Foundations of Applied Math
HW #3 Due Thursday

Reminder You need to turn in a .zipped folder that contains your .tex file, your image files and the tex file must compile. Rename the .tex file: HW3_YourLastName.tex and call the folder which you will compress: HW3_YourLastName

Please make sure you learn the examples in 1.4 before you do this HW.

1. 1.4) #3

   (a)

$$\begin{aligned} B_{n+1} &= B_n - .05F_n \\ F_{n+1} &= F_n - .15B_n \\ B_0 &= 27 \\ F_0 &= 33 \end{aligned}$$

   (b) Under the new assumptions, we see the British win the battle quite easily, as there are no French/Spanish ships remaining after only 10 encounters as seen in Figure 1.

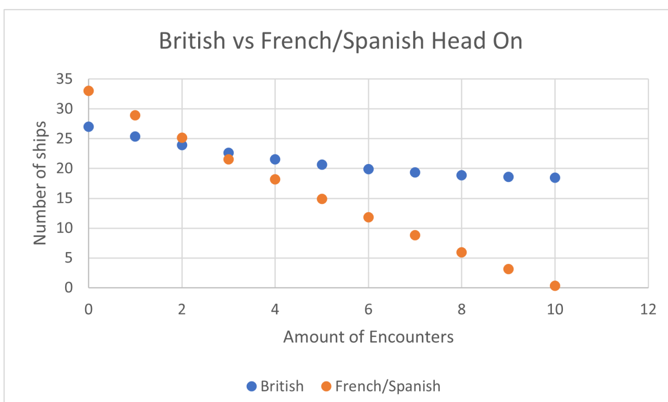


Figure 1: British vs French/Spanish Head On

(c) Applying the divide and conquer strategy, we see the British win by an even more extraordinary amount.

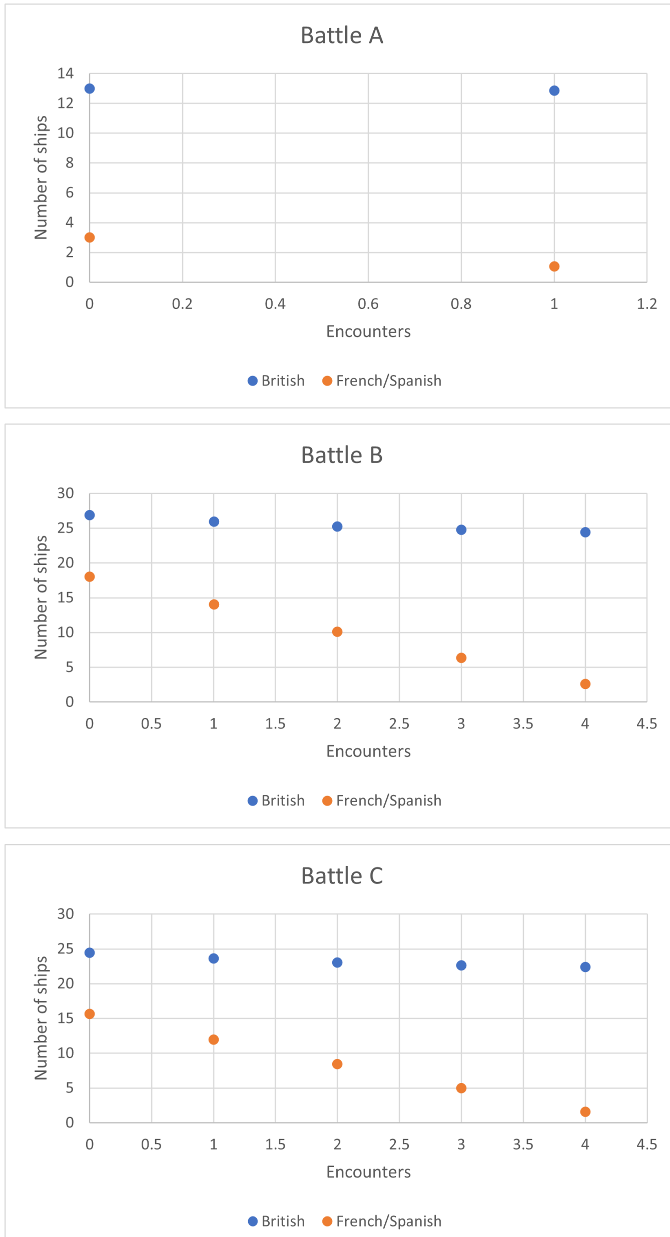


Figure 2: British vs French/Spanish Head On

2. 1.4) #4

(a) The owls in example 3 and the mice in this problem are modelled exactly the same. However, the hawks in example 3 and the owls in this problem are modelled differntly. In example 3, since the coefficient in front of the H_n term was greater than 1, the hawk's population would increase if there were no interactions between the owls and the hawks. However, in this problem, the coefficient infront of the O_n term is less than 1, meaning if there are no interactions between the owls and the mice, the owl population would decrease. The signs in the numbers 1.2, 0.7,

