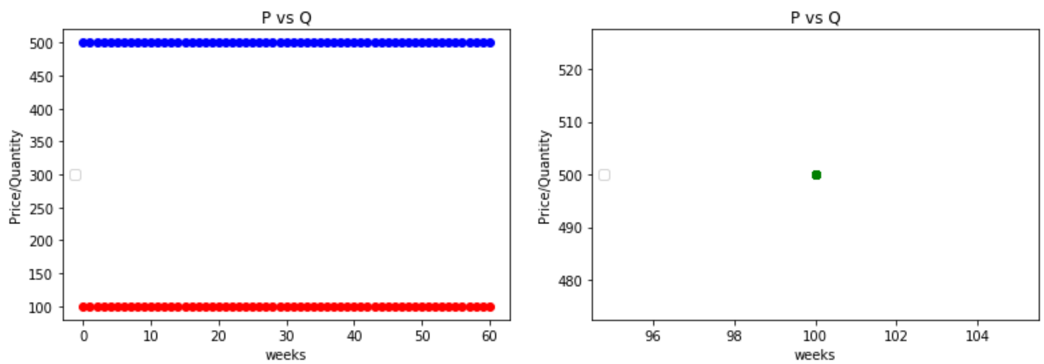
Foundations of Applied Math
HW #4 Due Friday, Oct. 9

Reminder You need to turn in a .zipped folder that contains your .tex file, your image files, your python file and the tex file must compile. Rename the .tex file: HW4_YourLastName.tex and call the folder which you will compress: HW4_YourLastName

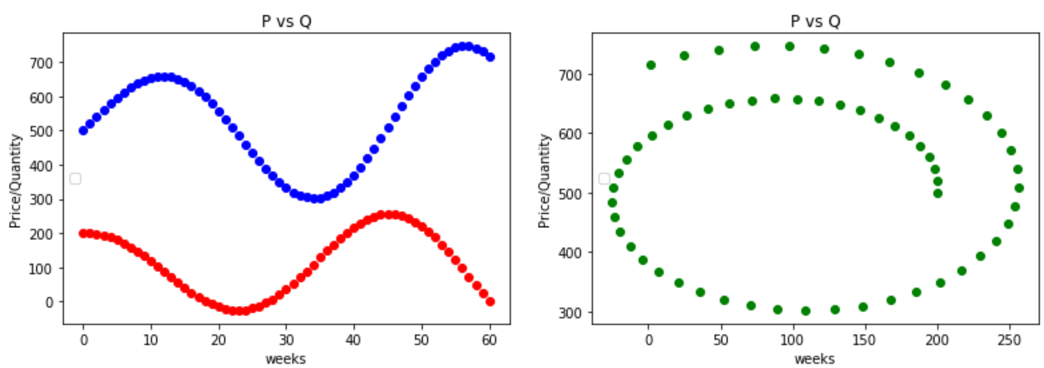
Please make sure you learn the examples in 1.4 before you do this HW.

1. 1.4) Do #6b this time using python. For each of the four cases please make sure you generate a plots like the ones we generated in HW 3 using Excel. (On Friday I will show you how to generate plots using arrays in python and how to generate a legend). Include your python file so that I can run it and paste your 8 graphs here. (Each case should have 2 graphs.)

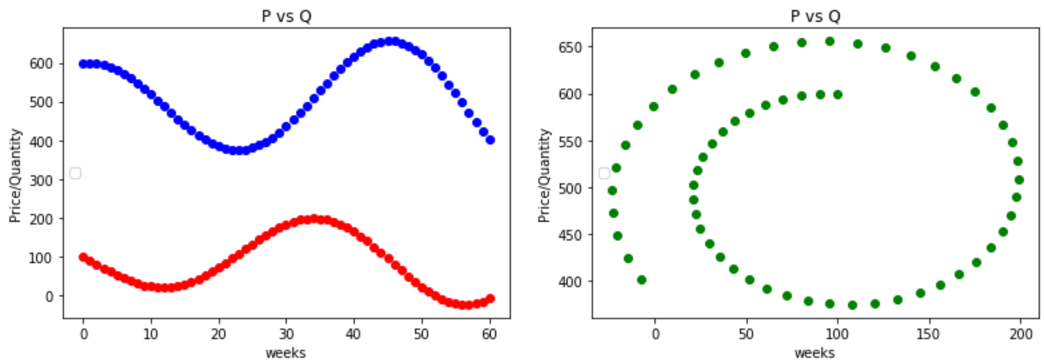
   (a) Case A:

