1. Let $f(x) = x^3 + 2x - 1$, $\delta_0 = \frac{1}{2}$ and $|x-1| < \delta$. Then, given $\epsilon > 0$, we obtain $|f(x) - f(1)| = |x^3 + 2x - 3| = |x-1||x^2+x+3|$. Since $x \in (\frac{1}{2}, \frac{3}{2})$, $|x-1||x^2+x+3| \leq |x-1||\frac{27}{4}|$. If we let $\delta_1 = \frac{4\epsilon}{27}$, $|f(x) - f(1)| < |x-1||\frac{27}{4}| < \frac{4\epsilon}{27}\frac{27}{4} = \epsilon$ Thus, if we let $\delta\ min(\frac{1}{2}, \frac{4\epsilon}{27})$, $|f(x) - f(1)| < \epsilon$. Thus, $f(x)$ is continuous at $x = 1$.

2. Let $f(x) = x^2$, $\delta_0 = \frac{a}{2}$ and $|x-a| < \delta$. Since f is an even function, we can assume $a, x \geq 0$. Since $|x-a| < \delta$, $x \in (\frac{a}{2}, \frac{3a}{2})$. Given $\epsilon > 0$: $|f(x) - f(a)| = |x^2 - a^2| = |x-a||x+a| < |x-a||\frac{5a}{2}|$. If we let $\delta_1 = \frac{2\epsilon}{5a}$, $|x-a||\frac{5a}{2}| < \frac{2\epsilon}{5a}\frac{5a}{2} = \epsilon$. Thus, if we let $\delta = min(\frac{a}{2}, \frac{2\epsilon}{5a})$ f is continuous at any $x \in \mathbb{R}$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by:
$$\begin{cases} \frac{1}{x} & x \neq 0 \\ c & x = 0 \end{cases}$$
Let x_n be the sequence defined by $x_n = \{\frac{1}{n}\}$. Thus, as $n \to \infty$, $x_n \to 0$. Since $x_n \to 0$, $f(x_n) \to \infty$. However, $f(0) = c$. Therefore, f is discontinuous at $x = 0$.

4. Let $g(x) = \frac{h(x)}{f(x)} = \frac{x^2+2x-1}{x^2+1}$. Since h and f are both polynomials, they are both continuous on all of $\mathbb{R}$. Thus, we must only check if f is non zero everywhere. Since $x^2 >= 0$ for all $x \in \mathbb{R}$, $x^2 + 1 >= 1 > 0$ for all $x \in \mathbb{R}$. Thus, f is never zero anywhere on $\mathbb{R}$. Thus, g is continuous everywhere.

5. Let $f(x) = x^2 - x$ and $x^2 < x$. Since $x^2 < x$, $x^2 - x < 0$. Since $x^2 - x < 0$, $0 < x < 1$. Thus, S is the open set $(0, 1)$ and thus, S is not closed. f does not obtain a maximum value as the maximum values are attained at 0 and 1 which are not in the set S. However, f does attain a minimum at .5.

6. Since the domain is closed and the function is defined on the entire interval, this function reaches a minimum at $x = 0$ where $f(0) = 0$ and a maximum at $x = \frac{2}{\pi}$ where $f(\frac{2}{\pi}) = \frac{2}{\pi} + \frac{2}{\pi}sin(\frac{\pi}{2})$.

7. For all $x, y \in \mathbb{R}$, $|cos(x) - cos(y)| = 2|sin(\frac{x+y}{2})sin(\frac{x-y}{2})| \leq 2|sin(\frac{x-y}{2})| < 2|\frac{x-y}{2}| = |x-y|$. Thus, if we let $\delta = \epsilon$, $|f(x) - f(y)| < \epsilon$ if $|x-y| < \delta$. Thus, $f(x) = cos(x)$ is uniformly continuous on all of $\mathbb{R}$.

8. Let $f(x) = 5x + 8$, $\delta = \frac{\epsilon}{5}$ and $|x-y| < \delta$. Then, $|f(x) - f(y)| = |5x + 8 - 5y - 8| = 5|x-y| < 5\frac{\epsilon}{5} = \epsilon$ Thus, f is uniformly continuous on $[0, \infty)$.

9. Let $f(x) = x^3$ on $[0, \infty)$ and let $\epsilon = \frac{1}{2}$. Let $x_n = \{\sqrt[3]{n}\}$ and $y_n = \{\sqrt[3]{n+1}\}$. Then, $|x_n - y_n| \to 0$ as $n \to 0$. However, $|f(x_n) - f(y_n)| = |n - n - 1| = 1$ for all $n \in \mathbb{N}$. Thus, by the proposition in class, $lim_{n\to\infty}|x_n - y_n| = 0$ and $|f(x_n) - f(y_n)| \geq \epsilon$. Thus, we conclude that $f(x) = x^3$ is not uniformly continuous.

10. Let $f(x) = cos(x)$ where $x \in \mathbb{R}$. Then, $f'(x) = -sin(x)$ Since $sin(x)$ is bounded by $[-1, 1]$, the derivative is bounded below by -1 and above by 1. Thus, $|f'| \leq 1$. Since f is a function with a bounded derivative, $cos(x)$ is continuous on all of $\mathbb{R}$.