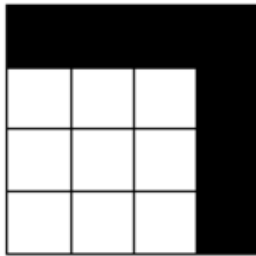


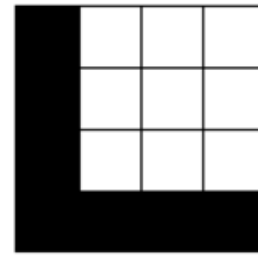
# Hopfield Network

Sungchul Lee

Training Image



[1 1 1 1 -1 -1 -1 1 -1 -1 -1 1 -1 -1 -1 1]



[1 -1 -1 -1 1 1 -1 -1 -1 1 1 -1 -1 1 1 1]

Training using the formula

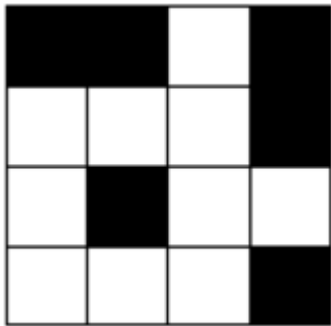
$$W = Y_1 Y_1^T + Y_2 Y_2^T - 2I$$

$$W = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} [1 1 1 1 -1 -1 -1 1 -1 -1 -1 1 -1 -1 -1 1] + \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [1 -1 -1 -1 1 1 -1 -1 -1 -1 -1 1 1 1 1] - 2I$$

## Weighted Matrix

0	0	0	0	0	-2	-2	0	0	-2	-2	0	0	0	0	2
0	0	2	2	-2	0	0	2	-2	0	0	2	-2	-2	-2	0
0	2	0	2	-2	0	0	2	-2	0	0	2	-2	-2	-2	0
0	2	2	0	-2	0	0	2	-2	0	0	2	-2	-2	-2	0
0	-2	-2	-2	0	0	0	-2	2	0	0	-2	2	2	2	0
-2	0	0	0	0	0	2	0	0	2	2	0	0	0	0	-2
-2	0	0	0	0	2	0	0	0	2	2	0	0	0	0	-2
0	2	2	2	-2	0	0	0	-2	0	0	2	-2	-2	-2	0
0	-2	-2	-2	2	0	0	-2	0	0	0	-2	2	2	2	0
-2	0	0	0	0	2	2	0	0	0	2	0	0	0	0	-2
-2	0	0	0	0	2	2	0	0	2	0	0	0	0	0	-2
0	2	2	2	-2	0	0	2	-2	0	0	0	-2	-2	-2	0
0	-2	-2	-2	2	0	0	-2	2	0	0	-2	0	2	2	0
0	-2	-2	-2	2	0	0	-2	2	0	0	-2	2	0	2	0
0	-2	-2	-2	2	0	0	-2	2	0	0	-2	2	2	0	0
2	0	0	0	0	-2	-2	0	0	-2	-2	0	0	0	0	0

## Test Image



[1 1 -1 1 -1 -1 -1 1 -1 1 -1 -1 -1 1 1]

$Y_n = [1 \ 1 \ -1 \ 1 \ -1 \ -1 \ -1 \ 1 \ -1 \ 1 \ -1 \ -1 \ -1 \ 1 \ 1] * \text{Weighted Matrix}$

$$Y_1 = 1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ -2 \\ 0 \\ 0 \\ -2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

$$Y_1 = 1 + (2+2-2+2+2) = 7$$

$$\text{Sign}(Y_1) = 1$$

$$Y_2 = 1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1] \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ -2 \\ -2 \\ 0 \end{bmatrix}$$

$$Y_2 = 1 + (-2+2+2+2+2-2+2+2+2) = 11$$

$$\text{Sign}(Y_2) = 1$$

$$Y_3 = -1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1] \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ -2 \\ -2 \\ 0 \end{bmatrix}$$

$$Y_3 = -1 + (2+2+2+2+2+2+2) = 13$$

$$\text{Sign}(Y_3) = 1 \quad \text{changed value } -1 \rightarrow 1$$

$$Y_4 = 1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1]$$

$$\begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ -2 \\ -2 \\ 0 \end{bmatrix}$$

$$Y_4 = 1 + 10 = 11$$

$$\text{Sign}(Y_4) = 1$$

$$Y_5 = -1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1]$$

$$\begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$$Y_5 = -11$$

$$\text{Sign}(Y_5) = -1$$

$$Y_6 = -1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1]$$

$$\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

$$Y_6 = -7$$

$$\text{Sign}(Y_6) = -1$$

$$Y_7 = -1 + [1\ 1 - 1\ 1 - 1 - 1 - 1\ 1 - 1\ 1 - 1 - 1 - 1 - 1\ 1]$$

$$\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

$$Y_7 = -7$$

$$\text{Sign}(Y_7) = -1$$

$$Y_8 = 1 + [1\ 1 - 1\ 1 - 1 - 1 - 1\ 1 - 1\ 1 - 1 - 1 - 1 - 1\ 1]$$

$$\begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ -2 \\ -2 \\ 0 \end{bmatrix}$$

$$Y_8 = 11$$

$$\text{Sign}(Y_8) = 1$$

$$Y_9 = 1 + [1\ 1 - 1\ 1 - 1 - 1 - 1\ 1 - 1\ 1 - 1 - 1 - 1 - 1\ 1]$$

$$\begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$$Y_9 = 7$$

$$\text{Sign}(Y_9) = 1$$

$$Y_{10} = 1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1] \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