.002 indicates these terms will add to their correlating population, while the -0.001 term will subtract from the total population.

(b) The graphs of the owl and mice populations with varying initial conditions

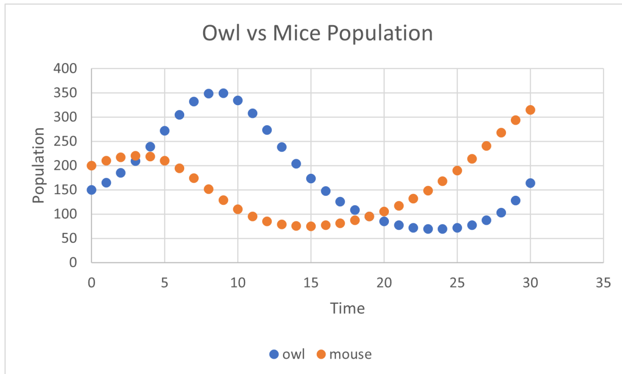


Figure 3: Owl vs Mouse Population graphs.

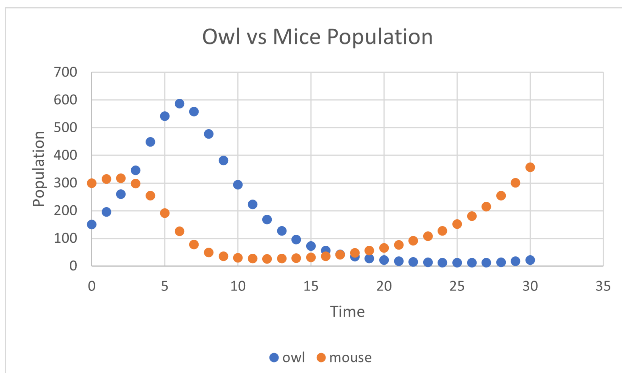


Figure 4: Owl vs Mouse Population graphs.

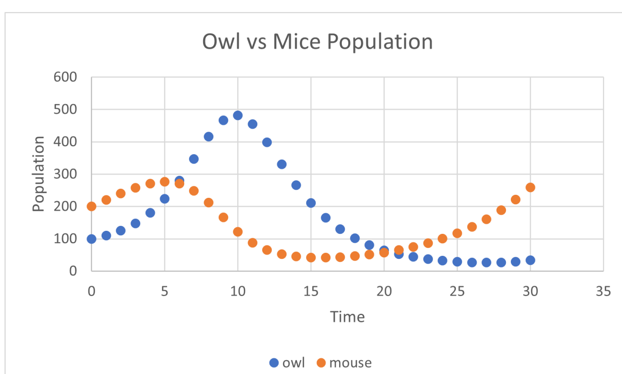


Figure 5: Owl vs Mouse Population graphs.

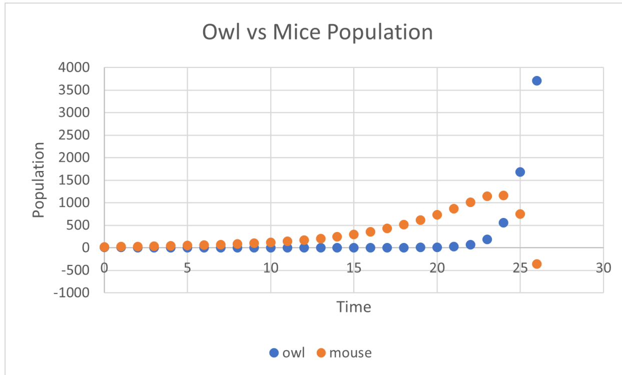


Figure 6: Owl vs Mouse Population graphs.

(c) Changing around our coefficients, we see this population model is sensitive to changes in the coefficients. By changing our coefficients while keeping our initial values the same, we see the graphs change drastically.

