Caron, 3.2 samples directed graph using urn-process

$$
\begin{aligned}
&1. \, W_\alpha^* \sim P_{W_\alpha^*} \\
&2. \, D_\alpha^* \mid W_\alpha^* \sim \text{Poisson}\left(W_\alpha^{*2}\right) \\
&3. \, (U_{kj})_{k=1,\ldots,D_\alpha^*; j=1,2} \mid W_\alpha^* \sim \text{ Urn process} \\
&4. \, D_\alpha = \textstyle\sum_{k=1}^{D_\alpha^*} \delta_{(U_{k1}, U_{k2})}
\end{aligned}
$$

Caron, 5.3 defines EPPF of $\mathrm{PK}(\rho \mid t)$ as

$$
\Pi_k^{(n)}\left(m_1, \ldots, m_k \mid t\right) = \frac{\sigma^k t^{-n}}{\Gamma(n - k\sigma) g_\sigma(t)} \int_0^t s^{n-k\sigma-1} g_\sigma(t-s) ds \left(\prod_{i=1}^k \frac{\Gamma\left(m_i - \sigma\right)}{\Gamma(1-\sigma)}\right)
$$

where g_σ is the pdf of the positive stable distribution.

In the urn-process,

$$
\begin{aligned}
\frac{\Pi_{n+1}^{(k+1)}\left(m_1, \ldots, m_k, 1 \mid W_\alpha^*\right)}{\Pi_n^{(k)}\left(m_1, \ldots, m_k \mid W_\alpha^*\right)} &= \frac{\sigma t^{-1} \gamma(n - k\sigma)}{\Gamma(n+1-(k+1)\sigma)} \frac{\int_0^t s^{n-(k+1)\sigma} g_\sigma(t-s) ds}{\int_0^t s^{n-k\sigma-1} g_\sigma(t-s) ds} \\
&= \frac{\sigma t^{-1} \gamma(n - k\sigma)}{\Gamma(n+1-(k+1)\sigma)} \frac{\mathbb{E}\left[(t-s)^{n-(k+1)\sigma} \mathbf{1}(0 \leq s \leq t)\right]}{\mathbb{E}\left[(t-s)^{n-k\sigma-1} \mathbf{1}(0 \leq s \leq t)\right]}
\end{aligned}
$$

However, the MC approximation of the value is not converged because $(t-s)^{n-k\sigma-1}$ becomes large for large urn samples.