Sunny Lee

1. Using the fixed point method, we find the function $g(x) = \pi + \frac{1}{2}\sin\frac{x}{2}$ equals x at the point $(3.627, 3.627)$

```
ans =

3.626995622438735475254086928317
```

.

Looking at the graph of $g\prime(x)$, we find the maximum value of $g\prime(x)$ are at the endpoints. So taking $g\prime(x)$ at $x = 0$, we find that:

$$|g\prime(x)| = |\frac{\cos\frac{x}{2}}{4}| = \frac{\cos(0)}{4} = \frac{1}{4}$$
$$|g\prime(x)| \leq \frac{1}{4} = k$$

To find the number of iterations theoretically required to reach an intersection with an accuracy of 10^{-2}:

$$10^{-2} \leq \frac{k^n}{1-k}|p_1 - p_0|$$

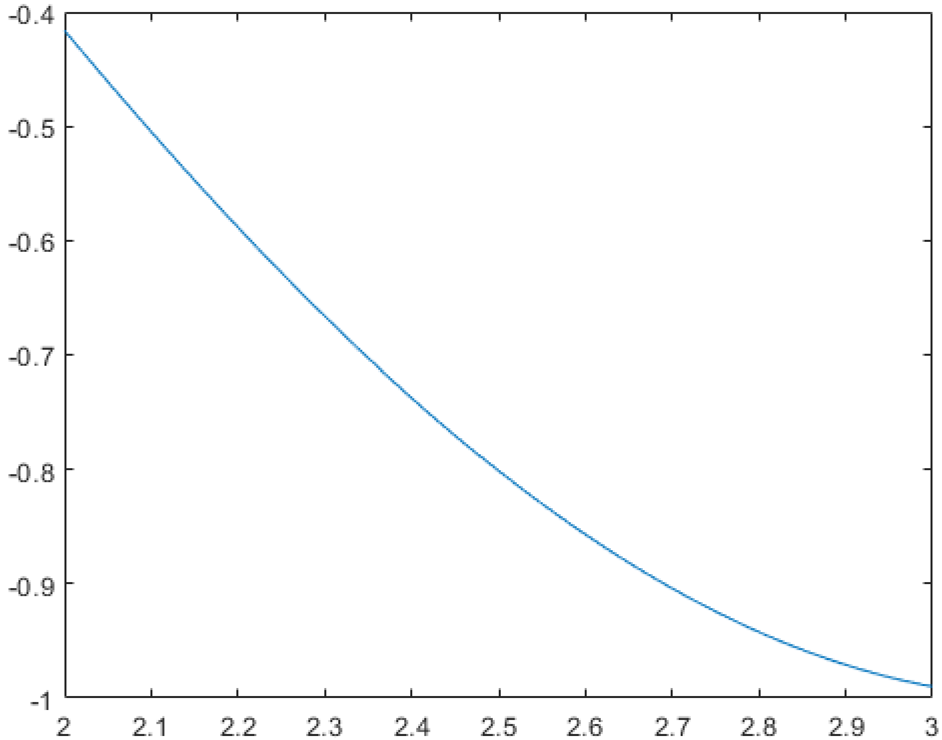
Using $p_0 = \pi$ and $k = .25$:

$$10^{-2} \leq \frac{.25^n}{1-.25}|(\pi + .5) - \pi| = \frac{.25^n}{.75}|.5|$$
$$\ln\frac{10^{-2}\cdot .75}{.5} \leq n \ln .25$$
$$\frac{\ln\frac{10^{-2}\cdot .75}{.5}}{\ln .25} \leq n$$
$$3.0294 \leq n$$

So, we expect to have 4 iterations to reach an approximation with an error 10^{-2}. Starting the fixed point iteration at $x = \pi$, we find it takes 3 iterations to reach an approximation within 10^{-2}, which fits closely with our theoretical number of iterations.

```
i =

     3
```

2. (a) Given the equation $2 + \sin(x) - x = 0$, we can add x to the rhs to get $2 + \sin(x) = x$ so we obtain $g(x) = 2 + \sin(x)$. Looking at the graph of $g'(x)$:



$g'(x)$ is a strictly decreasing function on this interval, so we take $g'(x)$ at $x = 3$.

$$|g'(3)| = |\cos(3)| \approx .99 = k$$

Taking $k = .99$:

