1. Using the Vandermonde matrix constructed from the $x$ vector, we can make an augmented matrix with the $y$ vector given. This augmneted matrix can then be solved through Gaussian Elimination, and we obtain the coefficients of the degree 9 function:

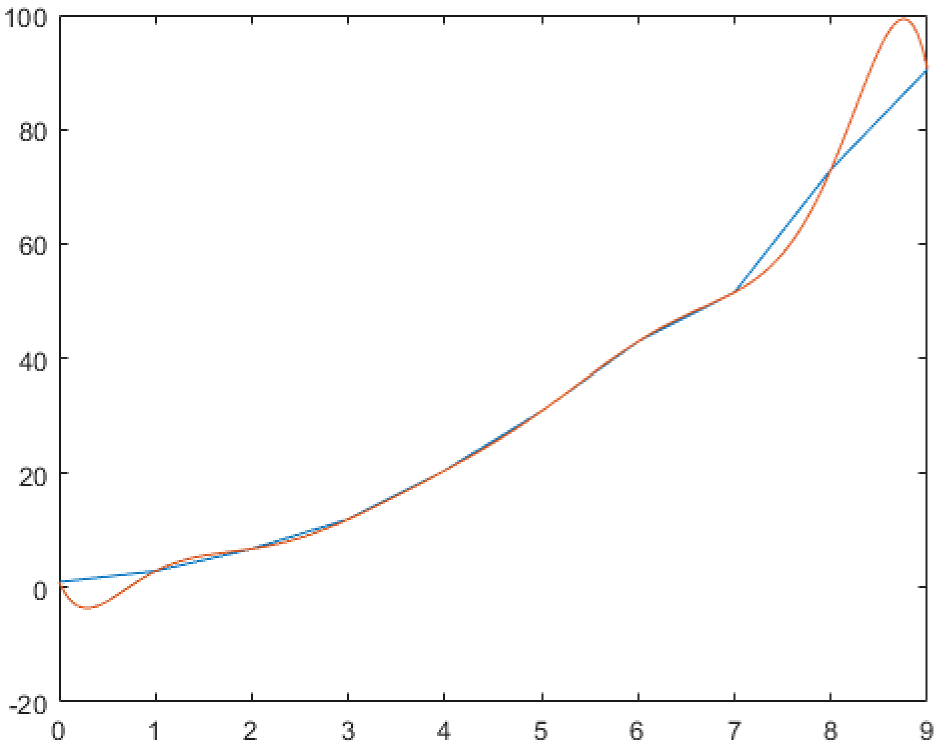
```
sol =

   1.0000  -38.0113  102.5482 -103.9042   55.8063  -17.5090    3.3224   -0.3750    0.0231   -0.0006
```

and thus, our Lagrange polynomial is:

$$x^9 - 38.0113x^8 + 102.5482x^7 - 103.9042x^6 + 55.8063x^5 - 17.5090x^4 + 3.3224x^3 - 0.3750x^2 + 0.0231x - 0.0006$$

When we graph the polynomial against our data, we can see there are definitely some overfitting problems happening:




2. (a) Using the normal equation where we take the inverse of $A$, we obtain the coefficients:

```
a =

    1.2000
    2.2000
    2.5000
    1.7000
```

and thus our multiple linear regression model is $1.2x_1 + 2.2x_2 + 2.5x_3 + 1.7x_4$

(b) Using the Cholesky factorization before solving for $x$, we obtain the lower triangular matrix:

```
L1 =

    1.0000         0         0         0
    0.3000    1.0000         0         0
    0.2000    0.0396    1.0000         0
    0.4000    0.0792    0.2859    1.0000
```

Solving using the lower triangular matrix, we see we obtain the same result:

```
a =

    1.2000
    2.2000
    2.5000
    1.7000
```

3. (a) Using the Vandermonde matrix to solve for the polynomial:

```
sol =

     3     2     1
```

And thus, using the Vandermonde matrix, we obtain the polynomial $3x^2 + 2x + 1$

(b) Using gradient descent, we first set our coefficients as $3.1x^2 + 1.9x + 1.1$. To estimate the coefficients we found in Part A, we must first find the gradient of the cost function. For our cost function, we will be using the mean squared error or MSE:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2$$

In our case, since $\hat{y} = ax^2 + bx + c$ and we are using 3 data points:

$$\frac{1}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)^2$$

To minimize this function using gradient descent, we must find the gradient of this function:

$$\frac{\partial MSE}{\partial a} = \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)x_i^2$$

$$\frac{\partial MSE}{\partial b} = \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)x_i$$

$$\frac{\partial MSE}{\partial c} = \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)$$

Thus, our iteration becomes:

$$\begin{bmatrix} a_{j+1} \\ b_{j+1} \\ c_{j+1} \end{bmatrix} = \begin{bmatrix} a_j \\ b_j \\ c_j \end{bmatrix} - .01 \begin{bmatrix} \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)x_i^2 \\ \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i)x_i \\ \frac{2}{3}\sum_{i=1}^{3}(ax_i^2 + bx_i + c - y_i) \end{bmatrix}$$

Iterating this in MATLAB:

```
coef =

    3.0000    1.9999    1.0000
```

Thus, we obtain the same results as the Vandermonde matrix.