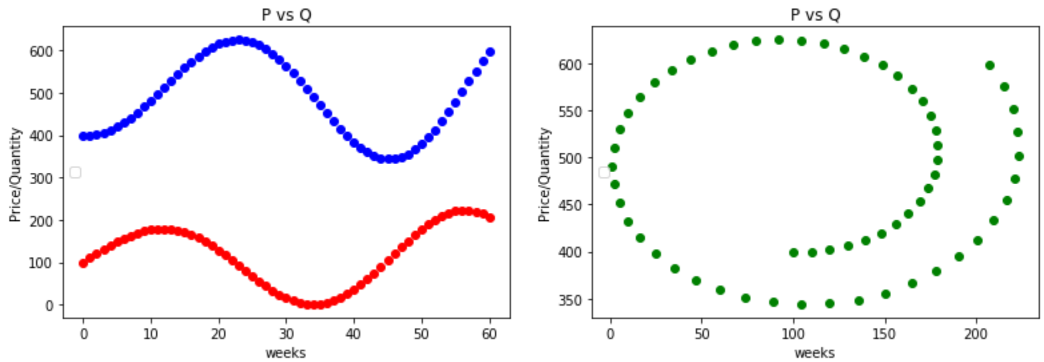
   (b) Case B:

   (c) Case C:



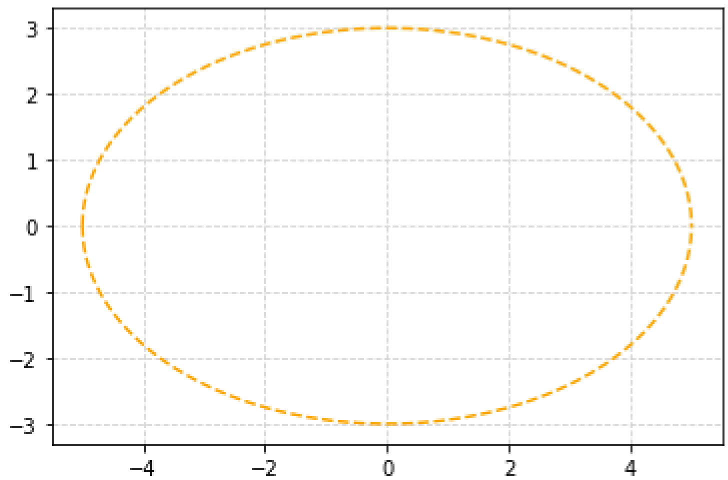
(d) Case D:



2. 1.4) Using arrays plot $\frac{x^2}{25} + \frac{y^2}{9} = 1$ using python. Make sure your plot is one color curve. Have your last two lines be:

```
plt.grid(color='lightgray',linestyle='--')
plt.show()
```

Include your python file so that I can run it and paste your graph here. Hint- you will need to solve for y.



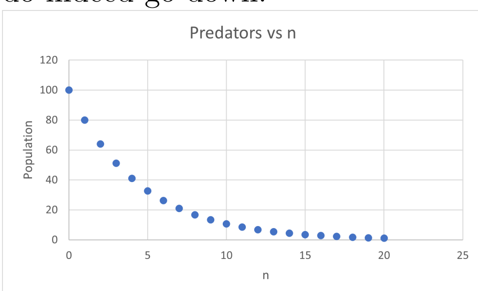
Python file is inlcuded in the zipped folder.

3. Consider the following model for two species.

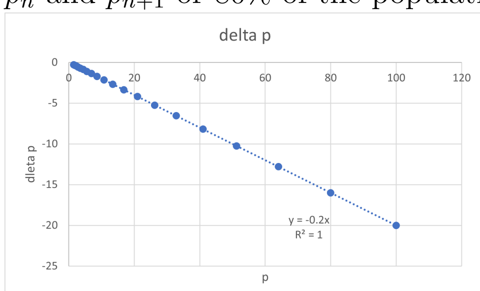
$$\begin{aligned} x_{n+1} &= 0.8x_n + 0.001x_n y_n \\ y_{n+1} &= y_n + 0.0001y_n(3000 - y_n) - 0.004x_n y_n \end{aligned}$$

a) This model is a predator-prey model. Explain why. Also state which species is the predator.
This model shows that the change in x_{n+1} increases depending on how many interactions there are between x and y. However, y decreases depending on how many interactions there are between x and y. Therefore, x is the predator and y is the prey.

b) According to the model, what happens to the population of the predator if there is no prey present? Support your answer with a graph of predators vs. n.
The population of the predators will only decrease if there are no prey. Specifically, only 80% of the predator population will survive in the next iteration if there are no prey. Looking at the graph of n vs p, we see that the population of the predators do indeed go down:



Plotting the Δp_n vs p graph, we also see that the change is indeed -20% between p_n and p_{n+1} or 80% of the population.



c) According to the model, will the population of the prey experience unlimited population growth if there are no predators present?
The population of the prey will not experience unlimited population growth because of the second term. This logistic term will force the maximum population to be 3000.

4. 2.2, #3

By Hooke's law:

$$\begin{aligned} F &= kS \\ 14 &= k(.37) \\ k &= \frac{14}{.37} \\ k &\approx 37.83784 \end{aligned}$$

Therefore, for 9-lb on the same spring:

$$\begin{aligned} F &= kS \\ S &= \frac{F}{k} \\ S &= \frac{9}{37.83784} \\ S &\approx .23786 \end{aligned}$$

So a 9-lb weight will stretch our spring by about .23876 inches.