3. 1.4) #6

(a) Intuitively, the system is saying the price will go up as long as the quantity is below 500 and the quantity will go down as long as the price is below 100. Once the price goes above 100, the quantity will start to go up, and once the quantity goes above 500, the price starts to go down. The coefficients represent the rate at which the price and quantity affect each other and the signs show whether the price or quantity increase or decrease with this term.

(b) i. Case A: These initial values make the second term in both systems disappear, therefore this will be an equilibrium solution.

ii. Case B:

Figure 7: Price vs Quantity

iii. Case C:

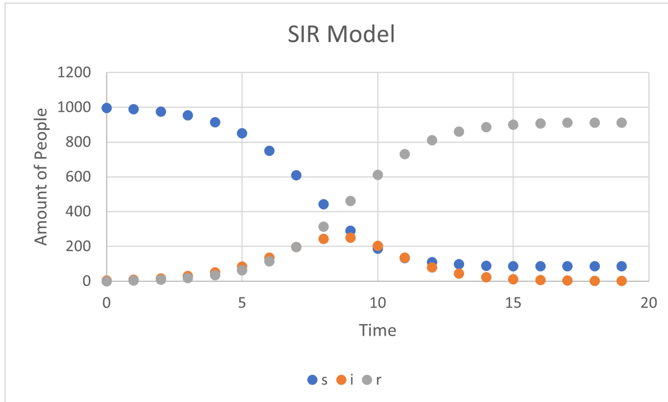
Figure 8: Price vs Quantity

iv. Case D:

Figure 9: Price vs Quantity

We see from Case B, C and D, we see that the price and the quantity of the product oscillate, with the price going below zero at certain points in the plot. The graphs also seem to oscillate around the same point: 500 for quantity, and 100 for price.

4. 1.4) #10 (SIR Problem) Our original model:



(a) With the initial conditions changed to $I_0 = 5, I_1 = 15$

$$
\begin{aligned}
I_0 = 5, I_1 &= I_0 - .6I_0 + aI_0S_0 \\
I_1 = 5 - .6(5) + a(5)(995) &= 15 \\
a &= \frac{15 - 5 + 3}{5(995)} \\
a &= 0.002613
\end{aligned}
$$

Therefore, our new model is:

$$
\begin{aligned}
S_{n+1} &= S_n - 0.002613S_nI_n \\
I_{n+1} &= I_n - .6I_n + 0.002613S_nI_n \\
R_{n+1} &= R_n + .6I_n \\
I_0 = 5, S_0 &= 995, R_0 = 0
\end{aligned}
$$

with the new model, our plot looks like: where the point of intersection between

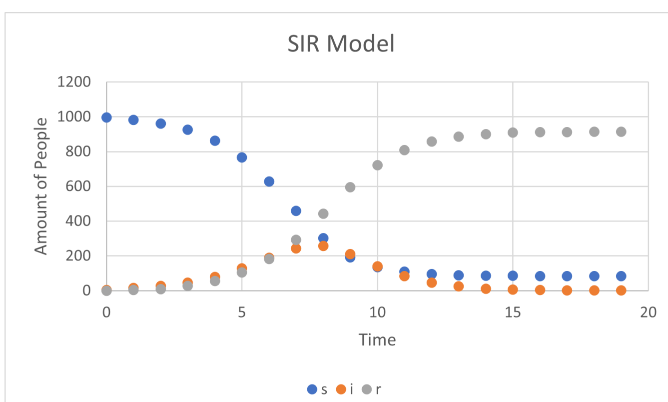


Figure 10: SIR model with $I_0 = 5, I_1 = 15$

the S, I, and R seem to intersect a bit earlier, but otherwise seem to be quite similar to our original.

