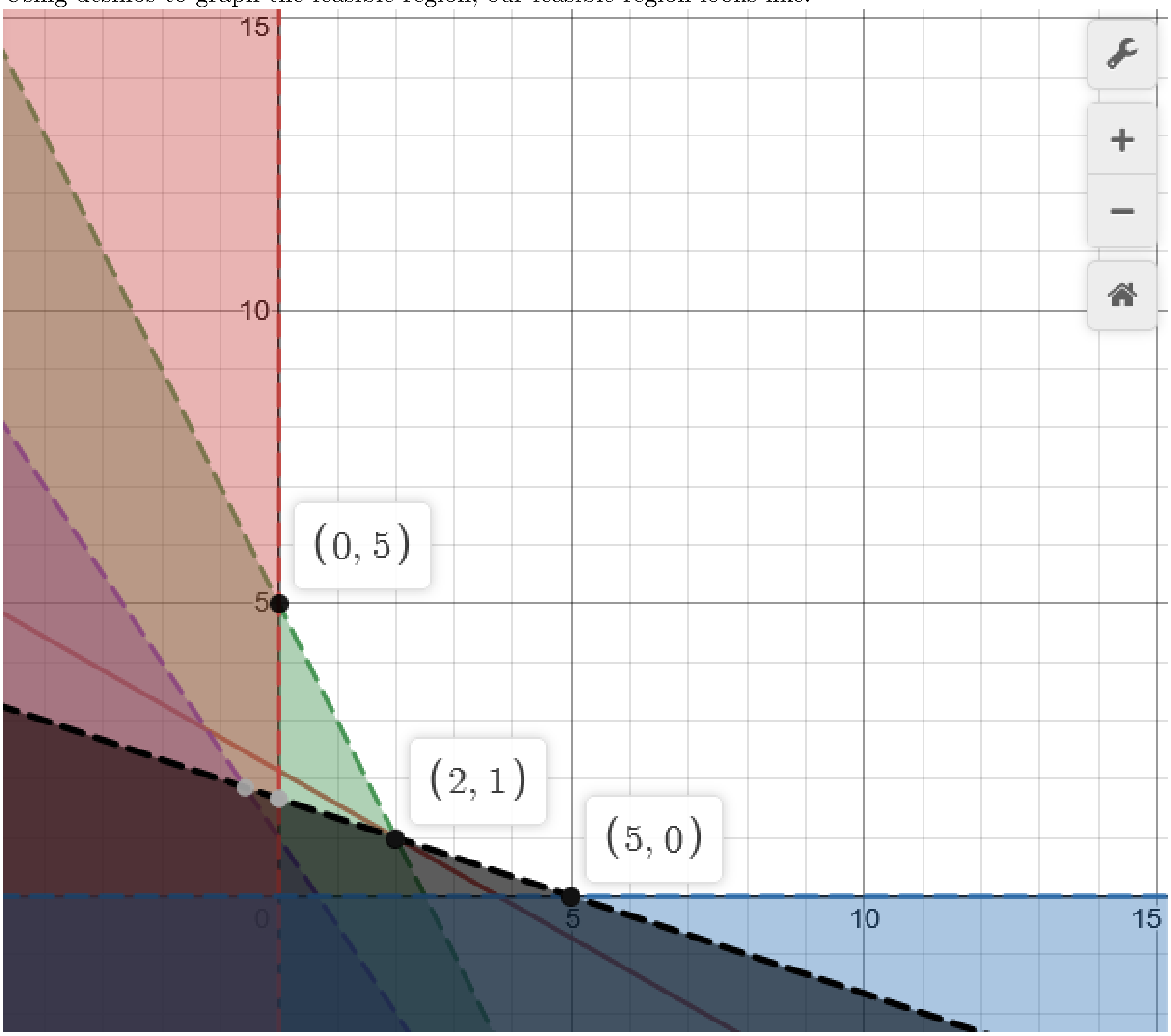
1.

$$\text{minimize } 4y_1 + 7y_2$$
$$\text{subject to } 2y_1 + y_2 \geq 5$$
$$3y_1 + 2y_2 \geq 2$$
$$y_1 + 3y_2 \geq 5$$
$$y_1, y_2 \geq 0$$

Using desmos to graph the feasible region, our feasible region looks like:



So we see that we have three points to check: $(0, 5)$ $(2, 1)$ $(5, 0)$. Using algebra and these three points:

$$4(0) + 7(5) = 35$$
$$4(2) + 7(1) = 15$$
$$4(5) + 7(0) = 20$$

Thus, we find that the point $(2, 1)$ minimizes our function with subject to the given restraints.

2. (a) $x_1 + 5x_2 + 3x_3 + 7x_4 + 5x_5$

   (b) $\begin{bmatrix} 48 \\ 45 \\ 26 \end{bmatrix}$

3.

$$\text{maximize } 5x_1 + 2x_2 + 5x_3$$
$$\text{subject to } 2x_1 + 3x_2 + x_3 \leq 4$$
$$x_1 + 2x_2 + 3x_3 \leq 7$$
$$x_1, x_2, x_3 \geq 0$$

Writing this problem in matrix-vector notation:

$$\text{maximize } z = \begin{bmatrix} 5 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
$$\text{subject to: } \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$