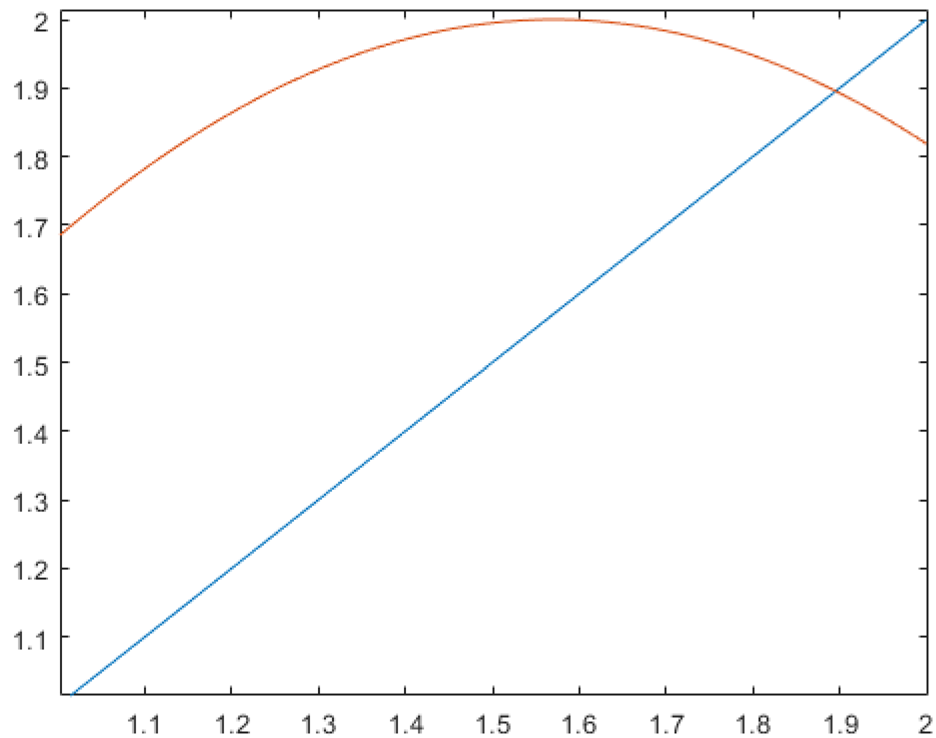
1. *Solution*

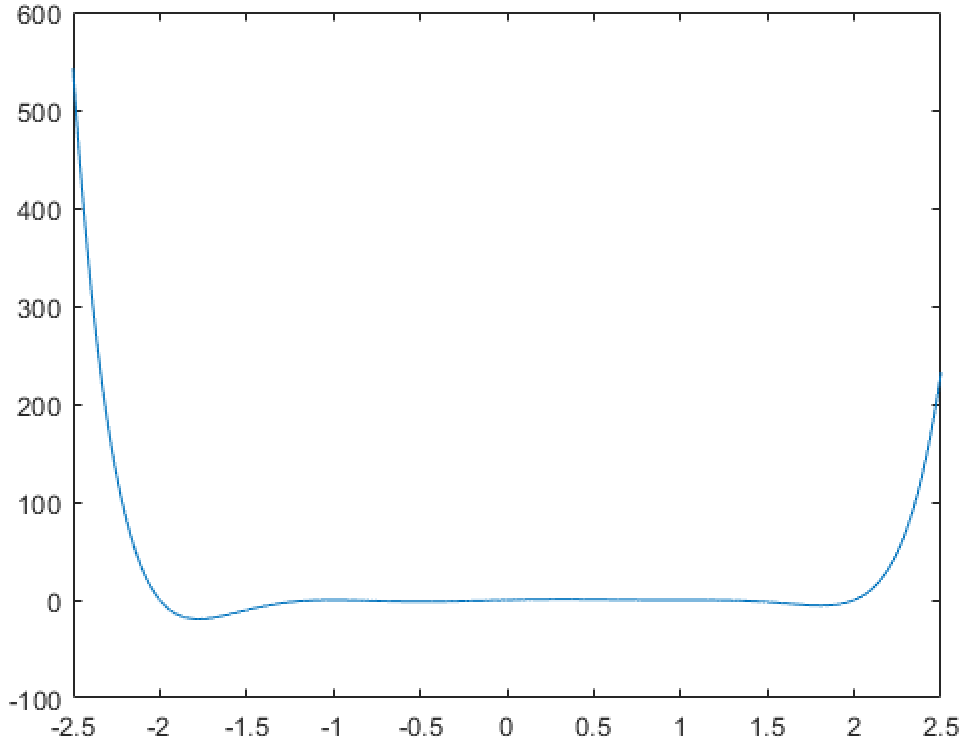
   (a) Graphing $f(x) = x$ and $g(x) = 2sin(x)$:

   (b) Using the bisection.m and the runbisection.m files, we find that the first positive root is at $1.895492553710938e + 00$ and converged in 16 iterations:

```
n2 =

     1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16


rr =

     1.895492553710938e+00

>>
```

2. *Solution.*
   Graphing the function:



(a) $[-2.5, -1.5]$

```
n2 =

     1


rr =

    -2
```

Here, the bisection method converges to -2

(b) $[-1.5, -0.5]$

```
n2 =

     []

Error using bisection (line 20)
f(a) and f(b) do not have opposite signs

Error in RunBisection_worksheet_4 (line 19)
rr=bisection(g, a, b, N, eps_step, eps_abs );
```

Here, the bisection did not work, as the function is negative on the whole interval and so $f(a)$ and $f(b)$ do not have different signs.

(c) $[-0.5, 0.5]$

```
n2 =

     1


rr =

     0
```

Here, the bisection method converges to 0

(d) $[0.5, 1.5]$

```
n2 =

     1


rr =

     1
```

Here, the bisection method converges to 1

(e) $[1.5, 2.5]$

```
n2 =

     1


rr =

     2
```

Here, the bisection method converges to 2

3. To find the number of iterations needed to achieve and approximation with an accuracy of at least 10^{-3} , we use the equation:

$$|p_n - p| = \frac{b-a}{2^n}$$
$$10^{-3} = \frac{4-1}{2^n}$$
$$2^n = 3(10^3)$$
$$\ln(2^n) = n\ln(2) = ln(3(10^3))$$
$$n = \frac{\ln(3(10^3))}{\ln(2)}$$
$$n \approx 11.55$$

   Thus, we find it takes about 12 iterations to find an approximation with an accuracy of 10^{-3}

   When we run the bisection method on this function, we find indeed that our theoretical result was spot on with our experimental result:

```
p1 =

  Columns 1 through 6

    2.500000000000000e+00    1.750000000000000e+00    1.375000000000000e+00    1.562500000000000e+00    1.468750000000000e+00    1.421875000000000e+00

  Columns 7 through 12

    1.398437500000000e+00    1.386718750000000e+00    1.380859375000000e+00    1.377929687500000e+00    1.379394531250000e+00    1.378662109375000e+00


n2 =

    1     2     3     4     5     6     7     8     9    10    11    12


rr =

    1.378662109375000e+00
```

4. Using the bisection method, we find that the first two roots are -5 and -3. The root where $x = 1$, however, we cannot find using the bisection method, as the function does not change signs near that specific root.

```
rr =

    -5
```

```
rr =

    -3
```

5. For the roots we can find with the bisection method:

```
rr =

    -5.323022460937500e+00
```

```
rr =

    -5
```

```
rr =

    -3
```

```
rr =

    -1.808105468750000e+00
```

```
rr =

    -8.049316406250000e-01
```

```
rr =

     1.936080932617188e+00
```

The root at $x = 1$ cannot be determined by the bisection method, as there is no sign change around that point. Numerical methods which require the first derivative would not always work, as this function is not everywhere differentiable. Since the function at $x = 1$ is the highest point around the root, $f(1)$ would have to one of the end points. Since $f(1)$ must be an endpoint, and the IVT only guarantees a root inside the interval, the IVT does not guarantee a root.