(b) With the initial conditions changed so the flu lasts 1 week:

$$\begin{aligned} S_{n+1} &= S_n - 0.001407 S_n I_n \\ I_{n+1} &= I_n - 1 I_n + 0.001407 S_n I_n \\ R_{n+1} &= R_n + 1 I_n \\ I_0 = 5, S_0 &= 995, R_0 = 0 \end{aligned}$$

with the new model, our plot looks like:

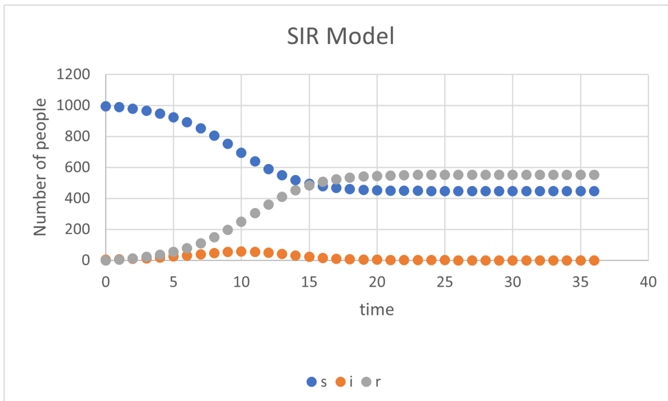


Figure 11: SIR model with flu lasts 1 week

We see if the flu only lasts one week, the amount of people who even get the flu is quite low, and since the amount of infected people is quite low, there are less chances to get the flu, resulting in people recovering faster than people getting the flu. Then, even though there are many people still susceptible, there are no people to catch the flu from, resulting in zero infected people within 20 weeks.

(c) With the initial conditions changed so the flu lasts 4 weeks:

$$\begin{aligned} S_{n+1} &= S_n - 0.001407 S_n I_n \\ I_{n+1} &= I_n - .25 I_n + 0.001407 S_n I_n \\ R_{n+1} &= R_n + .25 I_n \\ I_0 = 5, S_0 &= 995, R_0 = 0 \end{aligned}$$

with the new model, our plot looks like:

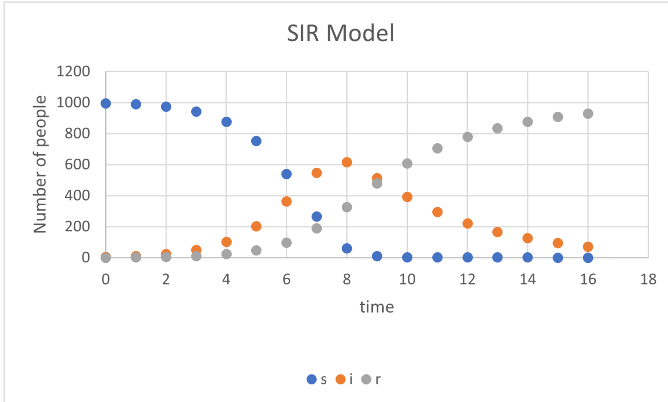


Figure 12: SIR model with flu lasts 4 weeks

Changing the flu to last 4 weeks, we see the number of infected people spikes very high compared to our original model. The number of susceptible people also goes down very quickly, while the removed seems to be somewhat slower than our original model. We see the flu seems to be a bit more dramatic when it lasts 4 weeks instead of our original 5/3.

(d) With 4000 students in the dorm, 5 infected and 30 more the next week:

$$I_0 = 5, I_1 = I_0 - .6I_0 + aI_0S_0$$
$$I_1 = 5 - .6(5) + a(5)(3995) = 30$$
$$a = \frac{30 - 5 + 3}{5(3995)}$$
$$a = 0.001402$$

Therefore, our new model is:

$$S_{n+1} = S_n - 0.001402 S_n I_n$$
$$I_{n+1} = I_n - .6I_n + 0.001402 S_n I_n$$
$$R_{n+1} = R_n + .6I_n$$
$$I_0 = 5, S_0 = 3995, R_0 = 0$$

with the new model, our plot looks like:

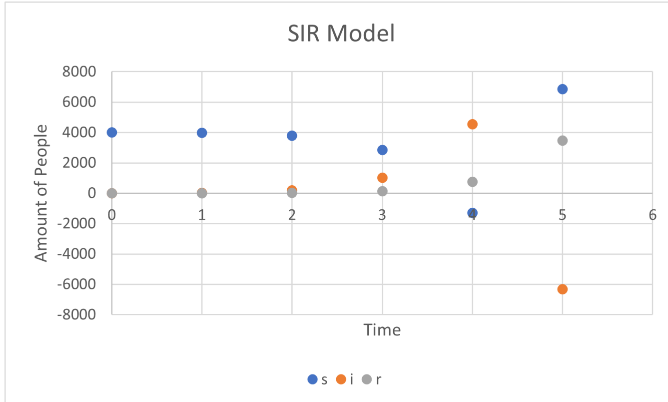


Figure 13: SIR model with $I_0 = 5, S_0 = 3995$

We see with the amount of students increased, there are more chances for interactions, even though the infection rate is quite similar. Because of this, we see the virus spreads much faster than before and weeks seem to be somewhat of an inadequate measure of time to model with this many students in dorms, for example, by week 4, the number of susceptible students is negative, meaning sometime between weeks 3 and 4, everyone susceptible should have been either infected or removed, but our model overshoots and becomes negative.