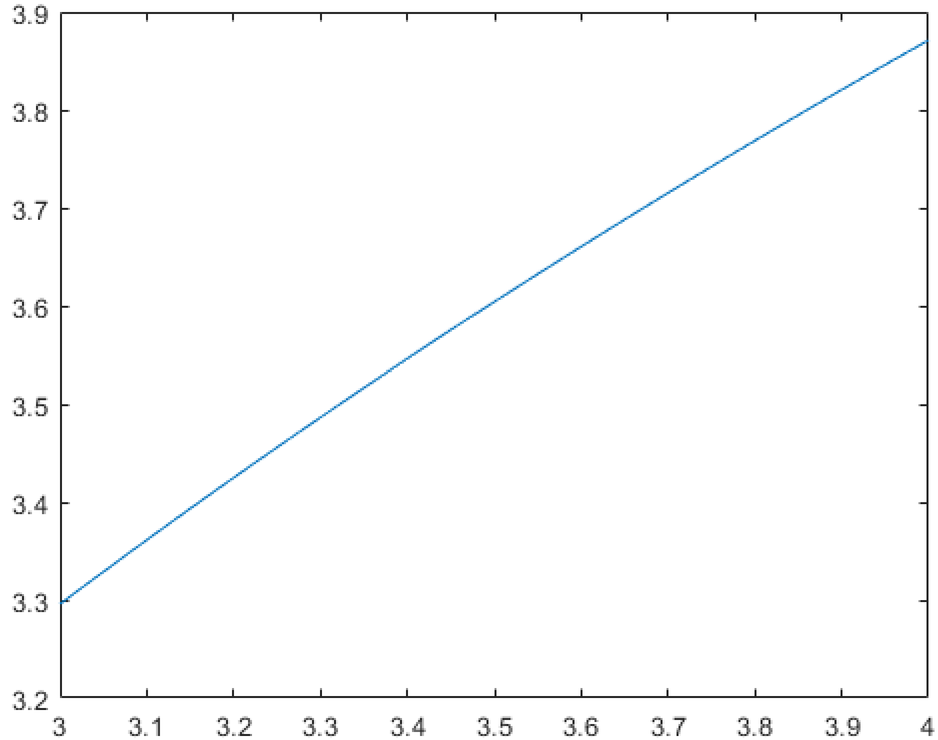
$$\frac{10^{-5}(1 - .99)}{g(2.5) - 2.5} \leq .99^n$$
$$1373.0998 \leq n$$

So it would take about 1374 iterations to reach an accuracy within 10^{-5}. Using MATLAB to estimate the root:

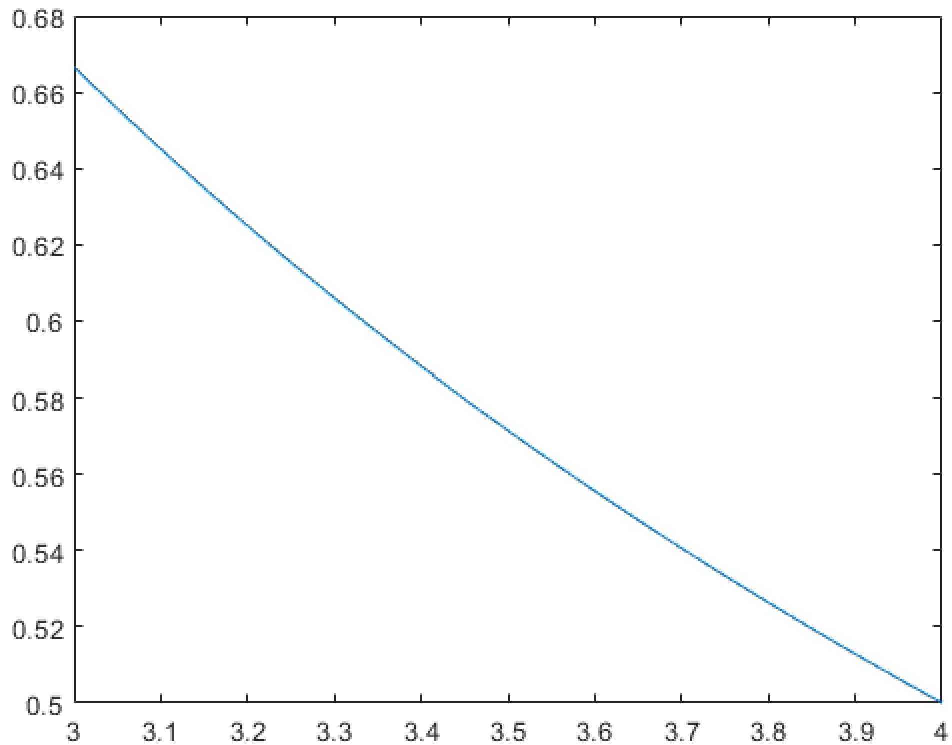
```
ans =

2.5541921027478671152262783690056


i =

     52
```