For a 22-lb force:

$$\begin{aligned} F &= kS \\ S &= \frac{F}{k} \\ S &= \frac{22}{37.83784} \\ S &\approx .58143 \end{aligned}$$

So a 22-lb weight will stretch our spring by about .58143 inches.

5. 2.2 #6

   By taking our data and fitting an OLS regression, we find through our summary:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  z^1/2   R-squared:                       0.998
Model:                            OLS   Adj. R-squared:                  0.997
Method:                 Least Squares   F-statistic:                     1492.
Date:                Wed, 07 Oct 2020   Prob (F-statistic):           3.82e-05
Time:                        11:55:07   Log-Likelihood:                 9.8452
No. Observations:                   5   AIC:                            -15.69
Df Residuals:                       3   BIC:                            -16.47
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0590      0.076      0.774      0.495      -0.183       0.301
y              0.4822      0.012     38.621      0.000       0.442       0.522
==============================================================================
Omnibus:                          nan   Durbin-Watson:                   1.823
Prob(Omnibus):                    nan   Jarque-Bera (JB):                0.506
Skew:                           0.237   Prob(JB):                        0.777
Kurtosis:                       1.516   Cond. No.                         24.4
==============================================================================
```

where we find that our P value is much greater than .05. Therefore, we fail to reject the null hypothesis that the y-intercept is zero. Since our R^2 value is also quite close to 1, we say that the data is indeed proportional. Therefore, running our model again forcing the y-intercept to be zero:

```
                                 OLS Regression Results
=======================================================================================
Dep. Variable:                  z^1/2   R-squared (uncentered):                   1.000
Model:                            OLS   Adj. R-squared (uncentered):              1.000
Method:                 Least Squares   F-statistic:                          2.629e+04
Date:                Thu, 08 Oct 2020   Prob (F-statistic):                    8.68e-09
Time:                        09:47:52   Log-Likelihood:                          9.3897
No. Observations:                   5   AIC:                                     -16.78
Df Residuals:                       4   BIC:                                     -17.17
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
y              0.4915      0.003    162.157      0.000       0.483       0.500
==============================================================================
Omnibus:                          nan   Durbin-Watson:                   1.673
Prob(Omnibus):                    nan   Jarque-Bera (JB):                0.367
Skew:                          -0.553   Prob(JB):                        0.832
Kurtosis:                       2.264   Cond. No.                         1.00
==============================================================================
```

We find from the model that our proportionality constant is .4915, therefore, our slope $k = .4915$

6. 2.2 #10 Using the data given and fitting an OLS regression:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  1.000
Method:                 Least Squares   F-statistic:                 7.348e+04
Date:                Thu, 08 Oct 2020   Prob (F-statistic):           3.84e-17
Time:                        10:53:30   Log-Likelihood:                -10.931
No. Observations:                  10   AIC:                             25.86
Df Residuals:                       8   BIC:                             26.47
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.3333      0.396      5.888      0.000       1.420       3.247
x              2.1342      0.008    271.072      0.000       2.116       2.152
==============================================================================
Omnibus:                        0.894   Durbin-Watson:                   2.146
Prob(Omnibus):                  0.640   Jarque-Bera (JB):                0.455
Skew:                          -0.483   Prob(JB):                        0.797
Kurtosis:                       2.602   Cond. No.                         78.2
==============================================================================
```

we see that the p-value is less than .05 therefore, we can conclude that in this case, y is not proportional to x^2.

7. 2.3), #2 Since we assume density is constant, we know weight is proportional to the volume:

$$\begin{aligned}
W &= kV \\
W &= k(L^3) \\
\frac{W}{L^3} &= k \\
k &= \frac{20}{3^3} \\
k &= \frac{20}{9} \\
k &\approx 2.2222
\end{aligned}$$

Therefore, for a 100-lb flamingo,

$$\begin{aligned}
100 &= 2.2222V \\
\frac{100}{2.2222} &= V \\
\frac{100}{2.2222} &= L^3 \\
L &= \sqrt[3]{\frac{100}{2.2222}} \\
L &\approx 3.5569
\end{aligned}$$

Which shows that a 100-lb flamingo would have length 3.5569ft. Since the leg height is also proportional to the length of the flamingo:

$$\begin{aligned}
L_{height} &= mL_{leg} \\
3 &= 2m \\
\frac{3}{2} &= m
\end{aligned}$$

and using our new height:

$$\begin{aligned}
3.5569 &= \frac{3}{2}L_{leg} \\
3.5569\frac{2}{3} &= L_{leg} \\
L_{leg} &\approx 2.3713
\end{aligned}$$

So we see height of the 100-lb flamingo is about 3.5569ft and the leg height of the 100-lb flamingo is 2.3713ft.