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1. Running Newton's method:

```
i =

    22


ans =

0.64116643077930320512863651852324
```

2. Running the Modified Newton's method: at $p = 0.64116643$. Using the first and second derivative:

```
i =

     3


ans =

0.6411857445050227323393377286296
```

   So we see that the Modified Newton's converges much faster than the regular Newton's method.

3. Using Newton's method:

```
i =

     4


ans =

4.5983520415341891789245837413861e-33
```

4. (a) Taking the errors for each iteration:

```
testError =

  Columns 1 through 4

    5.000000000000000e+00     1.996007984031936e-02     3.960458654313619e-02     7.679992372816707e-02

  Columns 5 through 8

    1.373924113248750e-01     1.994759647499278e-01     2.221566960406771e-01     2.236020134042795e-01
```

Comparing our last value with the actual value of the error:

```
error =

0.22360679774997896964091736687313
```

We see that the test error and the theoretical error are quite close.

(b) Running the errors on secant method, we find that the asymptotic error constant is about .78510:

```
testError =

  Columns 1 through 4

     3.585513856467077e-01     5.763903733098815e-01     6.075161300279640e-03     7.251733149410868e+03

  Columns 5 through 8

     6.486532273502557e-01     4.492728667228152e-01     1.383088181368798e+00     6.945140592284434e-01

  Columns 9 through 12

     7.248551288827954e-01     8.008492864672407e-01     7.837334822113763e-01     7.850992787254543e-01

>>
```

We also see from the errors that secant method took more iterations than Newton's method, confirming that secant method has a lower rate of convergence than Newton's method.

5.

$$e_{k+1} = x_{k+1} - x^* = g(x_k) - g(x^*)$$

Using a Taylor expansion around x^* and evaluated at x_k:

$$g(x_k) \approx g(x^*) + g'(x^*)(x_k - x^*) + \frac{1}{2}g''(x^*)(x_k - x^*)^2 + \frac{1}{6}g^{(3)}(x^*)(x_k - x^*)^3$$
$$+\frac{1}{24}g^{(4)}(c)(x_k - x^*)^4$$

Where $c \in [x_k, x^*]$. Since $g'(x^*) = g''(x^*) = g^{(3)}(x^*) = 0$ we can reduce our function to:

$$g(x_k) - g(x^*) = \frac{1}{24}g^{(4)}(c)(x_k - x^*)^4$$

Thus:

$$e_{k+1} = \frac{1}{24}g^{(4)}(c)e_k^4$$
$$\lim_{k\to\infty} \frac{|e_{k+1}|}{|e_k^4|} = |\frac{1}{24}g^{(4)}(c)|$$

Therefore, the order of convergence is 4, and our asymptotic error constant is $|\frac{1}{24}g^{(4)}(c)|$.