Write a python program that will sum the first 100 integers. *Solution.*

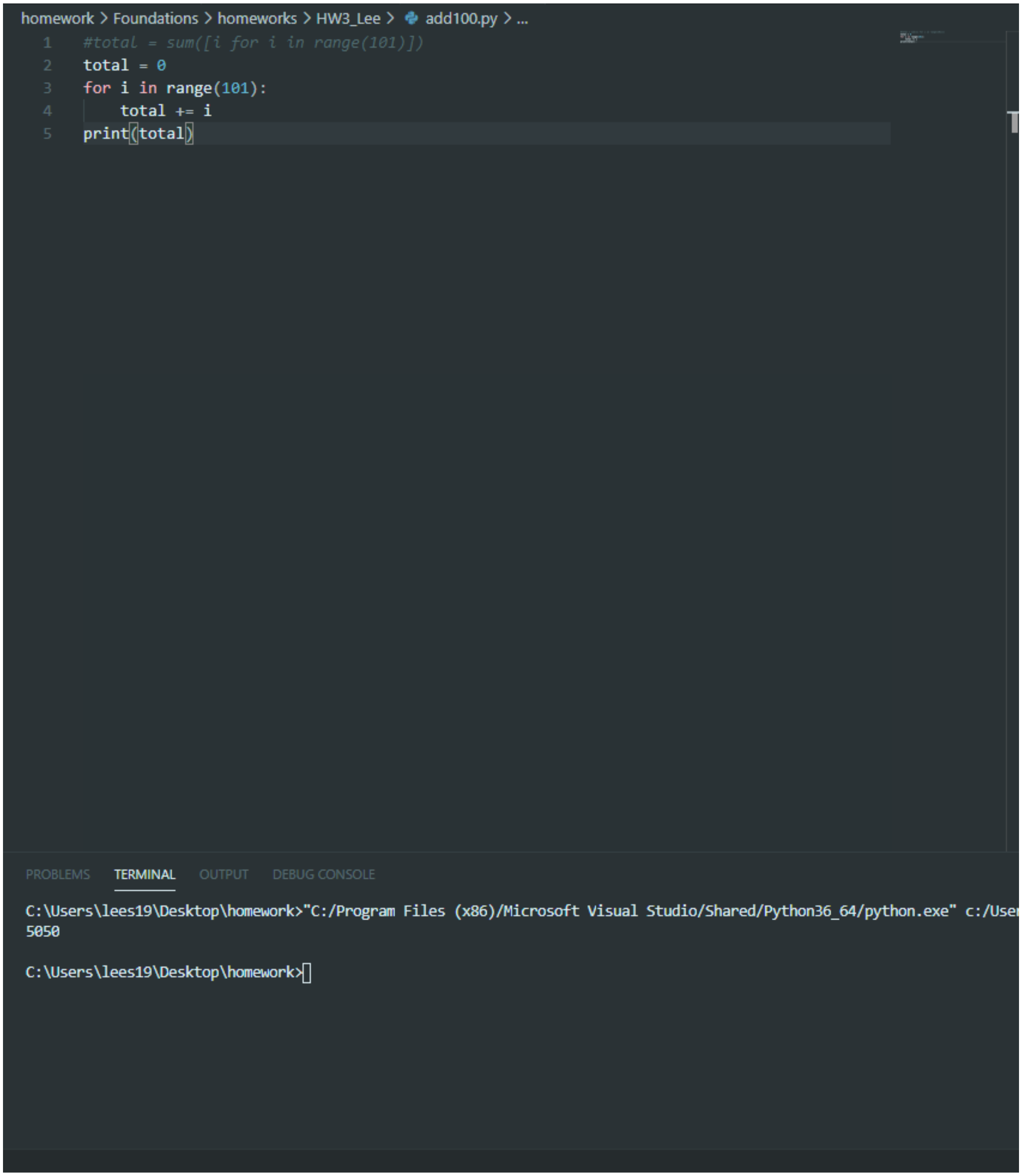


Figure 14: