1. Using the Gaussian Elimination method, we obtain the values $x_1 = 3.6480, x_2 = 6.0594, x_3 = -4.1321, x_4 = 4.1755, x_5 = -1.1407$:

```
x =

    3.6480    6.0594   -4.1321    4.1755   -1.1407
```

2. Using the Scaled Partial Pivoting algorithm, we obtain the values $x_1 = 3, x_2 = 1, x_3 = -2, x_4 = 1$.

```
x =

    3.0000    1.0000   -2.0000    1.0000
```

3. Using the LU decomposition:

```
L =

    1.0000         0         0         0         0
    0.9808    1.0000         0         0         0
    2.9778    0.5461    1.0000         0         0
    1.3422    1.2452    0.9402    1.0000         0
    0.9803    0.6163    0.0072    0.3323    1.0000


U =

    0.2028    0.0153    0.4186    0.8381    0.5280
         0    0.7318    0.4360   -0.8016    0.2169
         0         0   -0.9595   -1.3769   -1.1868
         0    0.0000         0    1.5472    0.4756
         0   -0.0000         0         0   -0.5864
```

Using this decomposition to solve for x:

```
ans =

    3.6480
    6.0594
   -4.1321
    4.1755
   -1.1407
```

which is the same vector we obtained in question 1.

4. In order to check to see if this matrix is positive definite, we can check if the determinant of its leading principal submatrices are all positive:

```
determinant =

  Columns 1 through 4

     8.094611368703402e+03     8.097136755739033e+03     3.238953060271237e+02     8.997117980600001e+00

  Columns 5 through 6

     1.110861100000000e-01     1.111000000000000e-01
```

Since all of the determinants above are positive, we conclude that A is a positive definite matrix. Using the Cholesky algorithm, we obtain the Upper triangular matrix:

```
U =

    4.7089    2.3909    3.1956   -2.4351   -0.0442    0.0177
         0    5.3480   -0.2203    2.2978   -0.5568   -0.0183
         0         0   11.7796   12.8444   -0.3501   -0.2315
         0         0         0    2.1960    0.2589   -0.1250
         0         0         0         0    0.8651    0.1255
         0         0         0         0         0    0.1596
```

and since $A = U^T U$, we can take the inverse of $U^T U$ and multiply them on both sides to solve our equation:

```
ans =

   30.9257
  -13.6848
  -20.0392
   22.8233
  -39.3435
  304.7686
```

5. Running the LDL factorization, we obtain these matrices:

```
L =

    1.0000         0         0         0         0         0
    0.5077    1.0000         0         0         0         0
    0.6786   -0.0412    1.0000         0         0         0
   -0.5171    0.4297    1.0904    1.0000         0         0
   -0.0094   -0.1041   -0.0297    0.1179    1.0000         0
    0.0038   -0.0034   -0.0197   -0.0569    0.1451    1.0000


D =

   22.1736         0         0         0         0         0
         0   28.6016         0         0         0         0
         0         0  138.7602         0         0         0
         0         0         0    4.8225         0         0
         0         0         0         0    0.7485         0
         0         0         0         0         0    0.0255
```

By taking the inverse of the LDL^T, we can find the solution to our equation:

```
ans =

   30.9257
  -13.6848
  -20.0392
   22.8233
  -39.3435
  304.7686
```

6. (a)

$$M_1^{-1}M_2^{-1} = (M_2M_1)^{-1}$$

$$M_2M_1 = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{32}m_{21} + m_{31} & m_{32} & 1 \end{bmatrix}$$

Calculating the matrix of cofactors:

$$\begin{bmatrix} 1 & -m_{21} & -m_{31} \\ 0 & 1 & -m_{32} \\ 0 & 0 & 1 \end{bmatrix}$$

Since the original matrix is lower triangular, we can multiply the diagonals of that matrix and we get $det(M_2M_1) = 1$. Thus, we transpose the matrix of cofactors:

$$\begin{bmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & -m_{32} & 1 \end{bmatrix}$$

(b) From the matrix given, we can perform elimination on the matrix to get the pivots:

$$\begin{bmatrix} 1 & a \\ 0 & 1 - a^2 \end{bmatrix}$$

To make the matrix positive definite, we must have $1 - a^2 \leq 0$. Thus, we see that the matrix A is not positive definite for all $|a| \geq 1$.

(c) Since the determinant of A is 55, A is invertible.

(d) Using the LDL factorization and looking at the diagonal matrix:

```
D =

    3.0000         0         0         0
         0    3.6667         0         0
         0         0    1.9091         0
         0         0         0    2.6190
```

Since all of the values are positive, we conclude that the matrix itself is positive definite.

7. (a) Finding the determinant of the matrix:

```
ans =

3685*x + 7370
```

and setting the determinant equal to zero, we find that $x = 2$.

(b) Since the determinant of the matrix is -1 which is not zero, we can find the inverse of the matrix:

```
ans =

     5    -2     1
    -4     1     0
     1     0     0
```

(c) Since $7 > 2+3$, $6 > 4+1$ and $5 > 2+1$, we can say that A is strictly diagonally dominant. Since A is strictly diagonally dominant, A is nonsingular. Gaussian Elimination cannot be performed on any linear system without row or column changes since there will be times where the current pivot is zero while other rows below have non zero values for that column.

(d) The inverse of A:

```
ans =

    9.0000  -36.0000   30.0000
  -36.0000  192.0000 -180.0000
   30.0000 -180.0000  180.0000
```

Calculating the determinant of A^{100}, we find that the determinant is not zero, and thus A^{100} is invertible.

8. Setting up the iterative form of the Jacobi Method:

$$x^{(k)} = \begin{bmatrix} 0 & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{2} & 0 & -\frac{1}{3} \\ -\frac{3}{7} & -\frac{3}{7} & 0 \end{bmatrix} x^{(k-1)} + \begin{bmatrix} \frac{1}{3} \\ 0 \\ \frac{4}{7} \end{bmatrix}$$

Using this method in MATLAB, we get these approximations for the first two iterations:

```
x =

    0.3333
         0
    0.5714


x =

    0.1429
   -0.3571
    0.4286
```

9. Running the Gauss Seidel method, our first two iterations yield:

```
x =

    0.3333
   -0.1667
    0.5000


x =

    0.1111
   -0.2222
    0.6190
```

10. To find $||A||_\infty$, we take the maximum of the sum of the absolute values of each row of A, thus: $||A||_\infty = max(3, 3, 3) = 3$. To find the $||A||_2$, we use the l_2 norm for vectors on each row, thus $||A||_2 = max(\sqrt{5}, \sqrt{5}, 3) = 3$.