1. (a)

$$\lim_{h\to 0} \frac{h - ln(1+h)}{h} = 0$$

Our third order taylor expansion for $ln(1+h)$ is:

$$ln(1+h) \approx \frac{h^3}{3} - \frac{h^2}{2} + h$$

Therefore:

$$\frac{h - (\frac{h^3}{3} - \frac{h^2}{2} + h)}{h} = \frac{h^2}{3} + \frac{h}{2}$$

So:

$$\lim_{h\to 0} \frac{h - ln(1+h)}{h} = O(h^2 + h) = O(h^2)$$

(b)

$$\lim_{h\to 0} \frac{1}{1-h} = 1$$

Then,

$$|\alpha_n - \alpha| = \frac{1}{1-h} - 1 = \frac{1}{1-h} - \frac{1-h}{1-h} = \frac{h}{1-h}$$

and since

$$|\frac{h}{1-h}| \leq |h|$$

for all positive integers h, we find that

$$\lim_{h\to 0} \frac{1}{1-h} = 1 + O(h)$$

(c)

$$\lim_{h\to 0} \frac{hcos(h) - tan^{-1}(h)}{h} = 0$$

Our third order taylor expansion for $hcos(h) - tan^{-1}(h)$ is:

$$hcos(h) - tan^{-1}(h) \approx \frac{-h^3}{6}$$

Then:

$$\frac{\frac{-h^3}{6}}{h} = \frac{-h^2}{6}$$

So:

$$\lim_{h\to 0} \frac{hcos(h) - tan^{-1}(h)}{h} = O(h^2)$$

2. *Solution.*
   The geometric series for $|r| < 1$ sums to:

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

   If we move the starting k and the n by 2:

$$\lim_{n+2\to\infty} \sum_{k=2}^{n+2} r^k = \lim_{n\to\infty} \frac{1-r^{n+3}}{1-r}$$

   Therefore:

$$\frac{1-r^{n+3}}{1-r} = \frac{1}{1-r} - \frac{r^{n+3}}{1-r} = \frac{1}{1-r} - \frac{r^{n+3}}{r} = \frac{1}{1-r} - r^{n+2}$$

   Thus:

$$\lim_{n+2\to\infty} \sum_{k=2}^{n+2} r^k = \lim_{n\to\infty} \frac{1-r^{n+3}}{1-r} = \frac{1}{1-r} + O(r^{n+2})$$

   So we see that we can generalize the convergence of $\sum_{k=i}^{\infty} r^k$ where $i = 3, 4, 5, \ldots$ as:

$$\lim_{n+i\to\infty} \sum_{k=i}^{n+i} r^k = \lim_{n\to\infty} \frac{1-r^{n+i}}{1-r} = \frac{1}{1-r} + O(r^{n+i})$$

3. (a) Multiplications: $\frac{n(n+1)}{2}$
   Additions: $\frac{n(n+1)}{2} - 1$

   (b) By taking the multiple a_i out of every iteration of i, we can sum all of the b_j and then multiply by a_i to drastically reduce the amount of multiplications:

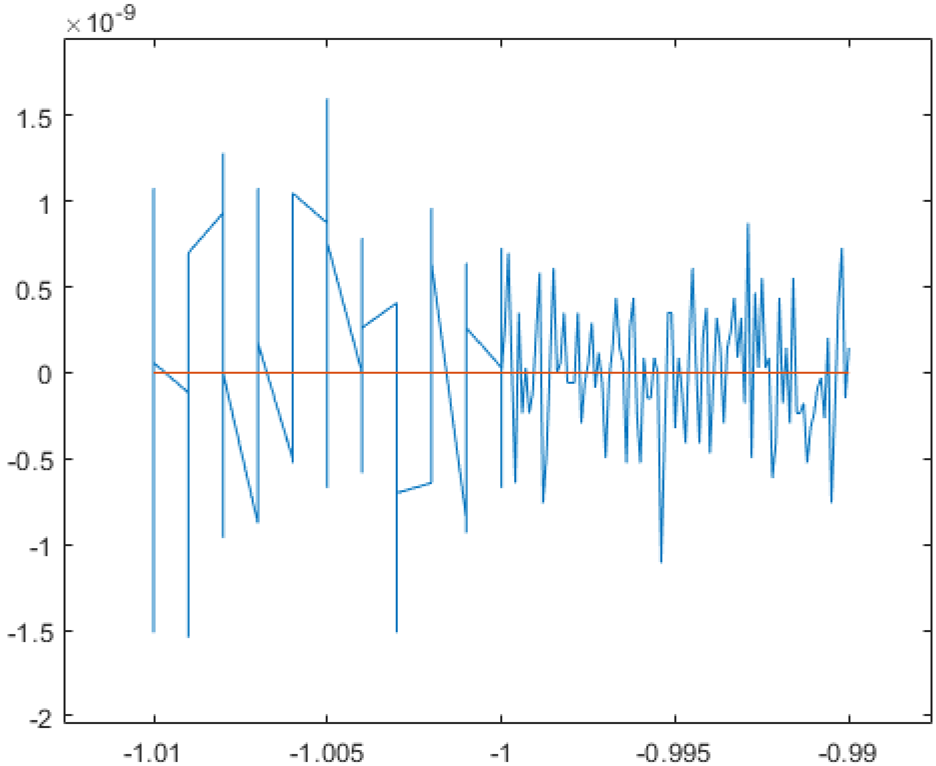
$$\prod_{i=1}^{n} a_i \sum_{j=1}^{i} b_j$$

   In this case, we still have $\frac{n(n+1)}{2} - 1$ additions, but we only have n multiplications.

4. Looking at the estimations where $P1$ corresponds to $1 + 20x + 45x^2 + 120x^3 + 210x^4 + 252x^5 + 210x^6 + 120x^7 + 45x^8 + 10x^9 + x^{10}$ and $P2$ corresponds to $(x+1)^{10}$:

x	P1	P2
-1.0100000000000000888	0.0000000096040000000	0.0000000000000000001
-1.0099000000000001990	-0.0000000015210000000	0.0000000000000000001
-1.0098000000000003091	0.0000000057120000000	0.0000000000000000001
-1.0097000000000004192	0.0000000082580000000	0.0000000000000000001
-1.0096000000000005294	-0.0000000002001000000	0.0000000000000000001
-1.0095000000000006395	-0.0000000004366000000	0.0000000000000000001
-1.0094000000000007496	-0.0000000013100000000	0.0000000000000000001
-1.0093000000000008598	-0.0000000015390000000	0.0000000000000000000
-1.0092000000000009699	-0.0000000008695000000	0.0000000000000000000
-1.0091000000000010800	0.0000000004256000000	0.0000000000000000000
-1.0090000000000011902	-0.0000000008731000000	0.0000000000000000000
-1.0089000000000013003	-0.0000000008877000000	0.0000000000000000000
-1.0088000000000014104	-0.0000000012220000000	0.0000000000000000000
-1.0087000000000015206	0.0000000001928000000	0.0000000000000000000
-1.0086000000000016307	0.0000000008222000000	0.0000000000000000000
-1.0085000000000017408	0.0000000008149000000	0.0000000000000000000
-1.0084000000000018510	-0.0000000007276000000	0.0000000000000000000
-1.0083000000000019611	-0.0000000009495000000	0.0000000000000000000
-1.0082000000000020712	-0.0000000007240000000	0.0000000000000000000
-1.0081000000000021814	0.0000000004475000000	0.0000000000000000000
-1.0080000000000022915	-0.0000000001928000000	0.0000000000000000000
-1.0079000000000024016	0.0000000001237000000	0.0000000000000000000
-1.0078000000000025118	0.0000000011680000000	0.0000000000000000000
-1.0077000000000026219	0.0000000002583000000	0.0000000000000000000
-1.0076000000000027320	0.0000000001128000000	0.0000000000000000000
-1.0075000000000028422	-0.0000000008658000000	0.0000000000000000000
-1.0074000000000029523	0.0000000010800000000	0.0000000000000000000
-1.0073000000000030624	0.0000000009313000000	0.0000000000000000000
-1.0072000000000031726	-0.0000000005202000000	0.0000000000000000000
-1.0071000000000032827	0.0000000000764000000	0.0000000000000000000
-1.0070000000000033928	0.0000000009059000000	0.0000000000000000000
-1.0069000000000035030	-0.0000000003565000000	0.0000000000000000000
-1.0068000000000036131	-0.0000000006148000000	0.0000000000000000000
-1.0067000000000037232	0.0000000000582100000	0.0000000000000000000
-1.0066000000000038334	0.0000000001892000000	0.0000000000000000000
-1.0065000000000039435	-0.0000000005020000000	0.0000000000000000000
-1.0064000000000040536	-0.0000000002692000000	0.0000000000000000000
-1.0063000000000041638	-0.0000000000254700000	0.0000000000000000000
-1.0062000000000042739	-0.0000000002801000000	0.0000000000000000000
-1.0061000000000043840	-0.0000000000909500000	0.0000000000000000000
-1.0060000000000044942	0.0000000002656000000	0.0000000000000000000
-1.0059000000000046043	-0.0000000005311000000	0.0000000000000000000
-1.0058000000000047145	0.0000000002074000000	0.0000000000000000000
-1.0057000000000048246	0.0000000000582100000	0.0000000000000000000
-1.0056000000000049347	0.0000000009313000000	0.0000000000000000000
-1.0055000000000050449	0.0000000007421000000	0.0000000000000000000
-1.0054000000000051550	0.0000000002874000000	0.0000000000000000000
-1.0053000000000052651	-0.0000000006767000000	0.0000000000000000000
-1.0052000000000053753	0.0000000014810000000	0.0000000000000000000
-1.0051000000000054854	-0.0000000001783000000	0.0000000000000000000
-1.0050000000000055955	-0.0000000002219000000	0.0000000000000000000
-1.0049000000000057057	-0.0000000004329000000	0.0000000000000000000

We see the $P2$ estimates are immediately zero, or extremely close to zero, while the $P1$ estimates float around the actual value of $P(-1)$. We also see this when we graph both equations:


$\times 10^{-9}$
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.01
-1.005
-1
-0.995
-0.99