1. *Solution.*

   (a) As x gets smaller and smaller, we see that the reformulated equation does much better than the original estimation. In fact, the original estimation seems to float around the actual value and then, because of loss of significance, seems to think that the numerator is so small, it is just zero.

```
>> Loss_sig
    n2         gg         gg2
  1.00 0.24845673   0.24845673
  2.00 0.24984395   0.24984395
  3.00 0.24998438   0.24998438
  4.00 0.24999844   0.24999844
  5.00 0.24999984   0.24999984
  6.00 0.24999998   0.24999998
  7.00 0.25000000   0.25000000
  8.00 0.24999998   0.25000000
  9.00 0.25000002   0.25000000
 10.00 0.25000002   0.25000000
 11.00 0.24997782   0.25000000
 12.00 0.25002223   0.25000000
 13.00 0.24868996   0.25000000
 14.00 0.22204460   0.25000000
 15.00 0.00000000   0.25000000
 16.00 0.00000000   0.25000000
 17.00 0.00000000   0.25000000
 18.00 0.00000000   0.25000000
 19.00 0.00000000   0.25000000
 20.00 0.00000000   0.25000000
>>
```

   (b) Here, as x gets smaller, we see that the original does quite well but again, the numerator loses much significance which the computer assumes is zero.

```
0.25 >> Loss_sig
  n2         gg        gg2
 1.00 0.16658335   0.16658335
 2.00 0.16666583   0.16666583
 3.00 0.16666666   0.16666666
 4.00 0.16666666   0.16666667
 5.00 0.16666728   0.16666667
 6.00 0.16665373   0.16666667
 7.00 0.17205357   0.16666667
 8.00 0.00000000   0.16666667
 9.00 0.00000000   0.16666667
10.00 0.00000000   0.16666667
11.00 0.00000000   0.16666667
12.00 0.00000000   0.16666667
13.00 0.00000000   0.16666667
14.00 0.00000000   0.16666667
15.00 0.00000000   0.16666667
16.00 0.00000000   0.16666667
17.00 0.00000000   0.16666667
18.00 0.00000000   0.16666667
19.00 0.00000000   0.16666667
20.00 0.00000000   0.16666667
>>
```

2. *Solution.*

   (a) For our approximation for π (22/7):

```
absError =

    -1.264489267349678e-03


relError =

    -4.024994347707008e-04

>>
```

   (b) For our approximation for e (2.718):

```
absError =

     2.818284590455633e-04


relError =

     1.036788960198905e-04

>>
```

(c) For our approximation for $\sqrt{2}$ (1.414):

```
absError =

     2.135623730952219e-04


relError =

     1.510114022219229e-04

>>
```

3. *Solution.*

(a)

$$\frac{|p^* - \pi|}{|\pi|} \leq .0001 \tag{1}$$
$$|p^* - \pi| \leq .0001\pi \tag{2}$$

Applying absolute value:

$$-.0001\pi \leq p^* - \pi \leq .0001\pi \tag{3}$$
$$-.0001\pi + \pi \leq p^* \leq .0001\pi + \pi \tag{4}$$
$$.9999\pi \leq p^* \leq 1.0001\pi \tag{5}$$

Therefore, our p^* value must be in the interval $(.9999\pi, 1.0001\pi)$

(b)

$$\frac{|p^* - e|}{|e|} = .0001 \tag{6}$$
$$|p^* - e| \leq .0001e \tag{7}$$

Applying absolute value:

$$-.0001e \leq p^* - e \leq .0001e \tag{8}$$
$$-.0001e + e \leq p^* \leq .0001e + e \tag{9}$$
$$.9999e \leq p^* \leq 1.0001e \tag{10}$$

Therefore, our p^* value must be in the interval $(.9999e, 1.0001e)$

(c)

$$\frac{|p^* - \sqrt{2}|}{|\sqrt{2}|} = .0001 \tag{11}$$
$$|p^* - \sqrt{2}| \leq .0001\sqrt{2} \tag{12}$$

Applying absolute value:

$$-.0001\sqrt{2} \leq p^* - \sqrt{2} \leq .0001\sqrt{2} \tag{13}$$
$$-.0001\sqrt{2} + \sqrt{2} \leq p^* \leq .0001\sqrt{2} + \sqrt{2} \tag{14}$$
$$.9999\sqrt{2} \leq p^* \leq 1.0001\sqrt{2} \tag{15}$$

Therefore, our p^* value must be in the interval $(.9999\sqrt{2}, 1.0001\sqrt{2})$

4. Using the nest.m function and the equivalent expression, we see that the two values are slightly different:

```
Using nest method: = 51.000000012750
Using equivalent expression: = 51.000000000000
Absolute error: = 0.000000012750
```