$$Y_{10} = -9$$

$$\text{Sign}(Y_{10}) = -1 \text{ changed value } 1 \rightarrow -1$$

$$Y_{11} = -1 + [1 \ 1 - 1 \ 1 - 1 - 1 - 1 \ 1 - 1 \ 1 - 1 - 1 - 1 - 1 \ 1] \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

$$Y_{11} = -7$$

$$\text{Sign}(Y_{11}) = -1$$

$$Y_{12} = -1 + [11-11-1-1-1-11-11-1-1-1-1-11] \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ -2 \\ -2 \\ -2 \\ 0 \end{bmatrix}$$

$$Y_{12} = 13$$

$$\text{Sign}(Y_{12}) = 1 \quad \text{changed value } -1 \rightarrow 1$$

$$Y_{13} = -1 + [11-11-1-1-1-11-11-1-1-1-1-11] \begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$$Y_{13} = -11$$

$$\text{Sign}(Y_{13}) = -1$$

$$Y_{14} = -1 + [11-11-1-1-1-11-11-1-1-1-1-11] \begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$Y_{14} = -11$$

$$\text{Sign}(Y_{14}) = -1$$

$$Y_{15} = -1 + [11-11-1-1-1-11-11-1-1-1-1-11] \begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 0 \\ 0 \\ -2 \\ 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$Y_{15} = -11$$

$$\text{Sign}(Y_{15}) = -1$$

$$Y_{16} = 1 + [11-11-1-1-1-11-11-1-1-1-1-11] \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ -2 \\ 0 \\ 0 \\ -2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Y_{16} = 7$$

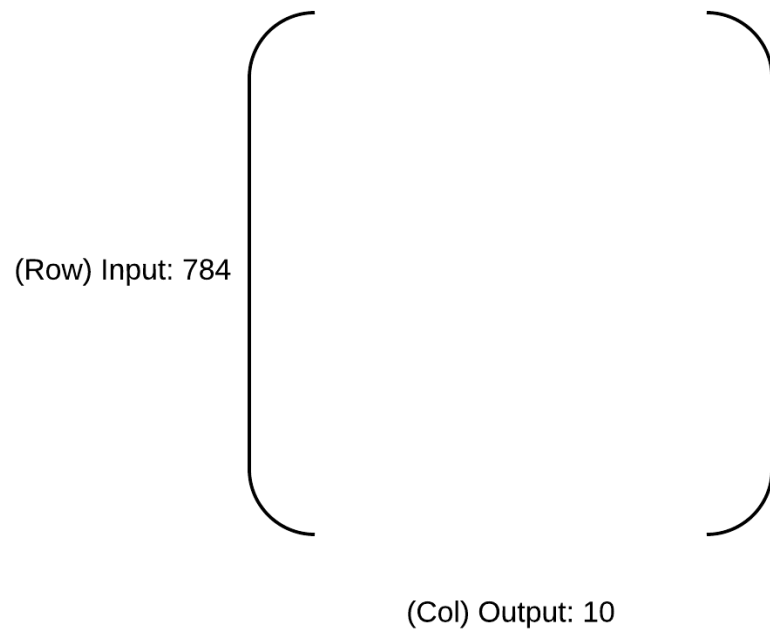
$$\text{Sign}(Y_{16}) = 1$$

Input value: [1 1 -1 1 -1 -1 -1 1 -1 1 -1 -1 -1 -1 1]

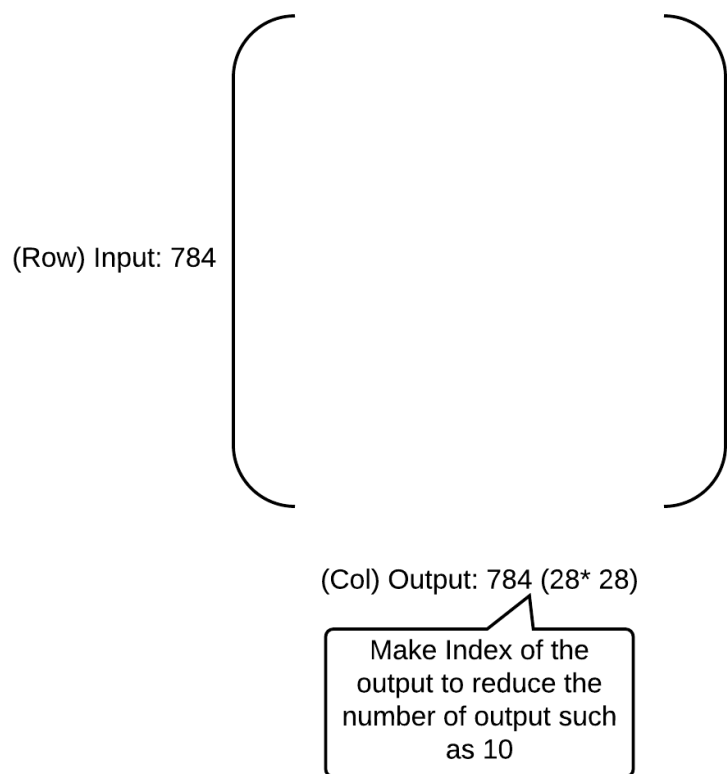
Output value: [1 1 **1** 1 -1 -1 -1 1 -1 **-1** -1 **1** -1 -1 1]

The output is same as the value of first train image. In the program, I do not reduce the number of output such as 10 output like below:





However, I recognize the handwriting image as the image like below.



Therefore, I make 784 outputs to find the image which is in training image and label. I make an index of the 784 outputs which are mapping the 10 outputs (0 ~9).