mathbb{E}\left[\exp\left(-tM^{\frac{1}{\sigma}}S_{\sigma,M^{1/\sigma}\tau}\right)\right]\end{aligned}$$

Hence, we can sample W_α^* as

1. Sample $T = S_{\sigma,M^{1/\sigma}\tau}$
2. Multiply $T = T \times M^{\frac{1}{\sigma}}$