We see it only takes 52 iterations to reach an accuracy of 10^{-5}.

(b) Given $3x^2 - e^x = 0$, we can reformulate the function as $x = \ln(3x^2)$ using the interval $[3, 4]$, and looking at the graph, we see it is a strictly increasing function.



Since it is strictly increasing, we only have to check the end points $x = 3, 4$ as our k value.



From our graph of the derivatve, we find that $g'(x)$ attains a maximum value at $x = 3$, so our k value is $\frac{2}{3}$. To obtain a number of iterations:

$$\frac{10^{-5}(1-\frac{2}{3})}{g(3.7)-3.7} \leq (\frac{2}{3})^n$$
$$20.79 \leq n$$

So it would take around 21 iterations to get an accuracy of 10^{-5}.

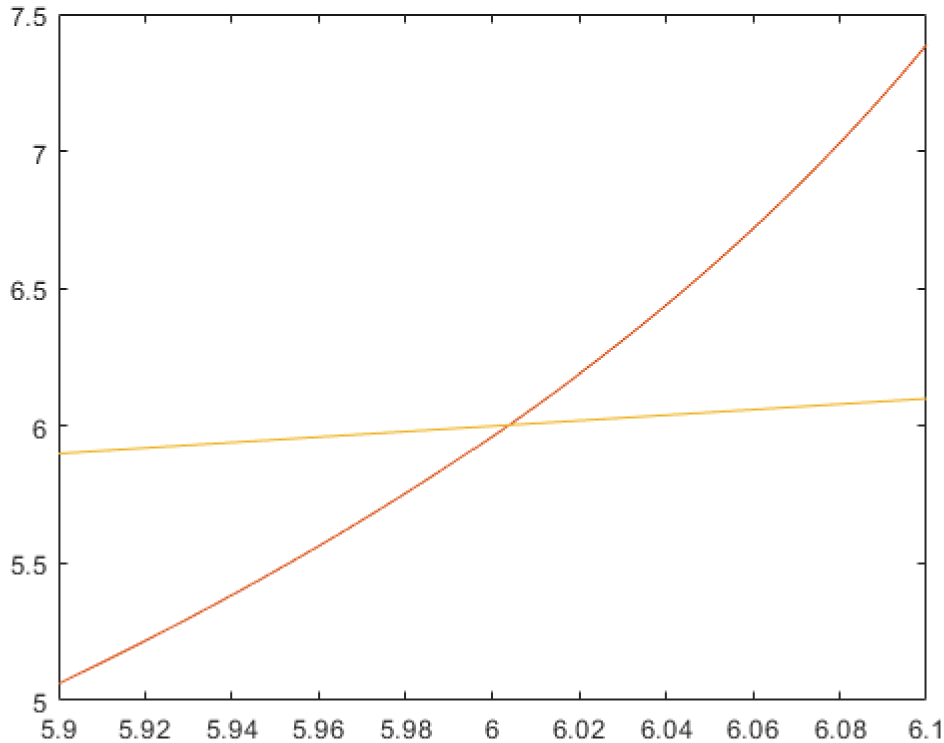
```
ans =

3.7330690231821785736726322612016


i =

    13
```

We see it only takes 13 iterations to reach an accuracy of 10^{-5}.

3. By reformulating the function to use with our fixed point method:

$$\ln(\frac{s_0 k^2}{m^2 g} + \frac{k}{m}t + 1) \cdot -\frac{m}{k} = t$$

Graphing the function $y = x$ and the reformulation:



We see they intersect at around $(6, 6)$, so taking our initial $p_0 = 6$ we find that the fixed point method converges to around 6.006:

```
ans =

6.0067495830685231605551884507214
```